Properties of heavy baryons in the relativistic quark model

D. Ebert[a], R. N. Faustov[b], V. O. Galkin[b]
[a] *Institut für Physik, Humboldt–Universität zu Berlin,*
Newtonstr. 15, D-12489 Berlin, Germany
[b] *Dorodnicyn Computing Centre, Russian Academy of Sciences,*
Vavilov Str. 40, 119991 Moscow, Russia

Abstract

The mass spectra of the ground state and excited heavy baryons consisting of two light (u, d, s) and one heavy (c, b) quarks are calculated. The heavy-quark–light-diquark picture is used within the relativistic quark model. The semileptonic heavy-to-heavy decay rates of these baryons are also calculated both in the heavy quark limit and with inclusion of first order $1/m_Q$ corrections. An overall good agreement of the obtained predictions with available experimental data is found.

During last few years a significant experimental progress has been achieved in studying heavy baryons with one heavy quark. At present masses of all ground states of charmed baryons as well as of their excitations are known experimentally with rather good precision [1]. The bottom sector is significantly less studied. Only half of the ground state bottom baryon masses are known now. The rate of the semileptonic decay $\Lambda_b \to \Lambda_c e\nu$ has been also measured. The Large Hadron Collider (LHC) will provide us with much more data on properties of ground state and excited bottom baryons. Here we review our studies of masses of the ground state and excited heavy baryons containing one heavy quark and their semileptonic decays. All calculations [2, 3, 4] are performed in the framework of the relativistic quark model based on the quasipotential approach in QCD. We use the heavy-quark–light-diquark approximation to reduce a complicated relativistic three-body problem to the subsequent solution of two more simple two-body problems. The first step is the calculation of the masses, wave functions and form factors of the diquarks, composed from two light quarks. Next, at the second step, a heavy baryon is treated as a relativistic bound system of a light diquark and heavy quark. It is important to emphasize that we do not consider a diquark as a point particle but explicitly take into account its structure through the diquark-gluon vertex expressed in terms of the diquark wave functions.

In the adopted approach the diquark is described by the wave function (Ψ_d) of the two-quark bound state and by the baryon is described by the wave function (Ψ_B) of the quark-diquark bound state, satisfying the quasipotential equations [5] of the Schrödinger type [6]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V_{d,B}(\mathbf{p},\mathbf{q};M)\Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \tag{2}$$

and the on-mass-shell relative momentum squared

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \tag{3}$$

Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		[2] our	[9] NJL	[10] BSE	[11] BSE	[12] Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

The kernel $V_{d,B}(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the QCD motivated operator of the quark-quark (d) or quark-diquark (B) interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [7]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the colour quark interaction. An important role in this construction is played by the Lorentz-structure of the nonperturbative confining interaction. In our analysis of mesons we adopted that the effective quark-antiquark interaction is the sum of the one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials with the vector confining potential containing the Pauli term. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. The explicit expressions for the quasipotential of the quark-quark (qq) interaction in the diquark and quark-diquark (Qd) interaction in the baryon are given in Refs. [2, 3]. The values of model parameters can be also found in these references.

At the first step, we calculate the masses and form factors of the light diquark. As it is well-known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment is required. To achieve this goal in describing light diquarks, we closely follow our recent consideration of the spectra of light mesons [8] and adopt the same procedure to make the relativistic quark potential local by replacing $\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \to E_{1,2} = (M^2 - m_{2,1}^2 + m_{1,2}^2)/(2M)$ (see discussion in Ref. [8]).

The quasipotential equation (1) is solved numerically for the complete relativistic potential which depends on the diquark mass in a highly nonlinear way [2]. The obtained ground state masses of scalar and axial vector light diquarks are presented in Table 1. These masses are in good agreement with values found within the Nambu–Jona-Lasinio model [9], by solving the Bethe-Salpeter equation with different types of the kernel [10, 11] and in quenched lattice calculations [12]. It follows from Table 1 that the mass difference between the scalar and vector diquark decreases from ~ 200 to ~ 120 MeV, when one of the u, d quarks is replaced by the s quark in accord with the statement of Ref. [13].

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor $F(r)$ entering the diquark-gluon vertex [2]. This form factor is expressed through the overlap integral of the diquark wave functions.

