ASYMPTOTICS AND LIGHT-CONE SINGULARITIES IN QUANTUM FIELD THEORY

by

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Abstract:

For a local amplitude we prove a one-to-one correspondence between properly defined scaling, the leading light-cone singularity and the asymptotic behaviour of the corresponding Jost-Lehmann spectral function in the sense of distribution theory.

1. Introduction

It has been claimed very often that Bjorken scaling of the imaginary part of the forward scattering amplitude for the virtual Compton process is explained in terms of the light-cone (LC-) singularities of the Fourier transform of that amplitude [1]. This claim has led to the development of the light-cone physics [1].

It is the aim of the present paper to discuss the connection between LC-singularities, Bjorken scaling and related asymptotics +). In particular we will show that there is a one-to-one correspondence between scaling, light-cone singularities and asymptotic behaviour of the Jost-Lehmann (JL) spectral function, if all these limits are understood as limits of sequences of distributions [2,3].

In order to avoid complications due to the photon spin we consider a model-amplitude

+) For comments on earlier attempts we refer to [2].

$$\tilde{T}(x,p) := \langle p|j(\tfrac{x}{2})j(-\tfrac{x}{2})|p\rangle$$

with a real scalar current $j(x)$ and a state $|p\rangle$ of one scalar particle of momentum p ($p^2 = 1$). Our assumptions are those of general quantum field theory, e.g. [4]

(A) $T(q,p)$ satisfies

A1) Lorentz-invariance:

$$T(\Lambda q,\Lambda p) = T(q,p) \qquad \forall\, \Lambda \in L_+^\uparrow$$

A2) Spectrum:

$$\text{supp}\, T(q,p) \subset \{(q,p)|q\cdot p \geq 0,\ q^2 + 2q\cdot p \geq 0\}$$

A3) Locality:

$$\tilde{T}(x,p) - \tilde{T}(-x,p) = 0 \quad \text{for} \quad x^2 < 0 \quad \forall\, p$$

A4)†) Temperedness:

$$T(q,p) \in \mathcal{S}'(\mathbb{R}^4_q) \quad \text{for fixed } p,\ p^2 = 1.$$

(B) Positivity:

$\tilde{T}(x,p)$ is of positive type, e.g.

$$\langle \tilde{T}(x-y,\ p), \varphi(x)\varphi^*(y)\rangle \geq 0 \quad \forall \varphi \in \mathcal{S}(\mathbb{R}^4) \forall\, p \ \text{fixed.}$$

Let us comment on condition A4). We think A4) to be natural in our context: On one hand, it is well known from the perturbative treatment of renormalizable field theories that the n-point-functions are tempered distributions. On the other hand, we don't know of any example for a nontrivial ordinary function $\tilde{T}(x,p)$ which satisfies A2) A3) and (B)††). Thus, in the general case, we cannot expect both $T(q,p)$ and $\tilde{T}(x,p)$ to be functions, and are thus forced to introduce a notion

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†) In the usual axiomatic framework temperedness of matrix elements of field operators holds if these are taken with respect to proper (wave packet) states. For reasons of simplicity we restrict ourselves to improper (plane wave) states.

††) In the special case supp $T(q,p) \subset \overline{V^+}$ it can be shown (Borchers [5]) by A3) and (B) that $\tilde{T}(x,p)$ cannot be an ordinary function different from a constant.

of asymptotic behaviour of (tempered) distributions for a characterization of Bjorken scaling.

At first it seems to be most natural to define the asymptotic behaviour of a distribution $F \in \mathcal{S}'(\mathbb{R}^1)$ via the asymptotic behaviour of its regularization [6], e.g.

Definition 1:

We say, $F \in \mathcal{S}'(\mathbb{R}^1)$ behaves asymptotically like $x^{-\gamma}$ (is of asymptotic degree $\gamma \in \mathbb{R}^1$) if

(i) $\lim\limits_{x \to +\infty} x^{\gamma} \quad (F * \varphi)(x) \exists = c_\gamma(\varphi) \qquad \forall \varphi \in \mathcal{D}(\mathbb{R}^1)$

(ii) $c_\gamma(\varphi) \neq 0$ for at least one φ, if $F \neq 0$.

