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ANTIMATTER AND COSMOLOGY

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ANTIMATTER AND COSMOLOGY

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PREFACE

This paper is an outgrowth of discussions at meetings of The Stockholm Cosmology Group, an informal gathering of people interested in charge-symmetrical and particularly the Alfvén-Klein metagalactic theory at which the author has been a regular guest. It is not supposed to be a penetrating analysis of the metagalactic theory but rather to illustrate some of the selected difficulties which the theory faces. Although many fruitful ideas were aired at the meetings, the paper is based only on published works.

Part 2 contains an attempt to alleviate one of the difficulties which at that time seemed a very promising and interesting undertaking. The world being as is and not as we wish it to be, it came to nought. The author is indebted to many people for interesting discussions and ideas and particularly to Dr. Bertel Laurent and Dr. Björn Roos who also critically read this manuscript.

A. PART I

INTRODUCTION

Cosmology occupies a somewhat unique position among the physical sciences in the sense that it is not subject to experiment. Even the observations are limited to observations at large distances and distant times and it is perhaps natural that cosmology has been a playground for various theories since these are not easily verifiable. After World War II, however, the refinement of observational techniques and accumulation of astronomical data (particularly since the invention of radio astronomy) shifted the emphasis in cosmology from attempts to interpret the meaning of Einstein's equations to attempts to account for what we can actually observe. A discovery, dated before the War, of the recession of distant galaxies, as evidenced by their redshift, led to the general acceptance of the idea of an expanding universe. Also the general relativity admits homogeneous models of an expanding universe; however, it is known that in following the expansion backward, the Einstein's equations lead to an inevitable singularity; a point of infinite curvature. It remains then to interpret this singularity. If it exists, then either the universe has its origin at this singularity, having been created, or we do not know how to interpret it. If it is only a mathematical feature, then either Einstein's equations hold no longer true in general, or at least near the singularity, or there is an unknown law of nature operating to prevent the singularity (as for instance the C-field in the steady state theory adding an extra term to Einstein's equations), or our description of the space-time manifold loses its meaning because of quantum effects. Despite many years of effort, the problem is unsolved. It was to overcome this difficulty that new approaches to cosmology were tried. One of these has been the idea of Klein who, in order to derive the Eddington relations, proposed the thought that the universe did not start from a singularity but from a real physically realizable state and expanded due to some real natural causes. With the contributions of Alfvén, the Alfvén-Klein metagalactic theory, as it became to be called, introduces no new physical laws and assumptions except those that have been verified in the laboratory; these are extended to be valid everywhere: thus antimatter, whose presence is a consequence of the charge symmetry principle verified in the laboratory on an elementary particle scale, is supposed to be present everywhere throughout the whole extent of the early metagalaxy. Another approach has been to avoid the question of singularity and study instead the following period, that of high density and temperature of which our knowledge is meager. This is a starting point of the Omnés theory [3]. As always in physics, both theories have their fathers and forefathers. The Alfvén-Klein theory goes back to the ideas of Charlier who attempted to explain the Olbert paradox by postulating larger and larger sets of systems nested within each other, ad infinitum, and the Alfvén-Klein theory is a description of one such system; the

Omnés theory is a modification of the Gamow theory, the modification being that the early (hadron) stage contains equal amounts of matter and antimatter and the theory is a description of a subsequent development. This paper deals only with the theory of Alfvén and Klein, and especially with the problems it encountered, the Omnés theory will be a subject of a future paper.

ALFVÉN-KLEIN COSMOLOGY

DESCRIPTION

The birth of the Alfvén-Klein metagalactic theory can be traced to ideas of Klein appearing in several articles since 1953 [1], and the theory appeared in its whole for the first time in 1962 [2]. The theory is as follows:

The metagalaxy which contains all the visible universe (and perhaps more) consisted in the initial stages of a large, extremely dilute cloud of protons, antiprotons, electrons and positrons, all at a temperature near zero. The constituents have been chosen on the basis of simplicity, these being considered as the most simple and stable elementary particles. No radiation was present at this time, although some primeval magnetic field was postulated to serve as a seed field for a mechanism to separate matter from antimatter which came into action later. Under its own gravitational attraction the metagalaxy began to contract (in essentially a free fall manner) and its increased density gave rise to increasing annihilation between matter and antimatter and thus to radiation. Although some radiation escaped from the metagalaxy, enough of it remained to generate a radiation pressure which finally came to be sufficiently high to halt the contraction and turn it into expansion. Thus the cause behind the present expansion of the universe is given. Although most of the content of the metagalaxy has been annihilated, sometimes during its history a mechanism for separating matter from antimatter came into being and the remaining content was saved from further annihilation by being separated into distinct regions interacting with each other only through their boundaries. Thus the present metagalaxy consists of matter and antimatter well separated, except perhaps in quasi-stellar objects [4]. Observable features of the present universe (such as stars, galaxies, etc) are supposed to have been born sometimes during its history, although no detailed mechanism is provided. The metagalaxy itself is considered as one of a large number of metagalaxies in accordance with the ideas of Charlier [5]. Let us then consider those features of the metagalaxy which have been subject to published detailed studies: dynamics and separation.

THE DYNAMICS OF THE METAGALAXY

There are at present two main investigations of the dynamical behavior of the metagalaxy, both originating at the Stockholm school: Laurent and Söderholm [6], thereafter called L-S, and Hellsten (unpublished). Of some interest is also a hydrodynamical study of large masses by Matsuda and Kato [7].

The Laurent-Söderholm calculation is a general relativistic treatment of a subsequent history of a large matter-antimatter cloud with a sharp boundary and a finite mass which contains no radiation in its initial stages. The energy-momentum tensor in the gravitational equations contains pressure-free matter and radiation given by

matter $T_{\mu\nu} = \epsilon U_\mu U_\nu$

radiation $S_{\mu\nu} = 2 \sqrt{g} \int \rho\,(p,x)\, p_\mu p_\nu\, \delta\,(p^2)\, \theta\,(p^0)\, d^4p$

and the transport of radiation is described by a transport equation which, when specialized to the local rest frame for ease of interpretation, reads

$$\frac{\partial}{\partial t}(p^0\rho) + \hat{p}\,\nabla\,(p^0\rho) = \frac{\alpha}{4\pi}\,\frac{\epsilon^2}{(p^0)^2}\, f\,(p^0) - \beta\,\epsilon\, p^0\,(\rho - \rho_0)$$

The meaning of the symbols is as follows·

$\rho(p,x)$	number of photons per phase space volume $d^3x d^3p$ with a four-momentum ρ
U	matter four-velocity
ϵ	matter density in a local rest frame
$f(x)$	frequency distribution such that $\int_0^\infty f(x)dx = 1$
$\theta(p_1)$	step function defined as $\theta(p_1) = $ 0 if $x < 0$; $\frac{1}{2}$ if $x = 0$; 1 if $x > 0$
α	annihilation rate divided by matter density
β	scattering cross section divided by matter density.

The meaning of the radiation transport equation is that the amount of radiation energy of a certain frequency p^0 and direction $\hat{p}$ formed per phase space volume is due to annihilation and independent of direction (first term on the right-hand side) and scattered radiation reradiated isotropically (second term). Numerical calculations are made for a range of the parameter k/k_0 $(0.5 \leq k/k_0 \leq 4)$ where k is given as

$$k = \sqrt{c\,\frac{\hat{\beta}}{\alpha}}$$

where $\hat{\beta}$ is a scattering cross section divided a proton mass and α an annihilation rate divided by a proton mass. The term k_0 is a particular value of k which is used as a standard and is given by

$$k_0 = \sqrt{c\,\frac{\hat{\beta}_0}{\alpha_0}}$$

where

$$\alpha_o = c\,\frac{\pi}{2}\,\frac{d_e^2}{m_p}$$

$$\beta_o = \frac{8\pi}{3}\,\frac{d_e^2}{m_p}$$

with d_e and m_p meaning the classical electron radius and the mass of a proton. The standard β_o is then the Thompson cross section divided by the proton mass and the standard α_o is the annihilation rate at the velocity of light likewise divided by the proton mass. The meaning of k is then the square root of the ratio of scattering to annihilation cross sections and the parameter k/k_o is therefore a measure of scattering to annihilation as they differ from the values accepted as standard and described by k_o. Thus for instance, under the assumption of unchanged scattering cross section (i.e. $\beta = \beta_o$), the decreasing k/k_o ratio means increasing annihilation and vice versa. The actual mass of the metagalaxy in grams is obtained from

$$m = 8.7\ 10^{53} \sqrt{\frac{\alpha\beta}{\alpha_o\beta_o}}\ M_o\ (g)$$

Note that both $\sqrt{\frac{\beta}{\alpha}}$ (embedded in the ratio k/k_o) and $\sqrt{\alpha\beta}$ and M_o (appearing in the scaling formula for mass but not in the differential equations) can be chosen freely. However, calculations set a limit on M_o since past a certain value of M_o (for each k/k_o) a gravitational collapse occurs. Having thus arrived at the possible range of M_o, we can then chose at will either the value $\sqrt{\alpha\beta}$ and so arrive at the total mass m of the metagalaxy in grams or chose the total mass m and so establish $\sqrt{\alpha\beta}$. However, since now both $\sqrt{\frac{\beta}{\alpha}}$ and $\sqrt{\alpha\beta}$ are known, we have uniquely established the values of α and β which, as has been said, are measures of annihilation and scattering. These in turn are limited by consideration of reasonableness since obviously, not all annihilation rates or scattering cross sections are possible. We can thus give certain criteria, as done later in this work, which annihilation rates and scattering cross sections are possible in order that the model metagalaxy might liken the presently observed universe. Having this in mind, let us consider the results of Laurent and Söderholm. The most important ones for this review are as follows:

a) The maximum outward (expansion) velocity increases with increased initial mass of the metagalaxy but reaches what seems to be an upper limit of 0.4 c.
b) Using the value of annihilation and scattering accepted as standard (i.e. $\alpha = \alpha_o$, $\beta = \beta_o$), the maximum value of the initial mass that escapes gravitational collapse is $2.18\ 10^{53}$ g. However, a large part of the mass has been annihilated and only $0.52\ 10^{53}$ g remains at the point of maximum outward velocity. Moreover, a significant part of the remaining mass is radiation, estimated by the authors to be about a half.

The results are summarized in Table I and also indicated in Figures 1-5 by "L-S".

For reference, it should be noted that the presently estimated matter content of the metagalaxy is about 10^{54} g and that quasistellar objects have been reported with a redshift of $z \approx 3$ representing an expansion velocity of .88 c, if the redshift is interpreted as a Doppler effect. The calculation has been made under the assumption of a constant k/k_o throughout the motion of the metagalaxy. It might be argued that due to the chemical development (recombination of protons and electrons into hydrogen, a similar formation of antihydrogen and the resulting annihilation of hydrogen-antihydrogen, for instance) or perhaps due to the formation of local irregularities, such as stars or galaxies, the parameter k/k_o would not be a constant but be a function of time and perhaps position.

