THE QCD VACUUM

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Abstract

The failure of perturbative QCD to describe low energy hadron physics reflects not the failure of QCD, but our failure to understand the nature of the QCD ground state. The QCD vacuum must be a complex medium. I advocate the position that the medium is a chromomagnetic superconductor. This model for the vacuum is very physical and relies on fairly well understood ideas from superconductivity. Recent calculations in lattice gauge theories lend support to this conjecture.

Quantum Chromodynamics (QCD) stands as one of the three pillars of the Standard Model. It has found widespread experimental support and general acceptance as the theory of the strong interactions. Nevertheless it is distinguished from its SU(2) and U(1) colleagues by not being readily amenable to perturbative analysis. While at large Q^2 perturbative QCD is very successful and is routinely used as a tool in experimental analysis, its application at low energies is fraught with danger. QCD is an elegant gauge theory for the interaction of quarks and gluons. Unfortunately the spectrum of strongly interacting particles, rich as it is, contains not a trace of a quark or a gluon. How can a theory which is so similar to its gauge brethren behave so differently? Why is the knowledge of the basic quark gluon interaction insufficient to directly tell us what the low energy theory looks like?

We find a clue to the mystery by looking at the SU(2) theory of weak interactions. Here, too, we find a dichotomy between the states of the bare Lagrangian and the states which we measure experimentally. The bare states are massless gauge bosons, while the physical states we observe are massive W's. The resolution of this paradox comes from the fact that the bare vacuum is the wrong place from which to study the theory. The physical vacuum is more complicated; it is a condensate of the Higgs field. What we naively consider empty, nothingness is in reality full of Higgs condensate. The fact that this condensate permeates all space is responsible for the masses of the known quarks and leptons. Quarks propagating in the bare vacuum have no mass. Quarks propagating in the true vacuum acquire mass. (It turns out that even in this more complicated Higgs vacuum perturbative expansions are valid once we realize the state we are to perturb about. In the familiar picture of the Mexican Hat potential we must perturb around the rim not the peak of the hat.) The lesson we want to abstract is that in order to extract the correct physics from the basic lagrangian we must understand the ground state of the system.

Another familiar example of the importance of the knowledge of the ground state is provided by superconductivity. Although, in principle, we can learn all there is to know about this system from study of the basic Schroedinger equation for interacting charged particles this is both technically infeasible and conceptually unenlightening. Even starting from traditional ideas of solid state physics, it was extremely difficult to understand the nature of superconductivity until it was appreciated that a new phenomena, the existence of Cooper pairs, was present and that superconductivity resulted from the condensation of these pairs. Nevertheless, many important properties of superconductivity were properly understood even before the advent of BCS theory by using the Landau Ginzberg theory. From the modern viewpoint, Landau and Ginzberg realized that superconductivity becomes simple when described in terms of the appropriate ground state—a condensate of Cooper pairs.

The Cooper pair condensate plays the role of the Higgs condensate of particle physics. For instance, the well known London equations for superconductivity result from the photon acquiring mass via the Higgs mechanism. The massive photon cannot sustain a magnetic field for a distance greater than the inverse of its mass, m. Thus magnetic fields cannot penetrate further into a superconductor than a distance proportional to $1/m$, the London penetration depth. This is the explanation for the Meissner effect, the expulsion of magnetic fields from a superconductor. The Meissner effect is an even more characteristic property of a superconductor than the ability to carry current unimpeded by resistance.

The crucial property of low energy QCD, the confinement of quarks and gluons, is not apparent from cursory study of the QCD Lagrangian

$$\mathcal{L} = -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + \overline{\psi}\,(i\mathcal{D}_{\mu}\gamma^{\mu} - m)\psi. \tag{1}$$

If we are to understand confinement and to develop a meaningful physical picture for it, we must understand the true ground state of the theory. We have to know what the QCD vacuum is. We again turn to superconductivity for inspiration.

Imagine that the most elusive of particles, a magnetic monopole, is discovered. Imagine placing it and its antiparticle, the anti monopole, inside a superconducting medium. We are immediately confronted by a major problem. What about the Meissner effect? What happens to the magnetic field attached to the monopole? Where does it go?

