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Generalized disformal invariance of cosmological perturbations with second-order field derivatives

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ABSTRACT

We investigate how the comoving curvature and tensor perturbations are transformed under the generalized disformal transformation with the second-order covariant derivatives of the scalar field, where the free functions depend on the fundamental elements constructed with the covariant derivatives of the scalar field with at most the quadratic order of the second-order covariant derivatives. Our analysis reveals that on the superhorizon scales the difference between the comoving curvature perturbations in the original and new frames is given by the combination of the time derivative of the comoving curvature perturbation, the intrinsic entropy perturbation of the scalar field, and its time derivative in the original frame. Thus, in the case that on the superhorizon scales (1) the intrinsic entropy perturbation and its time derivative vanish and (2) the comoving curvature perturbation in the original frame, $\mathcal{R}_c$, is conserved, the comoving curvature perturbation becomes invariant under the disformal transformation on the superhorizon scales. We also show that the tensor perturbations are also disformally invariant, in the case that the tensor perturbations in the original frame are conserved with time.

1. Introduction

Cosmological inflation is recognized as the most promising scenario about the history in the early Universe [1–6]. Although the latest observational data of the large-scale anisotropies of the Cosmic Microwave Background (CMB) [7–10] strongly favour the simplest single-field and slow-roll inflation models, models of inflation driven by various mechanisms, e.g., multiple scalar fields, other field species such as vector and spinor fields, modified kinetic terms, nonminimal (derivative) couplings to the spacetime curvature, and self-derivative interactions have also been explored [11–20].

For a long time, the scalar-tensor theories with the higher-derivative interactions have been thought to be problematic, since the equations of motion for the metric and the scalar field would generically contain derivatives higher than second-order, indicating the appearance of the Ostrogradsky ghosts [21]. Ref. [22] argued that an invertible frame transformation with the derivatives of the scalar field maps a class of the conventional scalar-tensor theories to a new class without the Ostrogradsky ghosts, despite the apparent higher-derivative features of the theory. More explicit studies have revealed that the appearance of the Ostrogradsky ghosts can be avoided by imposing the certain degeneracy conditions amongst the equations of motion with the highest order time derivatives. The scalar-tensor theories under the imposition of the degeneracy conditions have been developed and are currently recognized as the degenerate higher-order scalar-tensor (DHOST) theories [23–26]. The DHOST theories correspond to the most general scalar-tensor theories with the single scalar field without the Ostrogradsky ghosts and include all the previously known classes of the scalar-tensor theories, especially, the Horndeski theories [15,27,28] and the beyond-Horndeski theories [29–31].

In order to compare the inflationary models with the observational data, we consider the linear perturbations about the Friedmann-Lemâitre-Robertson-Walker (FLRW) spacetime

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -\left(1+2A(t,x^i)\right)dt^2 + 2a(t)\partial_i B(t,x^i)dtdx^i \\ &+ a(t)^2\Big[\left(1-2\psi(t,x^i)\right)\delta_{ij} + 2\partial_i\partial_j E(t,x^i) \\ &+ h_{ij}(t,x^i)\Big]dx^i dx^j, \end{aligned} \tag{1}$$

where t and $x^i (i=1,2,3)$ are the physical time and comoving spatial coordinates, $a(t)$ is the cosmic scale factor, A, B, ψ, and E are the scalar metric perturbation variables, and h_{ij} denotes the

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tensor metric perturbations obeying the transverse-traceless conditions $\delta^{ij}h_{ij}=0$ and $\partial^i h_{ij}=0$, respectively. We also consider the perturbation of the scalar field

$$\phi=\phi_0(t)+\phi_1(t,x^i), \tag{2}$$

ad neglect the vector metric perturbations. In order to see the dependence on the scales, from now on, we decompose any perturbation variable Q into the comoving Fourier modes

$$Q=\int d^3k Q_k e^{ik_i x^i}, \tag{3}$$

where k_i is the comoving momentum vector and $k^2:=\delta^{ij}k_i k_j$, although we will not show the subscript "k" explicitly. After the Fourier transformation, the spatial derivative ∂_i and the Laplacian term $\Delta:=\delta^{ij}\partial_i\partial_j$ are replaced by $(-ik_i)$ and $(-k^2)$ in the perturbation equations, respectively. Since in this paper we focus on the regime of the linearized perturbations, there will be no coupling of the different k modes.

While the tensor perturbations h_{ij} are gauge-invariant, the scalar perturbations A, B, ψ, E, and ϕ_1 are not. Thus, in order to compare with the observational data, we have to construct the gauge-invariant combinations of them [32–34]. The gauge-invariant perturbations relevant for the inflationary models in the scalar-tensor theories are given by the combinations of the metric and scalar field perturbations. The particularly important gauge-invariant quantity is the comoving curvature perturbation [35–38]

$$\mathcal{R}_c:=\psi+\frac{1}{\dot\phi_0}\frac{\dot a}{a}\phi_1, \tag{4}$$

where 'dot' denotes the derivative with respect to the time t. The spectral features of the comoving curvature perturbation are directly related to the data of the large-scale CMB anisotropies.

In developing the new inflationary models in the more general scalar-tensor theories, *frame invariance* of the cosmological observables should be very useful, since it allows us to evaluate the observables in the frame which is technically the most convenient. The conformal transformation $\tilde g_{\mu\nu}=\mathcal{C}(\phi)g_{\mu\nu}$, where ϕ denotes the scalar field, maps a class of the *conventional* scalar-tensor theories,

$$\mathcal{L}=\xi(\phi)R-\omega(\phi)\mathcal{X}-V(\phi), \tag{5}$$

with the potential $V(\phi)$, the nonminimal coupling to the Ricci curvature $\xi(\phi)R$, the kinetic function $\omega(\phi)$, and the kinetic term of the scalar field

$$\mathcal{X}:=\phi^\nu\phi_\nu, \tag{6}$$

where we have defined the shorthand notation for the covariant derivatives of the scalar field by $\phi_{\mu\nu\cdots\alpha}:=\nabla_\alpha\cdots\nabla_\nu\nabla_\mu\phi$ and $\phi^{\mu\nu\cdots\alpha}:=\nabla^\alpha\cdots\nabla^\nu\nabla^\mu\phi=g^{\rho\mu}g^{\sigma\nu}\cdots g^{\alpha\beta}\nabla_\beta\cdots\nabla_\sigma\nabla_\rho\phi$ represent the covariant derivatives of the scalar field associated with the metric $g_{\mu\nu}$, to another class in which the structure of the Lagrangian density (5) is preserved with the redefined functions of $\tilde\xi(\phi)$, $\tilde\omega(\phi)$, and $\tilde V(\phi)$. Interestingly, it has been shown that the gauge-invariant comoving curvature perturbation (4) is invariant under the conformal transformation [39–41], which allows us to evaluate observables in the Einstein frame obtained after eliminating nonminimal couplings. Similarly, the tensor metric perturbations, h_{ij}, are also manifestly conformally invariant.

Similarly, the disformal transformation [22,42,43]

$$\tilde g_{\mu\nu}=\mathcal{C}(\phi,\mathcal{X})g_{\mu\nu}+\mathcal{D}(\phi,\mathcal{X})\phi_\mu\phi_\nu, \tag{7}$$

where $\mathcal{C}$ and $\mathcal{D}$ are the free functions of the scalar field ϕ and the kinetic term $\mathcal{X}$ (see Eq. (6)), is known as the most general frame transformation which is composed of the scalar field and its first-order derivatives. The transformation (7) maps a class of the Class-^{2}N-I and Class-^{3}N-I DHOST theories to another class [24–26]. The Horndeski theories [15,27,28] are framed by the subclass of the disformal transformation (7), $\mathcal{C}=\mathcal{C}(\phi)$ and $\mathcal{D}=\mathcal{D}(\phi)$ [43], and the beyond-Horndeski theories are done by the subclass of the disformal transformation (7) $\mathcal{C}=\mathcal{C}(\phi)$ and $\mathcal{D}=\mathcal{D}(\phi,\mathcal{X})$ [29–31], respectively.

The invariance of the comoving curvature perturbation within the above class of the disformal transformation has been shown in Refs. [44–47]. The tensor perturbations are always shown to be disformally invariant. On the other hand, the invariance of the comoving curvature perturbation depends on the subclass of the disformal transformation. In the class of $\mathcal{C}=\mathcal{C}(\phi)$ and $\mathcal{D}=\mathcal{D}(\phi,\mathcal{X})$, the disformal invariance of the comoving curvature perturbation always holds [44,45]. In the most general class that $\mathcal{C}=\mathcal{C}(\phi,\mathcal{X})$ and $\mathcal{D}=\mathcal{D}(\phi,\mathcal{X})$, the invariance of the comoving curvature perturbation holds approximately on the superhorizon scales $k/(aH)\ll 1$ [46,47], where $H(t):=\dot a/a$ represents the Hubble expansion rate, whenever the gauge-invariant perturbation about the scalar field

$$\Sigma:=-\dot\phi_0\ddot\phi_0\left(\frac{\delta\mathcal{X}}{\dot{\mathcal{X}}_0}-\frac{\phi_1}{\dot\phi_0}\right)=A\dot\phi_0^2+\phi_1\ddot\phi_0-\dot\phi_0\dot\phi_1, \tag{8}$$

is suppressed on the superhorizon scales $k/(aH)\ll 1$, where $\mathcal{X}_0$ and $\delta\mathcal{X}$ represent the background and perturbation parts of $\mathcal{X}$ given, respectively, by

$$\mathcal{X}_0:=-\dot\phi_0^2,\qquad \delta\mathcal{X}:=2\dot\phi_0\left(A\dot\phi_0-\dot\phi_1\right). \tag{9}$$

Following the definition in Eq. (8), Σ represents the relative perturbation between the scalar field and its kinetic term. In the inflation models with the canonical/noncanonical kinetic terms, Σ is proportional to the intrinsic entropy perturbation of the scalar field

$$\delta\Gamma_{(\phi)}=\delta p_{(\phi)}-\left(\frac{\dot p_{0(\phi)}(t)}{\dot\rho_{0(\phi)}(t)}\right)\delta\rho_{(\phi)}, \tag{10}$$

where $\left(\rho_{0(\phi)}(t),p_{0(\phi)}(t)\right)$ and $\left(\delta\rho_{(\phi)},\delta p_{(\phi)}\right)$ represent the background and perturbation parts of the energy density and pressure of the scalar field, respectively [38,48]. For this reason, we call Σ the intrinsic entropy perturbation of the scalar field. In the models in the Horndeski theories with the canonical or noncanonical kinetic terms, it has been shown that when the comoving curvature perturbation is conserved on the superhorizon scales,

$$\dot{\mathcal{R}}_c\approx 0, \tag{11}$$

where '$\approx$' means that the equality holds only on the superhorizon scales, the perturbation Σ is suppressed on the superhorizon scales,

$$\Sigma\approx 0, \tag{12}$$

(see Refs. [38,44,46,48]). The results in Ref. [46] suggest that also in the Class-^{2}N-I and Class-^{3}N-I DHOST theories, when Σ is suppressed on the superhorizon scales, which are related to the Horndeski theory, $\mathcal{R}_c$ is conserved on the superhorizon scales via the disformal transformation (7) [24–26]. On the other hand, in the more general scalar-tensor theories with more than the third- order time derivatives, the correspondence between the conservation of $\mathcal{R}_c$ and the suppression of Σ on the superhorizon scales has not been clarified yet. Since the scalar-tensor theories with the third-order time derivatives mentioned below have not been formulated yet, for now we leave this subject for future work.

