DENSITY EFFECTS ON QUANTUM FLUCTUATION OF RADIATION IN SYNCHROTRONS

S.V. Koutin and A.N. Lebedev

P.N. Lebedev Physical Institute of RAS. 117924, Leninsky prosp. 53, Moscow, Russia

We discuss an influence of short-time electron correlation on quantum excitation of particle oscillations by synchrotron radiation. The effect should decrease the "natural" beam emittance and can be observable at reasonable intensities at 1-2 GeV energy.

Introduction. The excitation of synchrotron and betatron oscillations in the cyclic electron accelerators and storage rings due to quantum nature of radiation significantly affects their characteristics and operating mode. Together with radiation cooling, which is a pure classical effect, this effect defines equilibrium emittance of the beam. The emittance has a macroscopic value, although it is proportional to Compton wavelength of electron $\Lambda = \eta/mc$.

The effect of quantum excitation of oscillations similar to diffusion increase of beam phase volume has been calculated theoretically both in quantum and semi-classical theory and was confirmed in experiments. On the one hand, the semi-classical approach seems to be natural since quantum numbers, that correspond to macroscopic betatron and synchrotron oscillations are enormously high. On the other hand, it is based on "feasible", semi-insight statistical point of view, which have to be justified by experiments and quantum theory inside the bounds of the last. (The existing quantum theory uses single particle wave functions approach.) Semi-classical approach is based on single-particle theory as well, i.e. it ignores all correlation effects, connected to large number of radiating particles. Since the decrease of quantum excitation is of a large practical importance, we will try to study this approach and apply it to the case of high-density beam, when the correlation effects cannot be neglected. Quantum solution of this problem seems to be quite complicated but even the semi-classical approach leads us to the qualitatively new effects.

Main assumptions of single particle theory are:

- Quantum excitations are defined mostly by quanta with energy $\eta\omega_0\gamma^3$, which is a higher range of synchrotron radiation spectra. Here γ is electron's Lorentz factor, ω_0=c/R its angular frequency. Characteristic time of quantum radiation transition (radiation of one quantum) $\tau_{rad} \approx 2/\omega_0\gamma$. During this time electron pass distance of approximately $R/\gamma >> \lambda$, where $\lambda \approx R/\gamma^3$ is a wavelength of the characteristic quanta. Time τ_{rad} is significantly less than any of characteristic times of electron motion are. The recoil looks like a one-time kick, and sequence of such kicks is the white noise, that leads to diffusion of particles in the phase space.
- The average number of quanta, radiated during the time τ_{rad} has the order of the fine structure constant $\alpha \approx 1/137 << 1$. The minuteness of this number means that consecutive quanta are emitted statistically independently
- Phases of the fields radiated by the electron in the given direction in two consequent turns are random. This means that the spectra in short wave range in reality will be continuos, like the one of random process.
- The radiation of each quantum corresponds to loss of energy (and momentum) for one electron. For this to be perfectly true the distance between electrons should be significantly larger than wavelength, i.e. the density should be rather low. Otherwise, the quantum is radiated by a system of few particles, which accepts the recoil momentum of quantum.

The violation of the last assumption means the coherence of synchrotron radiation and lead to changes in its spectra and total intensity. However this spectra is coherent only if coherent position of two and more electrons is kept during several turns.

Let us consider a simple one-dimensional chain of N electrons, that are dispense by normal dispersion law on average distance Δ/N from each other with uncertainty of position δ, completely random on consequent turns. In a far-field zone, the field from such system is a pack of randomly distributed similar short pulses with length of $\approx 1/\omega_0\gamma^3$, which repeats (with other realization of distribution) after the time of $2\pi/\omega_0$. Simple calculation gives the following spectral intensity of synchrotron radiation of such system.

$$\frac{W(\omega)}{W_0(\omega)} = N + \left[\frac{\sin^2(\Delta\omega/2c)}{\sin^2(\Delta\omega/2Nc)} - N\right]\exp\left(-\frac{\omega^2\delta^2}{2c^2}\right), \quad (1)$$

where $W_0(0)$ - spectral radiation intensity of one electron (See Fig. 1). The first term corresponds to fully incoherent radiation, and the second describes coherent effects. The term in square bracket gives interference modulation of spectra. However, in the most significant high-frequency range of spectra, wavelengths are smaller than the position uncertainty δ and this modulation decreases exponentially; that means that radiation is completely incoherent. Note that the number of particles in radiation zone of length λ at certain incidental moments can be larger than one.