Table I

k/k_o	Initial mass giving a maximum expansion velocity $^{10}\log M_o$	Remaining mass %	Maximum expansion velocity v/c
0.5	-0.75	not given	.15
1.0	-0.65	30	.38
2.0	-1.0	not given	.4
4.0	-1.13	not given	.4

M_o is the designation for mass as used in the formula for mass scaling

1. LIMITATION ON MASS

The value of the annihilation cross section for the values of k/k_o as used by L-S implies an annihilation rate of

$$v\sigma_a \approx 10^{-15}\ cm^3\ sec^{-1}$$

As has been realized for a long time, this is the annihilation rate in the limit of high temperature. Since the metagalaxy has a temperature which is probably smaller than $10^2\ ^oK$, all interactions take place through S waves and due to the attractive influence of the Coulomb field between protons and antiprotons, the annihilation cross section follows rather a $1/v^2$ than the $1/v$ behavior as has been used. (See for instance, Landau and Lifshitz: Quantum Mechanics, 2nd ed., p. 549, Pergamon Press). Further, at such a low temperature most of the protons and electrons combine into hydrogen (see sec. 2) with similar formation of antihydrogen and the annihilation cross section for hydrogen-antihydrogen is even higher [8]. Puget [3, ref. cit.] gives the value for the $p\bar{p}$ annihilation cross section as

$$\sigma_a \approx \frac{\pi}{8}\lambda^2$$

(where λ is the deBroglie wavelength) which gives an annihilation rate of

$$v\sigma_a \approx 10^{-11}\ T^{-\frac{1}{2}}\ cm^3\ sec^{-1}$$

The annihilation rate is now dependent on temperature as $T^{-\frac{1}{2}}$ but the k/k_o ratio is affected only as $T^{1/4}$. The $H\bar{H}$ annihilation cross section is reported in ref. 8 as

$$\sigma_a = 1.06\ 10^7\ (\frac{c}{v})^{0.64}\ \pi\ r_o^2$$

leading to an annihilation rate of

$$v\sigma_a \approx 10^{-10}\ cm^3\ sec^{-1}$$

Further, the scattering cross section for high energy (180 MeV) photons and 100 MeV electron-positron pairs (which constitute the main source of the metagalactic opacity) is a fraction ($\approx 1/40$) of the Thompson cross section. Taking these values into account, we arrive at a more realistic scattering to annihilation ratio as

$$k/k_o \approx 10^{-3}$$

Since this ratio is so much smaller than the one previously used as standard, the calculations of L-S have been extended to encompass a wider range of the parameter k/k_o as $20 > k/k_o > 0.01$. The purpose of the calculations was to arrive at possible limits of the k/k_o values, to investigate the range of expansion velocity for different initial masses M_o and particularly to establish the maximum mass the model allows for various rates of annihilation and scattering or conversely, given the mass (as for instance, the presently observed mass of 10^{54} g) to give the criteria for annihilation and scattering rates so that the model may agree with observations. The following conclusions can be drawn:

a) If we accept the value of $k/k_o = 0.001$ as the more realistic ratio, we are clearly in a region of a radiation catastrophe. The annihilation rate is now so large that most of the mass gets annihilated and the rest collapses into the Schwarzschild radius. Indeed, the limit of existence when the metagalaxy halts just outside the Schwarzschild radius but achieves no appreciable outward velocity occurs at $k/k_o = 3.0$ for one particular mass of $M_o = 0.019$. All other masses considered collapse at this k/k_o.

b) At large value of k/k_o, the annihilation decreases rapidly providing less radiation to generate pressure with the result that the maximum initial mass M_o able to escape gravitational collapse gets rapidly smaller (Fig 5).

c) Since the temperature range of the metagalaxy is probably not larger than 100^o K, the temperature dependence of the annihilation rate as $T^{-\frac{1}{2}}$ is immaterial for the results because k/k_o is increased only as $T^{1/4}$ which is too small to displace k/k_o out of the region of gravitational collapse.

d) If we specify that the observed mass of the universe (10^{54} g) is to be attained, we can uniquely determine the annihilation rate and scattering cross section as illustrated in Fig 1, or if we specify the scattering cross section, we can determine the maximum mass the metagalaxy can attain, as done in Fig 2. Since the scattering cross section is probably smaller than the Thompson cross section, the observed mass cannot be attained.

One can see that the difficulty encountered is not only the large annihilation which prevails but also the insufficient opacity of the metagalaxy, that is, a large portion of the radiation escapes without providing the necessary pressure. We can think of decreasing the annihilation rate by separating matter from antimatter into separate cells which would react only through their boundaries; to increase the opacity is a more difficult task. We will probably have to abandon the radiation pressure as leading to the expansion of the metagalaxy and provide pressure by other means, perhaps by having a magnetic field act on the electrons and thus provide a sort of magnetic pressure. Whether this is possible, remains to be worked out.

2. EXPANSION VELOCITY

Since the theory attempts to provide an explanation of the recession velocity as being caused by radiation pressure, it is worthwhile to investigate what is in fact the maximum velocity permissible by the model, and secondly, what is the cause of the apparent limit of 0.4 c as reported by L-S. Since a force in a tensor notation is

$$F_\alpha = A\, S^{(p)}_{\alpha\beta}\, \hat{p}^\beta$$

$$= A\, p_\alpha p_\beta \frac{p^\beta}{|\bar{p}|}$$

$$= A\, \overset{(p)}{S^o_\alpha}$$

$$F_1 = A \sum_{\substack{\text{all} \\ \text{directions}}} \overset{(p)}{S^o_\alpha}$$

(p) denoting plane waves crossing an area A of a small sphere from all directions, we have to investigate whether the S^o_1 component of the radiation tensor used in the L-S work attains a zero value in a frame moving at some particular velocity, and if so, what is such a velocity. The tensor is

$$S_{\mu\nu} = 2\sqrt{g}\int \rho\,(p, x)\, p_\mu p_\nu\, \delta\,(p^2)\, \theta\,(p^o)\, d^4p$$

or in a local Lorentz frame

$$S_{\mu\nu} = \int \rho\, p_\mu p_\nu \frac{d^3p}{p^o}$$

For more details the reader should consult the original work by L-S. Note that the phase space density is Lorentz invariant in the sense that

$$\rho\,(p^\mu, x^\nu) = \rho'\,(p^{\mu'}, x^{\nu'})$$

the primes indicating components transformed by Lorentz transformation. Thus we shall calculate the tensor

$$S^{o'}_1 = \int p'_1 \left(\frac{p^{o'}}{|\bar{p}'|}\, \rho'\right) d^3p'$$

and see whether it can attain the value zero. Now the density is given as

$$\rho = N\,\delta\,(\bar{p}^2 - P^2)\,\theta\,(p_1)$$

where P represents a photon momentum of a particular frequency and $\theta\,(p_1)$ is a step function described earlier. Taking the x axis as the direction of propagation and transforming the density, the radiation tensor in a moving frame assumes the form

$$S_1^{o'} = N \int \bar{p}' \cos\theta\, \delta\,[\bar{p}'^2 \{\frac{(\cos\theta + v)^2}{1 - v^2} + \sin^2\theta\} - P^2]\, \theta\,(\frac{\cos\theta + v}{\sqrt{1 - v^2}})]$$

$$\bar{p}'^2 \sin\theta\, d\theta\, d\phi\, d\,|\bar{p}'|$$

where

$$d^3\bar{p}' = \bar{p}'^2 \sin\theta\, d\theta\, d\phi\, d\,|\bar{p}'|$$

$$p_1' = \bar{p}' \cos\theta + \bar{p}' v$$

$$p_2^{2'} + p_3^{2'} = \bar{p}^{2'} \sin^2\theta$$

Calling now

$$A^2 = \frac{(\cos\theta + v)^2}{1 - v^2} + \sin^2\theta$$

and writing the argument of the delta function as

$$(\bar{p} - \frac{P}{A})\,(\bar{p} + \frac{P}{A})$$

and integrating ϕ and the $\bar{p}$, one obtains

$$S_1^{o'} = 2\pi N \int \frac{(\frac{P}{A}) \cos\theta\, (\frac{P}{A})^2 \sin\theta\, d\theta}{2PA}$$

$$= \pi N P^2 (1 - v^2) \int \frac{\cos\theta \sin\theta\, d\theta}{(1 + v\cos\theta)^4}$$

Changing the variables by $\cos\theta = x$ and establishing the integration limits from the θ step function, one obtains the integral

$$\int_{-v}^{1} \frac{x\,dx}{(1 + vx)^4} = 0$$

which then, under the assumption that momentum density may be zero in a moving frame, should equal zero. A straightforward calculation shows that indeed the momentum density and therefore radiation pressure ceases when

$$v = \frac{-1 + \sqrt{7}}{3}\, c \approx 0.45\, c$$

We thus see that the maximum expansion velocity of the metagalaxy, as described by the radiation tensor used in L-S work is slightly above 0.4 c and the apparent limit as reported by the authors finds its explanation. Although the above velocity has been derived under the assumption that the source emitting radiation is at rest, calculations show that the maximum outward velocity is achieved immediately after turning and the assumption is thus not without support. This limit is compatible with the largest observed redshift of the galaxies, the theory seems to imply, however, that objects with larger redshifts, as for instance the quasistellar sources, own their large redshift to other causes than their velocity.

3. SEPARATION OF MATTER AND ANTIMATTER

Aside from considerations of dynamics a more realistic determination of the annihilation rate has also a bearing on matter-antimatter separation. Since it has been thought at one time that a decreased annihilation rate would enable the metagalaxy to attain a mass compatible with the observed mass (see sec. 2), the separation of matter and antimatter into cells of opposite kind, where annihilation would take place only at the boundaries, would accomplish such a purpose. The previous section shows, however, that the observed mass is difficult to attain; nevertheless it is of interest to consider what effect the new annihilation rate as described by $k/k_o = 10^{-3}$ would have on the separation process. The electrolysis mechanism as described by Alfvén [9] would separate regions with radius

$$R = 5.5\ 10^{9}\ \left(\frac{Bt}{N_o}\right)^{\frac{1}{2}}$$

where t is the life time of protons and B the magnetic field. Taking

$$N_o = 10^{-3}\ \text{protons-cm}^{-3}$$

$$B = 10^{-5}\ \text{gauss}$$

$$t = \frac{1}{N_o v \sigma_a} \approx 10^{14}\ \text{sec}$$

the radius of separated cells would be

$$R \approx 5.5\ 10^{15}\ \text{cm}$$

Since, however, the annihilation rate of $10^{-11}\ \text{cm}^{-3} - \text{sec}^{-1}$ would imply the mean free path of protons to be, at $T = 10^{o}$ K

$$\lambda = \frac{v}{N_o(v\sigma_a)} = 10^{18}\ \text{cm}$$

the much larger mean free path would cause an immediate mixing of the regions. A magnetic field is therefore required to prevent it. Since a charged particle entering a magnetic field is reflected approximately when

$$Br = \frac{c}{2e}\ \sqrt{2 m_p k T}$$

this is easily accomplished. With the same values for B and T as above, the thickness of the annihilation zone would be

$$r \sim 2\ 10^{5}\ \text{cm}$$

Thus we see that a magnetic field is not only necessary to separate matter from antimatter but also to keep them separated. However, as Alfvén pointed out, since the mechanism is not able to separate even a mass of one star, we need therefore another mechanism to enable individual cells of matter or antimatter to merge into larger units. Since the magnetic field apparently plays a central role in the theory, additional questions arise such as how did the magnetic field arise, what is its topology, how did such a topology arise and how does coalescence take place? These are unsolved questions.