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Nature resolves this seeming contradiction by channeling the magnetic field into a thin flux tube extending from the monopole to the anti monopole. Thus a reconciliation is achieved between the magnetic version of Gauss' law, requiring the existence of a magnetic field, and the Meissner effect, which tries to eliminate magnetic fields from the superconductor. If we try to separate the monopole, anti-monopole ($m\bar{m}$) pair we find that the energy required is proportional to the distance between them. They are held together by an effective linear potential. If the two particles are brought together to a distance smaller than the characteristic scales of the superconductor (i.e. closer than the width of the flux tube or the London penetration depth), the monopole-antimonopole interaction is dominated by a coloumbic potential. If the m–$\bar{m}$ pair is set spinning at relativistic speeds, the relation between the total angular momentum J and the energy E would be $J \sim E^2$ characteristic of a linearly rising Regge trajectory. The $m - \bar{m}$ system behaves just like a $q - \bar{q}$ meson as successfully described by the string or flux tube model of hadrons!

A single monopole in a superconducting medium has a flux tube or ''string'' running from it to the boundary where the magnetic field can escape. If the medium fills all of space, the energy of the monopole, proportional to the length of the string, is infinite. We can say monopoles are confined and cannot exist as single particle states. The reader cannot have missed the close analogy between the properties of a monopole in a superconductor and a quark in our physical vacuum. Nature has provided us with a mechanism for confining monopoles. Does an analogous mech- anism work for quarks or has nature seen fit to invent a new, unrelated mechanism for QCD? If Nature used a good idea once, why not use it again? Can the QCD vacuum be a superconducting medium?

This attractive, but naive expectation immediately fails since in QCD, quarks are analogs of electric charge not analogs of magnetic charge. What we need for QCD are magnetic supercurrents and a (dual) Meissner effect which excludes electric field. This would confine electric (here color) charges. Instead of a condensate of electrically charged Cooper pairs we need a condensate of colored monopoles. This Duality is easy enough to imagine with electric and magnetic quantities everywhere interchanged. If the analogy with superconductivity is meaningful the QCD vacuum must be a chromomagnetic superconductor.

A much more serious concern is the difference between abelian QED and non-abelian QCD. The nonabelain nature of QCD makes the theory much more complicated. Duality is much less easily imagined in a nonabelian theory and the nonlinear gauge couplings open up the possibility of more complex confining mechanisms. Nevertheless I find the analogy between quark confinement in our physical world and monopole confinement in superconductors to be a compelling one. It is hard for me to believe Nature has developed one model for confinement and then not used it again, especially when the superconducting analogy produces the exact physical properties (e.g. strings, short range coulomb interactions, possibility for bag like models etc.) observed to hold for QCD. It strikes me as both inelegant and contrary to the economy of Nature to invent two physically close but conceptually distant confinement mechanisms.

Since its initial espousal by t'Hooft [1] and Mandelstam [2], the concept of a Chromomagnetic Superconducting Vacuum has been pursued along two different paths. One path takes as its point of departure the observation that the conventional electric superconductor is most transparently studied by means of electrical quantities, i.e. photons A_μ, that couple simply to electrical charges. This observation seems so obvious as to be a truism, but actually contains more physics than appears at first blush. It is only because we live in a world that lacks magnetic monopoles that this approach is ''obvious.'' In a world containing both electrically and magnetically charged particles, the proper variables to use would not be so obvious. That electrical variables are ''better'' (for studying monopole confinement) is demonstrated by the following observation [3]. In the conventional description of superconductivity the London equations follow from the fact that a (electric) photon A_μ propagating in the superconductor acquires a mass (the Higgs mechanism). If we attempted to formulate a theory of monopole confinement in terms of Dual variables (C_μ rather than A_μ) which couple simply to monopoles, C_μ has a propagator behaving as $1/k^4$. This is very singular and difficult to work with and interpret. Surely a description couched in terms of electric variables A_μ is both more familiar and more appropriate.

For QCD we expect just the opposite situation. The dual of magnetic charge (i.e. electric charge) is confined. A long line of investigators [4] starting from Feynman has suggested that the gluon propagator is at least as singular as $1/k^4$. By analogy with the discussion in the previous paragraph we expect that the appropriate variables for describing confinement in low energy QCD should be the gauge variables C^a_μ dual to the gluon fields A^a_μ. [3,5] Again by analogy with the Meissner effect, the C^a_μ should acquire mass.