The last can significantly change the recoil momentum of each particular electron during radiation of one quantum. Really, n particles, which find themselves simultaneously in near-field zone (radiation zone) and keep their relative position at least during the time τ_{rad}, behave like organic whole, and share recoil momentum and energy loss. The fact that the number of quanta is increased proportionally to n^2, does not play significant role, since this happens, as shown above, due to soft coherent part of spectra, when the number of high-

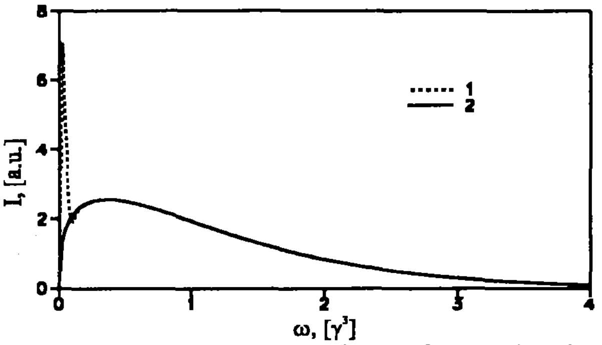

Fig. 1 1-Spectrum intensity per one electron of an one-dimensional chain of electrons with fixed average separation and uncertainty of position; 2 - spectra of single electron.

energy incoherent quanta is proportional to n only. For assurance we consider n to be less than 137, so the probability of radiation of two coherent consequent high-energy quanta in the same direction is negligible small.

Excitation of oscillations by radiation fluctuation. Let us, for example, consider linear synchrotron oscillation of value u — deviation of energy from its equilibrium value E_s [2].

During the radiation du/dt does not change and u jumps down on the value ε — quantum energy, that gives the change of amplitude:

$$\Delta A = -\frac{u}{A}\varepsilon + \frac{\varepsilon^2}{2A}\left(1 - \frac{u^2}{A^2}\right);\quad (\Delta A)^2 = \frac{u^2}{A^2}\varepsilon^2 \tag{2}$$

Let us average this expression with probability of radiation of quantum ηω per unit time P(u,ω), which in the quasi-classical approximation is equal to :

$$P(u,\omega) = \frac{1}{\eta\omega}\left[W^n_{coh}(\omega) + W^n_{incoh}(\omega)\right] \tag{3}$$

The coherent part of the spectral intensity $W^n_{coh}(\omega)$ should not depend on single particle energy, and incoherent part is equal to:

$$W^n_{incoh} = np_n W_0(u,\omega) = \left(nW_s(\omega) + n\cdot u\frac{\partial W_s(\omega)}{\partial E_s}\right)\cdot p_n \tag{4}$$

where n is number of electrons, participating in the act of emission, p_n — probability of such realization. By averaging we get:

$$V = <\Delta A\cdot P(u,\omega)> = -\frac{A}{2}\frac{\partial W_s}{\partial E_s}\frac{\varepsilon n}{\eta\omega} + \frac{1}{4A}\frac{\varepsilon^2}{\eta\omega}\left(W_{coh}(\omega) + nW_s(\omega)\right); \tag{5}$$

$$D = \frac{1}{2}<(\Delta A)^2\cdot P(u,\omega)> = \frac{1}{4}\frac{\varepsilon^2}{\eta\omega}\left(W_{coh}(\omega) + nW_s(\omega)\right). \tag{6}$$

For estimation we can put $\varepsilon = \eta\omega/n$ and neglect the terms with $W_{coh}(\omega)$ which are significant only at low frequencies, where ε^2 is very small. Then by average over possible realizations (number n) and spectra, for the diffusion coefficients we get:

$$<V> = -\frac{A}{2}\Gamma_s + \frac{<\varepsilon>}{4A}W_s;\quad <D> = \frac{<\varepsilon>}{4}W_s, \tag{7}$$

where W_s is total intensity of synchrotron radiation of a single particle, $<\varepsilon> \approx <\eta\omega/n>$ - average loss of energy per one particle, and $\Gamma_s = \partial W_s/\partial E_s$ is a well-known constant of radiation dumping of synchrotron oscillation [3].

