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Strings with a different tension as dark matter

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Abstract The string and brane tensions do not have to be put in by hand, they can be dynamically generated, as in the case when we formulate string and brane theories in the modified measure formalism. Here we take the measure as a metric independent lagrange multiplier that forces some lagrangian density to be zero. Then string tension appears, but as an integration constant. It can be seen however that these string tensions are not universal, but rather each string and each brane generates its own tension. To make the string tension fully dynamical, a bulk field (the tension field) is introduced. As we have seen in previous publications, world sheet conformal invariance in the case of two different species of strings with different tension can produce braneworlds, Swampland constraints may be avoided. Here we introduce braneworld scenarios without singularities of the string tensions, although they still can grow to very large values, this is done by demanding a certain periodicity in one light like coordinate. Now we add another crucial observation; Dark matter to us may consist of matter made out of strings with different tensions because of decoupling of standard string interactions for strings with different tensions, although interactions mediated by the tension field can exist between strings of different tensions.

1 Introduction

Strings are thought by many to be the microscopic description of particles and gravity [1,2]. Even accepting this, there is the question of whether string theory in its usual formulation is the last word, or whether we should go beyond and modify it, Theoretically, modifications may better describe nature. Hopefully the theory improvements help the corresponding predictions of the theory to explain the universe, this is very non trivial and has to be shown however.

We can start from the theoretical side; string theory has a dimensionful parameter, the tension of the string, in its standard formulation, the same is true for brane theories in their better known formulations. This is in spite of the fact that the string theory in a critical dimension enjoys world sheet conformal invariance, but this is not a scale symmetry that is realized in the embedding or target space time, that involves the embbeding metric, etc..

To circumvent this, in the framework of a Modified Measure Theory, a formalism originally used for gravity theories, see for example [3–10], the tension was derived as an additional degree of freedom [11–17]. See also the treatment by Townsend and collaborators [18,19]. In the approach we will follow in this paper the Measure will be just a Lagrange Multiplier, that fixes something to zero. This simplifies greatly the theory, since no additional degrees of freedom are required to define the measure.

A floating cosmological is a generic feature of the modified measure theories of gravity [3–10], including the covariant formulation of the unimodular theory [20], which is in fact a particular case of a modified measure theory, as reviewed in [21].

The tension of the string plays a very similar role to the cosmological constant in four dimensional gravity, but the analogous situation and the role of the cosmological constant is quite different to that of the string tension, because while several world sheets of strings with different tensions can exist in the same universe and in this way many strings can probe at the same region of space time, but that is not the case for the cosmological constant, where every cosmological constant defines necessarily a different universe.

This paper is organized as follows. After this section, the introduction, in Sect. 2 we introduce a new, simpler approach, as compared to what we have done in previous publications,

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the lagrange multiplier measure approach for the dynamical string tension theory. There is the need to also introduce an internal gauge field in the strings world sheet. The equations of motion of this world sheet gauge field leads to, when integrated to an arbitrary Tension for the string. In Sect. 3 we discuss the fact that this tension generation, an integration constant, could take place independently for each world sheet separately and that strings with a different tension to ours could be the origin of the Dark Matter. This means that the string tension is not a fundamental coupling in nature and it could be different for different strings or branes and in addition to this, since conventional string theories are designed only for strings with the same tension, this suggests the idea that strings with a different tension to that of ours could constitute the dark matter. In Sect. 4 we discuss, the possibility that nevertheless of interactions between strings with different tensions might exist, although they cannot be the conventional string interactions, to proceed in this direction we introduce currents in the world sheet can be that change locally the value of the tension of the string world sheet . We present then the coupling of the world sheet gauge fields to currents in the world sheet of the string that couple to the world sheet gauge fields, and as a consequence this coupling induces variations of the tension along the world sheet of the extended object. Then in Sect. 5 we consider a bulk scalar, the tension scalar, and see how this scalar naturally can induce these world sheet current that couples to the internal gauge fields, the equation of motion of the internal gauge field lead to the remarkably simple equation that the local value of the tension along the string is given by $T = e\phi + T_i$, where e is a coupling constant that defines the coupling of the bulk scalar to the world sheet gauge fields and T_i is an integration constant which can be different for each string in the universe. In Sect. 6 we introduce the target space scale invariance or space time scale symmetry of the theory, which does not exist in the standard string theory . Then, in Sect. 7, each string is considered as an independent system that can be quantized. We take into account the string generation by introducing the tension as a function of the scalar field as a factor inside a Polyakov type action with such string tension, then the metric and the factor $e\phi + T_i$ enter together in this effective action, so if there was just one string the factor could be incorporated into the metric and the condition of conformal invariance will not say very much about the scalar ϕ , but if many strings are probing the same regions of space time, then considering a background metric $g_{\mu\nu}$, for each string the "string dependent metrics" $(e\phi + T_i)g_{\mu\nu}$ appears and in the absence of other background fields, like dilaton and antisymmetric tensor fields, Einstein's equations apply for each of the metrics $(e\phi+T_i)g_{\mu\nu}$, considering two types of strings with $T_1{\neq}T_2$. We call $g_{\mu\nu}$ the universal metric, which in fact does not necessarily satisfy Einstein's equations. We find the tension that the two metrics are related by by a conformal transformation and in the case of two species of strings, the conformal transformation determines the tension field and specifying the conformal transformation between the two metrics determines the tensions of the two species of strings. In Sect. 8 we give an example using the metric of flat space in Minkowski coordinates and the same metric after special conformal transformation, configurations determines string tensions we call this effect we call this effect a correlation string interaction for multi strings. In an previous example. using cases where the individual string tension go to infinity, we have argued that this effect to produce the result that Swampland constraints may be avoided, now in Sect. 9 we introduced braneworld scenarios without singularities of the string tensions, although they still can grow to very large values, this is done by demanding a certain periodicity in one light like coordinate. Now in Sects. 10 and 11 we go back to our crucial observation discussed before in the paper before we introduced the tension field; Dark matter to us may consist of matter made out of strings with different tensions because of decoupling of standard string interactions for strings with different tensions, now even in the presence of interactions mediated by the tension field that can exist between strings of different tensions,

