

# CYCLOTRON WITH SECTIONED MAGNET SYSTEM \*

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## 1. Introduction

The energy of particles accelerated in a cyclotron is limited by their relativistic mass increase which changes the period of their revolution. The principle of phasing discovered by Veksler<sup>1,2)</sup> permitting the parameters of the accelerator to be changed with the time made it possible to overcome that restriction. The development of accelerators whose parameters do not change with the time was recently initiated. In the electronic model of the cyclotron by McMillan and his collaborators<sup>3,4)</sup>, constant frequency of revolution is obtained by increasing the magnetic field with the radius. The focusing of particles is ensured by azimuthal variation of the magnetic field according to Thomas's method<sup>5,6)</sup>. However, the application of increasing fields is not the only method whereby constant frequency of revolution can be maintained.

In 1950, one of the present authors<sup>7)</sup> suggested a direct current magnet consisting of a number of similar wedge-shaped sections for use in ring accelerators in which the period of particle revolution depends on the energy. However, a magnet consisting of wedge-shaped sections can also be used in a cyclotron. This method permits the maintenance of constant frequency of particle revolution in the absence of radial increase of the magnetic field and even if they decrease in radius—a condition which may be sometimes useful.

The motion of particles in cyclotrons with a sectioned magnet system with homogeneous (or almost homogeneous) fields  $H$  in the sections and fields  $\kappa H$  in the intermediate spaces between them is discussed in the present paper. Some of the results of the calculations set forth below are probably in some relation to the clover leaf type accelerator suggested by McMillan et al.,<sup>3,4)</sup> the theory of which has not yet been published.

## 2. Conditions of cyclotron resonance

In a cyclotron, the period  $T_\lambda$  of the r.f. voltage applied to the dees must equal the period of revolution of a particle round its orbit

$$T = S/\beta c \quad (2.1)$$

where  $S$  is the length of the orbit and  $\beta c$  is the velocity of the particle.

Introducing the mean field term

$$\bar{H} = \frac{\oint H ds}{\oint ds} = \frac{\oint H r d\nu}{S} = \frac{E\beta}{eS} \oint d\nu = \frac{2\pi E\beta}{eS} \quad (2.2)$$

where  $\nu$  is the angle of the trajectory bend and  $E$  is the particle energy, the equation (2.1) may be written in the form:

$$T = 2\pi E / c e \bar{H} \quad (2.3)$$

Thus to ensure constancy of the period of revolution, the mean magnetic field round the orbit must grow with increasing energy. If the magnet consists of sections of different magnetic field values which do not increase along the radius, an increase of the mean field can be obtained if suitable boundaries of magnet sections are selected. This means that the part of the trajectory which passes sections of greater intensity of the magnetic field must increase, while the part of the trajectory passing sectors where the values of the magnetic field are low must decrease. Let us examine a magnet composed of  $N$  sections with  $H$  field. In the intermediate spaces between them, the magnetic field equals  $\kappa H$  (where  $\kappa < 1$ ). For a closed orbit, the following relations will be correct in such a magnet:

$$N(\nu_1 + \nu_2) = 2\pi \quad (2.4)$$

where  $\nu_1$  is the angle of trajectory bend in a sector with field  $H$  and  $\nu_2$  is the angle of the bend in the intermediate space with a field  $\kappa H$ ,

$$T = Nr(\nu_1 + \nu_2/\kappa)/\beta c \quad (2.5)$$

where the radius  $r$  of the orbit curvature in a sector with field  $H$  is

$$r = E\beta/eH \quad (2.6)$$

\* This paper was presented in title only.

