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HOLOGRAPHIC THEORY OF GRAVITY AND COSMOLOGY

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Abstract

According to the holographic principle, the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram. We show that the holographic principle can be understood heuristically as originated from quantum fluctuations of spacetime. Applied to cosmology, this consideration leads to a dynamical cosmological constant Λ of the observed magnitude, in agreement with the result obtained by using unimodular gravity and causal-set theory for the present and recent cosmic epochs. By generalizing the concept of entropic gravity, we find a critical acceleration parameter related to Λ in galactic dynamics, and we construct a phenomenological model of dark matter which we call "modified dark matter" (MDM). We provide successful observational tests of MDM at both the galactic and cluster scales. We also discuss the possibility that the quanta of both dark energy and dark matter obey the quantum Boltzmann statistics or infinite statistics as described by a curious average of the bosonic and fermionic algebras.

1 Introduction and Summary

In Vulcano 2004, in a talk titled "Space-time fluctuations," I discussed some aspects of "space-time foam" – a foamy structure of spacetime arising from quantum fluctuations. [1] To examine how large the fluctuations are, I considered a gedankan experiment in which a light signal is sent from a clock to a mirror (at a distance l away) and back to the clock in a timing experiment to measure l. From the jiggling of the clock's position alone, the Heisenberg uncertainty principle yields $\delta l^2 \gtrsim \frac{\hbar l}{mc}$, where m is the mass of the clock. On the other hand, the clock must be large enough not to collapse into a black hole; this requires $\delta l \gtrsim \frac{Gm}{c^2}$. We conclude that the fluctuation of a distance l scales as $\delta l \gtrsim l^{1/3} l_P^{2/3}$ (where $l_P = \sqrt{\hbar G/c^3}$ is the Planck length). [2] I also showed that this scaling of δl is what the holographic principle [3] demands.

The present talk is a continuation of the talk I gave twelve years ago. I will start (in Section 2) by rederiving this scaling of δl by another method [4] which can be generalized to the case of an expanding universe for which a dyamical cosmological constant is shown to emerge, [5] a result that was earlier obtained [6] by a consideration (in Section 3) of unimodular gravity [7] and Sorkin's causal-set theory. This led me to my more recent work with Ho and Minic, and later also with Edmonds, Farrah and Takeuchi. We found it natural (see Section 4) to generalize Verlinde's formulation [8] of entropic gravity/gravitational thermodynamics to de-Sitter space with a positive cosmological constant. The result was a dark matter model which we call modified dark matter (MDM). [9] Recently we have successfuly tested MDM (see Section 5) with 30 galactic rotation curves and a sample of 93 galactic clusters. [10]

The take-home message from this talk is this: It is possible that the dark sector (dark energy and dark matter) has its origin in quantum gravity. And if the scenario to be sketched in Section 6 is correct, then we can expect some rather novel particle phenomenologies, for the quanta of the dark sector obey not the familiar Bose or Fermi statistics, but an exotic statistics that goes by the name infinite statistics [11] or quantum Boltzmann statistics. [12]

I would like to take this opportunity to make a disclaimer: In a recent paper "New Constraints on Quantum Gravity from X-ray and Gamma-Ray Observations" by Perlman et al. (ApJ. 805, 10 (2015)), it was claimed that detections of quasars at TeV energies with ground-based Cherenkov telescopes seem to have ruled out the holographic spacetime foam model (with δl scaling as $l^{1/3}l_P^{2/3}$). But now I (one of the authors) believe this conclusion is conceivably premature when proper averaging is carried out (though presently there is no formalism yet for carrying out such averages.)