In this paper, we consider the generalized disformal transformation with the second-order covariant derivatives of the scalar

field. First, we list the most fundamental scalar quantities constructed with the covariant derivatives of the scalar field with at most the quadratic order of the second-order derivatives. By "fundamental", we mean that all the other scalar quantities constructed with the derivatives of the scalar field can be expressed as the products of them. For instance, only the fundamental scalar quantity constructed with the first-order derivatives of the scalar field is given by the kinetic term $\mathcal{X}$, Eq. (6). The other scalar quantities with the first-order derivatives can be expressed in terms of the nonlinear combination of $\mathcal{X}$, e.g., $\phi^\mu\phi^\nu(\phi_\mu\phi_\nu)=\mathcal{X}^2$.

At the linear order of the second-order covariant derivatives of the scalar field, there are the two fundamental scalar quantities composed of the covariant derivatives of the scalar field given by

$$\Box\phi := g^{\mu\nu}\times\phi_{\mu\nu}, \tag{13}$$

$$\mathcal{Y} := 2\phi^\mu\phi^\nu\times\phi_{\mu\nu}=g^{\mu\nu}\phi_\mu\mathcal{X}_\nu, \tag{14}$$

where we have introduced the shorthand notation for the covariant derivative of the kinetic term $\mathcal{X}_\mu := \nabla_\mu\mathcal{X}$. At the quadratic order of the second-order covariant derivatives of the scalar field, there are also two the fundamental scalar quantities composed of the covariant derivatives of the scalar field given by

$$\mathcal{Z} := 4g^{\mu\nu}\phi^\rho\phi^\sigma\times\left(\phi_{\mu\rho}\phi_{\nu\sigma}\right)=g^{\mu\nu}\mathcal{X}_\mu\mathcal{X}_\nu, \tag{15}$$

$$\mathcal{W} := g^{\mu\nu}g^{\rho\sigma}\times\left(\phi_{\mu\rho}\phi_{\nu\sigma}\right). \tag{16}$$

The other combinations of the covariant derivatives of the scalar field with the quadratic order of the second-order covariant derivatives can be expressed in terms of the fundamental quantities at the linear order (13)-(14), e.g., $\phi^\rho\phi^\mu\phi^\sigma\phi^\nu\times\left(\phi_{\mu\rho}\phi_{\nu\sigma}\right)=\mathcal{Y}^2/4$ and $g^{\mu\rho}\phi^\nu\phi^\sigma\times\left(\phi_{\mu\rho}\phi_{\nu\sigma}\right)=(\Box\phi)\,\mathcal{Y}/2$. We assume that the free functions in the general disformal transformation are the functions of the fundamental elements (6), and (13)-(16), as well as the scalar field itself ϕ.

We also consider the tensors constructed with the covariant derivatives of the scalar field

$$\phi_\mu\phi_\nu,\quad \phi_{\mu\nu},\quad \phi_{(\mu}\mathcal{X}_{\nu)},\quad \mathcal{X}_\mu\mathcal{X}_\nu,\quad \phi^\rho{}_\mu\phi_{\rho\nu}, \tag{17}$$

whose contraction gives rise to the above fundamental scalar quantities $\mathcal{X}$, $\Box\phi$, $\mathcal{Y}$, $\mathcal{Z}$, and $\mathcal{W}$ (Eqs. (6), and (13)-(16)), respectively, and assume that Eqs. (17) constitute the nonconformal part of the generalized disformal transformation. By multiplying the free scalar functions of ϕ, $\mathcal{X}$, $\Box\phi$, $\mathcal{Y}$, $\mathcal{Z}$, and $\mathcal{W}$ to the conformal and nonconformal parts, we arrive at the disformal transformation considered in this paper

$$\begin{aligned}\tilde{g}_{\mu\nu} :=\ & \mathcal{F}_0 g_{\mu\nu}+\mathcal{F}_1\phi_\mu\phi_\nu+\mathcal{F}_2\phi_{\mu\nu}+\mathcal{F}_3\phi_{(\mu}\mathcal{X}_{\nu)}\\ &+\mathcal{F}_4\mathcal{X}_\mu\mathcal{X}_\nu+\mathcal{F}_5 g^{\rho\sigma}\phi_{\rho\mu}\phi_{\sigma\nu},\end{aligned} \tag{18}$$

where $\mathcal{F}_I := \mathcal{F}_I[\phi,\mathcal{X},\Box\phi,\mathcal{Y},\mathcal{Z},\mathcal{W}]$ $(I=0,1,2,3,4,5)$. Eq. (18) manifestly includes all the classes of the disformal transformation constructed with the covariant derivatives of the scalar field with at most the first-order covariant derivatives (7). On the other hand, Ref. [49] considered the disformal transformation with the second-order covariant derivatives of the scalar field given by

$$\tilde{g}_{\mu\nu}=\mathcal{G}_0 g_{\mu\nu}+\left(\mathcal{G}_1\phi_\mu+\mathcal{G}_2 X_\mu\right)\left(\mathcal{G}_1\phi_\nu+\mathcal{G}_2 X_\nu\right), \tag{19}$$

where $\mathcal{G}_J := \mathcal{G}_J[\phi,\mathcal{X},\mathcal{Y},\mathcal{Z}]$ $(J=0,1,2,3)$. The correspondence between Eqs. (18) and (19) is given by $\mathcal{F}_0=\mathcal{G}_0$, $\mathcal{F}_1=\mathcal{G}_1^2$, $\mathcal{F}_2=0$, $\mathcal{F}_3=2\mathcal{G}_1\mathcal{G}_2$, $\mathcal{F}_4=\mathcal{G}_2^2$, and $\mathcal{F}_5=0$. Thus, the transformation (18) generalizes Eq. (19), in terms of the additional dependence on $\Box\phi$ and $\mathcal{W}$, and the additional nonconformal parts $\phi_{\mu\nu}$ and $\phi^\rho{}_\mu\phi_{\rho\nu}$ in Eq. (17). As we will see in Sec. 4, these new terms result in the difference in the tensor perturbations between the frames, when the tensor perturbations are not conserved.

A crucial difference of the generalized disformal transformation with the second-order covariant derivatives of the scalar field from the disformal transformation only with the first-order derivatives (7) is that there will be infinite number of terms that constitute the transformation. For instance, we may add the more than the cubic order powers of the second-order covariant derivative of the scalar field, e.g., $\phi_\mu{}^\rho\phi_\rho{}^\alpha\phi_{\alpha\nu}$, $\phi_\mu{}^\rho\phi_\rho{}^\alpha\phi_\alpha{}^\beta\phi_{\beta\nu}$, and so on in Eq. (18). In order to scan the full space of the disformal transformation with the second-order covariant derivatives of the scalar field, a step-by-step analysis by adding a higher-order power of the second-order covariant derivatives of the scalar field would be necessary. On the other hand, higher-order derivative couplings to the matter and gravity sectors in the cosmological backgrounds would lead to anomalous behaviours, e.g., the partial breaking of the screening mechanism inside the stars [50–52] and the decay of gravitational waves [53,54], which would severely constrain the theory from the observational viewpoints, although these constraints may not be applied to the models of inflation and early Universe. Thus, to what extent we should extend the framework of the disformal transformation is indeed the matter of interests. We would like to emphasize that the framework of the generalized disformal transformation (18) is self-contained, and sufficient to see the essential features of the disformal transformation with the second-order covariant derivatives of the scalar field.

After the generalized disformal transformation (18), the action of the scalar-tensor theory written in terms of the new frame metric $\tilde{g}_{\mu\nu}$ and the scalar field ϕ would contain the third-order covariant derivatives of the scalar field $\tilde{\nabla}_\mu\tilde{\nabla}_\nu\tilde{\nabla}_\alpha\phi$, where $\tilde{\nabla}_\mu$ denotes the covariant derivative associated with the new metric $\tilde{g}_{\mu\nu}$, and after the Arnowitt-Deser-Misner (ADM) decomposition [55], the third-order time derivative terms such as $\mathcal{H}(\phi,\dot{\phi},\ddot{\phi})\dddot{\phi}^2$, which would lead to the equations of motion with the sixth-order time derivatives, and hence the two Ostrogradsky ghosts. To our knowledge, the degenerate scalar-tensor theories with the third- and higher order covariant derivative terms of the scalar field have not been constructed yet. Although the construction of these scalar-tensor theories is not our main purpose, we would like to mention the properties of the theory with the third-order time derivatives within analytical mechanics.