Baker, Ball, and Zachariesen [6] have vigorously pursued this program and constructed an effective low energy theory for QCD using gauge fields (magnetically) dual to the conventional variables. At low energies the effective theory is written as

$$\mathcal{L} = -\frac{\alpha}{4} G_{\mu\nu} G^{\mu\nu}. \tag{2}$$

Since the vacuum is viewed as a medium a factor of μ, a momentum dependent magnetic permeability, is included. The gluon is weakly coupled at low energy and $G_{\mu\nu}$ can be approximated by an abelian field C

$$G^{\mu\nu} = \partial^\mu C^\nu - \partial^\nu C^\mu. \tag{3}$$

In the confining phase C_μ should be massive (just as the photon in a superconductor).

$$\mu(q^2) = -\frac{M^2}{q^2} + 1 \qquad (4)$$

produces a massive propagator for C_μ. Baker, Ball and Zachariesen (BBZ) then extend this lagrangian by imposing dual gauge invariance and adding a new set of fields to restore locality ($\mu(q^2)$ behaves like M^2/∂^2 in configuration space and is nonlocal) to write down an effective low energy theory for QCD in terms of dual variables. The new Lagrangian is a nonabelian, renormalizable, (dual) gauge invariant theory [6].

$$\mathcal{L}(C) = \mathrm{Tr}\left\{\frac{1}{2}\tilde{F}_{\mu\nu}\mathcal{D}^2\tilde{F}^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - W(\tilde{F})\right\}$$
$$\mathcal{D} \equiv \partial_\mu - ig[C_\mu, \ \ ] \qquad (5)$$

The field tensor $\tilde{F}^{\mu\nu}$, introduced in order to make the theory local, is the field tensor for the Dual electric and magnetic fields **E** and **B** whereas G is the field tensor for the Dual fields **D** and **H**. Recall that the langrangian eq. 5 is for a medium with nontrivial constitutive relations. This is why *BBZ* need **E**, **B** and **D**, **H** to be independent fields. $W(\tilde{F})$ contains counterterms necessary to make the theory renormalizable. Two undetermined parameters which set the scales for the physics are in W.

The effective lagrangian has a flux tube solution which is identifiable with the hadronic string [6]. Equating the resulting string tension with the phenomenologically determined tension fixes one of the parameters in (5). The remaining parameters are fixed from:

1) the QCD sum rule determination of the strength of the gluon condensate; and

2) the coulomb part of the short range potential which appears in heavy quark potential models.

These parameters produce a flux tube width ~1 Fermi. The effective superconducting medium described by this parameterization is on the borderline between a type I and type II superconductor. This is satisfying because it is just such a critical, effective superconductor which can simultaneously account for the success of both bag models (type I) and string models (type II) for hadrons [7].

Including quarks in the dual QCD theory of BBZ is nontrivial but can be done at tree level. The flux tube then reproduces a very good potential between heavy quark, anti quark pairs (see Fig. 1). At high temperature the theory described by Eq. (5) undergoes a phase transition to a non confining phase [6,9]. The deconfining phase transitions is also coincident with a chiral symmetry restoring phase transition [6]. In the confining phase the Dual QCD Lagrangian spontaneously breaks chiral symmetry [6]. It is gratifying that such a physically evocative model as a chromomagnetic superconducting vacuum tailored to confine can naturally accommodate the other striking feature of the QCD vacuum, the spontaneous breaking of chiral symmetry [10]. Thus the Lagrangian of eq. (5) offers a reasonable candidate for an effective low energy theory of QCD.

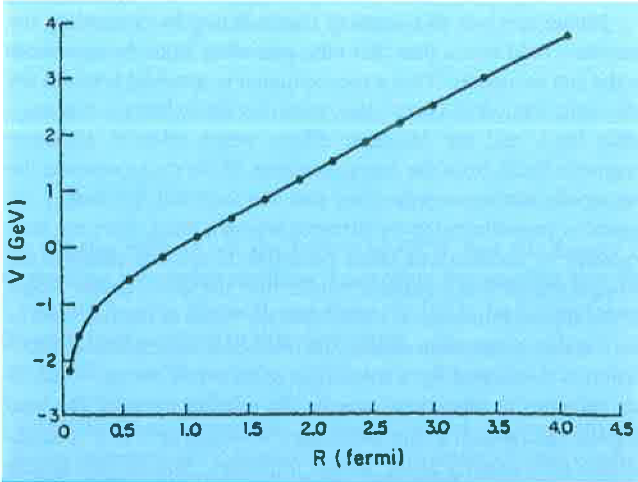

FIGURE 1

Comparison of quark potential as calculated by BBZ with the phenomenological Cornell potential (From Ref. 8).