But we prefer another definition [7]:

Definition 2:

We say, $F \in \mathcal{S}'(\mathbb{R}^1)$ behaves asymptotically like $x^{-\gamma}$ (is of asymptotic degree $\gamma \in \mathbb{R}^1$) if

(i) $\lim\limits_{\lambda \to +\infty} \lambda^{\gamma} F(\lambda x) \exists = F_\gamma(x)$ in $\mathcal{S}'(\mathbb{R}^1)$

(ii) $F_\gamma \neq 0$ if $F \neq 0$

There are some essential differences between both definitions:

(a) The asymptotic degree of $F \in \mathcal{S}'(\mathbb{R}^1)$ may be well-defined in the sense of both definitions but may differ: take F of compact support, then the asymptotic degree γ of F in the sense of definition 1 is $\gamma_1 = +\infty$, but $\gamma = \gamma_2$ is finite in the sense of definition 2.

(b) The asymptotic degree may be defined in the sense of one definition but not in the other:
$F(x) = e^{ix}$ has the asymptotic degree $\gamma_2 = +\infty$ in the sense of definition 2 but is not defined in the sense of definition 1.

(c) There are classes of tempered distributions on which both definitions agree, for instance on those homogeneous distributions which don't have compact support.

(d) Both definitions agree especially for

$$F \in \mathcal{C}(\mathbb{R}^1), \quad \lim_{x \to \pm\infty} x^{\gamma} F(x) \exists = c_{\gamma} \neq 0, \quad \gamma < 1. \tag{1}$$

Such a situation we want to cover: If we identify q and p with the photon and nucleon momentum in the virtual Compton process, we expect $T(q,p)$ to be a continuous function at least for $q^2 \leq 0$, because it is measurable there. The experimentally observed behaviour of $T(q,p)$, $q^2 \leq 0$, suggests a structure for $T(q,p)$ which can be expressed via a condition like (1).

We prefer definition 2, because

(α) we think it to be sufficiently good adapted to the experimental situation (see above).

(β) it is not far away from the most general situation we have in quantum field theory, e.g. each tempered distribution is almost power-behaved in the sense of definition 2:

Lemma 1 (Steinmann [7]):

There exists a function $\gamma : \mathcal{S}'(\mathbb{R}^1) \to (-\infty, +\infty]$ such that for all $F \in \mathcal{S}'(\mathbb{R}^1)$:

$$\left.\begin{array}{lll} \text{(i)} & \lim_{\lambda \to +\infty} \lambda^{\delta} F(\lambda x) \exists = 0 & \forall \delta < \gamma(F) \\ \text{(ii)} & \lim_{\lambda \to +\infty} \lambda^{\delta} F(\lambda x) \nexists & \forall \delta > \gamma(F) \end{array}\right\} \text{ in } \mathcal{S}'(\mathbb{R}^1)$$

(γ) Definition 2 is rather symmetric with respect to Fourier transformation:

$$\lim_{\lambda \to +\infty} \lambda^{\gamma} F(\lambda p) \exists \quad \text{in } \mathcal{S}'(\mathbb{R}^1_p) \quad \Leftrightarrow$$

$$\lim_{\lambda \to +\infty} \lambda^{\gamma - 1} \tilde{F}(x/\lambda) \exists \quad \text{in } \mathcal{S}'(\mathbb{R}^1_x)$$

and thus seems to be an appropriate notion for characterizing Bjorken scaling of

$$V(q,p) := T(q,p) - T(-q,p) \tag{2}$$

and the LC-singularity structure of $\tilde{V}(x,p)$.

We say that V satisfies condition A (B) if T does.

2. Asymptotics in Minkowski Space

In the following we want to define for V the properties of strong scaling$^{\dagger)}$ and weak scaling respectively in accordance with definition 2 and lemma 1.

Due to condition A1), V is a $O_+(3)$-invariant tempered distribution in the rest system $p = (1,\underline{0})$. We define

$$V(q) := V(q, (1,\underline{0})) \tag{3}$$

Hence $V(q) \in \mathcal{S}'(\mathbb{R}^4, O_+(3))$.