At the second step, we calculate the masses of heavy baryons as the bound states of a heavy quark and light diquark. For the potential of the heavy-quark–light-diquark interaction we use the expansion in p/m_Q. Since the light diquark is not heavy enough it should be treated fully relativistically. To simplify the potential we follow the same procedure, which was used

Table 2: Masses of the Λ_Q ($Q = c, b$) heavy baryons (in MeV).

		$Q = c$		$Q = b$		
$I(J^P)$	Qd state	M	$M^{\rm exp}$ [1]	M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ [15]
$0(\frac{1}{2}^+)$	$1S$	2297	2286.46(14)	5622	5624(9)	5619.7(2.4)
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930		
$0(\frac{3}{2}^-)$	$1P$	2628	2628.1(6)	5947		
$0(\frac{1}{2}^+)$	$2S$	2772	2766.6(2.4)?	6086		
$0(\frac{3}{2}^+)$	$1D$	2874		6189		
$0(\frac{5}{2}^+)$	$1D$	2883	2882.5(2.2)?	6197		
$0(\frac{1}{2}^-)$	$2P$	3017		6328		
$0(\frac{3}{2}^-)$	$2P$	3034		6337		

for light quarks in a diquark, and replace the diquark energies $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2} \to E_d = (M^2 - m_Q^2 + M_d^2)/(2M)$ in expressions for the quark-diquark quasipotential. This substitution makes the Fourier transform of the potential local. At leading order in p/m_Q the resulting quasipotentials can be presented in the following forms:
for the scalar diquark

$$V^{(0)}(r) = \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r), \tag{4}$$

and for the axial vector diquark

$$\begin{aligned} V^{(0)}(r) &= \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{M_d(E_d + M_d)}\frac{1}{r}\left[\frac{M_d}{E_d}\hat{V}'_{\rm Coul}(r)\right. \\ &\left. -V'_{\rm conf}(r) + \mu_d \frac{E_d + M_d}{2M_d} V'^{V}_{\rm conf}(r)\right]\mathbf{L}\mathbf{S}_d, \end{aligned} \tag{5}$$

$$\hat{V}_{\rm Coul}(r) = -\frac{4}{3}\alpha_s \frac{F(r)}{r}, \qquad V_{\rm conf}(r) = Ar + B, \qquad V^{V}_{\rm conf}(r) = (1-\varepsilon)(Ar + B),$$

where $\hat{V}_{\rm Coul}(r)$ is the smeared Coulomb potential (with the account of the diquark structure). Note that both the one-gluon exchange and confining potential contribute to the diquark spin-orbit interaction. In this limit the heavy baryon levels are degenerate doublets with respect to the heavy quark spin, since the heavy quark spin-orbit and spin-spin interactions arise only at first order in p/m_Q. Solving Eq. (1) numerically we get the heavy quark spin-independent part of the baryon wave function Ψ_B. Then the total baryon wave function is a product of Ψ_B and the spin-dependent part U_B [14].

The leading order degeneracy of heavy baryon states is broken by the p/m_Q corrections. The explicit expression for the quark-diquark potential up to the second order in p/m_Q is given in Ref. [3].

The calculated values of the ground state and excited baryon masses are given in Tables 2-5 in comparison with available experimental data [1, 15, 16, 17, 18, 19, 20, 21, 22, 23]. In the first two columns we put the baryon quantum numbers and the state of the heavy-quark–light-diquark bound system (in usual notations nL), while in the rest columns our predictions for the masses and experimental data are shown.

At present the best experimentally studied quantities are the mass spectra of the Λ_Q and Σ_Q baryons, which contain the light scalar or axial vector diquarks, respectively. They are presented in Tables 2, 3. Masses of the ground states are measured both for charmed and bottom Λ_Q and Σ_Q baryons. Recently the masses of the ground state Σ_b and Σ_b^* baryons were first reported by CDF [18]: $M_{\Sigma_b^+} = 5807.5^{+1.9}_{-2.2} \pm 1.7$ MeV, $M_{\Sigma_b^-} = 5815.2^{+1.0}_{-0.9} \pm 1.7$ MeV,

Table 3: Masses of the Σ_Q ($Q=c,b$) heavy baryons (in MeV).