2 String theory with a metric independent lagrange multiplier measure

The standard world sheet string sigma-model action using a world sheet metric is [22–24]

$$S_{sigma-model} = -T \int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (1)$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^μ we get the following equations of motion:

$$T_{ab} = (\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu)g_{\mu\nu} = 0, \quad (2)$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0, \quad (3)$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection for the external metric.

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. The modified-measure theory is an example of such a theory.

In previous publications in the framework of this theory we have considered two additional worldsheet scalar fields

$\varphi^i (i = 1, 2)$ are introduced. The new measure density was then formulated first in [11] and latter discussed and generalized also in [12] ,

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j. \tag{4}$$

A simpler formulation that gives equivalent results classically but without the need to consider additional degrees of freedom to define the measure Φ consists of simply considering Φ as an independent density, a Lagrange multiplier in fact. Then the modified bosonic string action is

$$S = -\int d^2\sigma\,\Phi\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)), \tag{5}$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a: $F_{ab} = \partial_a A_b - \partial_b A_a$.

It is important to notice that the action (5) is invariant under conformal transformations of the intrinsic measure combined with a diffeomorphism of the measure fields,

$$\gamma_{ab} \to J\gamma_{ab}, \tag{6}$$

and

$$\Phi \to \Phi' = J\Phi \tag{7}$$

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (5).

The variation with respect to Φ leads to the following equation :

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0. \tag{8}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0. \tag{9}$$

We see that these equations are the same as in the sigma-model formulation (2), (3). By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}$ from (8) we obtain (2).

A most significant result is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0. \tag{10}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{11}$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc.The variation with respect to X^μ leads to the second sigma-model-type equation (3). The idea of modifying the measure of integration proved itself effective and profitable. This can be generalized to incorporate super symmetry, see for example [12–15]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [18,19].

3 String tension generation as a world sheet effect

The fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before and it is exactly this feature that leads us to contemplate the possibility that there could be other strings in the universe that have a different string tension,

3.1 Dark matter from strings with different tensions

As discussed before, the process of string tension generation takes place in a given string, we know however that string interactions are defined only for strings with the same tension.

This does not mean exactly that each string has a different tension as any other string, because strings can multiply from a primordial string to many more strings with the same tension, but it could perfectly be the case that there are sets of sf strings, each with its own tension. There will not be however ordinary string interactions between two of these sets, each of them having a different tension relative to the other.

This raises the idea that the strings that constitute the Dark Matter are strings with a different tension to those from which we are build up.

In the next sections we generalize the dynamical string tension theories so that the tension can be truly dynamical and vary locally along the world sheet of the string, we then come back and ask again whether the notion of strings with a different tension to those that we are build of survives this generalization.

4 Making the string tension truly dynamical

As discussed by Polchinski for example in [25–27], gravity can be introduced in two different ways in string theory. One way is by recognizing the graviton as one of the fundamental excitations of the string, the other is by considering the effective action of the embedding metric, by integrating out the string degrees of freedom and then the embedding metric and other originally external fields acquire dynamics which is enforced by the requirement of a zero beta function. These equations fortunately appear to be string tension independent for the critical dimension $D = 26$ in the bosonic string for example, so they will not be changed by introducing different strings with different string tensions, if these tensions are constant along the the world sheet .

