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PAPER

Anomaly in the electron orbital gyromagnetic factor: QED and radiative interactions

M Awobode

Department of Physics, University of Massachusetts, Boston, MA 02125, United States of America

E-mail: awobode@gmail.com

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Abstract

Using standard Quantum Electrodynamics (QED), we have calculated the anomaly in the electron orbital g-factor, which is in reasonable agreement with experimental data. For second-order corrections, the anomaly in the orbital g-factor ($g_L - 1$) is of the order $\alpha/3\pi$, while for approximations to the same order, the well-known anomaly in the electron spin g-factor $(g_S - 2)/2 = \alpha/2\pi$, where $\alpha = 1/137$ is the structure constant. Unlike the spin-g anomaly, the orbital-g anomaly is not intrinsic, but depends on the atomic state, and on Z, the atomic mass. Thus, the orbital anomaly is actually $(\alpha/3\pi)$D, where D includes the effect of the magnetic field and/or the atomic properties. Hence, we have shown that radiative interactions, which are the sources of the spin-g anomaly, also produce an orbital anomaly. It is physically reasonable to expect that radiative interactions that modify many physical properties of electrons in electromagnetic fields will also modify the orbital magnetic moment of the electron.

1. Introduction: the electron gyromagnetic factors in the search for new physics

The energy W of a charged particle in a magnetic field $\boldsymbol{B}$ is given by the equation $W = -\boldsymbol{\mu} \cdot \boldsymbol{B}$ where $\boldsymbol{\mu}$ is the magnetic dipole moment which depends on the angular momentum vector $\boldsymbol{K}$, such that $\boldsymbol{\mu} \propto \boldsymbol{K}\mu_0$ and $\mu_0 = e\hbar/2mc$ is the Bohr magneton. The generic angular momentum $\boldsymbol{K}$ can take the specific form, $\boldsymbol{L}$, $\boldsymbol{S}$ or $\boldsymbol{J}$, where $\boldsymbol{L}$ corresponds to the orbital, $\boldsymbol{S}$ the spin and $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, the total angular momentum. The ratio $\mu/K = g_K\mu_0$ is observed to be in multiples of the Bohr magneton, where g_K is the gyromagnetic factor. Alternatively, the ratio μ/K is equal to g_K in the units of μ_0.

The g-factor is physically significant because it describes the extent to which the magnetic dipole moment contributes to the energy of a particle in a magnetic field. For example, the anomaly in the spin g-factor implies that an energy of the order of $(\alpha/2\pi)\mu_0 B$ is contributed by the vacuum. The anomaly in the spin g-factor manifests significantly in physical situations such as the relativistic laser-assisted Mott scattering [1]. For the spin angular momentum $\boldsymbol{S}$, the g-factor was earlier given as $g_S = 2$, whereas for the orbital angular momentum $\boldsymbol{L}$, $g_L = 1$. However, subsequent observations have shown that g_S is corrected by the radiative interactions, namely the self-energy, vertex correction and vacuum polarization, such that $g_s = 2(1 + \delta)$ where $\delta = 0.001159783287$, measured to a precision of 1 part in 10^{12} [2] and calculated to comparable accuracy and precision [3]. This is perhaps the best result in all of physics, providing stringent tests of QED, which hopefully will lead to the discovery of new physics. Although the radiative interactions do not act selectively, influencing only the spin degree of freedom, their effects on the orbital degree of freedom have been ignored and the electron orbital g-factor is considered exactly equal to 1, that is, $g_L = 1$, prompting the question: Why are the radiative interactions that correct the spin g-factor g_S ineffective in modifying the orbital g-factor g_L? We shall show in what follows that the radiative interactions also modify the orbital g-factor g_L for an electron in a magnetic field, confined in a circular orbit.

2. Experimental measurement of the spin and orbital g-factors and their anomalies

The measurement of the gyromagnetic factors (or magnetic moments) of an electron has a very long history. Einstein & de Haas, and Barnett, by various magneto-mechanical experiments attempted to measure the electron g-factor, on the assumption that material magnetism was due entirely to the orbital motion of charged particles, characterized by the orbital angular momentum **L**. The results obtained varied between $g = 1.02 \pm 0.10$ and $g = 2.02 \pm 0.02$, and with improved accuracy and precision, to a value $g = 1.919 \pm 0.002$ [4]. The realization that the electron has, in addition to the orbital angular momentum **L**, an intrinsic spin angular momentum **S** to which an additional magnetic moment should be ascribed, made possible a better interpretation of the results of these early experiments of Einstein, de-Haas, Barnett, Scott [5] and others.