In Appendix A, we show a simple example of the degenerate theory with the third-order time derivative in analytical mechanics. Refs. [56,57] discussed more general properties of analytical mechanics with the third- and higher-order time derivatives, and obtained the conditions to avoid the Ostrogradsky ghosts. In analytical mechanics with the second-order time derivatives, eliminating the linear momentum terms in the Hamiltonian by the secondary constraints that ensure the time evolution of the primary constraints arising from the degeneracy conditions is enough to remove all the Ostrogradsky ghosts [23,58–60]. On the other hand, in analytical mechanics with more than the third-order time derivatives, eliminating all the linear momentum terms from the Hamiltonian is not enough and more secondary constraints are necessary to remove all the Ostrogradsky ghosts. The extension of analytical mechanics with more than the third-order time derivatives to the scalar-tensor theories would also be the nontrivial issue from both the theoretical and technical aspects. In this paper, we will not focus on the construction of the scalar-tensor theories with more than the third-order time derivative terms without the Ostrogradsky ghosts, and simply assume that these scalar-tensor theories exist and the subclass of them is related via the generalized disformal transformation (18) as in the case of analytical mechanics.

In Sec. 2, we will derive the generalized disformal transformation of the scalar perturbations. In Sec. 3, we will derive the gen-

eralized disformal transformation (18) of the comoving curvature perturbation and the conditions under which the comoving curvature perturbations are disformally invariant on the superhorizon scales. In Sec. 4, we discuss the generalized disformal transformation (18) of the tensor perturbations. The last Sec. 5 will be devoted to giving a brief summary and conclusion.

2. The scalar perturbations

First, we consider the disformal transformation of scalar perturbations. Following the decomposition (1) and (2), the fundamental scalar quantities in the generalized disformal transformation (18) are decomposed into the background and perturbation parts as Eq. (9), and

$$\begin{aligned}\Box\phi =& -\left(\ddot{\phi}_0+3\frac{\dot{a}}{a}\dot{\phi}_0\right)\\&+2\left(\ddot{\phi}_0+3\frac{\dot{a}\dot{\phi}_0}{a}\right)A+\left(-\ddot{\phi}_1-3\frac{\dot{a}}{a}\dot{\phi}_1\right)\\&+\dot{\phi}_0\dot{A}+3\dot{\phi}_0\dot{\psi}-k^2\frac{1}{a^2}(\delta_g\phi),\end{aligned}\tag{20}$$

$$\begin{aligned}\mathcal{Y} =& 2\dot{\phi}_0^2\ddot{\phi}_0\\&+2\dot{\phi}_0\left(-4A\dot{\phi}_0\ddot{\phi}_0-\dot{\phi}_0^2\dot{A}+2\ddot{\phi}_0\dot{\phi}_1+\dot{\phi}_0\ddot{\phi}_1\right),\end{aligned}\tag{21}$$

$$\begin{aligned}\mathcal{Z} =& -4\dot{\phi}_0^2\ddot{\phi}_0^2\\&+8\dot{\phi}_0\ddot{\phi}_0\left(3A\dot{\phi}_0\ddot{\phi}_0+\dot{\phi}_0^2\dot{A}-\ddot{\phi}_0\dot{\phi}_1-\dot{\phi}_0\ddot{\phi}_1\right),\end{aligned}\tag{22}$$

$$\begin{aligned}\mathcal{W} =& \left(\ddot{\phi}_0^2+3\frac{\dot{a}^2}{a^2}\dot{\phi}_0^2\right)\\&-4A\left(\ddot{\phi}_0^2+3\frac{\dot{a}^2}{a^2}\dot{\phi}_0^2\right)-2\dot{\phi}_0\ddot{\phi}_0\dot{A}+6\frac{\dot{a}^2\dot{\phi}_0\dot{\phi}_1}{a^2}\\&-6\frac{\dot{a}\dot{\phi}_0^2\dot{\psi}}{a}+2\ddot{\phi}_0\ddot{\phi}_1+2k^2\frac{\dot{a}\dot{\phi}_0}{a^3}(\delta_g\phi),\end{aligned}\tag{23}$$

where we have introduced the gauge-invariant scalar field perturbation in the longitudinal gauge ($B=E=0$),

$$\delta_g\phi := \phi_1-a^2\dot{\phi}_0\left(\dot{E}-\frac{B}{a}\right).\tag{24}$$

Accordingly, the functions in Eq. (18) can be written as

$$\mathcal{F}_I = F_I(t)+\delta\mathcal{F}_I(t,x^i),\tag{25}$$

where $F_I(t)=\mathcal{F}_I[\phi_0,\mathcal{X}_0,\Box_0\phi_0,\mathcal{Y}_0,\mathcal{Z}_0,\mathcal{W}_0]$ with the index "0" representing the background part of Eqs. (2), (9), (20)-(23), and

$$\begin{aligned}\delta\mathcal{F}_I :=& F_{I,\phi}\delta\phi+F_{I,\mathcal{X}}\delta\mathcal{X}+F_{I,\Box\phi}\delta(\Box\phi)\\&+F_{I,\mathcal{Y}}\delta\mathcal{Y}+F_{I,\mathcal{Z}}\delta\mathcal{Z}+F_{I,\mathcal{W}}\delta\mathcal{W},\end{aligned}\tag{26}$$

with $\delta\phi$, $\delta\mathcal{X}$, $\delta(\Box\phi)$, $\delta\mathcal{Y}$, $\delta\mathcal{Z}$, and $\delta\mathcal{W}$ being the perturbation parts of Eqs. (2), (9), and (20)-(23), and $F_{I,\phi}:=\partial_\phi\mathcal{F}_I$, $F_{I,\mathcal{X}}:=\partial_\mathcal{X}\mathcal{F}_I$, $F_{I,\Box\phi}:=\partial_{\Box\phi}\mathcal{F}_I$, $F_{I,\mathcal{Y}}:=\partial_\mathcal{Y}\mathcal{F}_I$, $F_{I,\mathcal{Z}}:=\partial_\mathcal{Z}\mathcal{F}_I$, and $F_{I,\mathcal{W}}:=\partial_\mathcal{W}\mathcal{F}_I$ evaluated at the background $[\phi_0,\mathcal{X}_0,\Box_0\phi_0,\mathcal{Y}_0,\mathcal{Z}_0,\mathcal{W}_0]$.

Under the generalized disformal transformation (18), the background part of Eq. (1) is mapped to the in the form of the FLRW metric

$$d\tilde{s}_0^2=\tilde{g}_{0,\mu\nu}dx^\mu dx^\nu=-d\tilde{t}^2+\tilde{a}(\tilde{t})^2\delta_{ij}dx^idx^j,\tag{27}$$

where we have defined

$$d\tilde{t}:=\sqrt{\mathcal{A}(t)}dt,\qquad \tilde{a}(\tilde{t}):=\sqrt{\mathcal{B}(t)}a(t)\tag{28}$$

with the requirements $\mathcal{A}(t)>0$ and $\mathcal{B}(t)>0$, respectively, with

$$\begin{aligned}\mathcal{A}(t) :=& F_0-F_1\dot{\phi}_0^2\\&-\ddot{\phi}_0\left\{F_2-2F_3\dot{\phi}_0^2+\left(4F_4\dot{\phi}_0^2-F_5\right)\ddot{\phi}_0\right\},\end{aligned}\tag{29}$$

$$\mathcal{B}(t) := F_0-\frac{\dot{a}}{a}\dot{\phi}_0F_2+\frac{\dot{a}^2}{a^2}\dot{\phi}_0^2F_5,\tag{30}$$

and the perturbed part of Eq. (1) is disformally transformed as

$$\delta\tilde{g}_{\mu\nu}dx^\mu dx^\nu=\delta\tilde{g}_{tt}dt^2+2\delta\tilde{g}_{ti}dtdx^i+\delta\tilde{g}_{ij}dx^idx^j,\tag{31}$$

where

$$\begin{aligned}\delta\tilde{g}_{tt} :=& -\delta\mathcal{F}_0+\dot{\phi}_0^2\delta\mathcal{F}_1+\ddot{\phi}_0\delta\mathcal{F}_2-2\dot{\phi}_0^2\ddot{\phi}_0\delta\mathcal{F}_3\\&+4\dot{\phi}_0^2\ddot{\phi}_0^2\delta\mathcal{F}_4-\ddot{\phi}_0^2\delta\mathcal{F}_5\\&-2AF_0+2\dot{\phi}_0\dot{\phi}_1F_1+\left(-\dot{\phi}_0\dot{A}_2+\ddot{\phi}_1\right)F_2\\&+2\dot{\phi}_0\left(2\dot{\phi}_0\ddot{\phi}_0A+\dot{\phi}_0^2\dot{A}-2\ddot{\phi}_0\dot{\phi}_1-\dot{\phi}_0\ddot{\phi}_1\right)F_3\\&-8\dot{\phi}_0\ddot{\phi}_0\left(2A\dot{\phi}_0\ddot{\phi}_0+\dot{\phi}_0^2\dot{A}-\ddot{\phi}_0\dot{\phi}_1-\dot{\phi}_0\ddot{\phi}_1\right)F_4\\&+2\ddot{\phi}_0\left(\ddot{\phi}_0A+\dot{\phi}_0\dot{A}-\ddot{\phi}_1\right)F_5,\end{aligned}\tag{32}$$

$$\begin{aligned}\delta\tilde{g}_{ti} :=& a(t)\partial_i\Bigg[BF_0+\frac{\dot{\phi}_0}{a}\phi_1F_1\\&+\left\{-\dot{\phi}_0\left(\frac{A}{a}+\frac{\dot{a}}{a}B\right)-\frac{\dot{a}}{a^2}\phi_1+\frac{\dot{\phi}_1}{a}\right\}F_2\\&+\frac{\dot{\phi}_0}{a}\left(\dot{\phi}_0^2A-\ddot{\phi}_0\phi_1-\dot{\phi}_0\dot{\phi}_1\right)F_3\\&-4\frac{\dot{\phi}_0^2\ddot{\phi}_0}{a}\left(\dot{\phi}_0A-\dot{\phi}_1\right)F_4\\&+\frac{1}{a}\Bigg(\frac{\dot{a}\dot{\phi}_0^2}{a}A+\dot{\phi}_0\ddot{\phi}_0A+\frac{\dot{a}^2\dot{\phi}_0^2}{a}B+\frac{\dot{a}^2\dot{\phi}_0}{a^2}\phi_1\\&+\frac{\dot{a}\ddot{\phi}_0}{a}\phi_1-\frac{\dot{a}\dot{\phi}_0}{a}\dot{\phi}_1-\ddot{\phi}_0\dot{\phi}_1\Bigg)F_5\Bigg],\end{aligned}\tag{33}$$