Due to the topological isomorphism between $\mathcal{S}'(\mathbb{R}^3, O_+(3))$ and $\mathcal{S}'(\mathbb{R}^1_+)$ we may define uniquely a tempered distribution V_1 of the $O_+(3)$-invariants only:

$$\langle V_1(q_0,\rho), \phi f(q_0,\rho)\rangle = \langle V(q), f(q)\rangle \quad \forall f \in \mathcal{S}(\mathbb{R}^4) \tag{4}$$

with $\phi f(q_0,\rho) := (4\pi)^{-1} \int_{|\underline{\omega}|=1} d\underline{\omega}\; f(q_0,\sqrt{\rho}\,\underline{\omega})$.

We have [8] $\phi f \in \mathcal{S}(\mathbb{R}^1\times\mathbb{R}^1_+)$ and therefore $V_1(q_0,\rho) \in \mathcal{S}'(\mathbb{R}^1\times\mathbb{R}^1_+)$.

There exists a unique extension of V_1 to a distribution $V_2 \in \mathcal{S}'(\mathbb{R}^2)_-{}^{\dagger\dagger)}$:

$$\langle V_2(q_0,w), \varphi(q_0,w)\rangle = \langle V_1(q_0,\rho), \frac{1}{\sqrt{\rho}}\, \varphi(q_0,\sqrt{\rho})\rangle \tag{5}$$

$$\forall \varphi \in \gamma(\mathbb{R}^2)_-$$

and

$$V_2(q_0,w) = 0 \quad \text{on} \quad \mathcal{S}(\mathbb{R}^2) \setminus \mathcal{S}(\mathbb{R}^2)_- .$$

For the following it is advantageous to define a tempered distribution $F(u,v)$ by means of the relation$^{\dagger\dagger\dagger)}$

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†) Here and in the following 'scaling' always means 'Bjorken scaling'.

††) By $\mathcal{S}(\mathbb{R}^n)_-$ we denote the space consisting of testfunctions from $\mathcal{S}(\mathbb{R}^n)$ which are antisymmetric in all variables.

†††) There is a slight difference in the definition of (u,v) compared to [2].

$$F(u,v) := V_2(v+u,\ v-u). \qquad (6)$$

From the antisymmetry of V_2 we obtain the symmetry relations

$$F(u,v) = -F(v,u), \qquad (7a)$$

$$F(u,v) = F(-u,\ -v). \qquad (7b)$$

We are now prepared to state the scaling condition in two alternative forms.

<u>Definition 3</u>: (Strong scaling)

$V(q)$ shows strong scaling of degree β, if there exists a real constant β, such that

$$\lim_{\lambda\to+\infty} \lambda^{\beta-1} F(\lambda u,v) = F_\beta(u,v) \qquad \text{in } \mathcal{S}'(\mathbb{R}^2),$$

$$F_\beta \neq 0 \quad \text{if} \quad F \neq 0.$$

We conclude that $F_\beta(u,v)$ is a homogeneous distribution of degree $1-\beta$ on $\mathbb{R}^1$ with respect to u. With that, the symmetry relation (7b) and the support of F we obtain [8]

$$F_\beta(u,v) = u_+^{1-\beta} F_\beta(v) + u_-^{1-\beta} F_\beta(-v)$$
$$\text{for} \quad \beta \neq 2,3,4,\ldots \quad \text{with} \quad \operatorname{supp} F_\beta(v) \subset [-1/2,\ +\infty). \qquad (8)$$

Later on we will show that (8) may be continued into the points $\beta = 2,3,4,\ldots$ by means of locality leading to

$$F_n(u,v) = u^{1-n}\, F_n(v) \qquad (9)$$

We note that the form of the scaling limit (8), (9) is just the naively expected result.

It is well known that strong scaling is violated for renormalizable interactions by logarithmic terms in finite order of perturbation theory. Recent investigations even suggest that this is true for the exact amplitude in asymptotic free theories.

In this situation we consider in accordance with lemma 1 a weak form of the scaling limit which always exists. Weak scaling of degree

β is a consequence of strong scaling of degree β, but not vice versa.

Our formulation of strong scaling agrees with the point limit of F for $F \in \mathcal{C}(R^2)$ if $\beta < 2$ as shown in the introduction. With the usual variables (ν,ω) we have the asymptotic correspondence $u \to \nu$ $\omega: = -q^2/2\nu \to -2v$.