$I(J^P)$	Qd state	$Q=c$				$Q=b$		
		M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ [16]	$M^{\rm exp}$ [17]	M	$M^{\rm exp}$ [18]	$M^{\rm exp}$ [18]
$1(\frac12^+)$	$1S$	2439	2453.76(18)			5805	5807.5(2.6)	5815.2(2.0)
$1(\frac32^+)$	$1S$	2518	2518.0(5)			5834	5829.0(2.4)	5836.7(2.6)
$1(\frac12^-)$	$1P$	2805				6122		
$1(\frac12^-)$	$1P$	2795				6108		
$1(\frac32^-)$	$1P$	2799	$2802(^4_7)$			6106		
$1(\frac32^-)$	$1P$	2761	2766.6(2.4)?			6076		
$1(\frac52^-)$	$1P$	2790				6083		
$1(\frac12^+)$	$2S$	2864		2846(13)		6202		
$1(\frac32^+)$	$2S$	2912		2939.8(2.3)?	$2938(^3_5)$?	6222		
$1(\frac12^+)$	$1D$	3014				6300		
$1(\frac32^+)$	$1D$	3005				6287		
$1(\frac32^+)$	$1D$	3010				6291		
$1(\frac52^+)$	$1D$	3001				6279		
$1(\frac52^+)$	$1D$	2960				6248		
$1(\frac72^+)$	$1D$	3015				6262		

Table 4: Masses of the Ξ_Q ($Q=c,b$) heavy baryons with scalar diquark (in MeV).

$I(J^P)$	Qd state	$Q=c$			$Q=b$	
		M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ [19]	M	$M^{\rm exp}$[21]
$\frac12(\frac12^+)$	$1S$	2481	2471.0(4)		5812	5792.9(3.0)
$\frac12(\frac12^-)$	$1P$	2801	2791.9(3.3)		6119	
$\frac12(\frac32^-)$	$1P$	2820	2818.2(2.1)		6130	
$\frac12(\frac12^+)$	$2S$	2923			6264	
$\frac12(\frac32^+)$	$1D$	3030			6359	
$\frac12(\frac52^+)$	$1D$	3042		3054.2(1.5)	6365	
$\frac12(\frac12^-)$	$2P$	3186			6492	
$\frac12(\frac32^-)$	$2P$	3199			6494	

$M_{\Sigma_b^{*+}} = 5829.0^{+1.6}_{-1.7} \pm 1.7$ MeV, $M_{\Sigma_b^{*-}} = 5836.7^{+2.0}_{-1.8} \pm 1.7$ MeV. CDF also significantly improved the precision of the Λ_b mass [15]. For charmed baryons the masses of several excited states are also known. It is important to emphasize that the J^P quantum numbers for most excited heavy baryons have not been determined experimentally, but are assigned by PDG on the basis of quark model predictions. For some excited charm baryons such as the $\Lambda_c(2765)$, $\Lambda_c(2880)$ and $\Lambda_c(2940)$ it is even not known if they are excitations of the Λ_c or Σ_c. [1] Our calculations show that the $\Lambda_c(2765)$ can be either the first radial ($2S$) excitation of the Λ_c with $J^P=\frac12^+$ containing the light scalar diquark or the first orbital excitation ($1P$) of the Σ_c with $J^P=\frac32^-$ containing the light axial vector diquark. The $\Lambda_c(2880)$ baryon in our model is well described by the second orbital ($1D$) excitation of the Λ_c with $J^P=\frac52^+$ in agreement with the recent spin assignment [17] based on the analysis of angular distributions in the decays $\Lambda_c(2880)^+ \to \Sigma_c(2455)^{0,++}\pi^{+,-}$. Our model suggests that the charmed baryon $\Lambda_c(2940)$, recently discovered by BaBar[16] and then also confirmed by Belle [17], could be the first radial ($2S$) excitation of the Σ_c with $J^P=\frac32^+$ which mass is predicted slightly below the experimental value. If this state proves

[1] In Tables 2, 3 we mark with ? the states which interpretation is ambiguous.

Table 5: Masses of the Ξ_Q ($Q=c,b$) heavy baryons with axial vector diquark (in MeV).