and

$$\begin{aligned}\delta\tilde{g}_{ij} :=& a(t)^2\Bigg[\delta\mathcal{F}_0-\frac{\dot{a}\dot{\phi}_0}{a}\delta\mathcal{F}_2+\frac{\dot{a}^2\dot{\phi}_0^2}{a^2}\delta\mathcal{F}_5\\&-2\psi F_0+\left(2\frac{\dot{a}\dot{\phi}_0}{a}A+2\frac{\dot{a}\dot{\phi}_0}{a}\psi-\frac{\dot{a}}{a}\dot{\phi}_1+\dot{\phi}_0\dot{\psi}\right)F_2\\&-\left(4\frac{\dot{a}^2\dot{\phi}_0^2}{a^2}A+2\frac{\dot{a}^2\dot{\phi}_0^2}{a^2}\psi-2\frac{\dot{a}^2\dot{\phi}_0}{a^2}\dot{\phi}_1+2\frac{\dot{a}\dot{\phi}_0^2}{a}\dot{\psi}\right)F_5\Bigg]\delta_{ij}\\&+a(t)^2\partial_i\partial_j\Bigg[2EF_0\\&+\left(\frac{\dot{\phi}_0}{a}B-2\frac{\dot{a}\dot{\phi}_0}{a}E+\frac{\phi_1}{a^2}-\dot{\phi}_0\dot{E}\right)F_2\\&+\left(-2\frac{\dot{a}\dot{\phi}_0^2}{a^2}B+2\frac{\dot{a}^2\dot{\phi}_0^2}{a^2}E-2\frac{\dot{a}\dot{\phi}_0}{a^3}\phi_1+2\frac{\dot{a}\dot{\phi}_0^2}{a}\dot{E}\right)F_5\Bigg].\end{aligned}\tag{34}$$

By changing the time coordinate from t to $\tilde{t}$, $\delta\tilde{g}_{\tilde{t}\tilde{t}}=\frac{1}{\mathcal{A}}\delta\tilde{g}_{tt}$ and $\delta\tilde{g}_{\tilde{t}i}=\frac{1}{\sqrt{\mathcal{A}}}\delta\tilde{g}_{ti}$, we define the metric perturbations after the generalized disformal transformation (18) in the same manner as Eq. (1) by attaching 'tilde' to all the perturbation variables in the new frame. Thus, the metric perturbations in the new frame are given by

$$\tilde{A} := \frac{1}{\mathcal{A}}\left[\frac{1}{2}\delta\mathcal{F}_0 - \frac{1}{2}\dot{\phi}_0^2\delta\mathcal{F}_1 - \frac{1}{2}\ddot{\phi}_0\delta\mathcal{F}_2 + \dot{\phi}_0^2\ddot{\phi}_0\delta\mathcal{F}_3 - 2\dot{\phi}_0^2\ddot{\phi}_0^2\delta\mathcal{F}_4 + \frac{1}{2}\ddot{\phi}_0^2\delta\mathcal{F}_5 + AF_0 - \dot{\phi}_0\dot{\phi}_1F_1 - \frac{1}{2}\left(-\dot{\phi}_0\dot{A}_2 + \ddot{\phi}_1\right)F_2 - \dot{\phi}_0\left(2\dot{\phi}_0\ddot{\phi}_0A + \dot{\phi}_0^2\dot{A} - 2\ddot{\phi}_0\dot{\phi}_1 - \dot{\phi}_0\ddot{\phi}_1\right)F_3 + 4\dot{\phi}_0\ddot{\phi}_0\left(2A\dot{\phi}_0\ddot{\phi}_0 + \dot{\phi}_0^2\dot{A} - \ddot{\phi}_0\dot{\phi}_1 - \dot{\phi}_0\ddot{\phi}_1\right)F_4 - \ddot{\phi}_0\left(\ddot{\phi}_0A + \dot{\phi}_0\dot{A} - \ddot{\phi}_1\right)F_5\right], \tag{35}$$

and

$$\tilde{\psi} = \frac{1}{\mathcal{B}}\left[-\frac{1}{2}\delta\mathcal{F}_0 + \frac{a\dot{\phi}_0}{2a}\delta\mathcal{F}_2 - \frac{\dot{a}^2\dot{\phi}_0^2}{2a^2}\delta\mathcal{F}_5 + \psi F_0 - \left(\frac{\dot{a}}{a}\dot{\phi}_0A + \frac{\dot{a}}{a}\dot{\phi}_0\psi - \frac{\dot{a}}{2a}\dot{\phi}_1 + \frac{1}{2}\dot{\phi}_0\dot{\psi}\right)F_2 - \left(-2\frac{\dot{a}^2\dot{\phi}_0^2}{a^2}A - \frac{\dot{a}^2\dot{\phi}_0^2}{a^2}\psi + \frac{\dot{a}^2\dot{\phi}_0}{a^2}\dot{\phi}_1 - \frac{\dot{a}\dot{\phi}_0^2}{a}\dot{\psi}\right)F_5\right]. \tag{36}$$

3. The comoving curvature perturbations

We define the comoving curvature perturbation in the new frame, given by

$$\tilde{\mathcal{R}}_c = \psi + \frac{1}{\phi_{0,\tilde{t}}}\frac{\tilde{a}_{,\tilde{t}}}{\tilde{a}}\phi_1, \tag{37}$$

which is shown to be gauge-invariant. A straightforward computation shows that the difference between the comoving curvature perturbations in the original and new frames, Eqs. (4) and (37), is given by the combination of the gauge-invariant perturbations in the original frame

$$\tilde{\mathcal{R}}_c - \mathcal{R}_c = \frac{1}{a^2F_0 - a\dot{a}\dot{\phi}_0F_2 + \dot{a}^2\dot{\phi}_0^2F_5} \times \left(\mathcal{Q}_1(t)\dot{\mathcal{R}}_c + \mathcal{Q}_2(t)\Sigma + \mathcal{Q}_3(t)\dot{\Sigma} - k^2\mathcal{Q}_4(t)\left(\delta_g\phi\right)\right), \tag{38}$$

where the background-dependent functions $\mathcal{Q}_1(t)$, $\mathcal{Q}_2(t)$, $\mathcal{Q}_3(t)$, and $\mathcal{Q}_4(t)$ are, respectively, given in Eqs. (B.1)-(B.4) in Appendix B. Thus, in the case that

- (1) the adiabaticity holds on the superhorizon scales, $\Sigma \approx 0$ and $\dot{\Sigma} \approx 0$,
- (2) the comoving curvature perturbation in the original frame $\mathcal{R}_c$ is conserved on the superhorizon scales, $\dot{\mathcal{R}}_c \approx 0$,

we find that the comoving curvature perturbations in both the frames coincide on the superhorizon scales $k/(aH) \ll 1$,

$$\tilde{\mathcal{R}}_c \approx \mathcal{R}_c, \tag{39}$$

and the equivalence of the comoving curvature perturbations on the superhorizon scales. This is a direct extension of the results obtained in Ref. [49].

As mentioned in Sec. 1, in the various single-field inflation models, the conditions (1) and (2) hold at the same time on the superhorizon scales [38,44,46,48]. In these models, when $\Sigma \approx 0$ and $\dot{\Sigma} \approx 0$, the decaying mode among the two independent solutions of $\mathcal{R}_c$ is negligible on the superhorizons scales and $\dot{\mathcal{R}}_c \approx 0$. On the other hand, in the more general scalar-tensor models, e.g., in the DHOST theories and the theories with more than the third–order time derivatives, the correspondence between the conditions (1) and (2) has not been clarified yet. The exceptional case is that the background-dependent coefficients vanish coincidently, $\mathcal{Q}_1(t) = 0$, $\mathcal{Q}_2(t) = 0$, $\mathcal{Q}_3(t) = 0$, and $\mathcal{Q}_4(t) = 0$, where the exact frame invariance $\tilde{\mathcal{R}}_c = \mathcal{R}_c$ holds at any scale, irrespective of the adiabaticity of the scalar field. Since there are the six free functions $\mathcal{F}_I$ ($I = 0, 1, 2, 3, 4, 5$), it is able to achieve this condition.

It is also straightforward to confirm that the gauge-invariant intrinsic entropy perturbation in the new frame, denoted by $\tilde{\Sigma}$, can be written in terms of the gauge-invariant perturbations in the original frame

$$\tilde{\Sigma} := \tilde{A}\phi_{0,\hat{t}}^2 + \phi_1\phi_{0,\hat{t}\hat{t}} - \phi_{1,\hat{t}}\phi_{0,\hat{t}} = \frac{\alpha_1(t)\Sigma + \alpha_2(t)\dot{\Sigma} + \alpha_3(t)\dot{\mathcal{R}}_c - k^2\alpha_4(t)\left(\delta_g\phi\right)}{\mathcal{A}^2}, \tag{40}$$

where $\alpha_1(t)$, $\alpha_2(t)$, $\alpha_3(t)$, and $\alpha_4(t)$ are the background-dependent coefficients given in Eqs. (B.5)-(B.8) in Appendix B, respectively. Thus, whenever conditions (1) and (2) hold in the original frame, $\dot{\mathcal{R}}_c \approx 0$, the adiabaticity also holds in the new frame $\tilde{\Sigma} \approx 0$.

Now, we confirm that the previous results for the disformal transformation with the first-order derivative of the scalar field (7) can be reproduced.