An indication of the reality of the spin angular momentum came from the Stern-Gerlach experiment which showed that, in addition to the quantized orbital angular momentum **L**, the intrinsic angular momentum **S** is also quantized, thus providing a means of measuring the electron orbital and spin magnetic moments and their corresponding g-factors using atoms. However, due to the relativity of motion, the angular momenta **L** and **S** of an electron in an atom, for example, are coupled. Hence, the corresponding gyromagnetic factor can only be separately measured in special cases where either **L** or **S** vanishes, thereby making the spin-orbit coupling $\boldsymbol{S} \cdot \boldsymbol{L} = 0$. For example, one of the earliest measurements of g_S was by Phipps and Taylor [6], who used the Stern-Gerlach method on the hydrogen atom in the ground state, where l = 0, to show that $g_S = 2$ to within 10% precision. Atoms in the singlet state with S = 0 have also been used to measure the orbital g-factor g_L using the Stern-Gerlach method. Another means of measuring the electron g-factors is the use of the Zeeman effect in selected atomic states. Normal Zeeman effect observed for Cd and Zn show that $g_L = 1$. These above-mentioned results are consistent with those obtained from optical spectra [7]. Other techniques used in the measurement of the electron g-factors (such as the Landé g-factor g_J), from which other g-factors may be derived, include quantum beat spectroscopy [8], laser microwave/rf double resonance spectroscopy [9], and absorption saturation spectroscopy [10], etc. Before the introduction of newer techniques, the discrepancy in the hyperfine measurements on hydrogen indicated that the spin g-factor of the electron may be anomalous. This observation prompted Kusch *et al* [11] to perform more precise and accurate measurements of the electron spin g-factor, using the atomic beam magnetic resonance method.

Questions concerning the possible modification of both *g*-factor g_S and g_L were first clearly posed and addressed experimentally by Kusch and Foley who measured the ratio of the g-factors g_{J1}/g_{J2}, from which the quantity $\delta_S - 2\delta_L = a_{SL}$ may be extracted, where δ_S and δ_L are corrections to the spin and orbital *g*-factors respectively [12]. While attention was focused on δ_S, the matter could not be conclusively decided regarding δ_L, because additional data were required. Over the span of several decades, δ_S has been accurately and precisely measured, making it possible to determine δ_L from the equation $\delta_S - 2\delta_L = a_{SL}$ where a_{SL} is known. Starting with the experiments of Louisell *et al* [13], new methods of measuring the spin-g anomaly δ_S of the electron without using atoms have provided results with increasing accuracy and precision. Van Dyck *et al* used a Penning Trap in which was confined a single electron, to determine δ_S in a continuous Stern-Gerlach experiment [14]. Odom *et al* used a one-electron quantum cyclotron to measure the spin-g anomaly to one part per trillion [15]. Using these measured, highly precise values of δ_S and the available data on $\delta_S - 2\delta_L = a_{SL}$, non-vanishing values of δ_L have been determined from the ratios of g_J measured on atomic states in alkali atoms having a single valence electron [16] and on some noble gases with closed shells [17].

Thus, it has been observed from reliable data, that there is a clear deviation from 1 because, for the $^3P_2$ state of Ne, $\delta_L = (-4.22 \pm 0.01)$ x 10^{-4} to a very high precision, well above ten standard deviations [17]. Therefore, it is reasonable to assume that radiative interactions that correct the spin *g*-factor g_S may also correct the orbital *g*-factor g_L. Hence, deviations from $g_L = 1$, that is, $(g_L - 1)$ should be calculated from Quantum Field Theory, just as $(g_S - 2)/2$ was calculated from Quantum Electrodynamics (QED).

It is worth noting that a nonvanishing deviation $(g_L - 1)$ has already been calculated from the quantum theory of the relativistic electron considered as a particle [18]. Using an extension of the Dirac equation coupled to an external magnetic field *B*, an extra term dependent on the orbital angular momentum appears which implies a correction to the magnetic energy and hence the *g*-factor. Moreover, from a semi-classical point of view, it has been shown that the electron 'zitterbewegung' implies a correction to the orbital *g*-factor [19]. These approaches are valuable and useful for the conceptual development and design of new experiments for measuring $(g_L - 1)$. It will now be shown in what follows that a deviation from the exact value 1 of the orbital *g*-factor also follows from QED. First, we shall review the method of obtaining the spin-g anomaly and then follow similar steps to derive the orbital anomaly.

3. Second-order QED correction of the spin and orbital g-factor anomalies

In this section, we calculate the anomaly in the g-factor corresponding to the orbital angular momentum **L** using the formalism of QED, described by Greiner and Reinhardt [20]. There is a preference for this formalism over others because of its clarity, simplicity and elegance. For example, the derivation of the spin g-factor anomaly described below, proceeds rigorously. The formalism is based on perturbation theory and the use of propagators.

The system under consideration is that of an electron interacting with an electromagnetic field given as $A_\mu = (A_0, A_k)$, where A_0 is the electric scalar potential and A_k is the magnetic vector potential, with k = 1, 2, 3; in an atom, A_0 is the Coulomb potential. When $A_0 = 0$ and $A_k \neq 0$, the system described is that of an electron in a magnetic field, moving in a cyclotron orbit in free space. When $A_0 \neq 0$ and $A_k \neq 0$, the system described is that of an electron in a Coulomb field in the presence of an external magnetic field. In field theory, the interaction of an electron with an external electromagnetic field is described as a scattering process, where an electron with momentum **p** is scattered by the external field into a state with momentum $\boldsymbol{p'}$, with the absorption of a virtual photon of momentum **q**, such that $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$. In higher order perturbation, the vertex of the Feynman diagram is also corrected by the emission of a virtual photon [21].