2 Spacetime Foam and the Cosmological Constant Λ

We can rederive the scaling of δl by another argument. Let us consider mapping out the geometry of spacetime for a spherical volume of radius l over the amount of time $2l/c$ it takes light to cross the volume. [4] One way to do this is to fill the space with clocks, exchanging signals with the other clocks and measuring the signals' times of arrival. The total number of operations, including the ticks of the clocks and the measurements of signals, is bounded by the Margolus-Levitin theorem which stipulates that the rate of operations cannot exceed the amount of energy E that is available for the operation divided by $\pi\hbar/2$. This theorem, combined with the bound on the total mass of the clocks to prevent black hole formation, implies that the total number of operations that can occur in this spacetime volume is no bigger than $2(l/l_P)^2/\pi$. To maximize spatial resolution, each clock must tick only once during the entire time period. If we regard the operations as partitioning the spacetime volume into "cells", then on the average each cell occupies a spatial volume no less than $\sim l^3/(l^2/l_P^2) = ll_P^2$, yielding an average separation between neighboring cells no less than $\sim l^{1/3}l_P^{2/3}$. [5] This spatial separation can be interpreted as the average minimum uncertainty in the measurement of a distance l, that is, $\delta l \gtrsim l^{1/3}l_P^{2/3}$.

It is straightforward to generalize [5, 12] the above discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by the substitution of l by H^{-1} in the expressions for energy and entropy densities, where H is the Hubble parameter. (Henceforth we adopt $c = 1 = \hbar$ for convenience unless stated otherwise for clarity.) Applied to cosmology, the above argument leads to the prediction that (1) the cosmic energy density has the critical value $\rho \sim (H/l_P)^2$, and (2) the universe of Hubble size R_H contains $I \sim (R_H/l_p)^2$ bits of information. It follows that the average energy carried by each particle/bit is $\rho R_H^3/I \sim R_H^{-1}$. Such long-wavelength constituents of dark energy give rise to a more or less spatially uniform distribution of cosmic energy density and act as a dynamical cosmological constant with the observed small but nonzero value $\Lambda \sim 3H^2$.

3 Quantum (Generalized Unimodular) Gravity and (Dynamical) Λ

The dynamical cosmological constant we have just obtained will be seen to play an important role in our subsequent discussions. So let us "rederive" it by using another method based on quantum gravity. The idea makes use of the theory of unimodular gravity [7, 6], more specifically its generalized action given by $S_{unimod} = -(16\pi G)^{-1}\int[\sqrt{g}(R+2\Lambda) - 2\Lambda\partial_\mu\mathcal{T}^\mu](d^3x)dt$. In this theory, Λ/G plays the role of "momentum" conjugate to the "coordinate" $\int d^3x\mathcal{T}_0$ which can be identified as the spacetime volume V. Hence the fluctuations of Λ/G and V obey a quantum uncertainty principle, $\delta V\delta\Lambda/G \sim 1$.

Next we borrow an argument due to Sorkin, drawn from the causal-set theory, which stipulates that continous geometries in classical gravity should be replaced by "causal-sets", the discrete substratum of spacetime. In the framework of the causal-set theory, the fluctuation in the number of elements N making up the set is of the Poisson type, i.e., $\delta N \sim \sqrt{N}$. For a causal set, the spacetime volume V becomes $l_P^4 N$. It follows that $\delta V \sim l_P^4\delta N \sim l_P^4\sqrt{N} \sim l_P^2\sqrt{V} = G\sqrt{V}$, and hence $\delta\Lambda \sim V^{-1/2}$. By following an argument due to Baum and Hawking, we argued [6] that, in the framework of unimodular gravity, Λ vanishes to the lowest order of approximation and that its first order correction is positive (at least for the the cosmic epoch corresponding to redshift $z \lesssim 1$. See the second paper of Ref. [6].) We conclude that Λ is positive with a magnitude of $V^{-1/2} \sim R_H^{-2}$, contributing a cosmic energy density ρ given by: $\rho \sim \frac{1}{l_P^2 R_H^2}$, which is of the order of the critical density as observed!