$I(J^P)$	Qd state	$Q=c$				$Q=b$
		M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ [22]	$M^{\rm exp}$ [23, 19]	M
$\frac12(\frac12^+)$	$1S$	2578	2578.0(2.9)			5937
$\frac12(\frac32^+)$	$1S$	2654	2646.1(1.2)			5963
$\frac12(\frac12^-)$	$1P$	2934				6249
$\frac12(\frac12^-)$	$1P$	2928				6238
$\frac12(\frac32^-)$	$1P$	2931			2931(6)	6237
$\frac12(\frac32^-)$	$1P$	2900				6212
$\frac12(\frac52^-)$	$1P$	2921				6218
$\frac12(\frac12^+)$	$2S$	2984		2978.5(4.1)	2967.1(2.9)	6327
$\frac12(\frac32^+)$	$2S$	3035				6341
$\frac12(\frac12^+)$	$1D$	3132				6420
$\frac12(\frac32^+)$	$1D$	3127				6410
$\frac12(\frac32^+)$	$1D$	3131				6412
$\frac12(\frac52^+)$	$1D$	3123			3122.9(1.4)	6403
$\frac12(\frac52^+)$	$1D$	3087		3082.8(3.3)	3076.4(1.0)	6377
$\frac12(\frac72^+)$	$1D$	3136				6390

to be an excited Λ_c, for which we have no candidates around 2940 MeV, then it will indicate that excitations inside the diquark should be also considered. [2] The $\Sigma_c(2800)$ baryon can be identified in our model with one of the orbital ($1P$) excitations of the Σ_c with $J^P=\frac12^-,\frac32^-$ or $\frac52^-$ which predicted mass differences are less than 15 MeV. Thus masses of all these states are compatible with the experimental values within errors.

Mass spectra of the Ξ_Q baryons with the scalar and axial vector light (qs) diquarks are given in Tables 4, 5. Experimental data here until recently were available only for charm-strange baryons. In 2007 the D0 Collaboration [20] reported the discovery of the Ξ_b^- baryon with the mass $M_{\Xi_b}=5774\pm11\pm15$ MeV. The CDF Collaboration [21] confirmed this observation and gave the more precise value $M_{\Xi_b}=5792.9\pm2.5\pm1.7$ MeV. Our model prediction $M_{\Xi_b}=5812$ MeV is in a reasonable agreement with these new data. In the excited charmed baryon sector we can identify the $\Xi_c(2790)$ and $\Xi_c(2815)$ with the first orbital ($1P$) excitations of the Ξ_c with $J^P=\frac12^-$ and $J^P=\frac32^-$, respectively, containing the light scalar diquark, which is in agreement with the PDG [1] assignment. Recently Belle [22] reported the first observation of two baryons $\Xi_{cx}(2980)$ and $\Xi_{cx}(3077)$, which existence was also confirmed by BaBar [23]. The $\Xi_{cx}(2980)$ can be interpreted in our model as the first radial ($2S$) excitation of the Ξ_c with $J^P=\frac12^+$ containing the light axial vector diquark. On the other hand the $\Xi_{cx}(3077)$ corresponds to the second orbital ($1D$) excitation in this system with $J^P=\frac52^+$. Very recently the BaBar Collaboration [19] announced observation of two new charmed baryons $\Xi_c(3055)$ with the mass $M=3054.2\pm1.2\pm0.5$ MeV and $\Xi_c(3123)$ with the mass $M=3122.9\pm1.3\pm0.3$ MeV. These states can be interpreted in our model as the second orbital ($1D$) excitations of the Ξ_c with $J^P=\frac52^+$ containing scalar and axial vector diquarks, respectively. Their predicted masses are 3042 MeV and 3123 MeV.

For the Ω_Q baryons only masses of the ground-state charmed baryons are known. The Ω_c^* baryon was very recently discovered by BaBar [24]. The measured mass difference of the Ω_c^* and Ω_c baryons of $(70.8\pm1.0\pm1.1)$ MeV is in very good agreement with the prediction of our model 70 MeV [2].

[2] The Λ_c baryon with the first orbital excitation of the diquark is expected to have a mass in this region.

The detailed comparison of our predictions for the heavy baryon mass spectra with results of other calculations can be found in Refs. [2, 3]. Recent theoretical estimates of the heavy baryon masses can be also found in Ref. [25].