The goal of this calculation is to account for any observed anomaly in the orbital g-factor, whether obtained using electrons confined in a cyclotron orbit or observed with electrons bound in an atom in the presence of a magnetic field. However, the currently available experimental data were obtained using atomic systems, hence comparison between theory and experiment have been made with the anomaly observed in atoms. The atoms considered were similar or identical to those used by Kusch et al. in their investigations. These consist of the states of atoms with a single valence electron outside of closed shells, which are known to be LS coupled, and are significantly free from perturbations and configuration mixing. Noble gas atoms with closed shells are also considered.

Below, we calculate the electron orbital g-factor anomaly by deducing the consequences of a term in the energy equation. However, we present first, a brief review of the calculation from QED, of the spin-g anomaly. The review has been highly compressed into six equations, that is, equations (3.1)–(3.6), described below in section 3.1. The starting point of the calculation of the spin-g anomaly is the second term in the expression for the energy W, described by Greiner and Reinhardt [20], and given below as equation (3.2). The anomaly in the orbital g-factor is shown to follow from the first term of the same equation.

3.1. The electron spin g-factor

When an electron interacts with an external electromagnetic field, radiative effects such as the vertex correction $\Gamma(p, p')$, self-energy S_F (p) and vacuum polarization iD_F, modify its physical behavior [20]. The self-energy can be omitted because, for a free particle, it contributes to charge and mass renormalization.

Putting the above into the equation describing the interaction of an electron with an external electromagnetic field A_μ^{ext}, the interaction energy is

$$W_{em} = \int d^3x\, j_\mu A^\mu_{ext} = e\int dx^3 \bar{\psi}_{p'}\left(\gamma_\mu + \Gamma^R_\mu(p', p) + \frac{i\pi^R_{\mu\nu}}{4\pi} iD^{\nu_0}_{F\gamma_0}\right)\psi_p A^\mu_{ext} \tag{3.1}$$

where j_μ is the current density, γ_μ are the gamma matrices derived from the Dirac matrices in the Pauli representation, p and p' are the initial and final momenta of the scattered electron, respectively, e is the electron charge, and $\mu, \nu = 0, 1, 2, 3$.

Inserting the respective terms Γ_μ and D_F as described by Greiner and Reinhardt [20], equation (3.1) gives, in the limit of small momentum transfer $q^2 \to 0$,

$$W_{em} \simeq e\int d^3x \bar{\psi}_{p'}\left\{\gamma_\mu\left[1 + \frac{\alpha}{3\pi}\frac{\boldsymbol{q}^2}{m^2}\left(\ln\frac{m}{\mu_p} - \frac{3}{8} - \frac{1}{5}\right)\right] + \frac{\alpha}{2\pi}\frac{i}{2m}\sigma_{\mu\nu}q^\nu\right\}\psi_p A^\mu_{ext} \tag{3.2}$$

where, in the above, **q** is the momentum transfer operator, m is the electron mass, μ_p is the 'photon mass', α is the fine structure constant, $\psi_{p'}$ *and* ψ_p are Dirac wavefunctions, and $\sigma_{\mu\nu}$ are the spin operators.

We may, in the alternative write,

$$W_{em} \simeq e\int d^3x \bar{\psi}_{p'}\left\{\frac{1}{2m}(p + p')_\mu\left[1 + \frac{\alpha}{3\pi}\frac{\boldsymbol{q}^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right] + \left(1 + \frac{\alpha}{2\pi}\right)\frac{1}{2m}\sigma_{\mu\nu}q^\upsilon\right\}\psi_p A^\mu_{ext} \tag{3.3}$$

where (3.3) is obtained from (3.2) by the Gordon decomposition of the Dirac current density $(\bar{\psi}_{p'}\gamma_\mu\psi_p)$ into convective and polarization currents. It is usual to interpret the second term in the above equation (3.3) as a correction to the electron spin g-factor $g_S = 2$, where the above equations have been written in natural units, $\hbar = c = 1$, and $\hbar$ is the reduced Planck's constant, c is the speed of light in vacuum. For a pure magnetic field, we derive from (3.3) above,

$$W_{spin} \simeq \frac{-e}{4m}\left(1 + \frac{\alpha}{2\pi}\right)2\int d^3x \bar{\psi}(x)\sum\psi(x)\boldsymbol{B} = -\langle\boldsymbol{\mu}\rangle\cdot\boldsymbol{B} \tag{3.4}$$

with $\langle\boldsymbol{s}\rangle = \frac{1}{2}\langle\boldsymbol{\Sigma}\rangle$, where $\langle\boldsymbol{s}\rangle$ is the expectation value of the spin, and $\langle\boldsymbol{\Sigma}\rangle$ is the expectation value of the four-dimensional operator $\boldsymbol{\Sigma}$, which is defined as the analog of the Pauli spin operator. It follows that the magnetic moment is given by

$$\langle \boldsymbol{\mu} \rangle = \frac{e\hbar}{2\,mc}\left(1 + \frac{\alpha}{2\pi}\right)2\langle \boldsymbol{s} \rangle = 2\left(1 + \frac{\alpha}{2\pi}\right)\mu_B\langle \boldsymbol{s} \rangle = g_s\mu_B\langle \boldsymbol{s} \rangle \tag{3.5}$$