Introducing the notations

$$\zeta = E/E_0, \quad \zeta_M = TceH/2\pi E_0 \quad (2.7)$$

where  $E_0 = m_0 c^2$  is the energy of a particle at rest, the following equations may be obtained for the angles of the bend and the radius of curvature:

$$v_1 = \frac{2\pi}{N} \frac{1 - \kappa \frac{\zeta_M}{\zeta}}{1 - \kappa} \quad (2.8)$$

$$v_2 = \frac{2\pi}{N} \frac{\kappa}{1 - \kappa} \left( \frac{\zeta_M}{\zeta} - 1 \right) \quad (2.9)$$

$$r_1 = r = \frac{E_0}{eH} \sqrt{\zeta^2 - 1}; \quad r_2 = \frac{r}{\kappa} \quad (2.10)$$

For positive values of the coefficient  $\kappa$ , the angles  $v_1$  and  $v_2$  must be positive, but for  $\kappa < 0$   $v_2$  must be  $< 0$ .

The following restrictions in the choice of parameters and the limiting energy are obtained from it:

$$\zeta < \zeta_M \quad (2.11)$$

$$\kappa < \zeta/\zeta \quad (2.12)$$

The values of maximum  $R_M$  and minimum  $R_m$  of the distances from the orbit to the centre of the accelerator can also be easily calculated:

$$R_M = r \left[ 1 + \left( \frac{1}{\kappa} - 1 \right) \frac{\sin \frac{\pi}{N} \frac{\kappa}{1 - \kappa} \left( \frac{\zeta_M}{\zeta} - 1 \right)}{\sin \frac{\pi}{N}} \right] \quad (2.13)$$

$$R_m = r \left[ \frac{1}{\kappa} - \left( \frac{1}{\kappa} - 1 \right) \frac{\sin \frac{\pi}{N} \frac{1 - \kappa \zeta_M/\zeta}{1 - \kappa}}{\sin \pi/N} \right] \quad (2.14)$$

In the particular case of  $\kappa = 0$ , when no magnetic field is present in the intermediate spaces between the sections and the motion of the particles in these spaces is linear, a number of formulae can be simplified:

$$\lim_{\kappa \rightarrow 0} v_1 = v = 2\pi/N \quad (2.8a)$$

$$\lim_{\kappa \rightarrow 0} v_2 = 0 \quad (2.9a)$$

$$\lim_{\kappa \rightarrow 0} r_2 = \infty \quad (2.10a)$$

$$\lim_{\kappa \rightarrow 0} R_M = R = r \left[ 1 + \left( \frac{\zeta_M}{\zeta} - 1 \right) \frac{\pi/N}{\sin \pi/N} \right] \quad (2.13a)$$

$$\lim_{\kappa \rightarrow 0} R_m = y = \left[ 1 + \left( \frac{\zeta_M}{\zeta} - 1 \right) \frac{\pi/N}{\sin \pi/N} \right] \quad (2.14a)$$

$$\lim_{\kappa \rightarrow 0} r_2 v_2 = l = r \cdot 2\pi/N \cdot (\zeta_M/\zeta - 1) \quad (2.15a)$$

where  $l$  is the length of the linear part of the trajectory and  $y$  is the distance from the middle point of that part to the centre of the accelerator.

### 3. Focusing

The influence on the particle of the magnetic field varying step-wise is equivalent to the influence of a thin lens of focal distances  $f_z$  for vertical motions and  $f_r$  for radial motions

$$\Delta \left( \frac{dz}{dS} \right) = \left( \frac{dz}{dS} \right)_2 - \left( \frac{dz}{dS} \right)_1 = -\frac{z}{f_z} \quad (3.1)$$

$$\Delta \left( \frac{d\rho}{dS} \right) = \left( \frac{d\rho}{dS} \right)_2 - \left( \frac{d\rho}{dS} \right)_1 = -\frac{\rho}{f_r} \quad (3.2)$$

where  $z$  and  $\rho$  are the vertical and radial deviations of the particle from the closed orbit at the point where the orbit intersects the edge of the magnetic sector and,  $S$  is the length counted along the trajectory

$$f_z = r \cot \gamma / (1 - \kappa) = -f_r \quad (3.3)$$

The angle  $\gamma$  between the line normal to the sector boundary and the line to the trajectory is considered positive if the part of the trajectory passing into the weaker field  $\kappa H$  is located on the same side of the normal line as that in which the curvature centre of the part of the trajectory passing into the stronger field  $H$  is located.