4. UNIVERSAL BLACK BODY RADIATION

The observational tests for cosmologies are rare, mainly because various cosmologies predict features which are out of our observational means. However, at times, a fortuitous prediction of a cosmology can be put to a test. One of these is the prediction of the Gamow theory that there should exist relics from earlier times detectable today. The relics are photons which marked the end of thermal equilibrium in an expanding universe. At this time, at $T = 3\ 10^{3\,o}$ K, electrons and protons combined to form atoms and because of a large difference between scattering cross sections of photons and free electrons and photons and neutral atoms, the photons effectively ceased to react with matter. Since they originated in a thermal equilibrium, their spectrum should have a marked black body character. Their effective temperature, however, decreased due to the expansion of the universe (since their number in a comoving volume remains constant) but their spectrum nevertheless kept the black body character. Such spectrum has been detected following to a high degree the Planck distribution and represents a challenge to any cosmology to explain it. The metagalaxy in the Alfvén-Klein version never attained thermal equilibrium and any equilibrium spectrum is therefore without explanation. * That is not to say that such a spectrum cannot be accomodated within the theory but only to illustrate that despite the number of years the theory has been in existence, much work remains to be done. After all, the birth of modern physics came about with the invention of the experimental method and the only reason for existence of any theory of physics, as distinct from philosophy, is to account for observations.

* O. Klein has recently calculated a temperature which is near the temperature of the black body background radiation, assuming an adiabatic expansion of the metagalaxy.

EXPERIMENTAL EVIDENCE

The evidence on which we base our knowledge of the extraterrestial world is almost exclusively derived from radiation of different wavelengths which reaches our telescopes. However, when we try to search for antimatter in the universe, this evidence is ambiguous since matter and antimatter cannot be distiquished spectroscopically. Thus hydrogen and antihydrogen, for instance, have exactly the same spectrum. We could infer the presence of antimatter by means of some effect, as for instance the splitting of spectral lines in a magnetic field but an independent way of determining the direction of the magnetic field would have to be known. Thus we are limited in our search for antimatter to look for other effects, such as the direct evidence of cosmic rays or some indirect evidence, as the Faraday rotation, among others.

1. COSMIC RAYS

Cosmic rays are high energy particles, from protons to heavier nuclei, that arrive from outer space. If then there is a universe containing (perhaps equal) amounts of antimatter, such antimatter should make itself evident in that some portion of cosmic rays should consist of antinuclei. Since a secondary production of antiprotons (by collisions of cosmic rays with interstellar gas) is estimated as $\bar{p}/p \approx 10^{-3} - 10^{-7}$ [10] and a secondary production of alpha particles as 10^{-13}, the presence of a heavier nucleus in cosmic rays would be a direct and most conclusive evidence for the presence of antimatter in the metagalaxy. None have been detected. The upper limit of relative amount of antinuclei in primary cosmic rays has been reported as low as $5 \cdot 10^{-4}$ [11]. Since, however, there is no agreement as to where the cosmic rays originate, whether they are of local, galactic or extragalactic origin, the boundary of the region of matter from which the (matter) cosmic rays come from, cannot be established. It has been said that the steepening of the cosmic ray spectrum between 10^{15} eV - 10^{18} eV (as $E^{-\gamma}$ with $\gamma \approx 2.2$) indicates a presence of an extragalactic component; since, however, such high energy cosmic rays can be observed only by means of air showers and the charge cannot be distinquished, a possible evidence for regions outside our own galaxy is, even in this view, not available. Our immediate neighborhood is obviously made of matter; how far this region extends would be, with our present knowledge of the origin of cosmic rays, a mere speculation.

FARADAY ROTATION

The interaction of an electromagnetic wave with a magnetic field $\bar{B}$ and leptons leads to a rotation of polarization planes by an angle given as

$$\Delta\theta = \int (N_e - N_{\bar{e}})\ B_{||}\ ds$$

where the integral is taken along the line of sight between the observer and the source and $(N_e - N_{\bar{e}})$ is a difference between densities of negative and positive leptons. It is thus a measure of an excess of electrons over positrons or the average value of the magnetic field component parallel to the wave vector. Thus for a regular distribution of matter and antimatter along the line of sight the rotation angle should disappear. The observation of radio sources show, however, quite a large value of the rotation measure and a dependence on redshift [12,13]. The rotation can originate either at the source, or in the intergalactic medium or in our own galaxy. The knowledge of the electromagnetic properties of these regions should be therefore available. In the absence of such knowledge it can be argued that separate cells of matter and antimatter contain magnetic field of opposite sense or statistics can be pressed into service to show that due to the small variance of the rotation measure as a function of redshift tends to show mixed matter and antimatter at $z > 1.85$ [14] or the same statistics can show that scatter of the data attributable solely to antimatter regions shows no matter-antimatter symmetry for $z < 2$ [13]. Thus the observational evidence for the presence of antimatter is either lacking, or at best, ambiguous.

PART II

INTRODUCTION

This work has its origin in the investigations pertaining to the Alfvén-Klein metagalactic theory [2, 9]. Since the dynamical studies [6] showed a need for greater understanding of annihilation and recombination rates, effort has been undertaken to clarify them. When it became apparent that formation of hydrogen and antihydrogen atoms plays a large part in such processes, it became necessary to study the interaction of these atoms. The first trial calculation showed that there may be a repulsive potential between them. At about the same time, an argument has been advanced [23] that indeed due to the repulsive nature of the electron-positron overlap charge, a potential barrier ought to exist. Somewhat later, calculations by perturbation method confirmed this [15]. This had a profound importance for the metagalactic theory, since the annihilation rate would markedly decrease and, as was thought at that time, alleviate one of the major problems of the theory. Since the first calculation lacked in accuracy and the degree of accuracy in the variational method chosen depends on an adequate description of the wave function in the Schrödinger equation, the wave function has been enlarged to include a set of twenty functions consisting of 1s and 2p Slater orbitals. It soon became apparent, however, that while the potential barrier still existed, it decreased steadily with each enlarged input set of functions. The question now presented itself whether the potential barrier would prevail for an arbitrarily large set. Since the computor program was limited to an input of twenty functions, a new program was written extending the input to as high as 5f hydrogen states, limited essentially only by the computor capability. While the new program was in the check-out stage, a revised article by Junker and Bardsley was published in the Physical Review Letters [22] showing that for an input of more than sixty-four functions, the potential barrier has disappeared. Since the operation of the new program was expensive and the question was considered to be settled, further work on the subject was discontinued. Recently, a private communication from Kolos, Wolniewicz and Schrader, the scientists responsible for the most accurate calculation of the hydrogen molecule ever done, verified that by using a method superior in accuracy to both of the author and of Junker and Bardsley, the potential barrier between hydrogen and antihydrogen atoms does not exist. However, few other problems in quantum chemistry have been of such a fundamental importance (a repulsive potential would enable matter and antimatter in atomic form to coexist at low temperature, leading to far reaching consequences) while at the same time depended on so subtle effects requiring an extreme accuracy in computations.

ANNIHILATION RATES

Since annihilation and scattering within the metagalaxy have a strong influence upon its dynamical behavior [6], it is worthwhile to clarify what happens in the metagalaxy on the atomic and elementary particle scales. It is the purpose of this section to give an estimate of such processes and in what degree they influence the total annihilation rate and the optical depth. These processes as as follows:

$e + \bar{e} \rightarrow 2\gamma$	direct annihilation
$e + \bar{e} \rightarrow Ps$	annihilation from a bound state (positronium)
$p + \bar{p} \rightarrow (x)\,\pi^0 + (y)\,\pi^+ + (z)\,\pi^-$	reaction giving rise to 5-6 pions on the average which react as
$\pi^0 \rightarrow 2\gamma$	
$\pi^+ \rightarrow \mu^+ + \nu_\mu$	
$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$	which in turn disintegrate as
$\mu^+ \rightarrow \bar{e} + \bar{\nu}_\mu + \nu_e$	
$\mu^- \rightarrow e + \nu_\mu + \bar{\nu}_e$	

Since the life time of pions is small on the scale of metagalactic history, we shall consider only the final products

$p + \bar{p} \rightarrow 4\gamma + 2e + 2\bar{e}$ + neutrinos	
$p + e \rightarrow H$	
$\bar{p} + \bar{e} \rightarrow \bar{H}$	
$H + \bar{H} \rightarrow Pn + e + \bar{e}$	annihilation from a bound state of protonium
$\rightarrow Pn + Ps$	and positronium or a free $e\bar{e}$ pair

The energy of the final product is:

photons:	$E = m_e c^2 \approx 0.51$ MeV	from direct $e\bar{e}$ annihilation
	$E_{av} \approx 180$ MeV	for photons from $p\bar{p}$ reaction although the energy spectrum spans from several tens to several hundreds of MeV, 180 MeV being the average energy
$e\bar{e}$ pairs:	$E \approx 100$ MeV	for pairs from $p\bar{p}$ annihilation with an energy spectrum similar to that of 180 MeV photons
neutrinos:	$E \approx 800$ MeV	for all neutrinos from $p\bar{p}$ reaction, that is, about half of the available energy

DIRECT ANNIHILATION

The cross section for the $e\bar{e}$ reaction, in the region of intermediate energies when the plane wave approximation is acceptable, is given by Dirac [16] as

$$\sigma_{e\bar{e}} = \frac{c}{v} \pi r_o^2$$

where r_o is a classical electron radius and v the relative velocity. At the low energies pertinent in the metagalaxy the plane wave approximation is no longer valid and the effect of the Coulomb field has to be accounted for by a correction factor [17]

$$\frac{|\psi^+_{\bar{k}} (r=0)|^2}{|\psi_{\bar{k}}|^2} = \frac{2\pi}{k(1-e^{-2\pi/k})}$$

where $\psi^+_{\bar{k}}$ and $\psi_{\bar{k}}$ are wave functions of a particle in a Coulomb field, respectively a free particle and k is a wave number. We thus get for the $e\bar{e}$ direct annihilation cross section

$$\sigma_{e\bar{e}} = \frac{2\pi k}{v\, k\,(1-e^{-2\pi/k})}$$

The cross section for direct annihilation of protons and antiprotons is governed by strong interactions, in addition to the Coulomb field. Since no satisfactory theory extending over the energy range of interest exists, the same plane wave approximation and the same correction factor due to the influence of the Coulomb field has been used, this time, however, to extend existing experimental curve to low energies for which the experimental data are not available. Thus the $p\bar{p}$ annihilation cross section has been simulated by the formula

$$\sigma_{p\bar{p}} = A \frac{c}{v} \pi r_o^2 \frac{2\pi}{k(1-e^{-2\pi/k})}$$

and the constant A has been determined from $p\bar{p}$ scattering experiments at low energies [18]. This procedure may be somewhat questionable; however, what is needed is only an estimate and therefore deemed satisfactory. Comparison with expression derived by Puget [3, cit.] shows that these two expressions differ only by a factor of three.