Thus, the correct spin g-factor, to second-order perturbation, is

$$g_s = 2\left(1 + \frac{\alpha}{2\pi}\right) \simeq 2(1 + 0.00116141) \tag{3.6}$$

which was in good agreement with earlier experimental measurements. Subsequent work has given more precise measurements which are also in excellent agreement with calculations, perhaps representing the most precise results in the whole of physics. The current experimental [2] and theoretical results [3] compared are as follows:

Experiment: 1.001 159 652 180 73 (28) [0.28 ppt]; Hanneke *et al* [2].

Theory: 1.001 159 652 182 79 (7.71) [0.08 ppt]; Aoyama *et al* [3].

3.2. The electron orbital g-factor

The first term in above equation (3.2) will be shown here to correspond to a correction in the orbital g-factor, $g_L = 1$. Let the first term be written as

$$W_{orb} \simeq e\int d^3x\bar{\psi}\,\gamma_\mu\hat{G}\psi A^\mu \tag{3.7}$$

where the operator

$$\hat{G} = \left[\mathrm{II} + \frac{\alpha}{3\pi}\frac{\boldsymbol{q}^2}{m^2}\left(\ln\frac{m}{\mu_p} - \frac{3}{8} - \frac{1}{5}\right)\right] \tag{3.8a}$$

contains the unit operator $\mathbb{I}$ and the momentum transfer operator $\boldsymbol{q}$, a four-vector operator q_μ. Assuming that ψ_p is an eigenfunction of $\boldsymbol{q}$, then the operator $\hat{G}$ acts on ψ_p to give the eigenvalue,

$$G = \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu_p} - \frac{3}{8} - \frac{1}{5}\right)\right] \tag{3.8b}$$

where q^2 is the eigenvalue of the operator $\boldsymbol{q}^2$. This is possible because the operators $\boldsymbol{p}$ and $\boldsymbol{p}'$ commute, and thus can have common eigenfunctions.

Following the above example in the calculation of the spin-g anomaly, we assume a small (but finite) momentum transfer, $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, such that $\psi_p \approx \psi_{p'}$. Dropping the indices, we have from the above and from equation (3.7),

$$W_{orb} \simeq eG\int d^3x\bar{\psi}\gamma_\mu\psi A^\mu \tag{3.9}$$

Now, consider the interaction with an external electromagnetic field, $A^\mu = (A_o,\ A_k)$. The Dirac current density in (3.9) can be expanded as

$$\bar{\psi}\gamma_\mu\psi A^\mu = \bar{\psi}\gamma_0\psi A_0 + \bar{\psi}\gamma_k\psi A_k \tag{3.10}$$

The above may also be rewritten as follows, using the Dirac-Pauli representation of the gamma matrices, $\gamma_0 = \beta, \gamma_k = \beta\alpha_k,\ \ k = 1,\ 2,\ 3$:

$$\bar{\psi}\gamma_\mu\psi A_\mu = \bar{\psi}\beta\psi A_o + \bar{\psi}(\beta\alpha_k)\psi A_k \tag{3.11}$$

where $\boldsymbol{\alpha}$ and β are the Dirac matrices, and the Einstein summation convention is assumed. Substituting into (3.7), we obtain

$$W_{orb} \simeq eG\int d^3x\,\{\bar{\psi}\beta\psi A_o + \bar{\psi}(\beta\alpha_k)\psi A_k\} \tag{3.12}$$

The above may be expressed as a sum of expectation values,

$$W_{orb} \simeq eG\{\langle\beta A_o\rangle + \langle(\beta\alpha_k)A_k\rangle\} \tag{3.13}$$

We can obtain the expectation values if we recall the Dirac Hamiltonian H for an electron in a magnetic field,

$$H = c\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + \beta mc^2 \tag{3.14}$$

where $\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}/c$ is the mechanical momentum and $\boldsymbol{p}$ is the linear momentum. From the above, we obtain the following identities,

$$\boldsymbol{\alpha}H + H\boldsymbol{\alpha} = 2c\boldsymbol{\pi};\ \ \beta H + H\beta = 2mc^2 \tag{3.15}$$