In order to calculate the exclusive semileptonic decay rate of the heavy baryon, it is necessary to determine the corresponding matrix element of the weak current between baryon states. In the quasipotential approach, the matrix element of the weak current $J^W_\mu = \bar{Q}'\gamma_\mu(1-\gamma_5)Q$, associated with the heavy-to-heavy quark $Q \to Q'$ ($Q = b$ and $Q' = c$) transition, between baryon states with masses M_{B_Q}, $M_{B_{Q'}}$ and momenta p_{B_Q}, $p_{B_{Q'}}$ takes the form [26]

$$\langle B_{Q'}(p_{B_{Q'}})|J^W_\mu|B_Q(p_{B_Q})\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}_{B_{Q'}\,\mathbf{p}_{B_{Q'}}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_Q\,\mathbf{p}_{B_Q}}(\mathbf{q}), \tag{6}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{B\,\mathbf{p}_B}$ are the baryon ($B = B_Q, B_{Q'}$) wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_B$.

The wave function of the moving baryon $\Psi_{B_{Q'}\,\mathbf{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{B_{Q'}\,\mathbf{0}} \equiv \Psi_{B_{Q'}}$ by the transformation [26]

$$\Psi_{B_{Q'}\,\mathbf{\Delta}}(\mathbf{p}) = D^{1/2}_{Q'}(R^W_{L_{\mathbf{\Delta}}})D^{\mathcal{I}}_d(R^W_{L_{\mathbf{\Delta}}})\Psi_{B_{Q'}\,\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1, \tag{7}$$

where R^W is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the baryon rest frame to a moving one, $D^{1/2}(R)$ and $D^{\mathcal{I}}(R)$ are rotation matrices of the heavy quark and light diquark spins, respectively.

The hadronic matrix elements for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ are parameterized in terms of six invariant form factors:

$$\begin{aligned}
\langle \Lambda_{Q'}(v',s')|V^\mu|\Lambda_Q(v,s)\rangle &= \bar{u}_{\Lambda_{Q'}}(v',s')\Big[F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu\Big]u_{\Lambda_Q}(v,s),\\
\langle \Lambda_{Q'}(v',s')|A^\mu|\Lambda_Q(v,s)\rangle &= \bar{u}_{\Lambda_{Q'}}(v',s')\Big[G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu\Big]\gamma_5 u_{\Lambda_Q}(v,s),
\end{aligned} \tag{8}$$

where $u_{\Lambda_Q}(v,s)$ and $u_{\Lambda_{Q'}}(v',s')$ are Dirac spinors of the initial and final baryon with four-velocities v and v', respectively; $q = M_{\Lambda_{Q'}}v' - M_{\Lambda_Q}v$, and

$$w = v\cdot v' = \frac{M^2_{\Lambda_Q} + M^2_{\Lambda_{Q'}} - q^2}{2M_{\Lambda_Q}M_{\Lambda_{Q'}}}.$$

In the heavy quark limit $m_Q \to \infty$ ($Q = b, c$) the form factors (8) can be expressed through the single Isgur-Wise function $\zeta(w)$ [27]

$$\begin{aligned}
F_1(w) &= G_1(w) = \zeta(w),\\
F_2(w) &= F_3(w) = G_2(w) = G_3(w) = 0.
\end{aligned} \tag{9}$$

At subleading order of the heavy quark expansion two additional types of contributions arise [28]. The first one parameterizes $1/m_Q$ corrections to the HQET current and is proportional to the product of the parameter $\bar{\Lambda} = M_{\Lambda_Q} - m_Q$, which is the difference of the baryon and heavy quark masses in the infinitely heavy quark limit, and the leading order Isgur-Wise function $\zeta(w)$. The second one comes from the kinetic energy term in $1/m_Q$ correction to the HQET Lagrangian and introduces the additional function $\chi(w)$. Therefore the form factors are given by [28]

$$\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \zeta(w)\right],\\
G_1(w) &= \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right],
\end{aligned}$$

$$
\begin{aligned}
F_2(w) &= G_2(w) = -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w),\\
F_3(w) &= -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w).
\end{aligned} \tag{10}
$$