4 From Λ to Modified Dark Matter (MDM)

The dynamical cosmological constant (originated from quantum fluctuations of spacetime) can now be shown to give rise to a critical acceleration parameter in galactic dynamics. The argument [9] is based on a simple generalization of Verlinde's recent proposal of entropic gravity [8] for $\Lambda = 0$ to the case of de-Sitter space with positive Λ. Let us first review Verlinde's derivation of Newton's second law $\vec{F} = m\vec{a}$. Consider a particle with mass m approaching a holographic screen at temperature T. Using the first law of thermodynamics to introduce the concept of entropic force $F = T\frac{\Delta S}{\Delta x}$, and invoking Bekenstein's original arguments concerning the entropy S of black holes, $\Delta S = 2\pi k_B\frac{mc}{\hbar}\Delta x$, Verlinde gets $F = 2\pi k_B\frac{mc}{\hbar}T$. With the aid of the formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer, Verlinde obtains $\vec{F} = m\vec{a}$. Now in a de-Sitter space with positive cosmological constant Λ for an accelerating universe like ours, the net Unruh-Hawking temperature, as measured by a non-inertial observer with acceleration a relative to an inertial observer, is $\tilde{T} = \frac{\hbar \tilde{a}}{2\pi k_B c}$ with $\tilde{a} \equiv \sqrt{a^2 + a_0^2} - a_0$, [13] where $a_0 \equiv \sqrt{\Lambda/3}$. Hence the entropic force (in de-Sitter space) is given by the replacement of T and a by $\tilde{T}$ and $\tilde{a}$ respectively, leading to $F = m[\sqrt{a^2 + a_0^2} - a_0]$. For $a \gg a_0$, we have $F/m \approx a$ which gives $a = a_N \equiv GM/r^2$. But for $a \ll a_0$, $F \approx m\frac{a^2}{2\,a_0} = mv^2/r$ for circular motions, so the observed flat galactic rotation curves (v being independent of r) now require $a \approx \left(2a_N\, a_0^3/\pi\right)^{\frac{1}{4}}$. But that means $F \approx m\sqrt{a_N a_c}$, the modified Newtonian dynamics (MoND) scaling [14], proposed by Milgrom. Thus, we have recovered MoND with the correct magnitude for the critical galactic acceleration parameter $a_c = a_0/(2\pi) \approx cH/(2\pi) \sim 10^{-8} cm/s^2$ (where we recall H is the Hubble parameter). As a bonus, we have also recovered the observed Tully-Fisher relation ($v^4 \propto M$).

Next we [9] can follow the second half of Verlinde's argument [8] to generalize Newton's law of gravity $a = GM/r^2$. The end result is given by $\tilde{a} = G\tilde{M}/r^2$, where $\tilde{M} = M + M_d$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$, with M_d being some unknown mass, i.e., dark matter. For $a \gg a_0$, consistency with the Newtonian force law $a \approx a_N$ implies $M_d \approx 0$. But for $a \ll a_0$, consistency with the condition $a \approx \left(2a_N\, a_0^3/\pi\right)^{\frac{1}{4}}$ requires $M_d \approx \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$. (Note the curious connections among M_d, Λ and M.) Thus dark matter indeed exists. And the MoND force law derived above, at the galactic scale, is simply a manifestation of dark matter!

5 Observational Tests of MDM

In order to test MDM with galactic rotation curves, we fit computed rotation curves to a selected sample of Ursa Major galaxies given in [15], using the mass-to-light ratio M/L as our *only* fitting parameter. For the CDM fits, we use the Navarro, Frenk & White density profile, employing *three* free parameters (one of which is the mass-to-light ratio.) We find that both models fit the data well (and more or less equally well)! But while the MDM fits use only 1 free parameter, for the CDM fits one needs 3 free parameters. Thus the MDM model is a more economical model than CDM in fitting data at the galactic scale. As for dark matter density, the profiles predicted by MDM and CDM agree well in the asymptotic (large R) regime. See Ref. [10] for details.