Evaluating the expectation values using equation (3.15), we obtain the following: $\langle\beta\rangle = mc^2/E$, $\langle\alpha_k\rangle = c\pi_k/E$ and $\langle\beta\alpha_k\rangle = (mc^2/E)(c\pi_k/E)$, where E is the energy, and the properties of expectation values have been applied in the latter; $\boldsymbol{\alpha}$ and β are assumed to be independent. Thus,

$$W_{orb} \simeq eG\frac{mc^2}{E}\left\{\langle A_o\rangle + \frac{c\pi_k}{E}A_k\right\} \tag{3.16}$$

In the absence of an electric field or Coulomb potential, $A = 0$, the above reduces to

$$W_{orb} \simeq eG\frac{mc^2}{E}\left\{\frac{c}{E}\langle \boldsymbol{p}\cdot\boldsymbol{A}\rangle\right\} \tag{3.17}$$

for a weak magnetic field, where the term in A^2 has been neglected. For an electron moving in a plane circular orbit perpendicular to the field $\boldsymbol{B}$, where the field is oriented in the z-direction, that is, $\boldsymbol{B} = B\hat{\boldsymbol{z}}$, the vector potential $\boldsymbol{A} = \frac{1}{2}\boldsymbol{B}\text{ x }\boldsymbol{r}$. Substituting $\boldsymbol{A}$ into (3.17) yields

$$W_{orb} \simeq \frac{eG}{2c}\frac{mc^2}{E}\frac{c^2}{E}\langle \boldsymbol{p}\cdot\boldsymbol{B}\times\boldsymbol{r}\rangle \tag{3.18}$$

or,

$$W_{orb} \simeq \frac{e\hbar G}{2mc}\frac{mc^2}{E}\frac{mc^2}{E}\langle \boldsymbol{B}\cdot\hat{\boldsymbol{L}}\rangle \tag{3.19}$$

where we have put $\boldsymbol{L} = \boldsymbol{r}\text{ x }\mathbf{p}$. Substituting for G, and noting that $\mu_B = e\hbar/2mc$, equation (3.19) becomes,

$$W_{orb} \simeq \mu_B\left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right]\boldsymbol{B}\cdot\langle\boldsymbol{L}\rangle\left(\frac{mc^2}{E}\right)^2 \tag{3.20}$$

In the low energy limit, $(mc^2/E) = \sqrt{1 - v^2/c^2} \approx 1$, thus from (3.20),

$$g_L' = \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right] \tag{3.21}$$

This implies that the electron orbital g-factor is not 1. It is corrected by a term which depends on the fine structure constant α, the transfer momentum q, the mass of the electron m, and μ_p, the photon 'mass'.

An approximate value of the correction

$$\Delta_L \cong \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu_p} - \frac{3}{8} - \frac{1}{5}\right) \tag{3.22}$$

may be determined if we note that the rest energy of the electron dynamically fluctuates [22, 23], and in view of the fluctuations $m \approx \mu$; also $q^2 \approx E^2 - p^2 = m^2$. Hence, from (3.20) we have

$$\Delta_L = -4.45\times 10^{-4} \tag{3.23}$$

which has the correct sign and order of magnitude as indicated by the currently available experimental data on selected atomic states, as shown in table 1 below.

By writing equation (3.22) as

$$\Delta_L = \frac{\alpha}{3\pi}D;\;\; D = \frac{q^2}{m^2}\left(\ln\frac{m}{\mu_p} - \frac{3}{8} - \frac{1}{5}\right) \tag{3.24}$$

where D depends on the interaction of the electron with photons, we can compare the correction to the electron orbital and spin g-factors.

Thus, to first order

$$g_S = 2 + \frac{\alpha}{\pi}\text{ while } g_L = 1 + \frac{\alpha}{3\pi}D \tag{3.25}$$

The spin g-factor g_S is corrected by a term of order $(\alpha/2\pi) \sim 10^{-3}$, while the orbital g-factor g_L is corrected by a term of order $(\alpha/3\pi)D \sim 8\times 10^{-4}D$, where the numerical part of the term has the correct order of magnitude when compared with experimental data. The term D describes the part of the orbital anomaly that is dependent on the transfer of momentum between the electron and the photon.

An extension of the above analysis to include the atomic potentials A_o, is described below in section 4, in order to understand the differences in the orbital anomaly from atom to atom, and its variation with the principal quantum number n and the orbital angular momentum l.

Table 1. The electron's orbital anomaly calculated for the states $^3P_2$ and $^3D_3$ of Ne and Ar with reference to the $^3S_1$ state of He.