In our model we obtain the following expressions for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ form factors up to subleading order in $1/m_Q$

$$
\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q}+\frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w)+\zeta(w)]\\
&\quad +4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}-\frac{\bar\Lambda}{2m_Q}(w+1)\right]\chi(w),\\
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q}+\frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w)+\frac{w-1}{w+1}\zeta(w)\right]\\
&\quad -4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}w\chi(w),\\
F_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w)\\
&\quad -4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}+\frac{\bar\Lambda}{2m_Q}w\right]\chi(w),\\
G_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}\chi(w),\\
F_3(w) &= -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}\chi(w),
\end{aligned} \tag{11}
$$

where the leading order Isgur-Wise function of heavy baryons

$$
\zeta(w) = \lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\Psi_{\Lambda_{Q'}}\left(\mathbf{p}+2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\;\mathbf{e_\Delta}\right)\Psi_{\Lambda_Q}(\mathbf{p}), \tag{12}
$$

and the subleading function

$$
\chi(w) = -\frac{w-1}{w+1}\lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\Psi_{\Lambda_{Q'}}\left(\mathbf{p}+2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\;\mathbf{e_\Delta}\right)\frac{\bar\Lambda-\epsilon_d(p)}{2\bar\Lambda}\Psi_{\Lambda_Q}(\mathbf{p}), \tag{13}
$$

here $\mathbf{e_\Delta} = \mathbf{\Delta}/\sqrt{\mathbf{\Delta^2}}$ is the unit vector in the direction of $\mathbf{\Delta} = M_{\Lambda_{Q'}}\mathbf{v}' - M_{\Lambda_Q}\mathbf{v}$. It is important to note that in our model the expressions for the Isgur-Wise functions $\zeta(w)$ (12) and $\chi(w)$ (13) are determined in the whole kinematic range accessible in the semileptonic decays in terms of the overlap integrals of the heavy baryon wave functions, which are known from the baryon mass spectrum calculations. Therefore we do not need to make any assumptions about the baryon wave functions or/and extrapolate our form factors from the single kinematic point, as it was done in most of previous calculations.

For $(1-\varepsilon)(1+\kappa)=0$ the HQET results (10) are reproduced. This can be achieved either setting $\varepsilon = 1$ (pure scalar confinement) or $\kappa = -1$. In our model we need a vector confining contribution and therefore use the latter option. This consideration gives us an additional justification, based on the HQET, for fixing one of the main parameters of the model κ. In the heavy quark limit the wave functions of the initial Ψ_{Λ_Q} and final baryon $\Psi_{\Lambda_{Q'}}$ coincide, and thus the HQET normalization condition $\zeta(1) = 1$ is exactly reproduced. The subleading function $\chi(w)$ vanishes for $w = 1$. The function $\chi(w)$ is very small in the whole accessible kinematic range, since it is roughly proportional to the ratio of the heavy baryon binding energy to the baryon mass.

The $\Lambda_b \to \Lambda_c$ differential decay rate near zero recoil [28]:

$$\lim_{w\to 1}\frac{1}{\sqrt{w^2-1}}\frac{d\Gamma(\Lambda_b\to\Lambda_c e\nu)}{dw} = \frac{G_F^2|V_{cb}|^2}{4\pi^3}M_{\Lambda_c}^3(M_{\Lambda_b}-M_{\Lambda_c})^2|G_1(1)|^2 \tag{14}$$

is governed by the square of the axial current form factor G_1, which near this point has the following expansion

$$G_1(w) = 1 - \hat\rho^2(w-1) + \hat c(w-1)^2 + \cdots, \tag{15}$$

where in our model with the inclusion of the first order heavy quark corrections (11)

$$\hat\rho^2 = 1.51, \qquad \text{and} \qquad \hat c = 2.03.$$

This value of the slope parameter of the Λ_b-baryon decay form factor is in agreement with the recent experimental value obtained by the DELPHI Collaboration [29]

$$\hat\rho^2 = 2.03 \pm 0.46^{+0.72}_{-1.00}$$

and lattice QCD [30] estimate

$$\hat\rho^2 = 1.1 \pm 1.0.$$

Our prediction for the branching ratio of the semileptonic decay $\Lambda_b \to \Lambda_c e\nu$ for $|V_{cb}| = 0.041$ and $\tau_{\Lambda_b} = 1.23\times 10^{-12}$s [1]

$$Br^{\rm theor}(\Lambda_b \to \Lambda_c l\nu) = 6.9\%$$

is in agreement with available experimental data

$$Br^{\rm exp}(\Lambda_b\to\Lambda_c l\nu) = \begin{cases} \left(5.0^{+1.1+1.6}_{-0.8-1.2}\right)\% & \text{DELPHI [23]}\\ \left(8.1\pm 1.2^{+1.1}_{-1.6}\pm 4.3\right)\% & \text{CDF [25]}\end{cases} \tag{16}$$

and the PDG branching ratio [1]