The number of vertical and radial oscillations of the particle per revolution can be found by means of methods developed for strong-focusing accelerators<sup>8)</sup> provided the focusing or defocusing influence of the edges is taken into account.

$$\frac{\omega_z}{\omega} = \frac{\omega_z T}{2\pi} = \frac{N\mu_z}{2\pi}; \quad \frac{\omega_r}{\omega} = \frac{N\mu_r}{2\pi} \quad (3.4)$$

In the case of homogeneous magnetic fields or fields which deviating slightly from homogeneity, the following equation is obtained for  $\mu_z$  and  $\mu_r$ :

$$\cos \mu = c_1 c_2 - \frac{1}{2} \left( \kappa \frac{\eta_2}{\eta_1} + \frac{1}{\kappa} \frac{\eta_1}{\eta_2} \right) S_1 S_2 - \frac{1}{2} \left( \frac{1}{\kappa} c_1 \frac{S_2}{\eta_2} + c_2 \frac{S_1}{\eta_1} \right) \times \left( \frac{r}{f_1} + \frac{r}{f_2} \right) + \frac{1}{2\kappa} \frac{S_1 S_2}{\eta_1 \eta_2} \frac{r^2}{f_1 f_2} \quad (3.5)$$

where

$$S_1 = \sin \eta_1 v_1; S_2 = \sin \eta_2 v_2; c_1 = \cos \eta_1 v_1; c_2 = \cos \eta_2 v_2 \quad (3.6)$$

$$\eta_1 = \begin{cases} \sqrt{n_1} & \text{for vertical motion} \\ \sqrt{1-n_1} & \text{for radial motion} \end{cases}; \eta_2 = \begin{cases} \sqrt{n_2} & \text{for vertical motion} \\ \sqrt{1-n_2} & \text{for radial motion} \end{cases} \quad (3.7)$$

$$n_1 = -\frac{r}{H} \frac{\partial H}{\partial \rho}; n_2 = -\frac{1}{\kappa^2} \frac{r}{H} \frac{\partial(\kappa H)}{\partial \rho} \quad (3.8)$$

For a particular case of homogeneous magnetic fields ( $n_1 = n_2 = 0$ ) we obtain

$$\cos \mu_z = 1 - (1-\kappa)(v_1 + v_2/\kappa) \frac{\text{tg} \gamma_1 + \text{tg} \gamma_2}{2} + \frac{(1-\kappa)^2}{2} v_1 \frac{v_2}{\kappa} \text{tg} \gamma_1 \text{tg} \gamma_2 \quad (3.9)$$

$$\begin{aligned} \cos \mu_r &= \cos v_1 \cos v_2 - \frac{1}{2}(\kappa + 1/\kappa) \sin v_1 \sin v_2 \\ &+ (1-\kappa)(\sin v_1 \cos v_2 + 1/\kappa \cos v_1 \sin v_2) \frac{\text{tg} \gamma_1 + \text{tg} \gamma_2}{2} \\ &+ \frac{(1-\kappa)^2}{2\kappa} \sin v_1 \sin v_2 \text{tg} \gamma_1 \text{tg} \gamma_2. \end{aligned} \quad (3.10)$$

By means of simple but tedious calculations which are omitted here, the connection of the angles  $\gamma_1$  and  $\gamma_2$  with the form of the line  $R_m(\theta)$  can be found representing the geometrical place of the middle points of the trajectory parts in the intermediate spaces between magnet sections, i.e. the points where the distances of the orbits from the centre of the accelerator are minimum.

$$\text{tg} \gamma_1 = \frac{A_1 - B_2 \xi}{A_2 + B_1 \xi}; \text{tg} \gamma_2 = \frac{A_1 + B_2 \xi}{A_2 - B_1 \xi} \quad (3.11)$$