Term symbol of atomic states	Δ_L
Ne^3P_2/He^3S_1	$(-4.22 \pm 0.1) \times 10^{-4}$
Ar^3P_2/He^3S_1	$(-2.77 \pm 0.1) \times 10^{-4}$
Ne^3D_3/He^3S_1	$(-12.9 \pm 3.8) \times 10^{-4}$
Ar^3D_3/He^3S_1	$(-7.62 \pm 1.2) \times 10^{-4}$

3.3. The electron orbital anomaly

Another method of deriving the electron orbital anomaly is to consider that (3.3) can be written in a different but equivalent form. Using Fourier transform, the equation is changed to

$$W_{em} \simeq e\int d^3x\bar{\psi}_{p'} \times \left\{\frac{1}{2m}(p+p')_\mu\left[1-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)\Box\right]+\left(1+\frac{\alpha}{2\pi}\right)\frac{1}{2m}\sigma_{\mu\nu}\partial^\upsilon\right\}\psi_p A^\mu_{ext} \tag{3.26}$$

where the momentum operators have been transformed as follows, $p_\mu \to -i\partial_\mu$, $p'_\mu \to -i\partial'_\mu$, $q^\upsilon \to \partial^\nu$, $q_\mu \to i\partial_\mu$ and $q^2 = q_\mu q^\mu \to -\partial_\mu\partial^\mu = -\Box$ where $\Box \equiv \nabla^2 - \frac{\partial}{\partial t^2}$ is the d'Alembertian and ∇^2 is the Laplacian. Again, focusing only on the first term in the above equation, we obtain

$$W_{em} \simeq e\int d^3x\bar{\psi}_{p'} \times \left\{\frac{1}{2m}(p+p')_\mu\left[\psi_p A^\mu_{ext}-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)\Box\psi_p A^\mu_{ext}\right]\right\} \tag{3.27a}$$

For convenience, we write the above as,

$$W_{em} \simeq \frac{e}{2m}\int d^3x\bar{\psi}_{p'} \times \left\{(p+p')_\mu\left[\psi_p A^\mu_{ext}-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)\Box\psi_p A^\mu_{ext}\right]\right\} \tag{3.27b}$$

where the d'Alembertian operator $\Box$ acts on the product $\psi_p A^\mu_{ext}$, such that

$$\Box(\psi_p A^\mu_{ext}) = A^\mu_{ext}\Box\psi_p + \psi_p\Box A^\mu_{ext} \tag{3.28}$$

Thus, substituting equation (3.28) into equation (3.27b), we have

$$W_{em} \simeq \frac{e}{2m}\int d^3x\bar{\psi}_{p'} \times \left\{(p+p')_\mu\left[\psi_p A^\mu_{ext}-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)[A^\mu_{ext}\Box\psi_p+\psi_p\Box A^\mu_{ext}]\right]\right\} \tag{3.29}$$

Recalling that for small momentum transfer, $p' \approx p$ and therefore $\psi_{p'} \approx \psi_p$, the above can be written as

$$W_{em} \simeq \frac{e}{2m}\int d^3x\bar{\psi}_p \times \left\{(2p)_\mu\left[\psi_p A^\mu_{ext}-\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)[A^\mu_{ext}\Box\psi_p+\psi_p\Box A^\mu_{ext}]\right]\right\} \tag{3.30}$$

which reduces to

$$W_{em} \simeq \frac{e}{m}\int d^3x\bar{\psi}_p\left\{p_\mu\left[\psi_p A^\mu_{ext}+\frac{\alpha}{3\pi}\frac{1}{m^2}\left(\ln\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)[A^\mu_{ext}q^2\psi_p+\psi_p\Box A^\mu_{ext}]\right]\right\} \tag{3.31}$$

where $-q^2$ is not an operator, but the eigenvalue of the d'Alembertian operator $\Box$ acting on the eigenfunction ψ_p.

Expanding the above equation (3.31) and re-arranging, we have

$$W_{em} \simeq \frac{e}{m}\int d^3x\bar{\psi}_{p'} \times \left\{\left[p_\mu \psi_p A^\mu_{ext} + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)(p_\mu A^\mu_{ext}\psi_p + p_\mu(\Box A^\mu_{ext})\psi_p\right]\right\} \tag{3.32}$$

Using the fact that the operators p_μ and $\Box$ commute, i.e., $[p_\mu, \Box] = 0$, the last term in (3.32) above vanishes because for a uniform, time-independent field,

$$p_\mu \Box A^\mu_{ext} = \Box p_\mu A^\mu_{ext} = \Box(\boldsymbol{p}\cdot\boldsymbol{A}_{ext}) \tag{3.33}$$

and the quantity $e(\boldsymbol{p}\cdot\boldsymbol{A})/m = \mathcal{E}$ has the unit of energy; in uniform magnetic field, the derivative of the energy vanishes. Alternatively, we can use Ampere's law $\Box A^\mu_{ext} = J^\mu_{ext}$, where J^μ_{ext} is the external current that produced the magnetic field, to write, from the above, $p_\mu \Box A^\mu_{ext} = -i\hbar\nabla\cdot\mathbf{J}_{ext} = 0$, for a uniform, time-independent current density; we have used the operator $\boldsymbol{p} = -i\hbar\nabla$.