However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [28]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings and Branes that is induced by such external scalar field. This is very much in accordance to the philosophy of Schwinger [29] that proposed long time ago that a field theory must be understood by probing it with external sources.

As we will see however, there will be a fundamental difference between this background field and the more conventional ones (the metric, the dilaton field and the two index anti symmetric tensor field) which are identified with some string excitations as well. Instead, here we will see that a single string does not provide dynamics for this field, but rather when the condition for world sheet conformal invariance is implemented for two strings which sample the same region of space time, so it represents a collective effect instead.

4.1 Introducing world sheet currents that couple to the internal gauge fields of strings

If to the action we add a coupling to a world-sheet current j^{a_2}, i.e. a term

$$S_{\text{current}} = \int d^2\sigma A_{a_2} j^{a_2}, \tag{12}$$

see [30–33] for different applications of this. Then the variation of the total action with respect to $A_{a_2 \dots a_{p+1}}$ gives

$$\epsilon^{a_1 a_2}\partial_{a_1}\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = j^{a_2}. \tag{13}$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

5 Coupling to a bulk scalar field, the tension field

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^{a_1} = e\partial_\mu\phi\frac{\partial X^\mu}{\partial\sigma^a}\epsilon^{aa_1} \equiv e\partial_a\phi\epsilon^{aa_1}, \tag{14}$$

where e is some coupling constant.

Then (13) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \tag{15}$$

The constant of integration T_i may vary from one string to the other.

6 Target space scale invariance and its spontaneous breaking for the modified measure dynamical string tension theory

Notice that the string theory, has world sheet conformal invariance at the classical level, and this world sheet conformal invariance requires to be extended to the quantum level.

At the classical level, the ordinary string theory does not have target space scale invariance, which is very much related to the fact that there is a definite scale in the theory, the string tension.

Indeed, in the ordinary string theory, a scale transformation of the background metric

$$g_{\mu\nu} \to \omega g_{\mu\nu}$$

where ω is a constant, is not a symmetry of the Polyakov action, but in the dynamical tension string theory, this transformation is a symmetry provided the world sheet gauge fields and the measure transforms as [34],

$$A_a \to \omega A_a$$
$$\Phi(\varphi) \to \omega^{-1}\Phi(\varphi)$$

and the tension field transforms in a similar way,

$$\phi \to \omega^{-1}\phi$$

As we have seen, the integration of the equations of motions leads to the spontaneous generation of the string tension and at the same time, the spontaneous generation of the target space global scale invariance, since for this case, (15) is satisfied. or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \tag{16}$$

Notice that the interaction is metric independent. Notice that in the absence of these constants of integration, i.e. , if $T_i = 0$ there is no breaking of the Target space scale invariance, since the measure and the tension field transform in the same way, but the introduction of non zero constants of integration introduces a spontaneous breaking of the Target space scale invariance. The role of the constants of integrations T_i is analogous to the role of the integrations M_i that we discussed in the context of the gravitational theories.

One may interpret (16) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i. The consequences of an independent quantization of many strings with different T_i covering the same region of space time will be studied in the next section.

A similar exercise can be considered for the target space scale invariance and its spontaneous breaking for the modified measure dynamical brane tension theory.

The Target space scale invariance, when imposed in the effective theory of the target space scale invariance string theory, that in principle could be in any dimension D, implies that $D = 4$ [35].

7 Constraints from quantum conformal invariance on the tension field, when several strings share the same region of space

If we have a scalar field coupled to a string or a brane in the way described in the sub section above, i.e. through the current induced by the scalar field in the extended object, according to Eq. (16), so we have two sources for the variability of the tension when going from one string to the other: one is the integration constant T_i which varies from string to string and the other the local value of the scalar field, which produces also variations of the tension even within the string or brane world sheet.

As we discussed in the previous section, we can incorporate the result of the tension as a function of scalar field ϕ, given as $e\phi + T_i$, for a string with the constant of integration T_i by defining the action that produces the correct equations of motion for such string, adding also other background fields, the anti symmetric two index field $A_{\mu\nu}$ that couples to $\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu$ and the dilaton field φ that couples to the topological density $\sqrt{-\gamma}R$

$$S_i = -\int d^2\sigma (e\phi + T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu + \int d^2\sigma \sqrt{-\gamma}\varphi R. \quad (17)$$

Notice that if we had just one string, or if all strings will have the same constant of integration $T_i = T_0$.