- In the case

$$\mathcal{F}_0 = \mathcal{F}_0(\phi), \qquad \mathcal{F}_1 = \mathcal{F}_1(\phi, \mathcal{X}), \qquad \mathcal{F}_I = 0, \tag{41}$$

($I = 2, 3, 4, 5$) with $\mathcal{A} = F_0 - F_1\dot{\phi}_0^2$ and $\mathcal{B} = F_0$, which relates a class of the Horndeski [15,27,28] and beyond-Horndeski [29–31] theories to another [43], we obtain $\mathcal{Q}_1(t) = \mathcal{Q}_2(t) = \mathcal{Q}_3(t) = \mathcal{Q}_4(t) = 0$ and hence the relation

$$\tilde{\mathcal{R}}_c = \mathcal{R}_c, \tag{42}$$

which shows that the disformal invariance of the comoving curvature perturbations exactly holds at any scale [44,45].

- In the case that

$$\mathcal{F}_0 = \mathcal{F}_0(\phi, \mathcal{X}), \qquad \mathcal{F}_1 = \mathcal{F}_1(\phi, \mathcal{X}), \qquad \mathcal{F}_I = 0, \tag{43}$$

($I = 2, 3, 4, 5$) with $\mathcal{A} = F_0 - F_1\dot{\phi}_0^2$ and $\mathcal{B} = F_0$, which relates a class of the Class-^{2}N-I and Class-^{3}N-I DHOST theories to another [24–26], we find that $\mathcal{Q}_1 = \mathcal{Q}_3 = \mathcal{Q}_4 = 0$ and $\mathcal{Q}_2 = -a^2F_{0\mathcal{X}}$, and hence the relation

$$\tilde{\mathcal{R}}_c - \mathcal{R}_c = -\frac{F_{0\mathcal{X}}}{F_0}\Sigma, \tag{44}$$

which confirms the results in Refs. [46,47]. Thus, in the Class-^{2}N-I and Class-^{3}N-I DHOST theories, the equivalence between $\mathcal{R}_c$ and $\tilde{\mathcal{R}}_c$ holds when the intrinsic entropy perturbation Σ is suppressed on the superhorizon scales.

Since $\mathcal{R}_c$, $\Sigma/\left(\dot{\phi}_0\right)^2$, and $\delta_g\phi/(-a^2\dot{\phi}_0)$ are all the gauge-invariant versions of the metric perturbations ψ, A, and $\dot{E} - B/a$ constructed as the combinations with the scalar field perturbation ϕ_1, we expect that the form of the relations between frames (38) and (40) would remain the same even for the more general disformal transformations than Eq. (18), although the background-dependent coefficients $\mathcal{Q}_1(t)$, $\mathcal{Q}_2(t)$, $\mathcal{Q}_3(t)$, $\mathcal{Q}_4(t)$, $\alpha_1(t)$, $\alpha_2(t)$, $\alpha_3(t)$, and $\alpha_4(t)$ are modified accordingly. Thus, whenever in the original frame on the superhorizon scales the conditions (1) and (2) are satisfied at the same time we expect that the comoving curvature perturbation is disformally invariant on the superhorizon scales, $\tilde{\mathcal{R}}_c \approx \mathcal{R}_c$ even for the more general disformal transformations.

4. The tensor perturbations

We then consider the tensor perturbations about a FLRW spacetime (1). The functions $\mathcal{X}$, $\Box\phi$, $\mathcal{Y}$, $\mathcal{Z}$, $\mathcal{W}$, and hence $\mathcal{F}_I$ ($I = 0, 1, 2, 3, 4, 5$) in the generalized disformal transformation (18) remain the same as the background against the tensor perturbations.

Defining the tensor perturbations in the new frame $\tilde{h}_{ij}$ as in the same manner of the scalar perturbations, $\delta\tilde{g}_{ij}dx^i dx^j = \tilde{a}^2(\tilde{t})\tilde{h}_{ij}(\tilde{t}, x^i)dx^i dx^j$, the difference between h_{ij} and $\tilde{h}_{ij}$ can be computed as

$$\tilde{h}_{ij} - h_{ij} = -\frac{a\dot{\phi}_0\left(aF_2 - 2\dot{a}\dot{\phi}_0 F_5\right)}{2\left(a^2F_0 - a\dot{a}\dot{\phi}_0F_2 + \dot{a}^2\dot{\phi}_0^2F_5\right)}\dot{h}_{ij}, \tag{45}$$

where the nonzero difference arises from the disformal elements $\phi_{\mu\nu}$ and $g^{\rho\sigma}\phi_{\rho\mu}\phi_{\sigma\nu}$ in Eq, (17), which were not discussed in Ref. [49]. This is because for instance the ϕ_{ij} term in the spatial components of Eq. (18) gives rise to the nonzero contribution to the difference as $\sim \dot{h}_{ij}\dot{\phi}_0$. Eq. (45) also confirms that the tensor perturbations in the new frame also obeys the transverse-traceless gauge conditions $\delta^{ij}\tilde{h}_{ij} = \partial^i\tilde{h}_{ij} = 0$.

Thus, the tensor perturbations are also disformally invariant, in the case that the tensor perturbations in the original frame h_{ij} are conserved with time, $\dot{h}_{ij} = 0$. The exceptional case is that the background-dependent coefficient $aF_2 - 2\dot{a}\dot{\phi}_0F_5(t) = 0$ or $\dot{\phi}_0 = 0$, where the exact frame invariance $\tilde{h}_{ij} = h_{ij}$ is obtained even if $\dot{h}_{ij} \neq 0$.

We expect that even for the more general disformal transformation than Eq. (18), the difference in the tensor perturbations between frames is proportional to $\dot{h}_{ij}$. This is because at the level of the linearized perturbations the nonzero difference between h_{ij} and $\tilde{h}_{ij}$ would always arise from the contributions as $\delta\Gamma^t_{ij}\dot{\phi}_0$ and $\delta\Gamma^j_{it}\dot{\phi}_0$, both of which are proportional to $\dot{h}_{ij}$. Thus, even in the case of the more general disformal transformation than Eq. (18), whenever the tensor perturbations are conserved on the superhorizon scales, $\dot{h}_{ij} \approx 0$, the tensor perturbations remain invariant on the superhorizon scales, $\tilde{h}_{ij} \approx h_{ij}$.

5. Conclusions

We have investigated how the comoving curvature perturbation and tensor perturbations are transformed under the generalized disformal transformation with the second-order covariant derivatives of the scalar field. To construct the generalized disformal transformation, we considered the fundamental elements (6) and (13)-(16) constructed with the covariant derivatives of the scalar field with at most the quadratic order of the second-order covariant derivatives of the scalar field, and the covariant tensors whose contraction gives rise to the above fundamental elements. The resultant general form of the disformal transformation was given by Eq. (18), which included all the models with the disformal transformation (7) studied previously.

We then defined the gauge-invariant comoving curvature perturbations both in the original and new frames defined by Eqs. (4) and (37), and computed their difference. While reproducing the previous results on the disformal invariance in Refs. [44–47,49], we have also shown that the difference between the comoving curvature perturbations in the original and new frames, $\mathcal{R}_c$ and $\tilde{\mathcal{R}}_c$, was given by the combination of the time derivative of the comoving curvature perturbation in the original frame $\dot{\mathcal{R}}_c$, the gauge-invariant perturbation Σ given by Eq. (8), which is related to the intrinsic entropy perturbation of the scalar field, and its time derivative. In the case that

- (1) the adiabaticity holds on the superhorizon scales, $\Sigma \approx 0$ and $\dot{\Sigma} \approx 0$,
- (2) the comoving curvature perturbation in the original frame $\mathcal{R}_c$ is conserved on the superhorizon scales, $\dot{\mathcal{R}}_c \approx 0$,

the equivalence of the comoving curvature perturbations under the generalized disformal transformation (18) holds on the superhorizon scales. While in the previously known scalar-tensor theories, whenever the condition (1) holds the condition (2) also hods, in the more general scalar-tensor theories with the third–order time derivatives, which are related via the generalized disformal transformation (18) the relationship between the conditions (1) and (2) has not been clarified yet. Thus, in this paper, we raise the conditions (1) and (2) as the independent ones.

We have also shown that the difference between the tensor perturbations was proportional to the time derivative of the tensor perturbations in the original frame. Thus, the tensor perturbations were also disformally invariant, whenever the tensor perturbations in the original frame were conserved with time.

We should emphasize again that the disformal transformation Eq. (18) is not the most general one, in terms of the power of the second-order covariant derivatives and the order of the highest-order derivatives in the transformation. It would be interesting and important to extend the analysis in this paper to these more general disformal couplings. We hope to come back to these issues in our future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. A degenerate theory with third-order time derivatives in analytical mechanics

We consider the theory with the third-order time derivative in analytical mechanics

$$L = \frac{a_1}{2}\dddot{\phi}^2 + \frac{a_2}{2}\ddot{\phi}^2 + \frac{a_3}{2}\dot{\phi}^2 + \frac{b_1}{2}\dot{q}^2 + b_2\dddot{\phi}\dot{q} - V(\phi, q), \tag{A.1}$$

where a_1, a_2, a_3, b_1, and b_2 are constants, and the potential is given by the quadratic terms

$$V(\phi, q) = \frac{g_1}{2}\phi^2 + g_2\phi q + \frac{g_3}{2}q^2, \tag{A.2}$$

with g_1, g_2, and g_3 being constants. The theory equivalent to Eq. (A.1) can be obtained by introducing the two auxiliary fields R and Q

$$L_2 = \frac{a_1}{2}\dot{Q}^2 + \frac{a_2}{2}Q^2 + \frac{a_3}{2}R^2 + \frac{b_1}{2}\dot{q}^2 + b_2\dot{Q}\dot{q} - V(\phi, q) + \xi\left(\dot{\phi} - R\right) + \lambda\left(\dot{R} - Q\right). \tag{A.3}$$

It is straightforward to recover the theory (A.3) after eliminating the auxiliary fields R and Q by varying the Lagrangian (A.3) with respect to the Lagrange multipliers λ and ξ.