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c l\nu + \text{anything}) = (9.1\pm 2.1)\%. \tag{17}$$

The comparison of our model predictions with other theoretical calculations [31, 32, 33, 34, 35, 36, 37, 38] is given in Table 6. In nonrelativistic quark models [31, 32, 33] form factors of the heavy baryon decays are evaluated at the single kinematic point of zero recoil and then different form factor parameterizations (pole, dipole) are used for decay rate calculations. The relativistic three-quark model [34], Bethe-Salpeter model [35] and light-front constituent quark model [36] assume Gaussian wave functions for heavy baryons. The authors of the nonrelativistic quark model [37] use for the form factor evaluations the set of variational wave functions, obtained from baryon spectra calculations without employing the quark-diquark approximation. Finally, Ref. [38] presents the recent QCD sum rule prediction. Calculations of Refs. [33, 34, 35] are done in the heavy quark limit only, while the rest include first order $1/m_Q$ corrections for the decays of Λ-type baryons. From Table 6 we see that all theoretical models give close predictions for the semileptonic decays of heavy baryons with scalar diquark ($\Lambda_b \to \Lambda_c e\nu$ and $\Xi_b \to \Xi_c e\nu$), which are consistent with the available experimental data (16) and (17) for the $\Lambda_b \to \Lambda_c e\nu$ semileptonic decay. The results for averaged asymmetries of these decays (see [4]) are also close in most of the considered approaches. Thus one can conclude that the precise measurement of the semileptonic $\Lambda_b \to \Lambda_c e\nu$ decay rate will allow an accurate determination of the CKM matrix element V_{cb} with small theoretical uncertainties.

All predictions for heavy baryon decays with the axial vector diquark listed in Table 6 were obtained in the heavy quark limit. Here the differences between predictions are larger. The nonrelativistic quark model [31] gives for these decay rates values more than two times larger than other estimates. Our model values for these decay rates are the lowest ones. Among the relativistic quark models the closest to our predictions is given in [35]. Unfortunately, it will

Table 6: Comparison of different theoretical predictions for semileptonic decay rates Γ (in $10^{10}\mathrm{s}^{-1}$) of bottom baryons.

Decay	this work	[31]	[32]	[33]	[34]	[35]	[36]	[37]	[38]
$\Lambda_b \to \Lambda_c e\nu$	5.64	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.29	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi'_b \to \Xi'_c e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi'_b \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			

be difficult to measure such decays experimentally. Only Ω_b will decay predominantly weakly and thus has sizable semileptonic branching fractions, since a scalar ss diquark is forbidden by the Pauli principle. All other baryons with the axial vector diquark will decay predominantly strongly or electromagnetically and thus their weak branching ratios will be very small.

In conclusion we emphasize that, in calculating the heavy baryon masses and semileptonic decays, we do not use any free adjustable parameters, thus all obtained results are pure predictions. Indeed, the values of all parameters of the model (including quark masses and parameters of the quark potential) were fixed in our previous considerations of meson properties. Note that the light diquark in our approach is not considered as a point-like object. Instead we use its wave functions to calculate diquark-gluon interaction form factors and thus take into account the finite (and relatively large) size of the light diquark. The other important advantage of our model is the completely relativistic treatment of light quarks in the diquark and of the light diquark in the heavy baryon. We use the v/c expansion only for heavy (b and c) quarks. The obtained heavy baryon wave functions are used for the determination of the baryonic Isgur-Wise functions in the whole kinematic range accessible in semileptonic decays. Therefore we do not need to make any assumptions about the form of the baryon wave functions or/and extrapolate the form factors from one point to the whole kinematic region using some ad hoc ansatz as it was done in most of the previous calculations.