Similarly we treat the leading LC-behaviour in the following. We define a distribution $\tilde{V}^1$ of the $0_+(3)$-invariants x_0 and $\underline{x}^2$ by means of the chain of relations $(\forall f \in \mathcal{S}(R^4))$

$$
\begin{aligned}
&<\tilde{V}^1(x_0,\sigma),\ \phi f(x_0,\sigma)> = <\tilde{V}(x_0,\underline{x}),\ f(x_0,\underline{x})\\
&= (2\pi)^4 <V(q_0,\underline{q}),\ f(q_0,\underline{q})> = (2\pi)^4 <V_1(q_0,\rho),\ \phi f(q_0,\rho)>.
\end{aligned}
\tag{10}
$$

In exactly the same way as in case of $V_1\ (q_0,\rho)$ we extend $\tilde{V}^1(x_0,\sigma)$ to a distribution $\tilde{V}^2 \in \mathcal{S}'(R^2)_-$. By means of eqs. (5) and (10) we obtain

$$
\tilde{V}^2(x_0,\chi) = -2\pi i\int dq_0\int dw\ \sin q_0x_0 \cdot \sin w\chi\ V_2(q_0,w). \tag{11}
$$

Due to crossing (2) and locality A3) we may define a distribution $\bar{V} \in (\mathcal{S}(R^1_+)\otimes\mathcal{S}(R^1)_-)'$ by

$$
\tilde{V}^2(x_0,\chi) = \varepsilon(x_0)\bar{V}(x_0^2-\chi^2,\chi). \tag{12}
$$

Two alternative forms of leading LC-behaviour are defined in terms of $\bar{V}$ now.

<u>Definition 4</u>: (Strong LC-behaviour)†)

$\bar{V}(\eta,\chi)$ shows strong LC-behaviour of degree γ, if there exists a real constant γ, such that

$$
\lim_{\lambda\to+\infty} \lambda^{\gamma-2}\bar{V}(\eta/\lambda,\chi)\exists = \bar{V}_\gamma(\eta,\chi) \text{ in } (\mathcal{S}(R^1_+)\otimes\mathcal{S}(R^1)_-)',
$$

$$
\bar{V}_\gamma \neq 0 \quad \text{if} \quad \bar{V} \neq 0.
$$

Definition 4 implies that $\bar{V}_\gamma(\eta,\chi)$ is a homogeneous distribution of degree $\gamma - 2$ on $[0,\infty)$ with respect to η, i.e. [8]

†) Compare ref. [3].

$$\bar{V}_\gamma(\eta,\varkappa) = \frac{(\eta)_+^{\gamma-2}}{\Gamma(\gamma-1)}\, g_\gamma(\varkappa). \tag{13}$$

This result is equal to the usually assumed form of the leading LC-singularity.

With the same arguments as given above and in accordance with lemma 1 we introduce the notion of weak LC-behaviour of degree γ.

Now we give a general representation formula for a distribution $\bar{V}$ satisfying strong LC-behaviour.

Lemma 2:†)

A distribution $\bar{V}(\eta,\varkappa)$ which satisfies condition A shows strong LC-behaviour of degree γ if and only if there exists a natural number n with $n + \gamma - 2 \geq 0$, such that

$$\bar{V}(\eta,\varkappa) = D_\eta^n\, \eta^{n+\gamma-2}\, \bar{V}^0(\eta,\varkappa), \tag{14}$$

where $\bar{V}^0(\eta,\varkappa)$ exhibits the following properties:

(i) it is continuous and polynomially bounded in

$(\eta,\varkappa) \in \mathbb{R}^1_+ \times \mathbb{R}^1$,

(ii) it is an odd entire function of exponential type 1 in

$\varkappa \in \mathbb{C}, \ \forall \eta \in \mathbb{R}^1_+, \quad \eta$ fixed.

Corollary

Sufficient for the validity of (14) is strong LC-behaviour of degree γ of $\bar{V}(\eta,\varkappa)$ on $\mathbb{R}^1_\varepsilon$, $\forall \varepsilon > 0$,

$$\mathbb{R}^1_\varepsilon := \{\varkappa \mid |\varkappa| > \varepsilon > 0\}$$

3. Equivalence of Scaling and Leading LC-behaviour

The equivalence of scaling and leading LC-behaviour is stated in our following main theorem 1 by means of definitions 3 and 4 given in section 2.

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†) For all proofs we refer to ref. [2].