In any case, it is not our purpose here to do a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields, instead, we will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that all the metrics

$$g^i_{\mu\nu} = (e\phi + T_i)g_{\mu\nu} \quad (18)$$

will satisfy simultaneously the vacuum Einstein's equations,

7.1 The case where not all string tensions are the same, with special emphasis of two types of strings with $T_1 \neq T_2$

The interesting case to consider is therefore many strings with different T_i, let us consider the simplest case of two strings, labeled 1 and 2 with $T_1 \neq T_2$, then we will have two Einstein's equations, for $g^1_{\mu\nu} = (e\phi + T_1)g_{\mu\nu}$ and for $g^2_{\mu\nu} = (e\phi + T_2)g_{\mu\nu}$,

$$R_{\mu\nu}(g^1_{\alpha\beta}) = 0 \quad (19)$$

and , at the same time,

$$R_{\mu\nu}(g^2_{\alpha\beta}) = 0 \quad (20)$$

These two simultaneous conditions above impose a constraint on the tension field ϕ, because the metrics $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ are conformally related, but Einstein's equations are not conformally invariant, so the condition that Einstein's equations hold for both $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ is highly non trivial.

Notice however that if the two metrics say conformally related and the conformal transformation factor is just a constant scale, then if one of the Einstein's equations above is satisfied say (19) , then the other (20) will also be, that would be relevant for the case of two species of strings with constant although different string tensions.

For the general situations, we have,

$$e\phi + T_1 = \Omega^2(e\phi + T_2) \quad (21)$$

which leads to a solution for $e\phi$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2} \quad (22)$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} \quad (23)$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} \quad (24)$$

Whether strings 1 or 2 are the ones with negative tensions depends on the sign of $T_2 - T_1$. If we want the strings with negative tension to exist only in the early universe, we must take $T_2 - T_1$ to be negative. At the same time there will not be positive tension strings in the early universe, but in the late universe approaches a constant value. The positive string tension are the strings 1, with zero tension in the early universe and the tension $T_1 - T_2$ in the late universe. The negative string tension are the strings 2, with $T_1 - T_2$ tension in the early universe and the tension zero tension in the late universe.

8 A correlation string interaction for multi strings configurations determines string tensions

We notice the quantum conformal invariance starts to give useful information concerning the tension field for multi string configurations only after at least two strings with different string tensions covering the same region of space are considered. Then the tension of one string is correlated with the other, this phenomenon can be characterized as a new type of interaction between strings with different string tensions. This correlation, achieved through quantum mechanics, can legitimately called a new kind of string interaction of a very different nature to those considered in the standard string theory.

Notice that in the standard string theory two strings with different tensions cannot interact, cannot split into two strings with different tensions, etc. By contrast, in the dynamical string tension theory, these interactions are only triggered when the string tensions are different. Since conventional string interactions cannot change the tension of the strings, since splitting or joining strings does not change the tension, these interaction do not play a role in the dynamics of the tensions of the strings, as opposed to the new interactions that arise at the multi string level considered here.

Branewolds from Flat space in Minkowski coordinates and Flat space after a special conformal transformation can appear if for example we consider two vacuum metrics,

The first being a flat spacetime in Minkowski coordinates is,

$$ds_1^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta \tag{25}$$

where $\eta_{\alpha\beta}$ is the standard Minkowski metric, with $\eta_{00} = 1$, $\eta_{0i} = 0$ and $\eta_{ij} = -\delta_{ij}$. This is of course a solution of the vacuum Einstein's equations.

and for the second we consider the conformally transformed metric, [32,33],

$$ds_2^2 = \Omega(x)^2 \eta_{\alpha\beta} dx^\alpha dx^\beta \tag{26}$$

where conformal factor coincides with that obtained from the special conformal transformation

$$x\prime^\mu = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2 x^2)} \tag{27}$$

for a certain D vector a_ν. which gives $\Omega^2 = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$ In summary, we have two solutions for the Einstein's equations, $g^1_{\alpha\beta} = \eta_{\alpha\beta}$ and

$$g^2_{\alpha\beta} = \Omega^2 \eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2} \eta_{\alpha\beta} \tag{28}$$

We can then study the evolution of the tensions using $\Omega^2 = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$. We will consider the cases where $a^2 \neq 0$.

The result is a situation where two hypersufaces where the string tension goes to infinity. The strings are then confined to be inside these surfaces, giving rise to a braneworld scenario. The very large string tensions are associated to a large Planck mass, which in turn implies the weakening of the swampland constraints [**?**].