ANNIHILATION FROM BOUND STATES OF PROTONIUM AND POSITRONIUM

A proton and an electron may recombine upon encounter. Likewise for an electron and a positron or proton and an antiproton. Since at the low densities prevailing within the metagalaxy, the annihilation from bound states proceeds much faster than encounters with particles tending to break up such a bound state, we may safely assume that once a particle and an antiparticle recombine, they will also annihilate. Hence the annihilation cross section for particles and antiparticles is equal to the recombination cross section. Although the total recombination cross section has been accurately given by Seton [19] as

$$\sigma_R = \frac{2^6}{3} \left(\frac{\pi}{3}\right)^{\frac{1}{2}} \alpha^4 c\, a_o^2\, \lambda^{\frac{1}{2}} \left(0.4288 + 0.5\, \ln \lambda + 0.469\, \lambda^{-\frac{1}{3}}\right)$$

where

$$\lambda = \frac{hRc}{kT}$$

with R, α meaning Rydberg and fine structure constants and a_o the Bohr radius, we have used the Kramer's formula instead as being sufficiently accurate at low energies, i.e.

$$\sigma_R^K = \frac{8\pi e^6}{3\sqrt{3}\, m\, c^3\, \hbar} \frac{1}{kT} \ln \frac{E_{ionization}}{E_{kinetic}}$$

Recombination rates have been calculated for protonium, positronium and hydrogen and the results are illustrated in Fig 6.

ANNIHILATION OF HYDROGEN AND ANTIHYDROGEN

Anticipating results of the following section, let us state that the potential between hydrogen and antihydrogen is attractive, at least at small separation.

As the hydrogen and antihydrogen atoms approach each other the binding energy of the electron and the positron successively decreases due to their mutal interactions. At the same time, the interaction energy (zero at $R \to \infty$) of the $H\bar{H}$ system increases and at a certain internuclear distance R, say R_c, becomes equal to the binding energy of the original system (at $R \to \infty$) while concurrently the binding energy of the electron and positron goes to zero. The electron and the positron therefore escape from the system. If the pair escapes as positronium (binding energy 6.8 eV), the critical R_c occurs earlier, at the interaction energy of 20.4 eV. If in addition, the kinetic energy of the nucleons is less than 20.4 eV, protonium will be formed and annihilation will take place from a bound state of protonium. The annihilation cross section is then πR^2, where R is the maximum impact parameter which gives the closest point of approach equal to the critical internuclear distance R_c. The classical trajectory, an approximation used in this work, is of course dependent upon the kinetic energy and the $H\bar{H}$ potential. The annihilation rate, as illustrated in Fig 6, is therefore of the same order of magnitude as annihilation rates of $p\bar{p}$ and $e\bar{e}$ (positronium). The argument is due to Morgan and Hughes [8]. The products of annihilation give rise in turn to the following processes:

a) pair production by collision of high energy photons with charged particles with cross section

$$\sigma = \alpha \pi r_o^2 \left(\frac{28}{9} \ln \frac{2 E_\gamma}{m_e c^2} - \frac{106}{9}\right)$$

b) Compton heating of electrons, with a cross section

$$\sigma = \pi \left(\frac{e^2}{4\pi m_e c^2}\right) \frac{m_e c^2}{E_\gamma} \left(\ln \frac{2 E_\gamma}{m_e c^2} + \frac{1}{2}\right)$$

c) bremsstrahlung of a wide energy range

d) photoionization of the newly-formed hydrogen which, however, does not effect the annihilation rate since the annihilation rate of hydrogen-antihydrogen, respectively positronium are comparable in magnitude. We can thus conclude in general that annihilation within the metagalaxy is the predominant process with a cross section of 10^{-14} - 10^{-15} cm^2 and all other processes are almost negligeble with a cross section which is smaller than 10^{-25} cm^2.

THE HYDROGEN-ANTIHYDROGEN POTENTIAL

Problems amenable to exact solutions using quantum mechanical approach are two-particle problems. A system, like the hydrogen-antihydrogen system containing an electron, a positron, a proton and an antiproton can be solved only approximately. The approximation in solving the Schrödinger equation describing such a system, that is

$$\left[-\sum_{i=1}^{2} \frac{\hbar^2}{2 M_i} \nabla_i^2 - \sum_{j=1}^{2} \frac{\hbar^2}{2 m_j} \nabla_j^2 + V(X_i, x_j)\right] \psi(X_i, x_j) = E \psi(X_i, x_j)$$

with X_i and x_j meaning the coordinates of nucleons and electrons, consists in arresting the motion of the nuclei and considering only the motion of the electrons. The Coulomb interaction of all four particles is, however, included. The Schrödinger equation reduces then to

$$\left[-\sum_{j=1}^{2} \frac{\hbar^2}{2 m_j} \nabla_j^2 + V(X_i, x_j)\right] u(X_i, x_j) = E(X_i)\, u(X_i, x_j)$$

where the wave function $u(X_i, x_j)$ is now a function different from the function $\psi(X_i, x_j)$. The Born-Oppenheimer approximation, as this method is called, consists in using the energy of the electron motion $E(X_i)$ as the potential energy for the motion of the nuclei according to the equation

$$\left[-\sum_{i=1}^{2} \frac{\hbar^2}{2M_i} \nabla_i^2 + E(X_i)\right] v(X_i) = E\, v(X_i)$$

The product of the wave functions u (X_i, x_i) and v (X_i) should be a reasonable approximation to the four-particle wave function ψ $(X_i x_j)$ as it indeed is. The error in this approximation is about 0.0148 eV in the equilibrium energy of the hydrogen molecule. We see that the feasibility of the method rests essentially on the much greater mass of the nuclei. In addition to the above approximation, no regard has been paid in this work to such small effects as the spin-orbit coupling, coupling with nuclear moments and relativistic corrections.

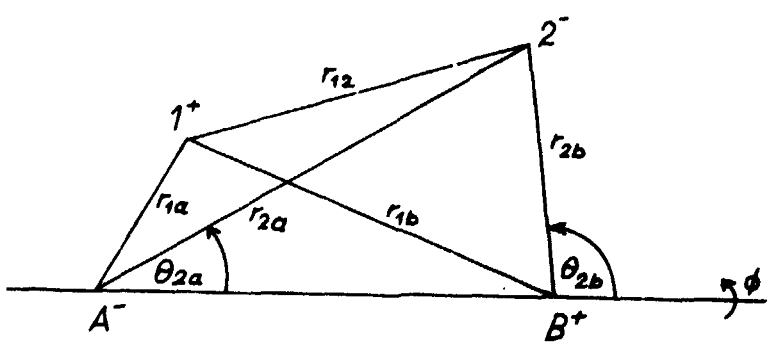
Since there are two fixed centers in the problem (nuclei), a suitable coordinate frame must be chosen. Such a frame is the prolate spheroid coordinate system, given by

$$\xi = \tfrac{1}{2} R\,(r_a + r_b)$$

$$\eta = \tfrac{1}{2} R\,(r_a - r_b)$$

$$\phi = \phi$$

with the meaning of various terms illustrated in the following figure:



The designation A^-, B^+, 1^+, 2^- denote in the same order an antiproton, a proton, a positron and an electron. R is the internuclear distance (held fixed) and the angle ϕ is an angle in the plane perpendicular to the axis connecting the two nuclei.

The Hamiltonian for the electronic wave function is

$$\hat{H} = -\tfrac{1}{2}\nabla_1^2 - \tfrac{1}{2}\nabla_2^2 - \frac{1}{r_{1a}} + \frac{1}{r_{2a}} + \frac{1}{r_{1b}} - \frac{1}{r_{2b}} - \frac{1}{r_{12}} - \frac{1}{R}$$

The expectation value of the first two terms describes the electron kinetic energy, of the next four terms their potential energy, the next term the interelectronic interaction energy, and the last term the internuclear interaction energy. It differs from a Hamiltonian for a hydrogen molecule only in signs due to the different charges of the particles.

THE WAVE FUNCTION

Since the system is composed of an atom and an antiatom, the wave function has to encompass both centers, subject only to symmetry along the axis connecting the two nuclei. No other symmetry has been put in although there is a symmetry upon simultaneous exchange of coordinates and charge. We therefore use as a wave function for the system a linear combination of products of two wave functions as

$$u = \sum_{ij} c_{ij}\, \phi_i(1)\, \phi_j(2) = \sum_k C_K\, \Phi_K$$

where the functions ϕ are wave functions for a hydrogen or an antihydrogen atom. The c_{ij} are unknown coefficients to be determined. Thus, for instance, for the 1s state, the wave function ϕ_{1s} is

$$\phi_{1s} = \left(\frac{\alpha^3}{\pi}\right)^{\frac{1}{2}} e^{-\alpha r}$$

These are also called Slater orbitals. Since we have two fixed centers, the product then refers either to one center or both. For instance,

$$u = C_1\, (1s)_A\, (1s)_A + C_2\, (1s)_A\, (2p)_B + \cdots$$

where the subscripts mean the nucleonic centers and the expression in parentheses are basic functions corresponding to quantum numbers (1s, 2p, etc). The symmetry along the line connecting the nuclei (the z axis) then places certain restrictions on the products. In this work, 1s and 2p states have been used giving a linear combination of twenty terms.

METHOD OF SOLUTION

Since the wave function is not exact, that is

$$\int \psi\, (\hat{H} - E)\, \psi^*\, dV \neq 0$$

the energy E must be minimized by

$$\delta \int \psi\, (\hat{H} - E)\, \psi^*\, dV = 0$$

where the variation δ means $\frac{\partial}{\partial C_i}$ with respect to the coefficients to achieve a minimum value of the energy. This leads to the eigenvalue problem

$$(F - ES)\, C = 0$$

where F is the matrix of the Hamiltonian, i.e.

$$F_{KL} = \int \Phi_K^*\, \hat{H}\, \Phi_L$$

and S is called the overlap matrix given as

$$S_{KL} = \int \Phi_K^* \Phi_L$$

Transformation methods are available for diagonalization. The eigenvector C for the lowest eigenvalue of the energy are then the coefficients C_k we wished to determine. The method gives energy eigenvalues which are upper bounds to the true values. Further improvement is attained by a numerical variation of the screening constants α appearing in each orbital. This, except for the calculation of the matrix elements, is the most time-consuming part of the problem. Since the atomic states contain each its own screening constant, these have to be varied separately.

MATRIX ELEMENTS

The use of Slater orbitals, which have been chosen for the ease of interpretation of the physical behaviour of the $H\bar{H}$ system, leads, however, to integrals which are not always straightforward. Consider the matrix element

$$\langle (1s)_A^1 \, (2p_z)_B^1 \left| \frac{1}{r_{12}} \right| (2p_z)_B^2 \, (2p_z)_B^2 \rangle$$

where the subscripts denote nucleons and the superscripts the leptons. In spheroidal coordinates, it reads

$$\left(\frac{\alpha_1^3}{\pi}\right)^{\frac{1}{2}} \quad \frac{\alpha_2^5}{3\pi}^{3/2} \quad \left(\frac{R}{2}\right)^9 \int_0^{2\pi} d\phi \int_{-1}^{1} d\mu_1 \, d\mu_2 \quad \text{(times)}$$

$$\int_1^{\infty} \exp\left[-\frac{R}{2}\left(\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_2\lambda_1-\alpha_2\mu_1\right)\right]\left(\lambda_1-\mu_1\right)\frac{1}{r_{12}} \quad \text{(times)}$$

$$\exp\left[-R\left(\alpha_2\lambda_2-\alpha_2\mu_2\right)\right]\left(\lambda_2-\mu_2\right)^2\left(\lambda_1^2-\mu_1\right)^2\left(\lambda_2^2-\mu_2^2\right)d\lambda_1\,d\lambda_2$$

The interelectronic distance $\frac{1}{r_{12}}$ has to be expanded in Neuman series, or

$$\frac{1}{r_{12}} = \frac{2}{R} \sum_{\tau=0}^{\infty} \sum_{\nu=0}^{\tau} \left[(-1)^{\nu} 2\,(2\tau+1)\left\{\frac{(\tau-\nu)!}{(\tau+\nu)!}\right\}\right] Q_\tau^\nu(\lambda_>) \quad \text{(times)}$$

$$P_\tau^\nu(\lambda_<)\, P_\tau^\nu(\mu_1)\, P_\tau^\nu(\mu_2) \cos\nu\,(\phi_1-\phi_2)$$

where the summation in square brackets starts from $\nu = 1$, for $\nu = 0$ being $[\;] = 2\tau + 1$ and the signs <, > mean larger or smaller of λ_1 or λ_2. The integrals thus lead to infinite series in most cases.