Theorem 1

For a distribution $V(q)$ which satisfies condition A, we have:

(a) Strong scaling of degree β implies strong LC-behaviour of degree β $\forall$ $\beta \in \mathbb{R}^1$.

(b) Strong LC-behaviour of degree $\beta > 0$ implies strong scaling of degree β.

In addition we may derive some interesting properties of the distribution $F_\beta(v)$ and $g_\beta(\varkappa)$ which occur in the asymptotic forms of F and $\bar{V}$ (eqs. (8), (13)) respectively.

Lemma 3:

The distribution $g_\beta(\varkappa)$ defined by means of eq. (13) is an odd entire function of exponential type 1 which is polynomially bounded for $\mathrm{Im}\varkappa = 0$.

Lemma 3 is an immediate consequence of the representation eq. (14) for $\bar{V}$.

Lemma 4:†)

The distribution $F_\beta(v)$ (eq. (8)) and $g_\beta(\varkappa)$ are related to each other according to ($\beta > 0$)

$$F_\beta(v) = -i\pi^{-1}\int_0^\infty d\varkappa g_\beta(\varkappa)\varkappa^{\beta-2}2^{\beta-1}\cos[2v\varkappa + \tfrac{\pi}{2}(\beta-1)] \qquad (15)$$

$$\text{for} \quad v \geq -\tfrac{1}{2}.$$

From the spectrum condition we know supp $F_\beta \subset [-1/2,\infty)$. On the other hand the r.h.s. of eq. (15) is symmetric (antisymmetric) under the substitution $v \to -v$, if β is equal to an odd (even) integer. Therefore, we obtain from eq. (15) the well-known fact, that the scaling function F_β has bounded support for integer $\beta = n$

$$\text{supp} \quad F_n \subset [-\tfrac{1}{2}, \tfrac{1}{2}].$$

This result proves eq. (9) as the continuation of eq. (8) to integer β.

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†) Compare Gatto, Menotti - ref. [1] and ref. [9].

Our theorem 1 may be extended immediately to the cases of weak scaling and weak LC-behaviour respectively.

Corollary 1a

For a distribution $V(q)$ which satisfies condition A, we have:

(a) Weak scaling of degree $\beta \geq 0$ implies weak LC-behaviour of degree β.

(b) Weak LC-behaviour of degree $\beta > 0$ implies weak scaling of degree β.

Corollary 1b

Singularities in the interior of the light cone do not contribute to the scaling limit.

4. Equivalence of Leading LC-behaviour and JL-asymptotic [3]

As a consequence of condition A the $0_+(3)$-invariant tempered distribution $V(q)$ satisfies a JL-representation.

Lemma 5 (Jost-Lehmann [10]):

A distribution $V(q)$ satisfies conditions A2) - A4), if and only if there exists an $0_+(3)$-invariant tempered distribution $\underline{\psi}(s,u)$ with

$$\text{supp } \underline{\psi} \subset \{(s,\underline{u}) \,|\, |\underline{u}| \leq 1,\ s \geq s_0(u) = (1 - \sqrt{1-\underline{u}^2}\,)^2\}$$

such that

$$\langle V(q),f(q)\rangle = \langle \underline{\psi}(s,\underline{u}),\ Tf(s,\underline{u})\rangle \ \forall f \in \mathcal{S}(\mathbb{R}^4) \tag{16}$$

with

$$Tf(s,\underline{u}) := \int d^4q f(q)\varepsilon(q_0)\delta(q_0^2 - (\underline{q}-\underline{u})^2 - s)$$

By means of the Fourier transform of $\underline{\psi}$ with respect to $\underline{u}$ we introduce a distribution $\underline{\psi}_2 \in (\mathcal{S}(\mathbb{R}^1_+) \otimes \mathcal{S}(\mathbb{R}^1)_-)'$ of the $0_+(3)$-invariants only:

$$<\underline{\tilde{\psi}}_2(s,\varkappa),\ \varkappa\phi\tilde{f}(s,\varkappa^2)> \; = \; (2\pi)^3 <\underline{\psi}(s,\underline{u}),\ f(s,\underline{u})>. \tag{17}$$

With that and the definition of $\bar{V}$ we obtain as an alternative form of the JL-representation

$$<\bar{V}(\eta,\varkappa),\ g(\eta,\varkappa)> \; = \; <\underline{\tilde{\psi}}_2(s,\varkappa),\ \varphi_g(s,\varkappa)>, \tag{18}$$

where the mapping

$$g(\eta,\varkappa) \mapsto \varphi_g(s,\varkappa) := i(2\pi)^2 \int_0^\infty d\eta J_0(\sqrt{\eta s})\frac{\partial}{\partial\eta}g(\eta,\varkappa) \tag{19}$$

is a topological isomorphism of $\mathcal{S}(\mathbb{R}_1^+)\otimes\mathcal{S}(\mathbb{R}^1)_-$.