9 Non singular braneworlds with periodic compactification

In previous research we have studied situations where there are two surfaces where the string tensions approach infinity, so we argue these situations represent the emergence of a braneworld scenario. Here, by introducing a periodic compactification of one dimension, instead of a segment compactification we achieve still a very big growth of the string tension up to a maximum value, which is not infinite however. This will help us also with our discussion of strings with a different tension as dark matter.

For this purpose, let us take $\Omega^2 = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$, with $a^\mu = (A, A, 0, 0, \ldots)$ so that $a^2 = 0$, The string tensions of the strings one and two are given by

$$\begin{aligned} e\phi + T_1 &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \end{aligned} \tag{29}$$

$$\begin{aligned} e\phi + T_2 &= \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \end{aligned} \tag{30}$$

using that $a^\mu = (A, A, 0, 0, \ldots)$, since $a^2 = 0$ we get that the tensions are

$$\begin{aligned} e\phi + T_1 &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu)^2}{(2a_\mu x^\mu)(2 + 2a_\mu x^\mu)} \\ &= \frac{(T_2 - T_1)(1 - 2A(x - t))^2}{-4A(x - t))(1 + A(x - t))} \end{aligned} \tag{31}$$

and in a similar fashion, we get,

$$e\phi + T_2 = \frac{(T_2 - T_1)}{-4A(x - t))(1 + A(x - t))} \tag{32}$$

Defining the light like variable $\Delta = x - t$, we see that if $A < 0$ and for $\frac{(T_2 - T_1)}{-A} > 0$, these tensions are positive and approach infinity at $\Delta = 0$ and $\Delta = -1/A > 0$.

We could avoid the infinite tension strings by starting the Δ interval not at zero, but at a small positive value and then ending it at a value a bit smaller so the tensions recover their initial value. This results in an interval which is a bit smaller, and can be made periodic, since at the start of the interval and at the end the tensions have the same values.

To see the length of the new, periodic interval, we see that the tension of the second string depends only on $\Delta(1 + A\Delta)$, and let us start with a positive value of $\Delta = \Delta_1 > 0$, and at a certain value $\Delta = \Delta_2$, the string tensions become of the same as those at $\Delta = \Delta_1$, the relevant equation so that the second string acquires again its initial value is,

$$\Delta_1(1 + A\Delta_1) = \Delta_2(1 + A\Delta_2)$$

which lead us to an equation that allow us to solve for Δ_2,

$$(\Delta_1 - \Delta_2)(1 + A(\Delta_1 + \Delta_2)) = 0$$

so the solution for $\Delta_2 \neq \Delta_1$ is

$$\Delta_2 = -1/A - \Delta_1 < -1/A$$

since by assumption $\Delta_1 > 0$ so that means no tension singularities appear in the new, now periodic interval. Notice that since the tension of the second string and that of the first string differ by a constant, both string tensions share then the same periodicity. Δ_1 is now interpreted as the ultraviolet regularization that avoids the infinite growth of the strings at the borders of the now periodic interval.

10 Disappearance of standard string interactions between strings with different tensions

We have see a new type of interactions between strings, in fact between string with different tensions, mediated by the tension field, but at the same time, the standard interactions of strings disappear. This is because these standard interactions have been formulated only for strings with the same tension, so these kind of interactions, disappear now, since they consist of splitting or joining, etc. of strings which only make sense for strings with the same tension. For example for the model of the non singular braneworlds with periodic compactification, the difference between the tensions is always a constant, so there will never be standard string interactions between these two species of strings.

Even a simpler model that does not involve a tension field, would show similar effect, the tension field should be necessary to show some interaction between strings with different tensions, although not the standard string interactions. Also the cosmological emergence of different tension strings should be explained. One should point out however that strings with all types of tensions contribute to the structure of space time. The space time string tension metrics are related by a conformal transformation, so , in this way the effects of one string type affects the common metric and this back reacts on the other space time metric. So both strings species gravitate,

11 Emergence of a new model for dark matter

We then look at what we know about our universe. There is indeed a big sector of our universe that does not share standard model interactions with us, the dark sector. But in the context of our findings here, we see that Dark matter to us may consist of matter made out of strings with different tensions because of the decoupling of standard string interactions for strings with different tensions.

The decoupling of the dark sector, or strings with different string tensions to ours, could have interesting and unexpected consequences, like for example a different reheating era for the conventional matter and for the dark sector. Such scenario was studied in [36] and the model of Dark Matter as strings with a different tension exactly fits this scenario. As pointed out in [36], the primordial nucleosynthesis (BBN) provides strong evidence that the early Universe contained a hot plasma of photons and baryons with a temperature T>MeV. However, the earliest probes of dark matter originate from much later times around the epoch of structure formation.

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