Well known methods for their solution are available. Defining new functions

$$G_{\tau}^{\nu}(m,\alpha) = \int_{-1}^{1} e^{-\alpha\mu} P_{\tau}^{\nu}(\mu)\, \mu^{m} (1-\mu^{2})^{\nu/2}\, d\mu$$

$$W_{\tau}^{\nu}(m,n;\alpha,\beta) = \int_{1}^{\infty}\int_{1}^{\infty} Q_{\tau}^{\nu}(\lambda_{>})\, P_{\tau}^{\nu}(\lambda_{<})\, e^{-\alpha\lambda_1-\beta\lambda_2}\, \lambda_1^{m}\lambda_2^{n} \quad \text{(times)}$$

$$(\lambda_1^2-1)^{\nu/2}(\lambda_2^2-1)^{\nu/2}\, d\lambda_1\, d\lambda_2$$

most integrals can be written as a combination of such functions and these functions have been tabulated [21]. Since, however, calculations have to be performed for each value of the internuclear distance R and various values of the screening constants α_1 and α_2 (for states 1s and 2p) these functions have been programmed in double precision on a computor and checked against the published values. The integrals themselves have been given in ref. 21, both as formulas and also for few values of the parameter R and α. Further, the values of the integrals have been checked against values calculated by using Gaussian type orbitals. Since infinite series are involved, the major problem encountered was the accuracy in the computation of integrals since small errors in their values led to unexpectedly large errors in the energy eigenvalues.

RESULTS AND COMMENTS

The results of the twenty function computation show a potential barrier at about R = 2.6 a.u. At the time when the computations have been completed, similar results have been reported [21]. However, calculation using 4, 8, 16 and 20 functions show also a steadily decreasing height of the potential. Since the program has been written to encompass calculations of a hydrogen molecule by a change of a single command and such calculations showed good agreement with published values, the behaviour of the potential has been deemed essentially correct. The question to settle was whether with an increased number of input base functions the potential barrier would prevail. The option was either to increase the input, or, since a variational calculation will yield a value larger than the true value, to determine the lower bound of the energy. Since the lower bound calculation entails determination of the matrix elements of a squared Hamiltonian and thus is notoriously difficult, the decision has been made to increase the input. Some time afterwards, a revised paper has been published by Junker and Bardsley [22] and the program has been discontinued. A twenty-term wave function consisting of 1s and 2p states cannot obviously describe the system adequately. However, what is evident in this approximation is that the exchange charge responsible for the binding of the hydrogen molecule plays almost no role and the effects are mostly electrostatic. Addition of more functions with m ≠ 0 describing higher states leads to a better description of correlation effects between the electron and the positron; this is brought out more clearly in the wave function of Kolos, Wolniewicz and Schrader which contains positron-electron inter-particle coordinates. If positronium is formed between the nuclei or the leptons escape as free particles, the wave function in such circumstances warrants a far better description that has been done in this work. The hydrogen-antihydrogen potential problem may by thus perhaps cited as a prime example of the importance of correlation effects. It would be of interest to determine the exact behaviour of the $H\bar{H}$ system by extending calculations up to the point when the potential becomes attractive and studying the corresponding wave function, but since the primary interest in the $H\bar{H}$ potential was in determination of the $H\bar{H}$ annihilation cross section, the disappearance of the barrier settled the question. What at the outset appeared as an extremely exciting problem with far reaching consequences not only for a certain cosmological theory but also for a general behaviour of coexistence of matter and antimatter in atomic form, proved in the end to be a battle of decimal places and fizzled out in an anticlimax.

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FIGURE CAPTIONS

Fig 1 Annihilation rates and scattering cross sections (multiplied by c) under the condition that the observed mass 10^{54} is attained.

Fig 2 Attained mass for various cross sections which are multiples of the Thompson scattering cross section

Fig 3 Percentage of initial mass annihilated

Fig 4 Maximum expansion velocity as a function of k/k_o. Triangles indicate at which value of R the contraction of the metagalaxy stops and is turned into expansion. R is a parameter in the scaling of the radius as

$$r = 1.3 \; 10^{26} \sqrt{\frac{\alpha\beta}{\alpha_o\beta_o}} \; R \text{ (cm)}$$

Fig 5 Maximum initial mass parameter M_o at which a maximum expansion velocity is attained. M_o is a parameter in the scaling equation for mass

$$m = 8.7 \; 10^{53} \sqrt{\frac{\alpha\beta}{\alpha_o\beta_o}} \; M_o \text{ (g)}$$

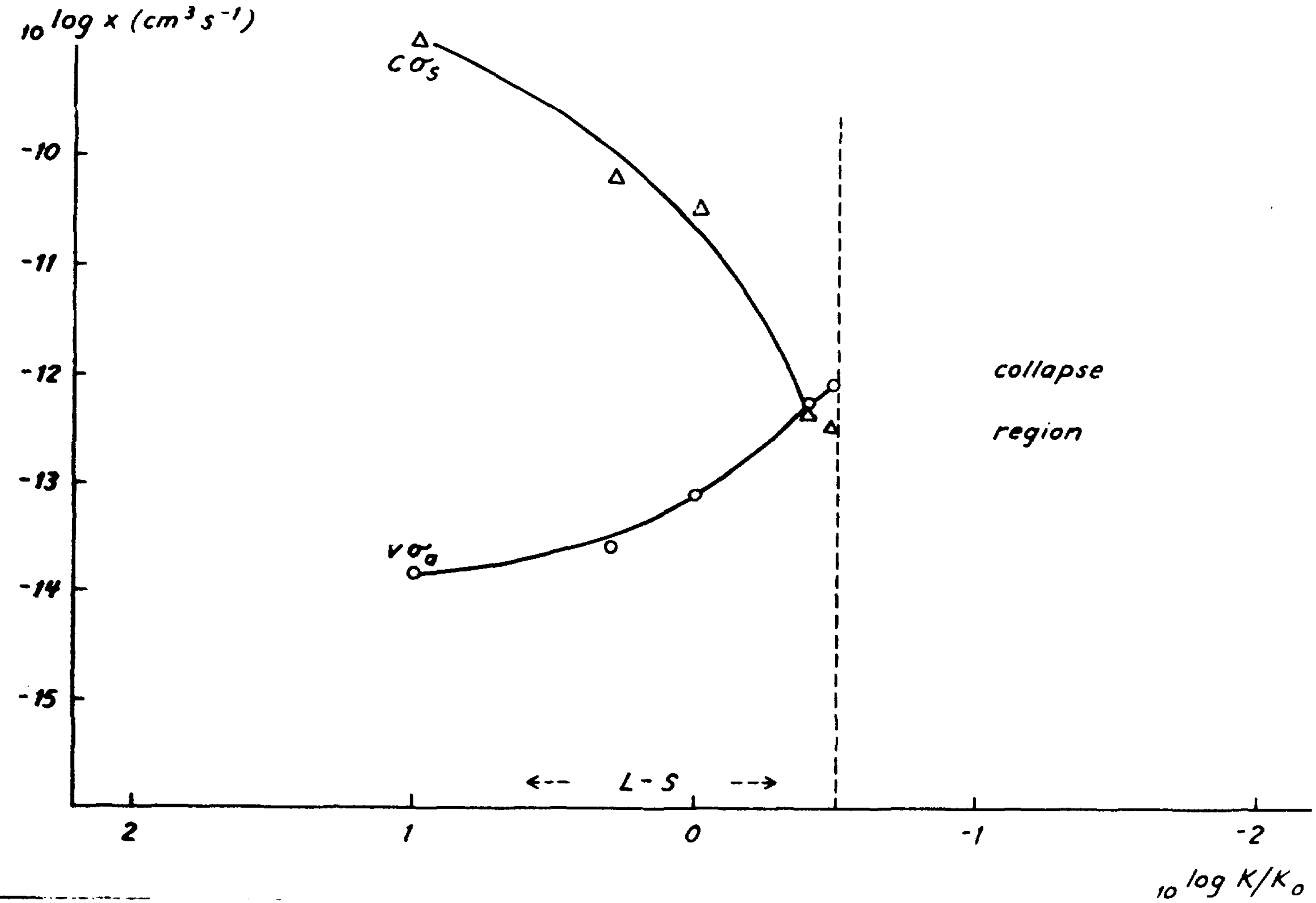
Fig 6 Annihilation and recombination rates. The fulldrawn lines indicate direct annihilation, the dotted lines annihilation from a bound state. Hydrogen formation is marked by H.

Fig 7 Hydrogen-antihydrogen potential. Numbers indicate the number of terms used in the wave function, JB means results reported by Junker and Bardsley [22] and KWS results by Kolos, Wolniewicz and Schrader (private communication).

Fig 8-9 Behaviour of coefficients of some terms in the 20-term wave function to illustrate exchange and hybrid effects. The letters designate the nuclear centers (proton and antiproton), the small letters for the atom in 1s state and the capital letters for the atom in 2p state, and the signs signify the charge on the leptons. Thus for instance (b^-A^+) describes a term of the wave function with the electron in 1s state located on center b (antiproton) and the positron in 2p state located on center a (proton), that is, an exchange term. Note the emergence of the positronium terms, especially in the 1s state.