where

$$\xi = R_m \quad d\theta/dR_m \quad (3.12)$$

$\theta$  is the azimuthal angle,

$$A_1 = \left( \frac{dr_2}{dR_m} - 1 \right) \sin \frac{v_2}{2} + \frac{r_2}{2} \frac{dv_2}{dR_m} = \frac{(1-\kappa) \sin \frac{v_1}{2} \sin \frac{v_2}{2} + \frac{\pi}{N} (\zeta^2 - 1) \frac{\zeta_M}{\zeta^3} \kappa \left( \cos \frac{v_1}{2} \sin \frac{v_2}{2} - \frac{1}{1-\kappa} \sin \frac{\pi}{N} \right)}{\sin \frac{\pi}{N} - (1-\kappa) \sin \frac{v_1}{2} - \frac{\pi}{N} (\zeta^2 - 1) \frac{\zeta_M}{\zeta^3} \kappa \cos \frac{v_1}{2}} \quad (3.13)$$

$$A_2 = \frac{dr_2}{dR_m} - \left( \frac{dr_2}{dR_m} - 1 \right) \cos \frac{v_2}{2} = \frac{\sin \frac{\pi}{N} - (1-\kappa) \sin \frac{v_1}{2} \cos \frac{v_2}{2} - \frac{\pi}{N} (\zeta^2 - 1) \frac{\zeta_M}{\zeta^3} \kappa \cos \frac{v_1}{2} \cos \frac{v_2}{2}}{\sin \frac{\pi}{N} - (1-\kappa) \sin \frac{v_1}{2} - \frac{\pi}{N} (\zeta^2 - 1) \frac{\zeta_M}{\zeta^3} \kappa \cos \frac{v_1}{2}} \quad (3.14)$$

$$B_1 = \left( \frac{r_2}{R_m} - 1 \right) \sin \frac{v_2}{2} = \frac{(1-\kappa) \sin v_1/2 \cdot \sin v_2/2}{\sin \pi/N - (1-\kappa) \sin v_1/2} \quad (3.15)$$

$$\begin{aligned} B_2 &= \frac{r_2}{R_m} - \left( \frac{r_2}{R_m} - 1 \right) \cos \frac{v_2}{2} \\ &= \frac{\sin \pi/N - (1-\kappa) \sin v_1/2 \cdot \cos v_2/2}{\sin \pi/N - (1-\kappa) \sin v_1/2} \end{aligned} \quad (3.16)$$

The left equalities (3.13-3.16), as well as the equations (3.1)-(3.12) apply not only to the cyclotron but also to the ring phasotron.

The expressions contained in the formulae (3.9) and (3.10) may be found from (3.11).

$$\frac{\text{tg} \gamma_1 + \text{tg} \gamma_2}{2} = \frac{A_1 A_2 + B_1 B_2 \xi^2}{A_2^2 - B_1^2 \xi^2}; \text{tg} \gamma_1 \text{tg} \gamma_2 = \frac{A_1^2 - B_2^2 \xi^2}{A_2^2 - B_1^2 \xi^2} \quad (3.17)$$

For symmetrical magnet sections

$$\text{tg} \gamma_1 = \text{tg} \gamma_2 = A_1/A_2 \quad (\xi = 0) \quad (3.18)$$

For a particular case ( $\kappa = 0$ ) where magnetic fields in the immediate spaces between sections are absent<sup>9)</sup>, formulae (3.13)-(3.16) become much simplified

$$\lim_{\kappa \rightarrow 0} A_1 = \frac{1}{2} \frac{dl}{dy} = \frac{\zeta_M/\zeta^3 - 1}{N/\pi \text{tg} \pi/N + \zeta_M/\zeta^3 - 1} \text{tg} \pi/N \quad (3.13a)$$

$$\lim_{\kappa \rightarrow 0} A_2 = 1 \quad (3.14a)$$

$$\lim_{\kappa \rightarrow 0} B_1 = \frac{1}{2} \frac{l}{y} = \frac{\zeta_M/\zeta - 1}{N/\pi \cdot \text{tg} \pi/N + \zeta_M/\zeta - 1} \text{tg} \pi/N \quad (3.15a)$$

$$\lim_{\kappa \rightarrow 0} B_z = 1 \quad (3.16a)$$

The use of asymmetrical magnet sections ( $\xi \neq 0$ ) permits an increase in vertical focusing and results at the same time in an increase in the number of radial oscillations per revolution. Some features of this method also occur in Kerst's method<sup>10,11</sup> suggested for accelerators with radially increasing magnetic fields.