Therefore,

$$W_{em} \simeq \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right]\frac{e}{m}\int d^3x\bar{\psi}_p\{p_\mu A^\mu_{ext}\}\psi_p \tag{3.34}$$

Noting again that $p_\mu A^\mu = \mathbf{p}\cdot\mathbf{A}$, we have

$$W_{em} \simeq \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right]\frac{e}{m}\int d^3x\bar{\psi}_p(\mathbf{p}\cdot\mathbf{A})\psi_p \tag{3.35}$$

For a constant magnetic field in the z-direction, we may, for simplicity, use the vector potential described as

$$\boldsymbol{A} = \frac{1}{2}\boldsymbol{B}\times\boldsymbol{r} \tag{3.36}$$

Thus, equation (3.35), gives

$$W_{em} \simeq \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right]\frac{e}{2m}\int d^3x\bar{\psi}_p(\mathbf{B}\cdot\mathbf{L})\psi_p \tag{3.37}$$

The above can be re-written as,

$$W_{em} \simeq \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right)\right]\frac{e}{2m}\mathbf{B}\cdot\mathbf{L} \tag{3.38}$$

where $\mathbf{L}$ is the expectation value of the orbital angular momentum operator $\mathbf{L}$. By inserting the constants $\hbar$ and c in the above, and writing the Bohr magneton $\mu_B = e\hbar/2mc$, we find that the above equation implies that

$$g_L = 1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right) \tag{3.39}$$

showing that the electron orbital g-factor g_L has an anomalous part,

$$\Delta_L = \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8} - \frac{1}{5}\right) \tag{3.40}$$

The derivation of the orbital anomaly as described above in sections 3.2 and 3.3, meets expectations, because the equations (3.2) and (3.26) from which the correction is derived, are equivalent. The Fourier transformation of equation (3.2) into equation (3.26) merely changed the momentum operators to gradients in configuration space.

The above remarks are further confirmed by again transforming the momenta to gradients, but instead of putting $q^2 = -\partial_\mu\partial^\mu = -\Box$, where $\Box$ is the d'Alembertian, we put the transfer momentum $q^2 = q\cdot q = q_\mu q^\mu \to -\tilde{\Box}$, where $q = p' - p$ and the operator $\tilde{\Box}$ is analogous to the d'Alembertian $\Box$. Replacing $\Box$ by $\tilde{\Box}$ in equation (3.26), and again noting that,

$$\tilde{\Box}\psi_p = -q^2\psi_p$$

where $-q^2$ is the eigenvalue of $\tilde{\Box}$ (because the momenta $p'_\mu = -i\hbar\frac{\partial}{\partial x'_\mu}$ and $p_\mu = -i\hbar\frac{\partial}{\partial x_\mu}$ commute and ψ_p is assumed to be their common eigenfunction), we note that the above procedure gives the same result equation (3.40), that describes the orbital anomaly.

3.4. Radiative correction to the orbital g-factor

There is yet another method, which offers some interpretational advantage, of deriving the orbital anomaly based on the formalism of QED described by Akhiezer and Berestetskii [24], who showed that the effective potential energy due to radiative interactions is,

$$e\delta\varphi(x) = \frac{-e^2}{(4\pi)^2 m}(\beta \cdot \Sigma \boldsymbol{H} - i\beta\boldsymbol{\alpha}\boldsymbol{E}) + \frac{e^3}{(4\pi)^2}\frac{4}{3m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)(\Box\varphi - \boldsymbol{\alpha} \cdot \Box\boldsymbol{A}) \tag{3.41}$$

where $\boldsymbol{\alpha}$ and β are the Dirac matrices, $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields respectively, and φ and $\boldsymbol{A}$ are the scalar and vector potentials, respectively, and $\Box$ is the d'Alembertian operator. The quantities m and λ are the electron mass and the photon 'mass' respectively. The first term in (3.41) is related to the spin anomaly, while the second term is related to the orbital anomaly, as will be shown here.

Expectation values are the measurable and empirically significant quantities in quantum theory. In the absence of potential φ, the term $\Box\varphi = 0$, and the expectation value of the second term in the effective potential energy becomes

$$\langle e\delta\varphi\rangle(x) = \int d^3x\bar{\psi}_p\left[\frac{-e^3}{(4\pi)^2}\frac{4}{3m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)(\boldsymbol{\alpha} \cdot \Box\mathbf{A})\right]\psi_p \tag{3.42}$$

The d'Alembertian operator acts on the product $\mathbf{A}\psi_p$, such that

$$\Box(\mathbf{A}\psi_p) = \mathbf{A}\Box\psi_p + \psi_p\Box\mathbf{A} \tag{3.43}$$

Thus,

$$\boldsymbol{\alpha} \cdot \Box\mathbf{A}\psi_p = (\boldsymbol{\alpha} \cdot \mathbf{A})\Box\psi_p + \psi_p(\boldsymbol{\alpha} \cdot \Box\mathbf{A}) \tag{3.44}$$

In general, for a time-independent potential $\mathbf{A}$, the second term vanishes, i.e, $\boldsymbol{\alpha} \cdot \Box\mathbf{A} = 0$, because $\boldsymbol{\alpha} \cdot \Box\mathbf{A} = \boldsymbol{\alpha} \cdot \nabla^2\mathbf{A} = \nabla^2(\boldsymbol{\alpha} \cdot \boldsymbol{A}) = -(\boldsymbol{p} \cdot \mathbf{p})(\boldsymbol{\alpha} \cdot \boldsymbol{A}) = (\mathbf{p} \cdot \boldsymbol{A})(\boldsymbol{p} \cdot \boldsymbol{\alpha}) = 0$, where we have used the momentum operator $\mathbf{p} = -i\nabla$, such that $\mathbf{p} \cdot \mathbf{p} = -\nabla \cdot \nabla = -\nabla^2$ and $\nabla \cdot \boldsymbol{\alpha} = 0$.