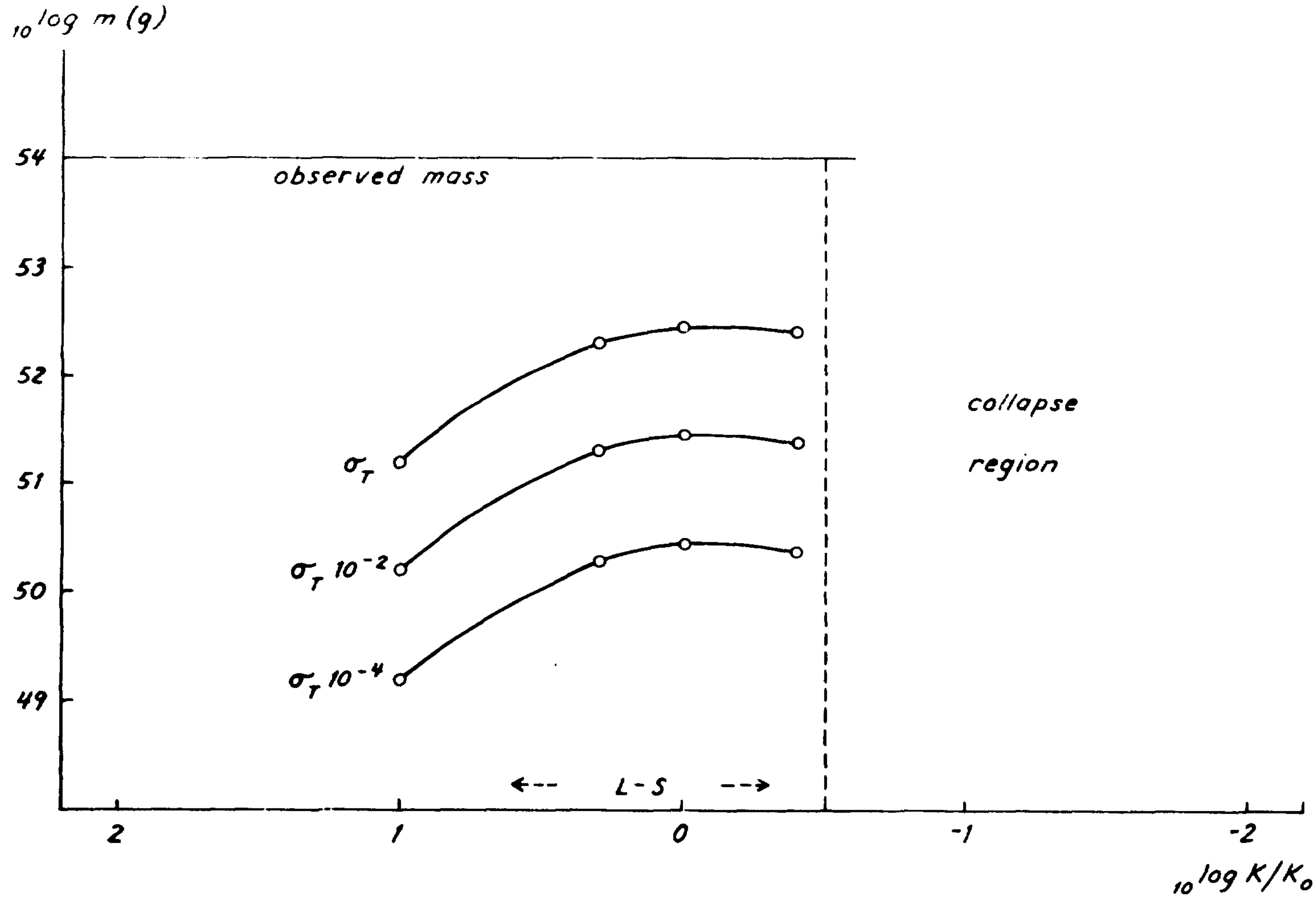
Fig 10 Behaviour of the screening constants α_1 for 1s state and α_2 for the 2p state.

Fig 1



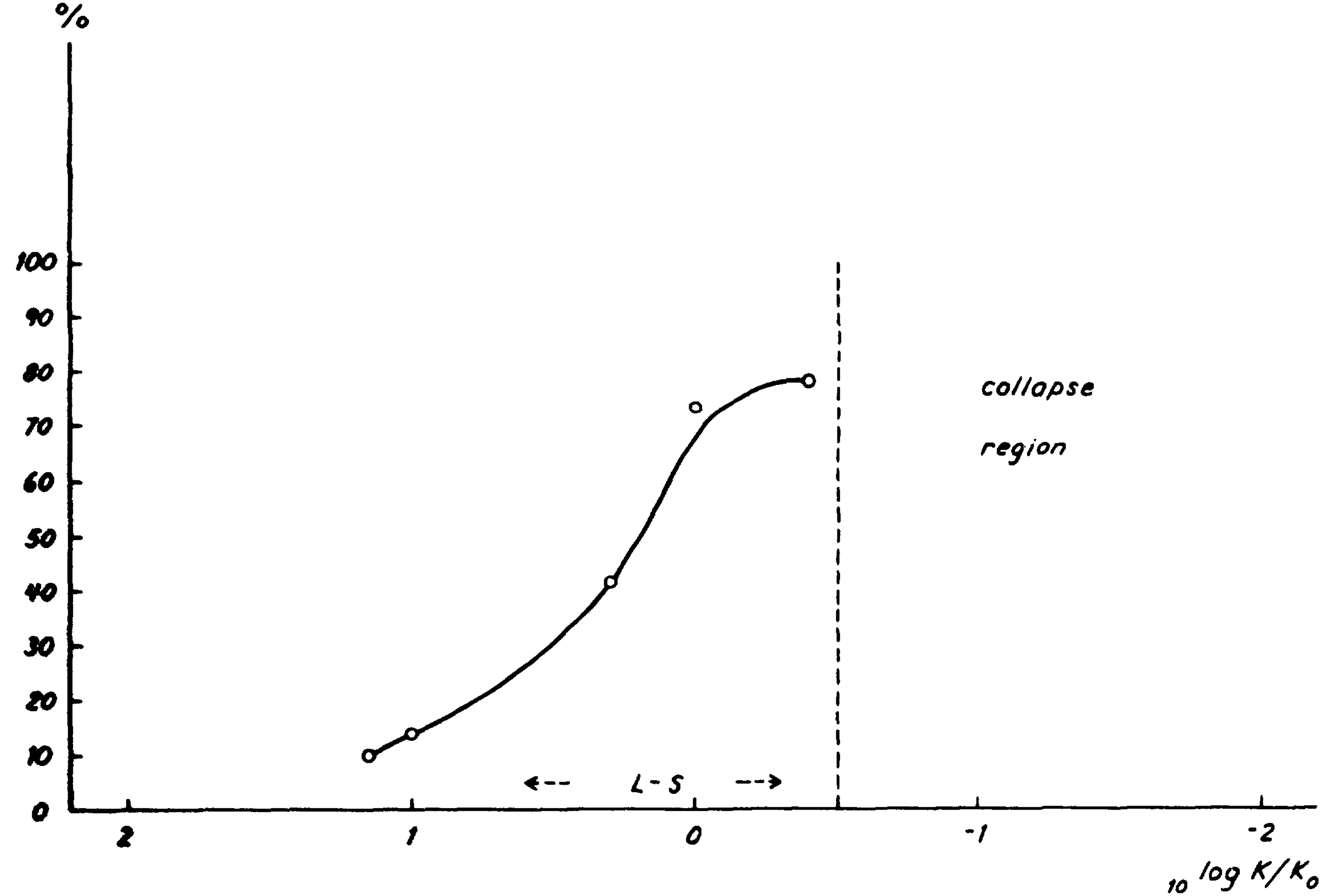
$_{10}\log x\ (cm^3 s^{-1})$
$c\sigma_s$
$v\sigma_a$
-10
-11
-12
-13
-14
-15
collapse
region
L - S
2
1
0
-1
-2
$_{10}\log K/K_o$

Fig 2



$_{10}\log m\ (g)$
54
53
52
51
50
49
observed mass
σ_T
$\sigma_T\ 10^{-2}$
$\sigma_T\ 10^{-4}$
collapse
region
← -- L - S -- →
2
1
0
-1
-2
$_{10}\log K/K_0$

Fig 3



%
100
90
80
70
60
50
40
30
20
10
0
2
1
0
-1
-2
10 log K/Ko
← - - L-S - - →
collapse
region