Calculated values of diffusion coefficients give the evolution in time of distribution function by amplitude F(A,t) defined by Fokker-Plank like equation:

$$\frac{\partial F}{\partial t} + \frac{1}{A}\frac{\partial}{\partial A}A\left(<V> - <D>\frac{\partial}{\partial A}\right)F = 0 \tag{8}$$

The stabilized distribution is: $F_{st} = a\cdot\exp(-a^2/2)$; $a = \sqrt{2\Gamma_s/W_s\varepsilon}$ and gives the mean-square stabilized amplitude:

$$A^2_{st} = W_s\varepsilon/\Gamma_s \tag{9}$$

Congregate radiation zone. By congregate radiation zone, we will understand space volume, such that when several electrons appear in this zone for a short time τ_{rad} they behave as one congregate radiator that accepts the total recoil momentum. One can get an idea about size and configuration of this zone, considering near zone fields. Let us consider one electron, moving along circumference; observation point is on fixed angular distance μ. (For simplicity we assume the motion to be plane). From general equation for retarded fields, we found force acting at point μ [1].

$$F_\tau(\mu) = \frac{e^2}{R^2}\gamma^{-2}\left(2\sin\frac{\mu'}{2} - \beta\sin\mu'\right)^{-3}\times \times\left\{\sin\mu' - 2\beta^2\cos\mu'\sin\frac{\mu'}{2} + \gamma^2\beta^2\left(\sin\mu' - 2\beta^2\sin\frac{\mu'}{2}\right)(\cos\mu' - 1)\right\} \tag{10}$$

where

$$\mu' - \mu = 2\left|\sin\frac{\mu'}{2}\right|. \tag{11}$$

At small μ' $\mu \approx \mu' - \beta|\mu'| + \mu'^3/24$ and

$$\frac{R^2}{e^2}F_\tau = \frac{|\mu|}{\gamma^2\mu^3} + \begin{cases} -4\gamma^4/3 & \text{for } \mu > 0 \\ 0 & \text{for } \mu < 0 \end{cases} \tag{12}$$

Anti-symmetrical and diverging at $\mu \to 0$ part we identify as Coulomb interaction of two electrons. It is not relevant to considered effect, at least since it does not change the average momentum of interacting particles. We will come back to this question later.

Radiation part of the force $F_\tau(\mu)$ is strongly asymmetric and in relativistic case it is practically equal to zero behind radiating particle. In the point where the particle is situated ($\mu \to 0$) self-interaction force is equal to the $4\gamma^4 e^2/3R^2$, well known value of the recoil momentum

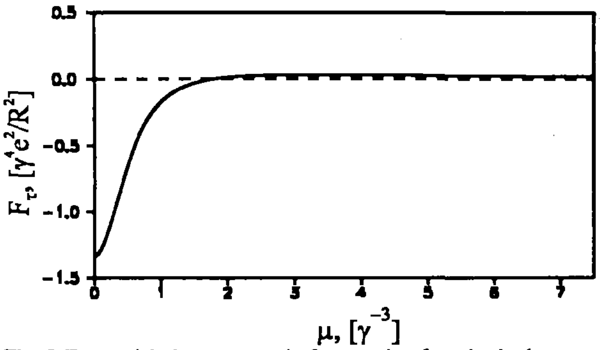

Fig. 2 Tangential electromagnetic force acting from back electron to forward one without Coulomb part vs. angular distance between electrons.

per unit time transferred by radiation. Radiation part of force $F_\tau(\mu)$ vs. angle μ is shown in the Fig. 2. One can see that this force became practically
equal to zero at μ about $(1.5\text{-}2)\cdot\gamma^{-3}$. Therefore, we might conclude that the electrons radiate independently, when distance between them is larger than $2\,\gamma^{-3}$.

When the pair radiates collectively back and fore electrons share recoil momentum in some proportion. However since to the moment of next photon radiation ($\approx 137R/c\gamma$) the given pair will decay with large probability, the "back" electron might become a "fore" one in the other pair; i.e. in average the electron in pair radiation will lose half of photon momentum. If there are n electrons simultaneously in the zone of congregate radiation, each of them will get one n^{th} of photon momentum. Transverse dimension of congregate radiation zone can be obtained from alike but more complicated calculations. However, it is quite easy to see that while radiation concentrated in a small angle of approximately γ^{-1} one can consider the wave as a plane one. Hence the transverse dimensions are γ times lager than longitudinal one and have an order of γ^{-2} in units of circumference radius. Thus, taking into consideration qualitative character of all relations above we will consider the volume of the congregated radiation zone to be equal to $R^3\gamma^{-7}$. It is interesting that zone of congregate radiation defined this way does not depends on radiation frequency and is defined only by upper bound of the radiation spectrum.

The average number of particles n, which are simultaneously in near-field zone at the moment of quantum emission, depends on their distribution over the bunch. For simplicity, we can use Poisson distribution with:

$$p_n = \exp(\bar{n})\bar{n}^n / n!\,;\ \langle n^{-1} \rangle = (1-\exp(\bar{n}))/\bar{n} \qquad (13)$$

Here $\bar{n} = \nu N$, (ν is ratio of this zone volume to the volume of the whole bunch, N is number of particles in bunch). The announced effect consists in decrease of value n^{-1} (averaged over spectra).