In terms of $\underline{\tilde{\psi}}_2$ we may formulate the JL-asymptotic now.

Definition 5: (Strong JL-asymptotic)

$\underline{\tilde{\psi}}_2(s,\varkappa)$ shows strong JL-asymptotic of degree δ, if there exists a real constant δ, such that

$$\lim_{\lambda\to+\infty} \lambda^\delta \underline{\tilde{\psi}}_2(\lambda s,\varkappa)\exists = \underline{\tilde{\psi}}_\delta(s,\varkappa) \quad \text{in } (\mathcal{S}(\mathbb{R}_+^1)\otimes\mathcal{S}(\mathbb{R}^1)_-)',$$

$$\underline{\tilde{\psi}}_\delta \neq 0 \quad \text{if} \quad \underline{\tilde{\psi}}_2 \neq 0.$$

Definition 5 implies that $\underline{\tilde{\psi}}_\delta(s,\varkappa)$ is a homogeneous distribution of degree $-\delta$ on $[0,\infty)$ with respect to s, i.e. [8]:

$$\underline{\tilde{\psi}}_\delta(s,\varkappa) = \frac{(s)_+^{-\delta}}{\Gamma(1-\delta)}\, \varphi_\delta(\varkappa), \tag{20}$$

Again we may introduce a weak form of JL-asymptotic in accordance with lemma 1.

From the properties of the mapping (19) we obtain the following theorem:

Theorem 2: (Zavialov [3])

Suppose $V(q)$ satisfies condition A. Then $\bar{V}(x^2,\varkappa)$ shows strong LC-behaviour of degree β if and only if $\underline{\tilde{\psi}}_2(s,\varkappa)$ shows strong JL-asymptotic of degree β, $\forall\beta\in\mathbb{R}^1$.

Corollary 2

Theorem 2 may be extended to the cases of weak LC-behaviour and weak JL-asymptotic respectively.

5. Equal-time Limits and LC-singularities

The connection between leading LC-behaviour and equal-time commutation, supposed by many authors†), can be put on a rigorous mathematical basis. Let us consider the most important example, the so-called "Schwinger term sum rule". By means of the representation eq. (14) and Lebesque's bounded convergence criteria one easily proves the following lemma:

Lemma 6:

Suppose $\bar{V}(x^2,\varkappa)$ shows strong LC-behaviour of degree 1, then

$$\lim_{\lambda\to+\infty} \lambda\frac{\partial}{\partial x_0} \tilde{V}(\frac{x_0}{\lambda}, \underline{x})\exists = \left.\frac{g_1(\varkappa)}{\varkappa}\right|_{\varkappa=0} \delta(\underline{x}) \quad \text{in } \mathcal{S}'(\mathbb{R}^4). \tag{21}$$

Inverting eq. (15) we obtain the desired sum rule

$$\left.\frac{g_1(\varkappa)}{\varkappa}\right|_{\varkappa=0} = -2i \int_{-1/2}^{+1/2} dv\, F_1(v),$$

where the integral on the r.h.s. has to be understood as a regularized one if necessary.

By the same method other equal-time sum rules including their generalization for $\beta \neq 1$ may be derived.

But the argumentation leading to lemma 6 cannot be reversed. The equal-time limit might exist in the sense of (21) even for an arbitrary singular behaviour of $\bar{V}$ on the light-cone [11].

6. Conclusions

The combination of theorems 1 and 2 leads to the supposed equivalence between scaling, leading LC-behaviour and JL-asymptotic in a region $\beta > 0$ which contains the physical relevant interval $1 \leq \beta \leq 2$. We suppose that zero is not a natural lower bound for β in this context, because quite recently Zavialov proved the same equivalence for a some-

†) Compare the first two papers of ref. [1].

what different definition of scaling without any restriction on β [3].

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