#### 4. Energy limit

The energy limit obtainable in a cyclotron is characterized by the point of intersection of the stability boundary  $\zeta$  ( $\zeta_M$ ) at which  $\cos \mu_z = 1$  with the boundary at which  $\cos \mu_r = -1$ . For the case  $\kappa = 0$ ,  $\xi = 0$ , we get:

$$\zeta_{\max}^2 = 1 + N/\pi \cdot \operatorname{ctg} \pi/N \quad (4.1)$$

With growing  $\xi$  the energy limit increases and approaches (for either value of  $\kappa$ ) the value corresponding to

$$\zeta_{\max}^2 = \frac{1}{1 - N/2\pi \cdot \sin 2\pi/N} \quad (4.2)$$

Thus, when the number of sections is greater the energy limit also increases and becomes a linear function of  $N$  when the values of  $N$  are large.

$$\zeta_{\max} \approx N/\pi \quad (4.3)$$

The stability region is intersected by resonance lines. The most harmful are those between the frequency of vertical oscillations and the frequency of revolution and those of coupling between the frequency of vertical and radial oscillations. They can be eliminated by various methods. One effective method is to choose the coefficient  $\kappa > 0.4$ , but the stability region and the energy limit decrease as  $\kappa$  increases. Values of  $\zeta_{\max}$  for  $N = 3$  and 4 and  $\xi = 3$  for various values of the coefficient  $\kappa$  are given in Table 1.

TABLE I.  
Dependence of  $\zeta_{\max}$  on  $\kappa$  values,  $\xi = 3$

$\kappa$	$\zeta_{\max}$	
	$N = 3 \rightarrow$	$N = 4$
0	1.34	1.67
0.5	1.20	1.60
0.9	1.027	1.027

Resonance lines between the frequency of radial oscillations and revolution frequency intersect the stability region only if  $N \geq 4$ . Another effective method of eliminating resonance is to use nonhomogeneous fields,

i.e. introduce variable coefficients ( $\zeta_M(\zeta)$  and  $\kappa(\xi)$  (Thomas-type cyclotron). In this way, no passage through resonance is needed to reach an energy of about 500 Mev for protons.

#### 5. Parameters tolerances

Two criteria determining parameter tolerances can be given.

##### (a) Stability criterion

Parameter tolerances should not disturb the stability of orbits or cause a passage through resonance.

The tolerance for the value of magnetic field homogeneity can be established from equations (3.4) and (3.5).

$$4 \left| \frac{\Delta \omega_z}{\omega} \right| = \frac{2\pi}{N} \left| \frac{\Delta \cos \mu_z}{\sin \mu_z} \right| = \frac{4\pi}{N} \frac{\zeta_M}{\zeta} \frac{1}{(1 - \kappa) \sin \mu_z} \times \left| \left[ n_1 (1 - \kappa) \frac{\zeta_M}{\zeta} + n_2 \kappa^2 \left( \frac{\zeta_M}{\zeta} - 1 \right) \right] \right| \ll 1 \quad (5.1)$$

Tolerances for the other parameters can be found in the same way.

A phasing criterion is of particular significance for a moderate amplitude of the accelerating voltage  $V$ .

##### (b) Phasing criterion

The phase shift  $\varphi$  of the electric field for particle passages through the accelerating slit, resulting from the tolerances for the parameters of the accelerator, must not exceed an accepted value depending on the values of phase  $\varphi_i$  at the beginning and  $\varphi_k$  at the end of acceleration. Let us examine the requirements for magnetic field stabilization. A variation of the magnetic field for a value  $\Delta H/H = h$  causes a change in the period of particle revolution by a value which does not depend upon the coefficients  $\kappa$  and  $\xi_M$

$$T = T_0 \zeta (\zeta^2 + 2h + h^2)^{-1/2} \quad (5.2)$$

The latter formula permits the determination of the velocity of phase variation of a particle.