An alternative approach to revealing the chromomagnetic superconducting properties of the QCD vacuum was inaugurated by t'Hooft [11]. If chromomagnetic monopoles are to condense and fill the vacuum, we must be able to see them. Their very existence, let alone the dynamics responsible for their condensation, is not explicit in the QCD Lagrangian eq. (1). Gauge invariance in complicated non-abelian theories like QCD is both a boon and a bane. t'Hooft's insight was that a clever choice of gauge can make the QCD monopoles manifest and help us in studying their dynamics. The importance of gauge choice is already familiar to us in the electroweak sector where it is only in a properly chosen gauge that the physical particles (massive gauge bosons) become clear. The art of proper gauge choice in QCD is even more subtle.

The simplest way to motivate t'Hooft's suggested choice is to remind ourselves of the emergence of the t'Hooft Polyakov monopole in SU(2) gauge theory. The pure SU(2) gauge sector is augmented by the addition of a Higgs scalar in the adjoint (vector) representation of SU(2). Spontaneous symmetry breaking then reduces the SU(2) symmetry to a residual U(1) symmetry generated by the σ_3 generator of SU(2). Charged gauge bosons acquire mass while the U(1) gauge boson remains massless and behaves like a photon. Monopole solutions exist in the broken theory.

Generalization to SU(3) or higher SU(N) is straightforward. By introducing Higgs in the adjoint representation we can break the SU(N) symmetry down to $U(1)^{N-1}$. The U(1)'s are generated by the N–1 diagonal generators of the SU(N) (i.e., the λ_3 and λ_8 Gellman matrices in SU(3)). N(N–1) nondiagonal gauge bosons, charged with respect to the U(1)'s, acquire mass, while the N–1 diagonal gauge bosons remain massless and behave like U(1), QED photons.

The quarks are also charged with respect to these U(1)'s. N–1 independent solitons, each with magnetic charge in one of the unbroken U(1)'s, exist. If we let the mass of the Higgs boson become infinite and thus disappear from the physics, we can consider this procedure to be a gauge fixing. The monopoles remain, except now they are singular. This unusual gauge fixing still leaves a $U(1)^{N-1}$ gauge invariance undetermined. Such gauge fixings are known as Abelian Projections [11]. There are many other, more elegant and technically superior ways, to fix an Abelian Projection. I have introduced the projection via an explicit symmetry breaking to make it concrete. There is no need for extra fields. An Abelian Projection can be accomplished entirely in a pure gauge theory. A common feature of these Abelian Projections is that we expect to see monopole configurations [11].

By clever choice of gauge, we have exposed monopole configurations in QCD. Their relevance for confinement remains to be seen. t'Hooft's conjecture was that the monopoles would indeed condense and trigger confinement for each of the U(1)s. Because confinement occurs in a U(1) sector, the superconducting analogy is very specific in this model. Except for the fact that the superconductivity and confinement occur for the dual quantities, the picture is completely analogous to the conventional electrodynamic superconductor. The nonabelian confinement mechanism results from confinement in several abelian subgroups. The picture is attractive, satisfying preconceptions about nature's frugality with important ideas and provides a strong physical model for confinement. Is it true?

The importance of monopole condensation as the mechanism for confinement is confirmed from studies in lattice field theory and from supersymmetry. While QED, as physically experienced, is obviously not a confining theory, QED on a lattice behaves completely differently; it is a confining theory. (QED becomes nonconfining when the continuum limit is taken). The confining mechanism for lattice QED is understood. It is the condensation of monopoles. This has been confirmed in many different studies in many different ways [12].

It has long been suspected that N=1 supersymmetric gauge theories exhibit confinement. It is now understood [13] that this same theory exhibits monopole condensation precisely at the phase transition when confinement is supposed to occur. As our knowledge of field theory deepens and broadens evidence for natures predilection to confine charges by condensing monopoles becomes more evident. What about QCD?

The major effort in proving t'Hooft's conjectured confinement mechanism for QCD has been carried out on the lattice. One particular gauge choice has been most popular in these studies. It is known as the maximal abelian gauge (MAG) [14]. In the continuum formulation MAG is imposed by minimizing, over all SU(2) gauge transformations, the functional

$$G = \int W_\mu^+ W_\mu^- d^4x \tag{6}$$

with $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^1 \pm A_\mu^2)$ This leads to the following condition.

$$(\partial_\mu + igA_\mu^3) = W_\mu^+(\partial_\mu - igA_\mu^3)W_\mu^- = 0. \tag{7}$$

On the lattice maximal abelian gauge is implemented by maximizing the lattice functional

$$G = \frac{1}{2}\sum_x \mathrm{Tr}\,[U_\mu^+(x)\tau_3 U_\mu(x)\tau_3 \tag{8}$$

where U(x) is the lattice link variable.

$$U_\mu(x) = e^{ig\vec{A}_\mu\vec{\tau}} \tag{9}$$

In this specific gauge there is mounting evidence to support t'Hoofts conjecture:

1) Early studies indicated that monopole condensation does occur in SU(2) gauge theory when working in MAG. This is inferred from the increased density of the monopoles in the confining phase [14].