We regard ϕ, R, Q, and q as the dynamical variables, and define the conjugate momenta by

$$P_Q := \frac{\partial L_2}{\partial \dot{Q}} = a_1 \dot{Q} + b_2 \dot{q}, \tag{A.4}$$

$$P_q := \frac{\partial L_2}{\partial \dot{q}} = b_1 \dot{q} + b_2 \dot{Q}, \tag{A.5}$$

$$P_R := \frac{\partial L_2}{\partial \dot{R}} = \lambda, \tag{A.6}$$

$$P_\phi := \frac{\partial L_2}{\partial \dot{\phi}} = \xi. \tag{A.7}$$

First, we consider the nondegenerate case

$$\frac{\partial^2 L_2}{\partial \dot{Q}^2}\frac{\partial^2 L_2}{\partial \dot{q}^2} - \left(\frac{\partial^2 L_2}{\partial \dot{Q}\,\partial \dot{q}}\right)^2 = b_1 a_1 - b_2^2 \neq 0. \tag{A.8}$$

In this case, by rewriting $\dot{Q}$ and $\dot{q}$ in terms of P_Q and P_q,

$$\dot{q} = \frac{a_1 P_q - b_2 P_Q}{b_1 a_1 - b_2^2}, \qquad \dot{Q} = \frac{-b_2 P_q + b_1 P_Q}{b_1 a_1 - b_2^2}, \tag{A.9}$$

we obtain the Hamiltonian

$$\begin{aligned} H &:= P_Q \dot{Q} + P_R \dot{R} + P_\phi \dot{\phi} + P_q \dot{q} - L_2 \\ &= \frac{1}{2(b_1 a_1 - b_2^2)}\left(a_1 P_q^2 - 2 b_2 P_q P_Q + b_1 P_Q^2\right) \\ &+ P_R Q + P_\phi R - \frac{a_2}{2} Q^2 - \frac{a_3}{2} R^2 + V(\phi, q). \end{aligned} \tag{A.10}$$

Thus, the Hamiltonian (A.10) is not bounded from below, because of the linear dependence on the momenta P_ϕ and P_R. In other words, the theory (A.1) contains the two Ostrogradsky ghosts.

Second, we consider the degenerate case

$$\frac{\partial^2 L_2}{\partial \dot{Q}^2}\frac{\partial^2 L_2}{\partial \dot{q}^2} - \left(\frac{\partial^2 L_2}{\partial \dot{Q}\,\dot{q}}\right)^2 = b_1 a_1 - b_2^2 = 0, \tag{A.11}$$

under which P_Q and P_q satisfy

$$P_q - \frac{b_2}{a_1} P_Q = 0. \tag{A.12}$$

Regarding

$$X_1 := P_q - \frac{b_2}{a_1} P_Q \approx 0, \tag{A.13}$$

as the primary constraint, the total Hamiltonian can be defined as

$$\begin{aligned} \tilde{H} &:= H + \mu Q = \left(P_Q \dot{Q} + P_R \dot{R} + P_\phi \dot{\phi} + P_q \dot{q} - L_2\right) + \mu X_1 \\ &= \frac{a_1}{2 b_2^2} P_q^2 + P_R Q + P_\phi R - \frac{a_2}{2} Q^2 - \frac{a_3}{2} R^2 + V(\phi, q) \\ &+ \mu X_1. \end{aligned} \tag{A.14}$$

The time evolution of the primary constraint X_1 then generates the secondary constraint

$$\begin{aligned} X_2 &:= \dot{X}_1 = \{X_1, \tilde{H}\} \\ &= -g_2 \phi - g_3 q + \frac{b_2}{a_1}(P_R - a_2 Q) \approx 0, \end{aligned} \tag{A.15}$$

where we define the Poisson bracket,

$$\begin{aligned} \{U_1, U_2\} &:= \left(\frac{\partial U_1}{\partial \phi}\frac{\partial U_2}{\partial P_\phi} - \frac{\partial U_1}{\partial P_\phi}\frac{\partial U_2}{\partial \phi}\right) \\ &+ \left(\frac{\partial U_1}{\partial R}\frac{\partial U_2}{\partial P_R} - \frac{\partial U_1}{\partial P_R}\frac{\partial U_2}{\partial R}\right) \\ &+ \left(\frac{\partial U_1}{\partial Q}\frac{\partial U_2}{\partial P_Q} - \frac{\partial U_1}{\partial P_Q}\frac{\partial U_2}{\partial Q}\right) \\ &+ \left(\frac{\partial U_1}{\partial q}\frac{\partial U_2}{\partial P_q} - \frac{\partial U_1}{\partial P_q}\frac{\partial U_2}{\partial q}\right), \end{aligned} \tag{A.16}$$

which relates P_R to the other phase space variables and eliminates the term linear in P_R in the total Hamiltonian (A.14).

There is still the other term linear in P_ϕ in the total Hamiltonian (A.14). We note that no further constraint is generated if $\{X_2, X_1\} \neq 0$, since the time evolution of X_2 fixes the Lagrange multiplier μ. In order to obtain enough constraints, we have to impose

$$\{X_1, X_2\} = g_3 - \frac{b_2^2}{a_1^2} a_2 \approx 0. \tag{A.17}$$

The time evolution of the secondary constraint X_2 provides the tertiary constraint

$$\begin{aligned} X_3 &:= \dot{X}_2 = \{X_2, \tilde{H}\} \\ &= -g_2 R - \frac{a_2}{a_1} P_q - \frac{b_2}{a_1}\left(P_\phi - a_3 R\right) \approx 0, \end{aligned} \tag{A.18}$$

which relates P_ϕ to the other phase space variables and eliminates the term linear in P_ϕ in the Hamiltonian (A.14). Since $\{X_3, X_1\} = 0$, the time evolution of X_3 provides the quaternary condition

$$\begin{aligned} X_4 &:= \dot{X}_3 = \{X_3, \tilde{H}\} \\ &= \frac{g_2 b_2 q}{a_1} + \frac{a_2^2 b_2^2 q}{a_1^3} - g_2 Q \\ &+ \frac{a_3 b_2 Q}{a_1} + \frac{a_2 g_2 \phi}{a_1} + \frac{g_1 b_2 \phi}{a_1} \approx 0. \end{aligned} \tag{A.19}$$

Since $\{X_4, X_1\} \neq 0$, the time evolution of X_4, $\dot{X}_4 = \{X_4, \tilde{H}\} \approx 0$, fixes the Lagrange multiplier μ and no further constraint is generated.

We note that all the constraints $X_i \approx 0$ $(i = 1, 2, 3, 4)$, (A.13), (A.15), (A.18), (A.19), are the second-class ones, since

$$\begin{aligned} &\{X_1, X_2\} = \{X_1, X_3\} = 0, \\ &\{X_1, X_4\} = -\{X_2, X_3\} = \frac{b_2}{a_1^3}\left(-2 a_1^2 g_2 - a_2^2 b_2 + a_1 a_3 b_2\right), \\ &\{X_2, X_4\} = 0, \\ &\{X_3, X_4\} = \frac{b_2}{a_1^4}\left(a_2^3 b_2 + a_1^2 (2 a_2 g_2 + g_1 b_2)\right). \end{aligned} \tag{A.20}$$

Starting from the 8-dimensional phase space $(\phi, R, Q, q, P_\phi, R_R, P_Q, P_q)$, the 4 second-class constraints leave $4 (= 2 \times 2)$ independent variables in the phase space, namely, 2 degrees of freedom, and hence all the Ostrogradsky ghosts are removed.

Appendix B. The coefficients in Eqs. (38) and (40)

The coefficients in Eq. (38) are given by

$$\begin{aligned} \mathcal{Q}_1(t) &:= -\frac{a^2}{2}\dot{\phi}_0 F_2 - \frac{3a^2}{2}\dot{\phi}_0 F_{0,\Box\phi} + a\dot{a}\dot{\phi}_0^2 F_5 \\ &+ 3a\dot{a}\dot{\phi}_0^2 F_{0,\mathcal{W}} + \frac{3a\dot{a}}{2}\dot{\phi}_0^2 F_{2,\Box\phi} - 3\dot{a}^2\dot{\phi}_0^3 F_{2,\mathcal{W}} \\ &- \frac{3\dot{a}^2}{2}\dot{\phi}_0^3 F_{5,\Box\phi} + \frac{3\dot{a}^3}{a}\dot{\phi}_0^4 F_{5,\mathcal{W}}, \\ \mathcal{Q}_2(t) &:= -a^2 F_{0,\mathcal{X}} + 2\dot{a}^2 F_5 + 6\dot{a}^2 F_{0,\mathcal{W}} + 3\dot{a}^2 F_{2,\Box\phi} \\ &- \frac{a\dot{a}F_2}{\dot{\phi}_0} - \frac{3a\dot{a}F_{0,\Box\phi}}{\dot{\phi}_0} + a\dot{a}\dot{\phi}_0 F_{2,\mathcal{X}} \\ &- \frac{6\dot{a}^3\dot{\phi}_0 F_{2,\mathcal{W}}}{a} - \frac{3\dot{a}^3\dot{\phi}_0 F_{5,\Box\phi}}{a} \\ &- \dot{a}^2\dot{\phi}_0^2 F_{5,\mathcal{X}} + \frac{6\dot{a}^4\dot{\phi}_0^2 F_{5,\mathcal{W}}}{a^2} + 2a^2\ddot{\phi}_0 F_{0,\mathcal{Y}} \end{aligned} \tag{B.1}$$