$$\rho = \frac{d\varphi}{dt} = \frac{eV \cos \varphi}{E_0 T} \frac{d\varphi}{d\xi} = \omega - \omega_0 = \frac{2\pi}{T_0} \left[ \frac{(\zeta^2 + 2h + h^2)^{1/2}}{\zeta} - 1 \right] \quad (5.3)$$

The expression determining the admitted value of  $h$ , obtained after integrating, is

$$\frac{eV}{E_0} \left| \frac{\sin \varphi_k - \sin \varphi_i}{2\pi} \right| = \left| \sqrt{\zeta_{\max}^2 + 2h + h^2} - (\zeta_{\max} + h) \right| \quad (5.4)$$

from which

$$|h| \approx \frac{\zeta_{\max}}{\zeta_{\max} - 1} \frac{eV}{E_0} \left| \frac{\sin \varphi_k - \sin \varphi_i}{2\pi} \right| \quad (5.5)$$

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## DISCUSSION

*V. Migulin:* R. B. Neal told us about the advantages of linear accelerators.

With reference to Livingston's paper, however, in the region of a few Bev circular machines have definite advantages. Would one of the groups compare the advantages of linear and circular machines?

*R. Hofstadter:* We easily have an external beam in Linacs and the question is, if one can get one out of circular machines. The energy can be easily varied from nearly 0 to maximum in linacs. It may be possible to do it with circular machines, but I do not know anything about it.

*J. J. Livingood:* The Russian results on the section magnet system are analogous to those obtained by M. H. Foss, L. C. Teng and E. A. Crosbie theoretically at Argonne. The fields are uniform radially and azimuthally except for the sudden change from H to H prime. This should permit a simpler construction than with the curved poles of Thomas' machine. If one is willing to put up with very weak vertical focusing, at least one or both limiting lines of the sectors can be linear. Both boundaries must be curved if one wants strong vertical focusing.

*A. A. Kolomenski:* We examined both homogeneous and inhomogeneous fields inside one sector. Homogeneous fields are easier to produce and will be used. Historically the idea was to have no field at all in the intermediate section.

*A. Roberts:* My results presented on Monday did not seem to be in agreement with these. Kolomenski said that with 4 sectors one could obtain up to 500 Mev, which I guess is the one-half integral resonance. Has he means to avoid the one-third resonance which occurs with 4 sectors?

*A. A. Kolomenski:* In the paper presented, only linear resonances are considered. Work done on non-linearities in Moscow are not reported. The avoidance of the vertical resonance only was outlined here.

*D. J. Judd:* I might remark that there is no difference in principle between the properties of cyclotrons having square-wave or discontinuous variations of magnetic field with azimuth, such as discussed in the preceding paper, and those having sinusoidal or other periodic variations, such as that proposed by Thomas and incorporated in the electron model cyclotrons built at Berkeley several years ago. In the pioneering theoretical work of McMillan at Berkeley in 1949, square-wave fields were considered; both the cases of uniform fields with radially varying azimuthal extent and of wedge sectors with rising fields were treated. Results similar to those described in the present paper were obtained. He dealt both with configurations having  $\alpha = 0$  and with

those having finite and varying  $\alpha$ . In the square-wave cases, one may obtain rigorous closed analytic expressions for the linearized betatron oscillation frequencies, and thus may study energy limits, which cannot be so conveniently obtained by extending the work of Thomas, treating sinusoidal fields by successive approximations.

There is one important difference between square and sinusoidal fields. The former must give way to the latter as one approaches the center of the cyclotron. I carried out detailed design calculations for the model cyclotrons at Berkeley, starting in 1950, and found that it was simpler, in calculating the required fields analytically to the required accuracy, to retain the sinusoidal form at all radii, so as to avoid the necessity of making this transition. An additional motivation was that I discovered an exact scalar magnetic potential function for the required field, which greatly simplified the construction of the magnet pole pieces.

The precision of the fields in the models may be judged by the fact that electrons were accelerated to about 70 Kev by r.f. voltages of 25 volts, so that the particles stayed in phase for over 3000 turns. This was not limited by field precision but by the energy gain required to enable the electrons to clear the source structure on the first turn.