Fig 4

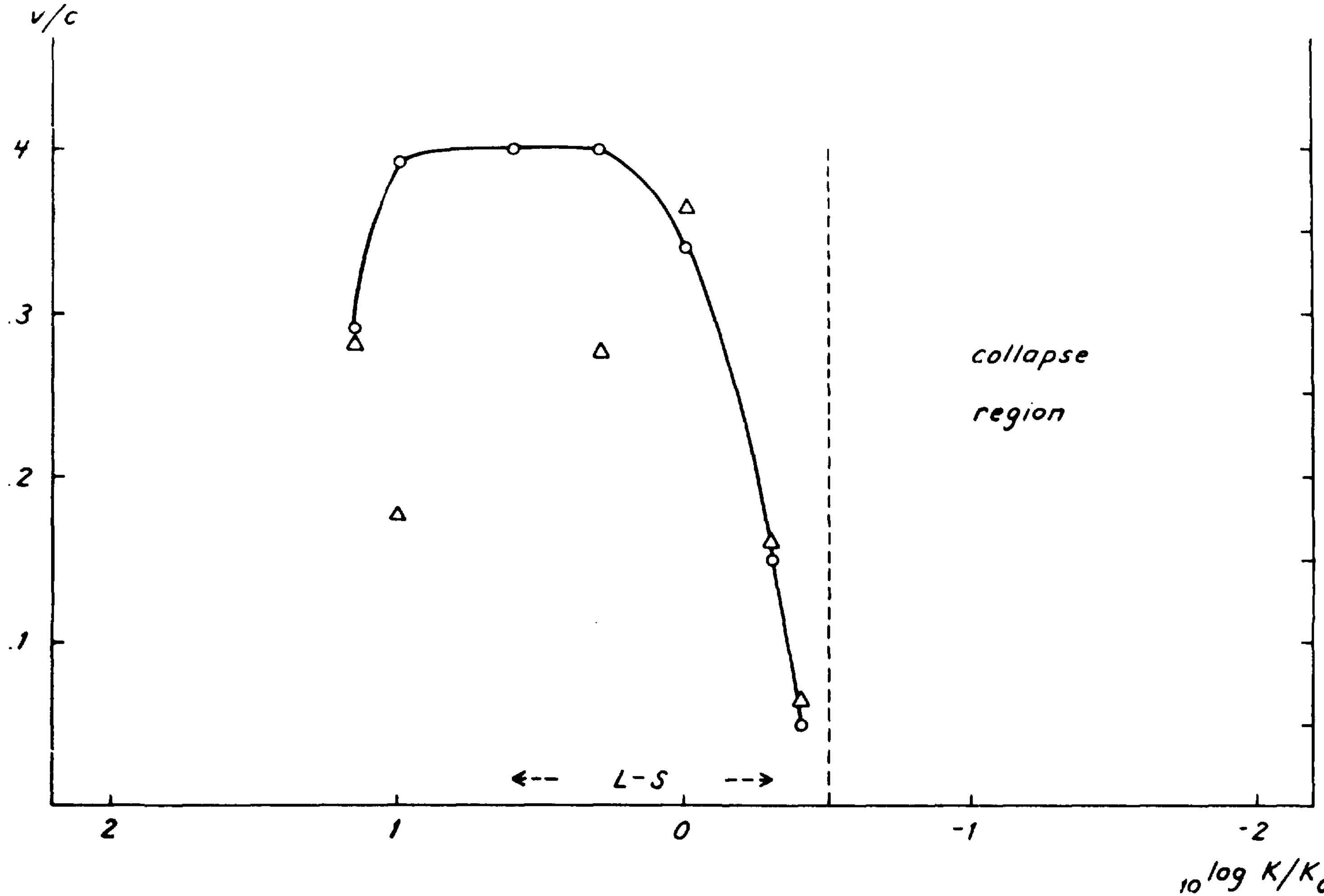
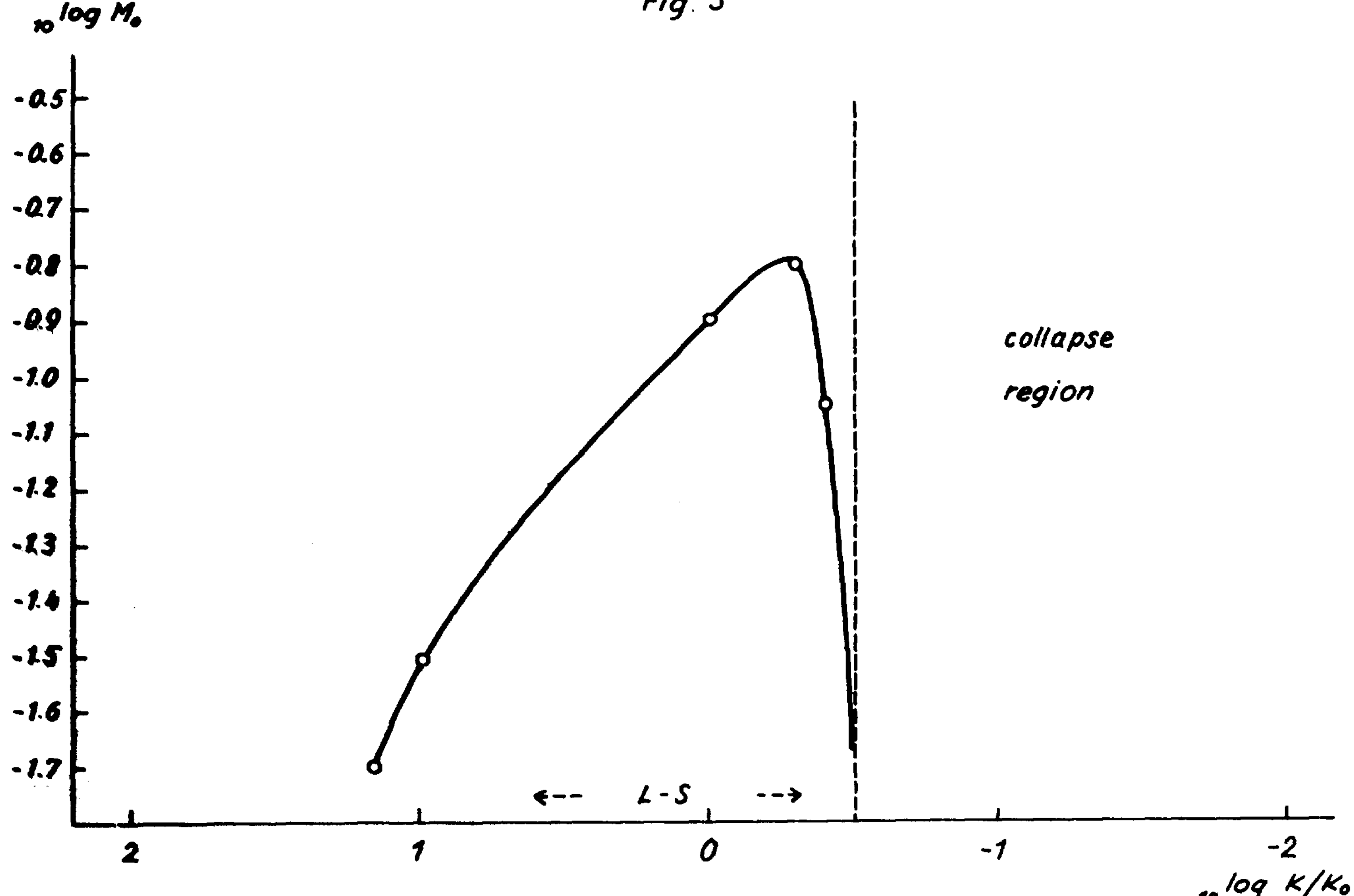
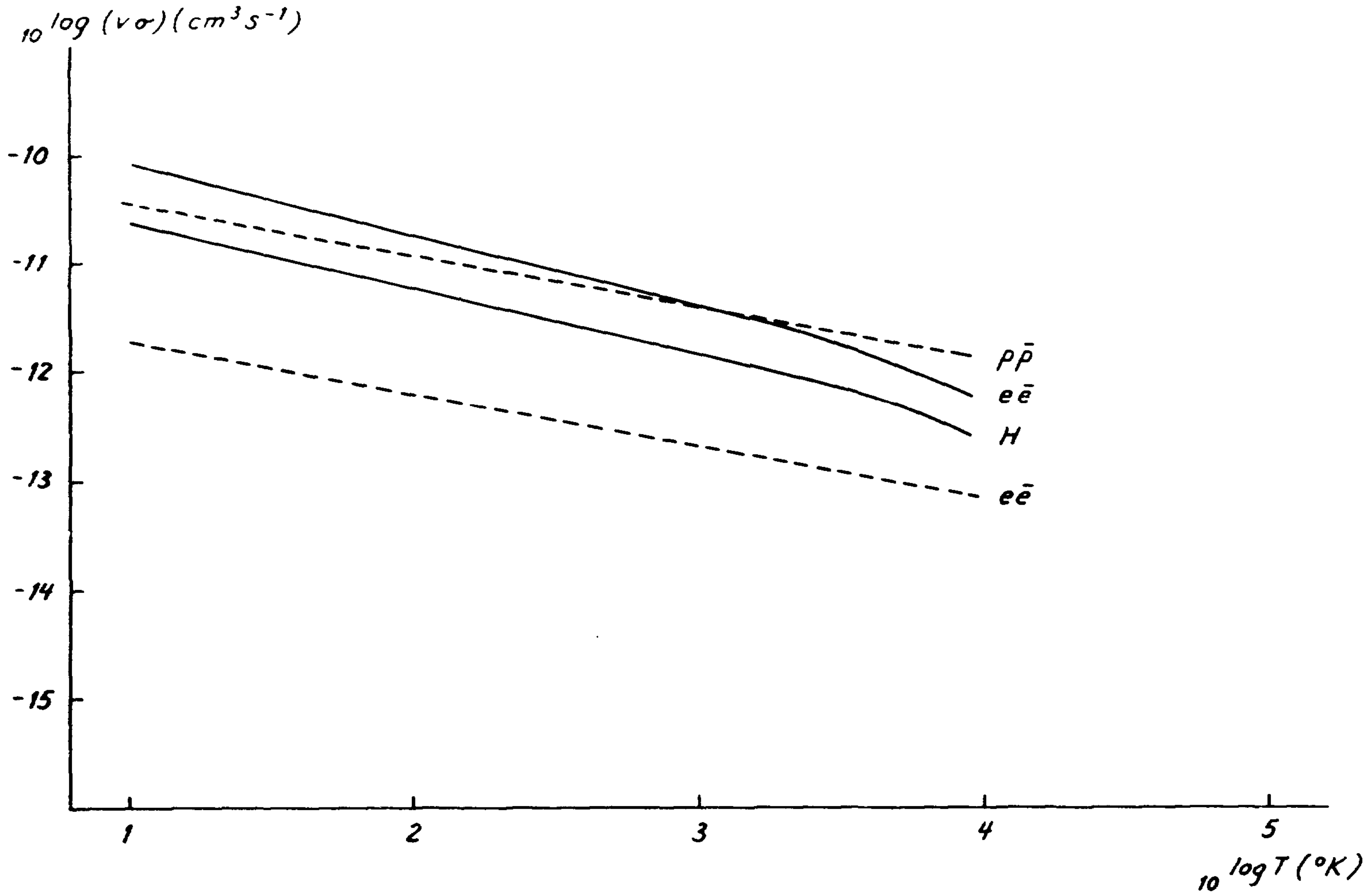


Fig. 5



10 log M₀
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
-1.7
2
1
0
-1
-2
10 log K/K₀
L-S
collapse
region

Fig 6



$_{10}\log(v\sigma)\,(cm^3 s^{-1})$
-10
-11
-12
-13
-14
-15
1
2
3
4
5
$_{10}\log T\,(°K)$
$p\bar{p}$
$e\bar{e}$
H
$e\bar{e}$