$$
\begin{aligned}
&- 2a\dot{a}\dot{\phi}_0\ddot{\phi}_0 F_{2,\mathcal{Y}} + 2\dot{a}^2\dot{\phi}_0^2\ddot{\phi}_0 F_{5,\mathcal{Y}} - 4a^2\ddot{\phi}_0^2 F_{0,\mathcal{Z}} \\
&+ 4a\dot{a}\dot{\phi}_0\ddot{\phi}_0^2 F_{2,\mathcal{Z}} - 4\dot{a}^2\dot{\phi}_0^2\ddot{\phi}_0^2 F_{5,\mathcal{Z}}, \qquad \text{(B.2)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{Q}_3(t) :=& \frac{a\dot{a}}{2}F_{2,\Box\phi} - \frac{a^2}{2\dot{\phi}_0}F_{0,\Box\phi} + a^2\dot{\phi}_0 F_{0,\mathcal{Y}} - \frac{\dot{a}^2\dot{\phi}_0}{2}F_{5,\Box\phi} \\
&- a\dot{a}\dot{\phi}_0^2 F_{2,\mathcal{Y}} + \dot{a}^2\dot{\phi}_0^3 F_{5,\mathcal{Y}} - a\dot{a}\ddot{\phi}_0 F_{2,\mathcal{W}} \\
&+ \frac{a^2\ddot{\phi}_0}{\dot{\phi}_0}F_{0,\mathcal{W}} - 4a^2\dot{\phi}_0\ddot{\phi}_0 F_{0,\mathcal{Z}} + \dot{a}^2\dot{\phi}_0\ddot{\phi}_0 F_{5,\mathcal{W}} \\
&+ 4a\dot{a}\dot{\phi}_0^2\ddot{\phi}_0 F_{2,\mathcal{Z}} - 4\dot{a}^2\dot{\phi}_0^3\ddot{\phi}_0 F_{5,\mathcal{Z}}, \qquad \text{(B.3)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{Q}_4(t) :=& -\frac{F_{0,\Box\phi}}{2} + \frac{\dot{a}\dot{\phi}_0 F_{0,\mathcal{W}}}{a} + \frac{\dot{a}\dot{\phi}_0 F_{2,\Box\phi}}{2a} \\
&- \frac{\dot{a}^2\dot{\phi}_0^2 F_{2,\mathcal{W}}}{a^2} - \frac{\dot{a}^2\dot{\phi}_0^2 F_{5,\Box\phi}}{2a^2} + \frac{\dot{a}^3\dot{\phi}_0^3 F_{5,\mathcal{W}}}{a^3}. \qquad \text{(B.4)}
\end{aligned}
$$

The coefficients in Eq. (40) are given by

$$
\begin{aligned}
\alpha_1(t) :=& F_0 + \frac{3\dot{a}\dot{\phi}_0}{a}F_{0,\Box\phi} + \dot{\phi}_0^2 F_{0,\mathcal{X}} - \frac{6\dot{a}^2\dot{\phi}_0^2}{a^2}F_{0,\mathcal{W}} \\
&- \frac{3\dot{a}\dot{\phi}_0^3}{a}F_{1,\Box\phi} - \dot{\phi}_0^4 F_{1,\mathcal{X}} + \frac{6\dot{a}^2\dot{\phi}_0^4}{a^2}F_{1,W} - \ddot{\phi}_0 F_2 \\
&- \frac{3\dot{a}\dot{\phi}_0\ddot{\phi}_0}{a}F_{2,\Box\phi} - 2\dot{\phi}_0^2\ddot{\phi}_0 F_{0,\mathcal{Y}} - \dot{\phi}_0^2\ddot{\phi}_0 F_{2,\mathcal{X}} \\
&+ \frac{6\dot{a}^2\dot{\phi}_0^2\ddot{\phi}_0}{a^2}F_{2,\mathcal{W}} + \frac{6\dot{a}\dot{\phi}_0^3\ddot{\phi}_0}{a^2}F_{3,\Box\phi} + 2\dot{\phi}_0^4\ddot{\phi}_0 F_{1,\mathcal{Y}} \\
&+ 2\dot{\phi}_0^4\ddot{\phi}_0 F_{3,\mathcal{X}} - \frac{12\dot{a}^2\dot{\phi}_0^4\ddot{\phi}_0}{a^2}F_{3,\mathcal{W}} + \ddot{\phi}_0^2 F_5 \\
&+ \frac{3\dot{a}\dot{\phi}_0\ddot{\phi}_0^2}{a}F_{5,\Box\phi} + 4\dot{\phi}_0^2\ddot{\phi}_0^2 F_{0,\mathcal{Z}} + 2\dot{\phi}_0^2\ddot{\phi}_0^2 F_{2,\mathcal{Y}} \\
&+ \dot{\phi}_0^2\ddot{\phi}_0^2 F_{5,\mathcal{X}} - \frac{6\dot{a}^2\dot{\phi}_0^2\ddot{\phi}_0^2}{a^2}F_{5,\mathcal{W}} - \frac{12\dot{a}\dot{\phi}_0^3\ddot{\phi}_0^2}{a}F_{4,\Box\phi} \\
&- 4\dot{\phi}_0^4\ddot{\phi}_0^2 F_{1,\mathcal{Z}} - 4\dot{\phi}_0^4\ddot{\phi}_0^2 F_{3,\mathcal{Y}} - 4\dot{\phi}_0^4\ddot{\phi}_0^2 F_{4,\mathcal{X}} \\
&+ \frac{24\dot{a}^2\dot{\phi}_0^4\ddot{\phi}_0^2}{a^2}F_{4,\mathcal{W}} - 4\dot{\phi}_0^2\ddot{\phi}_0^3 F_{2,\mathcal{Z}} - 2\dot{\phi}_0^2\ddot{\phi}_0^3 F_{5,\mathcal{Y}} \\
&+ 8\dot{\phi}_0^4\ddot{\phi}_0^3 F_{3,\mathcal{Z}} + 8\dot{\phi}_0^4\ddot{\phi}_0^3 F_{4,\mathcal{Y}} \\
&+ 4\dot{\phi}_0^2\ddot{\phi}_0^4 F_{5,\mathcal{Z}} - 16\dot{\phi}_0^4\ddot{\phi}_0^4 F_{4,\mathcal{Z}}, \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2(t) :=& \frac{\dot{\phi}_0}{2}F_2 + \frac{\dot{\phi}_0}{2}F_{0,\Box\phi} - \dot{\phi}_0^3 F_3 - \frac{\dot{\phi}_0^3}{2}F_{1,\Box\phi} - \dot{\phi}_0^3 F_{0,\mathcal{Y}} \\
&+ \dot{\phi}_0^5 F_{1,\mathcal{Y}} - \dot{\phi}_0\ddot{\phi}_0 F_5 - \dot{\phi}_0\ddot{\phi}_0 F_{0,\mathcal{W}} - \frac{1}{2}\dot{\phi}_0\ddot{\phi}_0 F_{2,\Box\phi} \\
&+ 4\dot{\phi}_0^3\ddot{\phi}_0 F_4 + \dot{\phi}_0^3\ddot{\phi}_0 F_{1,\mathcal{W}} + \dot{\phi}_0^3\ddot{\phi}_0 F_{3,\Box\phi} + 4\dot{\phi}_0^3\ddot{\phi}_0 F_{0,\mathcal{Z}} \\
&+ \dot{\phi}_0^3\ddot{\phi}_0 F_{2,\mathcal{Y}} - 4\dot{\phi}_0^5\ddot{\phi}_0 F_{1,\mathcal{Z}} - 2\dot{\phi}_0^5\ddot{\phi}_0 F_{3,\mathcal{Y}} + \dot{\phi}_0\ddot{\phi}_0^2 F_{2,\mathcal{W}} \\
&+ \frac{\dot{\phi}_0\ddot{\phi}_0^2}{2}F_{5,\Box\phi} - 2\dot{\phi}_0^3\ddot{\phi}_0^2 F_{3,\mathcal{W}} - 2\dot{\phi}_0^3\ddot{\phi}_0^2 F_{4,\Box\phi} \\
&- 4\dot{\phi}_0^3\ddot{\phi}_0^2 F_{2,\mathcal{Z}} - \dot{\phi}_0^3\ddot{\phi}_0^2 F_{5,\mathcal{Y}} + 8\dot{\phi}_0^5\ddot{\phi}_0^2 F_{3,\mathcal{Z}} \\
&+ 4\dot{\phi}_0^5\ddot{\phi}_0^2 F_{4,\mathcal{Y}} - \dot{\phi}_0\ddot{\phi}_0^3 F_{5,\mathcal{W}} + 4\dot{\phi}_0^3\ddot{\phi}_0^3 F_{4,\mathcal{W}} \\
&+ 4\dot{\phi}_0^3\ddot{\phi}_0^3 F_{5,\mathcal{Z}} - 16\dot{\phi}_0^5\ddot{\phi}_0^3 F_{4,\mathcal{Z}}, \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3(t) :=& \frac{3\dot{\phi}_0^3}{2}F_{0,\Box\phi} - \frac{3\dot{a}\dot{\phi}_0^4}{a}F_{0,\mathcal{W}} - \frac{3\dot{\phi}_0^5}{2}F_{1,\Box\phi} + \frac{3\dot{a}\dot{\phi}_0^6}{a}F_{1,\Box\phi} \\
&- \frac{3\dot{\phi}_0^3\ddot{\phi}_0}{2}F_{2,\Box\phi} + \frac{3\dot{a}\dot{\phi}_0^4\ddot{\phi}_0}{a}F_{2,\mathcal{W}} + 3\dot{\phi}_0^5\ddot{\phi}_0 F_{3,\Box\phi} \\
&- \frac{6\dot{a}\dot{\phi}_0^4\ddot{\phi}_0}{a}F_{3,\mathcal{W}} + \frac{3\dot{\phi}_0^3\ddot{\phi}_0^2}{2}F_{5,\Box\phi} - \frac{3\dot{a}\dot{\phi}_0^4\ddot{\phi}_0^2}{a}F_{5,\mathcal{W}} \\
&- 6\dot{\phi}_0^5\ddot{\phi}_0^2 F_{4,\Box\phi} + \frac{12\dot{a}\dot{\phi}_0^6\ddot{\phi}_0^2}{a}F_{4,\mathcal{W}}, \qquad \text{(B.7)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_4(t) :=& \frac{\dot{\phi}_0^2}{2a^2}F_{0,\Box\phi} - \frac{\dot{a}\dot{\phi}_0^3}{a^3}F_{0,\mathcal{W}} - \frac{\dot{\phi}_0^4}{2a^2}F_{1,\Box\phi} + \frac{\dot{a}\dot{\phi}_0^5}{a^3}F_{1,\mathcal{W}} \\
&- \frac{\dot{\phi}_0^2\ddot{\phi}_0}{2a^2}F_{2,\Box\phi} + \frac{\dot{a}\dot{\phi}_0^3\ddot{\phi}_0}{a^3}F_{2,\mathcal{W}} + \frac{\dot{\phi}_0^4\ddot{\phi}_0}{a^2}F_{3,\Box\phi} \\
&- \frac{2\dot{a}\dot{\phi}_0^5\ddot{\phi}_0}{a^3}F_{3,\mathcal{W}} + \frac{\dot{\phi}_0^2\ddot{\phi}_0^2}{2a^2}F_{5,\Box\phi} - \frac{\dot{a}\dot{\phi}_0^3\ddot{\phi}_0^2}{a^3}F_{5,\mathcal{W}} \\
&- \frac{2\dot{\phi}_0^4\ddot{\phi}_0^2}{a^2}F_{4,\Box\phi} + \frac{4\dot{a}\dot{\phi}_0^5\ddot{\phi}_0^2}{a^3}F_{4,\mathcal{W}}. \qquad \text{(B.8)}
\end{aligned}
$$