I would like to inquire for details on the model for the 10 Gev Russian machine, specially the number of sectors, field gradients, number and position of the straight sections.

*A. M. Stolon:* 4 straight sections with the injector in one and the accelerating electrodes in the other 3. The field gradient  $n = 0.7$  to  $0.55$  is not constant with radius but in these limits the radius is  $R = 2$  metres in the magnet sectors. We have 600 mm. straight section, C shaped magnet cross section, 0.5 sec. rise time, 1 pulse in 3 sec.

*M. H. Blewett:* If you had C-shaped magnets in the model, why did you change to H-shaped magnets in the machine?

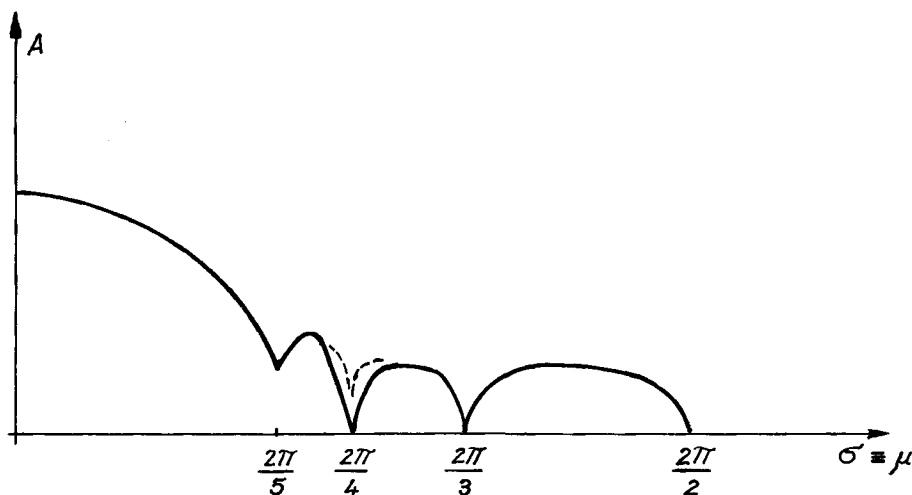
*A. M. Stolon:* It was not an engineering model, but a model to study particle orbits. In the large magnets we felt we would have a better field distribution if the magnets were symmetric and more stable mechanically.

*J. J. Livingood:* Where is this model with AG described?

*A. M. Stolon:* This model has weak focusing and was made several years ago.

*M. G. N. Hine:* I understood from E. G. Komar's talk that a 600 Mev AG model was made.

*A. M. Stolon:* These were two different things. This weak focusing model has been made. At the moment we build a model of an 200-600 Mev AG proton accelerator to be used to study Vladimirov's accelerator properties.



*T. A. Welton:* There is a disagreement as to correct variation of  $Q_r$  with energy in a fixed frequency cyclic accelerator.

I reduced the formulas of H. Roberts and E. M. Moroz given in standard form to smooth approximation and even performed the calculation outlined by Roberts and also performed by Moroz. The result is that my derivation disagrees with Roberts', agrees with Symon's and agrees with Moroz' and Rabinovich's. There is a clear-cut disagreement between Roberts and Teng and on the other hand Symon's, my own and Moroz' results.

The  $Q_r = 3/2$  resonance with 4 sectors comes quite low.

The first integral resonance comes as high as 350 Mev.

*D. W. Kerst:* An elementary difficulty encountered with spiral ridges are the non-linear stop bands. If in a non-linear machine one plots the permissible initial amplitude or radial betatron oscillations for not losing particles against our parameter  $\sigma = \mu$ , our experimental work on a digital computer gives us blanks (fig.) at

- $\sigma = \pi$  (linear stability limit),
- $2\pi/3$  (quadratic non-linearity),
- $2\pi/4$  (cubic non-linearity),

whereas according to P. A. Sturrock and J. Moser the cubic non-linearity is sometimes harmless.

For one spiral gap accelerator we are not sure we have no trouble at the cubic non-linearity resonance.