Equilibrium emittance. We will estimate the effect in a case when bunch dimensions are defined only by quantum fluctuation. Although the existing accelerators do not satisfy this condition, there are certain experiments on direct measurement of quantum limits. The equilibrium mean-square dimensions of the bunch can be presented as follows

$$\begin{aligned} A_x^2 &= C_x \alpha^2 R \Lambda \gamma^2 / n \\ A_z^2 &= C_z R\Lambda / Q^2 n \\ A_\tau^2 &= C_\tau 137\alpha \cdot \mathrm{ctg}\phi_s / q\gamma n \end{aligned} \qquad (12)$$

where Λ is the Compton wavelength, α — momentum compaction factor, q — harmonic number, Q — betatron tune and ϕ_s — equilibrium phase. Numeric coefficients C are of order of unity and are defined by structural functions and distribution of radiation damping decrements [3]. Since our calculations are qualitative only, one need not to define such coefficients with more precision.

In accordance with consideration above for an average number of electrons in the congregate radiation zone we get

$$n = 1 + \begin{cases} R^3 N n^{3/2}/\gamma^7 A_x A_z A_\tau & \text{for } A_z^2 > nR^2/\gamma^4 \\ R^3 N n/\gamma^5 A_x A_\tau & \text{for } A_z^2 < nR^2/\gamma^4 \end{cases} \qquad (14)$$

Last condition appears because the vertical size of the bunch can be smaller than congregate zone. For the other degrees of freedom this seems unrealistic (A_z has the lowest value due to the specific of vertical oscillations excitations by quanta coming away from orbit plane). For simplicity we replace above equation by a single one

$$N^2 = \frac{A_x^2 A_\tau^2}{R^6}\left(A_z^2 + \frac{R^2 n}{\gamma^4}\right)\frac{(n-1)^2}{n^3}\gamma^{14} \qquad (15)$$

By substituting, we get a relation between the number of particles in the bunch and a value of n that characterizes compression of the bunch due to the density effect.

$$N^2 = N_0^2 \frac{(n-1)^2}{n^3}(n + n_0), \qquad (16)$$

where

$$\begin{aligned} N_0^2 &= 137 C_x C_\tau \frac{\Lambda\alpha^2 \mathrm{ctg}\phi_s}{Rq}\gamma^{11}; \\ n_0 &= C_z \frac{\Lambda\gamma^4}{RQ^2} \end{aligned} \qquad (17)$$

In absence of other perturbations, the density effect amplifies itself: the smaller the bunch size for fixed number of particles is the higher density and the lower quantum excitations are. Formally, it leads to radiation collapse at $N>N_0$ i.e. $n \to \infty$ (See Fig. 3), although at large n many assumption made above are violated. Note the interesting hysteresis like behavior of the model at $n_0>2$. It can provide additional possibilities for achieving higher bunch density by optimal choosing of function N(t) and γ(t). The threshold value of particle number N_0 for the energy of about 1GeV is

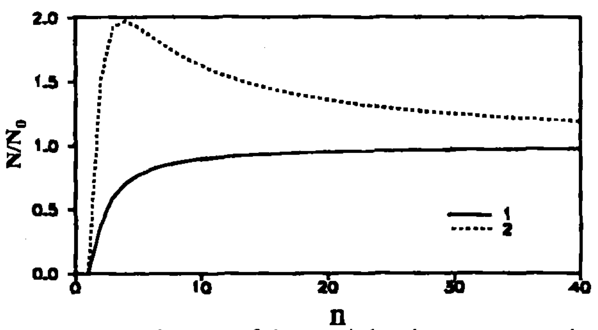

Fig. 3 Normalized number of electrons in bunch vs. average number of electrons in congregate radiation zone. 1 — for n_0=1; 2 — for n_0=10

practically obtainable and has an order of 10^{11}-10^{12}.

Unfortunately, for the higher energy the number N_0 is unachievable high.

Conclusion. Principal possibility to decrease a bunch size below limits, defined by quantum fluctuations will open interesting perspectives for cooler rings, for producing super-short electron bunches and for synchrotron radiation sources (possibly coherent). Therefore, despite of qualitative character of our arguments they give basis for more detailed study of density effect: quantum theoretical and experimental. Even in semi-classical considerations some moments need additional study. For example the Coulomb field in the congregate zone is comparable or exceeds the radiation field that requires consideration of intra-beam scattering. (We are grateful to A.N. Skrinsky). Concerning this question, we can note a somewhat different physical nature of these two effects, one of which is quantum one and another — pure classical and depending on velocity distribution of particles. In addition at n>>1 intra-beam scattering can not be considered as two-particle effect and need a special consideration.

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