We find that all presently available experimental data for the masses of the ground state and excited heavy baryons can be accommodated in the picture treating a heavy baryon as the composite system of the light diquark and heavy quark, experiencing orbital and radial excitations only between these constituents. The data on semileptonic decays of heavy baryons are also well described in our approach.

The authors are grateful to M. Ivanov, V. Matveev, M. Müller-Preussker and V. Savrin for support and discussions. This work was supported in part by the Russian Science Support Foundation (V.O.G.) and the Russian Foundation for Basic Research (RFBR) (grant No.08-02-00582) (R.N.F. and V.O.G.).

References

[1] Particle Data Group, , W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006).

[2] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **72**, 034026 (2005).

[3] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B **659**, 612 (2008).

[4] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **73**, 094002 (2006).

[5] A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).

[6] A. P. Martynenko and R. N. Faustov, Theor. Math. Phys. **64**, 765 (1985) [Teor. Mat. Fiz. **64**, 179 (1985)].

[7] D. Ebert, V. O. Galkin, and R. N. Faustov, Phys. Rev. D **57**, 5663 (1998); Phys. Rev. D **62**, 034014 (2000); Phys. Rev. D **67**, 014027 (2003).

[8] D. Ebert, R. N. Faustov and V. O. Galkin, Mod. Phys. Lett. A **20**, 1887 (2005); Eur. Phys. J. C **47**, 745 (2006).

[9] D. Ebert, T. Feldmann, C. Kettner, and H. Reinhardt, Z. Phys. C **71**, 329 (1996); Int. J. Mod. Phys. A **13**, 1091 (1998).

[10] C. J. Burden et al., Phys. Rev. C **55**, 2649 (1997).

[11] P. Maris, Few Body Syst. **32**, 41 (2002).

[12] M. Hess et al., Phys. Rev. D **58**, 111502 (1998).

[13] R. L. Jaffe, Phys. Rep. **409**, 1 (2005).

[14] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Phys. Rev. D **70**, 014018 (2004); Phys. Rev. D **66**, 014008 (2002).

[15] D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 202001 (2006).

[16] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **98**, 012001 (2007).

[17] R. Mizuk *et al.* [BELLE Collaboration], arXiv:hep-ex/0608043.

[18] I. V. Gorelov [CDF Collaboration], arXiv:hep-ex/0701056.

[19] B. Aubert *et al.* [BaBar Collaboration], arXiv:0710.5763 [hep-ex].

[20] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **99**, 052001 (2007).

[21] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **99**, 052002 (2007).

[22] R. Chistov *et al.* [BELLE Collaboration], Phys. Rev. Lett. **97**, 162001 (2006).

[23] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607042.

[24] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **97**, 232001 (2006).

[25] I. M. Narodetskii, M. A. Trusov and A. I. Veselov, arXiv:0801.1980 [hep-ph].

[26] R. N. Faustov, Ann. Phys. **78**, 176 (1973); Nuovo Cimento A **69**, 37 (1970).

[27] N. Isgur and M. B. Wise, Nucl. Phys. B **348**, 276 (1991).

[28] M. Neubert, Phys. Rep. **245**, 259 (1994); A. Falk and M. Neubert, Phys. Rev. D **47**, 2982 (1993).

[29] DELPHI Collaboration, J. Abdallah et al., Phys. Lett. B **585**, 63 (2004).

[30] K. C. Bowler et al., Phys. Rev. D **57**, 6948 (1998).

[31] R. Singleton, Jr., Phys. Rev. D **43**, 2939 (1991).

[32] H.-Y. Cheng and B. Tseng, Phys. Rev. D **53**, 1457 (1996).

[33] J. G. Körner, M. Krämer and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).

[34] M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, Phys. Rev. D **56**, 348 (1997).

[35] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **59**, 074016 (1999).

[36] F. Cardarelli and S. Simula, Phys. Rev. D **60**, 074018 (1999).

[37] C. Albertus, E. Hernández and J. Nieves, Phys. Rev. D **71**, 014012 (2005).

[38] M.-Q. Huang, H.-Y. Jin, J. G. Körner and C. Liu, Phys. Lett. B **629**, 27 (2005).