*M. G. N. Hine:* As a comment to M. G. White's paper about injecting the possible total amount into a conventional type of constant gradient or alternating gradient synchrotron, I have no definite figure to quote, but my impression is that the highest current determined eventually by the space charge can be obtained practically as easily with single turn injection as with multiturn injection.

Although at the start of multiturn injection the particles are lying on different radii, these radii come together after  $1/4$  wavelength of synchrotron oscillation, so that beam will look much as if it were injected on a single turn.

I feel that injecting a large current into a single turn will be easier than injecting a lower current in many turns.

*M. H. Hamermesh:* To miss the injector it is better to use multiturn injection on 24 turns.

*J. B. Adams:* We thought that the limitation is due to space charge.

*M. H. Hamermesh:* Yes, finally, but you cannot possibly get to it.

*M. G. N. Hine:* With the fairly large energy accelerators with small aperture milliamperes can be easily obtained for 1-turn injection and this takes you already into the space charge limited region.

*M. H. Hamermesh:* In our case we were not limited by space charge after 24 turns of injection.

*A. Roberts:* I would like to make a comment on M. G. White's proposal.

At the Rochester cyclotron we used various rotating condensers with ceramic blades. Our 1st condenser had no failure in 4 years. Now we had 3 failures last year, due to radiation damage. The radiation level is only a few hundred watts. White can get several kW of radiation, so he should inquire about the effect of such a radiation on the ceramic chamber. Neutron yields are very intense.

*M. G. White:* We hope to get the beam largely outside the chamber eventually.

*R. L. Thornton:* The experience of people with electron synchrotrons using ceramic vacuum chambers will rule out this eventuality.

*E. M. McMillan* : This is not really quite right.

*V. I. Veksler* : The neutron level in gamma-ray machines is much less than in proton machines so that experience might not be relevant.

*K. Johnsen* : Hine's point is important.

In a given machine with a given aperture, a given acceptance phase space is to be filled with a beam of the highest possible intensity. If the injected beam has a small emittance (small area in phase space), one gains by using multi-turn injection, and the beams corresponding to the various turns are aligned beside each other in the acceptable phase space. However, if one can increase the beam intensity by increasing the emittance of the beam until it is of the same size as the acceptance in phase space of the vacuum chamber without decreasing the density in phase space inside the beam, then nothing is gained by using multi-turn injection. What type of injector one has available or is prepared to choose therefore also has some influence on the problem in practice.

*J. P. Blewett* : Is the point raised by Hamermesh covered by Johnsen's comment?

*M. H. Hamermesh* : All we did was to take reasonable values for quality and intensity and to calculate the optimum number of turns which could be injected, which gave the figure of 24 turns I quoted.

*V. I. Veksler* : I would like to make some remarks about the question on beam density. I agree with Hamermesh, because one must consider not only the radial extent of the beam to be of importance, but also the vertical

direction extent of beam to be important. The net injection result would be that it is more advantageous to use multi-turn injection.

*G. K. Green* : In constant gradient machines one must consider 4 dimensional phase space. The radial position and velocity spread at betatron oscillation frequencies is much less important in AG machines, and we agree with Johnsen for the AG machine.

*F. C. Shoemaker* : With reference to Hine's comment, if the injector energy is modulated upwards to match the magnetic field increase, so that the beam is always injected on the equilibrium orbit, the entire area of phase space for synchrotron oscillations is uniformly covered with beam which will not bunch down to a small diameter in the chamber.

*W. M. Brobeck* : Regarding H versus C magnet shape, I must remark that C magnets use single plates.

In the Bevatron with a  $2 \times 6$  foot aperture planned, a single plate C magnet was impractical. Do you have the same situation with the Russian machine?

*A. M. Stolov* : With the C shaped magnet, if the aperture is small, one single sheet may be stable. But the 10 Bev machine with  $8 \text{ m}^2$  in cross section had to be made of component parts in its cross section. Also we have always weak focusing. The gradient tolerance for the machine is 0.036 gauss/cm. In this case the symmetry of the magnet system is of primary importance, otherwise high current on compensating windings to fix up field defects would be needed.