References

[1] A.D. Linde, Particle physics and inflationary cosmology, vol. 5, arXiv:hep-th/0503203, 1990.
[2] A.R. Liddle, D. Lyth, Cosmological Inflation and Large Scale Structure, 2000.
[3] V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, Oxford, 2005.
[4] D.H. Lyth, A. Riotto, Phys. Rep. 314 (1999) 1, arXiv:hep-ph/9807278.
[5] B.A. Bassett, S. Tsujikawa, D. Wands, Rev. Mod. Phys. 78 (2006) 537, arXiv:astro-ph/0507632.
[6] D. Langlois, Lectures on inflation and cosmological perturbations, pp. 1–57, arXiv:1001.5259 [astro-ph.CO], 2010.
[7] P. Ade, et al., Planck, Astron. Astrophys. 571 (2014) A22, arXiv:1303.5082 [astro-ph.CO].
[8] P. Ade, et al., Planck, Astron. Astrophys. 594 (2016) A20, arXiv:1502.02114 [astro-ph.CO].
[9] N. Aghanim, et al., Planck, arXiv:1807.06209 [astro-ph.CO], 2018.
[10] Y. Akrami, et al., Planck, arXiv:1807.06211 [astro-ph.CO], 2018.
[11] C. Armendariz-Picon, T. Damour, V.F. Mukhanov, Phys. Lett. B 458 (1999) 209, arXiv:hep-th/9904075.
[12] M. Alishahiha, E. Silverstein, D. Tong, Phys. Rev. D 70 (2004) 123505, arXiv:hep-th/0404084.
[13] T. Kobayashi, M. Yamaguchi, J. Yokoyama, Phys. Rev. Lett. 105 (2010) 231302, arXiv:1008.0603 [hep-th].
[14] C. Burrage, C. de Rham, D. Seery, A.J. Tolley, J. Cosmol. Astropart. Phys. 01 (2011) 014, arXiv:1009.2497 [hep-th].
[15] T. Kobayashi, M. Yamaguchi, J. Yokoyama, Prog. Theor. Phys. 126 (2011) 511, arXiv:1105.5723 [hep-th].
[16] H. Motohashi, W. Hu, Phys. Rev. D 101 (2020) 083531, arXiv:2002.07967 [hep-th].
[17] A. Golovnev, V. Mukhanov, V. Vanchurin, J. Cosmol. Astropart. Phys. 06 (2008) 009, arXiv:0802.2068 [astro-ph].
[18] S. Kanno, M. Kimura, J. Soda, S. Yokoyama, J. Cosmol. Astropart. Phys. 08 (2008) 034, arXiv:0806.2422 [hep-ph].
[19] F.L. Bezrukov, M. Shaposhnikov, Phys. Lett. B 659 (2008) 703, arXiv:0710.3755 [hep-th].
[20] C. Germani, A. Kehagias, Phys. Rev. Lett. 105 (2010) 011302, arXiv:1003.2635 [hep-ph].
[21] R.P. Woodard, Scholarpedia 10 (2015) 32243, arXiv:1506.02210 [hep-th].
[22] M. Zumalacárregui, J. García-Bellido, Phys. Rev. D 89 (2014) 064046, arXiv:1308.4685 [gr-qc].
[23] D. Langlois, K. Noui, J. Cosmol. Astropart. Phys. 1602 (2016) 034, arXiv:1510.06930 [gr-qc].
[24] J. Ben Achour, D. Langlois, K. Noui, Phys. Rev. D 93 (2016) 124005, arXiv:1602.08398 [gr-qc].
[25] J. Ben Achour, M. Crisostomi, K. Koyama, D. Langlois, K. Noui, G. Tasinato, J. High Energy Phys. 12 (2016) 100, arXiv:1608.08135 [hep-th].
[26] D. Langlois, Int. J. Mod. Phys. D 28 (2019) 1942006, arXiv:1811.06271 [gr-qc].
[27] G.W. Horndeski, Int. J. Theor. Phys. 10 (1974) 363.
[28] C. Deffayet, X. Gao, D. Steer, G. Zahariade, Phys. Rev. D 84 (2011) 064039, arXiv:1103.3260 [hep-th].
[29] J. Gleyzes, D. Langlois, F. Piazza, F. Vernizzi, J. Cosmol. Astropart. Phys. 1308 (2013) 025, arXiv:1304.4840 [hep-th].
[30] J. Gleyzes, D. Langlois, F. Piazza, F. Vernizzi, Phys. Rev. Lett. 114 (2015) 211101, arXiv:1404.6495 [hep-th].
[31] J. Gleyzes, D. Langlois, F. Piazza, F. Vernizzi, J. Cosmol. Astropart. Phys. 1502 (2015) 018, arXiv:1408.1952 [astro-ph.CO].
[32] J.M. Bardeen, Phys. Rev. D 22 (1980) 1882.
[33] H. Kodama, M. Sasaki, Prog. Theor. Phys. Suppl. 78 (1984) 1.
[34] V.F. Mukhanov, H. Feldman, R.H. Brandenberger, Phys. Rep. 215 (1992) 203.
[35] J.M. Bardeen, P.J. Steinhardt, M.S. Turner, Phys. Rev. D 28 (1983) 679.
[36] K.A. Malik, D. Wands, D.H. Lyth, A.R. Liddle, AIP Conf. Proc. 555 (2001) 289, arXiv:astro-ph/0010639.
[37] D.H. Lyth, K.A. Malik, M. Sasaki, J. Cosmol. Astropart. Phys. 05 (2005) 004, arXiv:astro-ph/0411220.
[38] C. Gordon, D. Wands, B.A. Bassett, R. Maartens, Phys. Rev. D 63 (2000) 023506, arXiv:astro-ph/0009131.
[39] N. Makino, M. Sasaki, Prog. Theor. Phys. 86 (1991) 103.

[40] J.-O. Gong, J.-c. Hwang, W.-I. Park, M. Sasaki, Y.-S. Song, J. Cosmol. Astropart. Phys. 1109 (2011) 023, arXiv:1107.1840 [gr-qc].
[41] T. Chiba, M. Yamaguchi, J. Cosmol. Astropart. Phys. 0810 (2008) 021, arXiv:0807.4965 [astro-ph].
[42] J.D. Bekenstein, Phys. Rev. D 48 (1993) 3641, arXiv:gr-qc/9211017 [gr-qc].
[43] S. Liberati, Class. Quantum Gravity 30 (2013) 133001, arXiv:1304.5795 [gr-qc].
[44] M. Minamitsuji, Phys. Lett. B 737 (2014) 139, arXiv:1409.1566 [astro-ph.CO].
[45] S. Tsujikawa, J. Cosmol. Astropart. Phys. 1504 (2015) 043, arXiv:1412.6210 [hep-th].
[46] H. Motohashi, J. White, J. Cosmol. Astropart. Phys. 1602 (2016) 065, arXiv:1504.00846 [gr-qc].
[47] G. Domenech, A. Naruko, M. Sasaki, J. Cosmol. Astropart. Phys. 1510 (2015) 067, arXiv:1505.00174 [gr-qc].
[48] A.J. Christopherson, K.A. Malik, Phys. Lett. B 675 (2009) 159, arXiv:0809.3518 [astro-ph].
[49] A.L. Alinea, T. Kubota, arXiv:2005.12747 [gr-qc], 2020.
[50] T. Kobayashi, Y. Watanabe, D. Yamauchi, Phys. Rev. D 91 (2015) 064013, arXiv:1411.4130 [gr-qc].
[51] K. Koyama, J. Sakstein, Phys. Rev. D 91 (2015) 124066, arXiv:1502.06872 [astro-ph.CO].
[52] R. Saito, D. Yamauchi, S. Mizuno, J. Gleyzes, D. Langlois, J. Cosmol. Astropart. Phys. 1506 (2015) 008, arXiv:1503.01448 [gr-qc].
[53] P. Creminelli, M. Lewandowski, G. Tambalo, F. Vernizzi, J. Cosmol. Astropart. Phys. 12 (2018) 025, arXiv:1809.03484 [astro-ph.CO].
[54] P. Creminelli, G. Tambalo, F. Vernizzi, V. Yingcharoenrat, J. Cosmol. Astropart. Phys. 05 (2020) 002, arXiv:1910.14035 [gr-qc].
[55] R.L. Arnowitt, S. Deser, C.W. Misner, Phys. Rev. 116 (1959) 1322.
[56] H. Motohashi, T. Suyama, M. Yamaguchi, J. Phys. Soc. Jpn. 87 (2018) 063401, arXiv:1711.08125 [hep-th].
[57] H. Motohashi, T. Suyama, M. Yamaguchi, J. High Energy Phys. 06 (2018) 133, arXiv:1804.07990 [hep-th].
[58] H. Motohashi, T. Suyama, Phys. Rev. D 91 (2015) 085009, arXiv:1411.3721 [physics.class-ph].
[59] H. Motohashi, K. Noui, T. Suyama, M. Yamaguchi, D. Langlois, J. Cosmol. Astropart. Phys. 1607 (2016) 033, arXiv:1603.09355 [hep-th].
[60] R. Klein, D. Roest, J. High Energy Phys. 07 (2016) 130, arXiv:1604.01719 [hep-th].