To test MDM with astronomical observations at a larger scale, we [10] compare dynamical and observed masses in a large sample of galactic clusters studied by Sanders [16] using the compilation by White, Jones, and Forman. Sanders [16] studied the virial discrepancy (i.e., the discrepancy between the observed mass and the dynamical mass) in the contexts of Newtonian dynamics and MoND. He found the well-known discrepancy between the Newtonian dynamical mass ($M_{\rm N}$) and the observed mass ($M_{\rm obs}$): $\left\langle \frac{M_{\rm N}}{M_{\rm obs}} \right\rangle \approx 4.4$. And for the sample clusters, he found $\langle M_{\rm MoND}/M_{\rm obs}\rangle \approx 2.1$.

We [10] have adapted Sanders' approach to the case of MDM. Noting that the argument used in Section 4 does allow M_d to include a term of the form $\xi \left(\frac{a_0}{a}\right) M$ with an undetermined universal parameter ξ, we (in some unpublished work) have decided to use a more general profile of the form $M_d = \left[\xi \left(\frac{a_0}{a}\right) + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right] M$. For $\xi \approx 0.5$, we get $\left\langle \frac{M_{\rm MDM}}{M_{\rm obs}} \right\rangle \approx 1.0$. (As an aside, we have refit the galaxy rotation curves using $\xi = 0.5$ and have found equally good fits.) Thus the virial discrepancy is eliminated in the context of MDM! At the cluster scale, MDM is superior to MoND.

6 The Dark Sector and Infinite Statistics

What is the essential difference between ordinary matter and dark energy from our perspective? To find that out, let us recall our discussion in Section 2, and liken the quanta of dark energy to a perfect gas of N particles obeying Boltzmann statistics at temperature T in a volume V. For the problem at hand, as the lowest-order approximation, we can neglect the contributions from matter and radiation to the cosmic energy density for the recent and present eras. Thus let us take $V \sim R_H^3$, $T \sim R_H^{-1}$, and $N \sim (R_H/l_P)^2$. A standard calculation (for the relativistic case) yields the partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$, where $\lambda = (\pi)^{2/3}/T$, and we get, for the entropy of the system, $S = -(\partial(-TlnZ_N)/\partial T)_{V,N} = N[ln(V/N\lambda^3) + 5/2]$.

The important point to note is that, since $V \sim \lambda^3$, the entropy S becomes nonsensically negative unless $N \sim 1$ which is equally nonsensical because N should not be too different from $(R_H/l_P)^2 \gg 1$. But the solution [12] is obvious: the N inside the log of S somehow must be absent. That is the case if the Gibbs $1/N!$ factor is absent from the partition function Z_N, implying that the "particles" are distinguishable and nonidentical!

Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor is infinite statistics (sometimes called "quantum Boltzmann statistics") [11]. Thus the "particles" constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics. [12]

To show that the quanta of MDM also obey this exotic statistics, we [9] first reformulate MoND via an effective gravitational dielectric medium, motivated by the analogy [17] between Coulomb's law in a dielectric medium and Milgrom's law for MoND. Ho, Minic and I then find that MoNDian force law is recovered if the quanta of MDM obey infinite statistics.

What is infinite statistics? Succinctly, a Fock realization of infinite statistics is provided by the commutation relations of the oscillators: $a_k a_l^\dagger = \delta_{kl}$. Curiously a theory of particles obeying infinite statistics cannot be local [11]. But the TCP theorem and cluster decomposition have been shown to hold despite the lack of locality [11]. Actually this lack of locality is not unexpected. After all, non-locality is also present in holographic theories, and the holographic principle is an important ingredient in the formulation of quantum gravity. Infinite statistics and quantum gravity appear to fit together nicely, and non-locality seems to be a common feature of both of them. [12] Perhaps it is the extended nature of the dark quanta that connects them to such global aspects of space-time as the Hubble parameter and the cosmological constant.

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