Fig 7

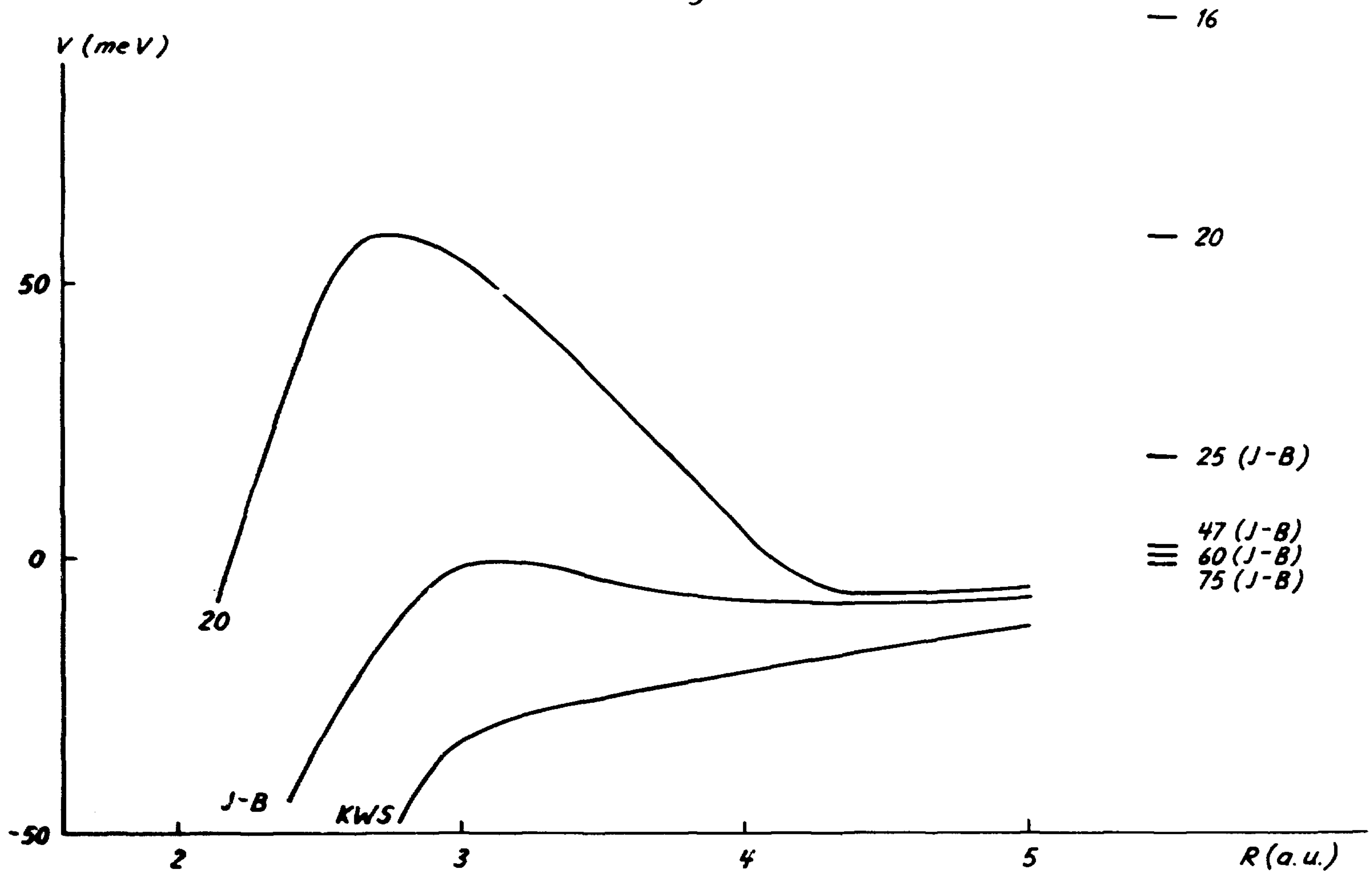


Fig 8

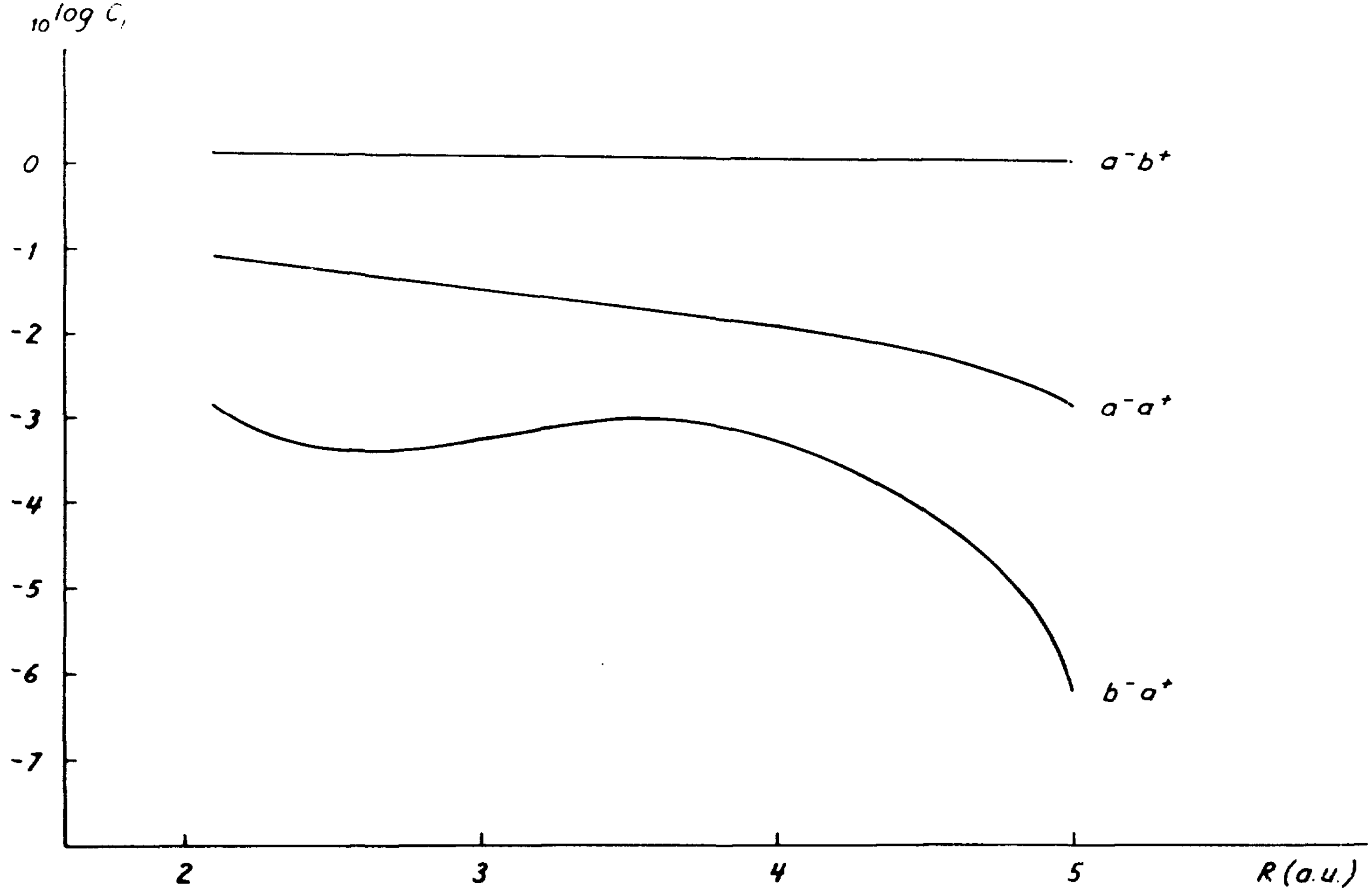


Fig. 9

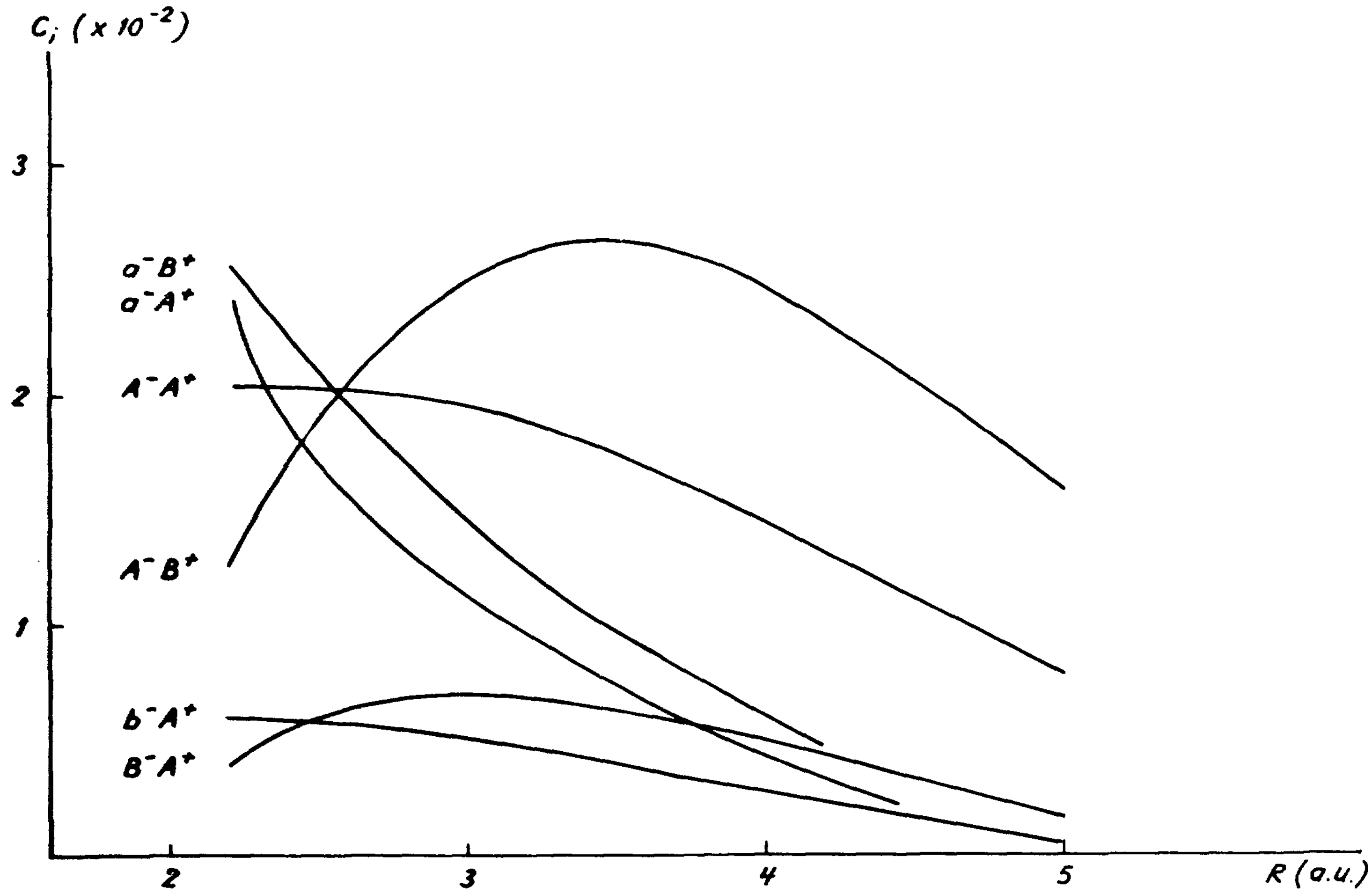
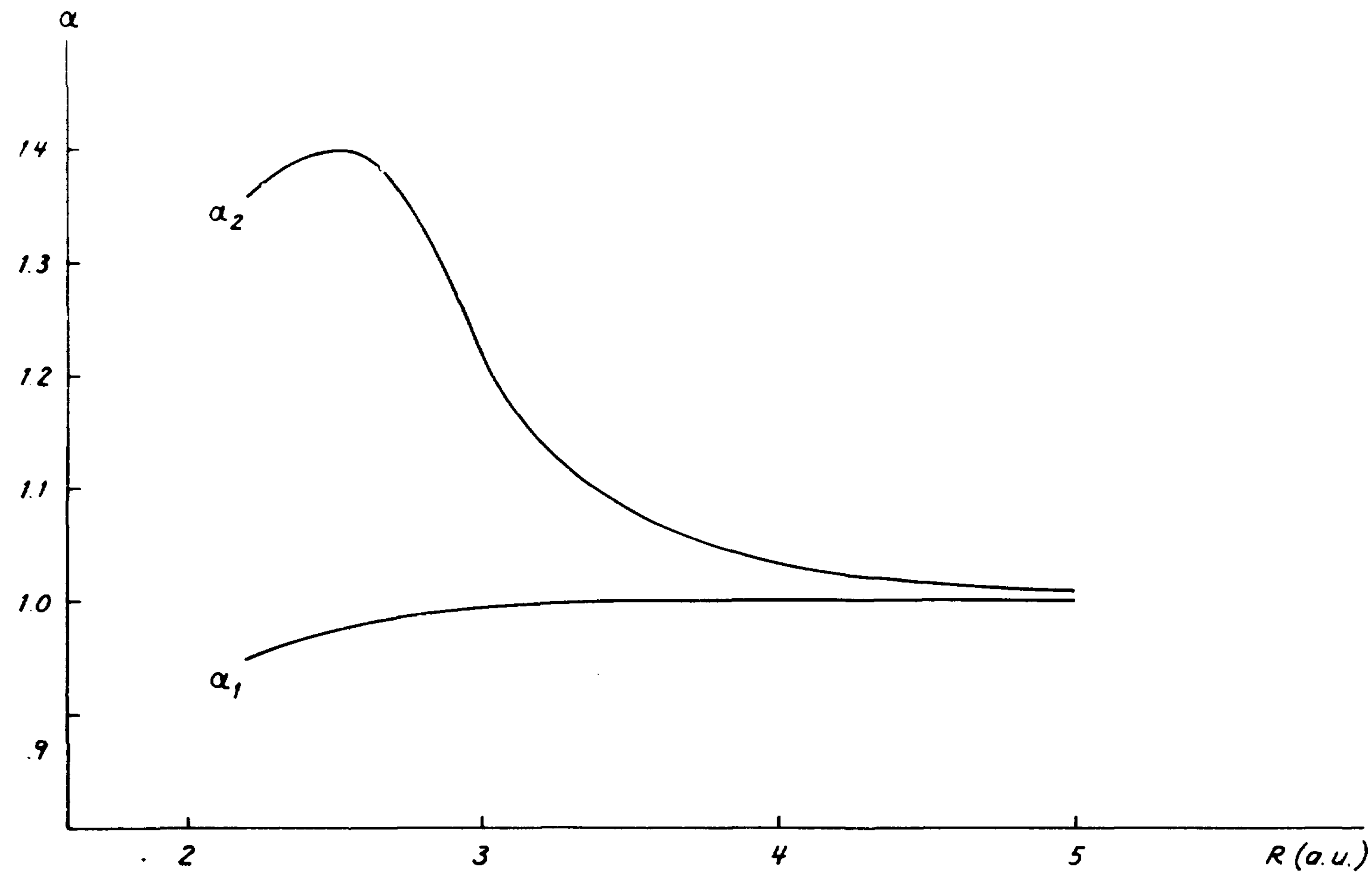


Fig 10



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