2) Abelian Wilson loops are defined by replacing the full SU(2) link variables by U(1) link variables. In MAG (and not in any other studied gauge) these Abelian Wilson loops are strongly enhanced [15]. The Creutz ratios for these Abelian Wilson loops agree with those calculated in full SU(2).

3) The contribution to the string tension coming from the monopoles in MAG SU(2) agrees very well with the string tension calculated from the full SU(2) theory [16]. The string tension is extracted from the heavy quark potential (see Figure 2).

4) There is direct evidence for a chromomagnetic Meissner effect [17,18]. The monopole currents circulate around the chromoelectric flux tubes. The chromoelectric field and the chromomagnetic current generated by monopoles obey the extended dual London equations. Interestingly enough the superconductor for both SU(2) and SU(3) gauge theories seem to be very close to the borderline between type 1 and type 2 superconductors.

5) A candidate for a good order parameter related to the monopole density has been proposed. The order parameter differs from zero when the U(1) symmetry, corresponding to monopole charge conservation, is spontaneously broken. Just as with Cooper pairs, this signals the existence of a dual superconductor, and characterizes the confining phase transition in SU(2) Lattice Gauge theory [19].

These lattice studies are very encouraging but not conclusive. We need to better understand the dependence of the confining mechanism on the gauge choice. Not all gauges that expose monopole configurations show a simple picture of confinement, only MAG. Could many of the success quoted above be no more than gauge artifacts of MAG, rather than dynamical signals of the confinement mechanism [20]? By choosing MAG, are we unveiling

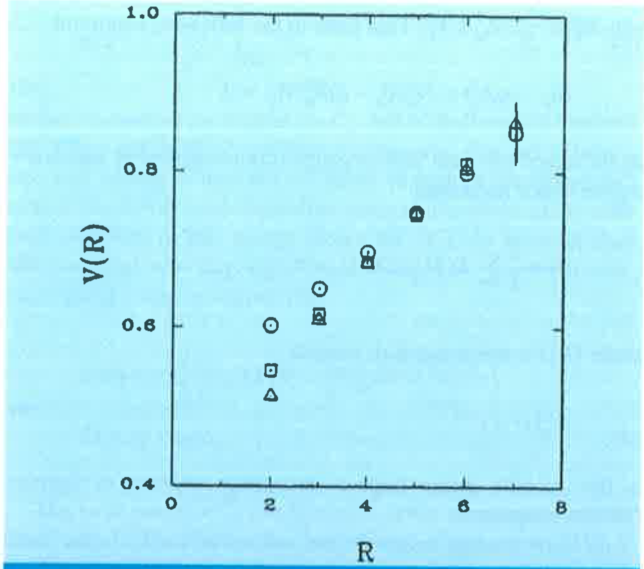

FIGURE 2

Comparison of the monopole potentials (circles), the U(1) approximation to full SU(2) potential (squares) and the full SU(2) potentials (triangles) at $\beta = 2.45$. From Ref. 16.

the true dynamical variables that are hidden in other gauges or are we being seduced by enticing, but imaginary, constructs. The large N limit of QCD provides ambiguous guidance. While to leading order the Abelian projection is consistent with large N limit [21], there is a discrepancy at next to leading order [20]. So the jury is still out.

To my mind, the picture of confinement as a chromomagnetic Meissner effect is very compelling. It is strikingly physical, and has its roots in important and profound ideas from condensed matter physics. The general properties of such a theory are in agreement with the insight that we have developed independently from our experience with hadrons. The string and bag models of hadrons were advanced to describe hadrons and only later where their close similarity to the well understood gedanken world of magnetic monopole confinement in superconductors appreciated. It is hard to imagine that the correspondence of the pictures is accidental rather a reflection of the correspondence of the physics.

The procedure of Abelian Projection seems to me the most promising technique of seeing the emergence of confinement from a non-abelian gauge theory. Maximal Abelian Gauge seems the most promising choice of gauge. Lattice studies offer the best hope of putting all the pieces together. We know that quarks are confined. We are confident that QCD is the correct theory for describing how quarks interact. The only way for both these facts to be true is if our vacuum has a highly non-trivial structure. The precise nature of this vacuum, which we so often take for granted because of its ubiquity, still eludes us. But I would bet that it is a chromomagnetic superconductor [22].

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