Hence, equation (3.42) reduces to

$$\langle e\delta\varphi(x)\rangle = \int d^3x\bar{\psi}_p\left[\frac{-e^3}{(4\pi)^2}\frac{4}{3m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)\right](\boldsymbol{\alpha} \cdot \mathbf{A})\Box\psi_p \tag{3.45}$$

The d'Alembertian acts on the eigenfunction ψ_p, such that $\Box\psi_p = -q^2\psi_p$. Therefore,

$$\langle e\delta\varphi(x)\rangle = \int d^3x\bar{\psi}_p\left[\frac{e^3}{(4\pi)^2}\frac{4q^2}{3m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)\right](\boldsymbol{\alpha} \cdot \mathbf{A})\psi_p \tag{3.46}$$

The above can be re-written as

$$\langle e\delta\varphi(x)\rangle = \left[\frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)\right]\int d^3x\bar{\psi}_p(e\boldsymbol{\alpha} \cdot \mathbf{A})\psi_p \tag{3.47}$$

It may be observed that for a magnetic vector potential $\mathbf{A}$, given by (3.36), describing a constant magnetic field, we have

$$\int d^3x\bar{\psi}_p(e\boldsymbol{\alpha} \cdot \mathbf{A})\psi_p = \frac{e\mathbf{B} \cdot \langle\mathbf{L}\rangle}{2mc} \tag{3.48}$$

where, based on the Dirac electron theory, we have put $c\boldsymbol{\alpha}$ as the velocity $\mathbf{v}$, $\mathbf{p} = m\mathbf{v}$ as momentum, and written $\mathbf{p} \cdot \mathbf{B} \times \boldsymbol{r} = \mathbf{B} \cdot \mathbf{L}$. Thus,

$$\langle e\delta\varphi(x)\rangle = \left[\frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)\right]\frac{e\mathbf{B} \cdot \langle\mathbf{L}\rangle}{2mc}\ ;\ \ \mu_B = \frac{e\hbar}{2mc} \tag{3.49}$$

which implies that the correction to the orbital g-factor g_L, is

$$\Delta_L = \left[\frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(ln\frac{m}{\lambda} - \frac{3}{8} - \frac{1}{5}\right)\right] \tag{3.50}$$

Thus, we arrive at the same conclusion that the orbital g-factor is anomalous.

The physical interpretation of the second term in equation (3.41) may provide some insight into the physical origin of the electron orbital anomaly. Akhiezer and Berestetskii [24] concluded that the term is of the order of magnitude of the fluctuations of the fields, which causes electron displacement. Such vacuum fluctuations, together with the zitterbewegung [19] and the fluctuations of the rest energy [22, 23], are probable origins of the energy that manifests as the anomalous part of the electron orbital magnetic moment. It should be noted that the zero-point fluctuations do not vanish [25], even for constant electromagnetic fields. Thus, the electron spin and orbital anomalies appear to arise from quantum fluctuations inherent in the fields.

4. Dependence of the orbital anomaly on the atomic potential and the orbital angular momentum

The influence of atomic potentials on the energy W_{orb} and the orbital anomaly Δ_L may be considered by evaluating the expectation values of the Dirac matrices, $\boldsymbol{\alpha}$ and β, using the Dirac Hamiltonian in the presence of both magnetic and Coulomb potentials. That is,

$$H = c\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + \beta(mc^2 + eA_o). \tag{4.1}$$

Analogous to what was done in section (3.2), we form the expressions,

$$\boldsymbol{\alpha}H + H\boldsymbol{\alpha} = 2c\boldsymbol{\pi} \text{ and } \beta H + H\beta = 2(mc^2 + eA_o) \tag{4.2}$$

From which follows the expectation values,

$$\langle\boldsymbol{\alpha}\rangle = \frac{c\langle\pi\rangle}{E} \text{ and } \langle\beta\rangle = \frac{\langle mc^2\rangle}{E} + \frac{\langle eA_o\rangle}{E} \tag{4.3}$$

Recall that the second term in (3.13) is a magnetic term, hence it is sufficient to consider it separately. If $W_{orb} = E_1 + E_2$, then,

$$E_2 = eG\langle\beta\alpha_k A_k\rangle = ecG\left[\frac{mc^2}{E^2} + \frac{\langle A_o\rangle}{E^2}\right]\langle\pi_k A_k\rangle \tag{4.4}$$

where G is the eigenvalue equation (3.8b). Substituting $\boldsymbol{\pi} = \boldsymbol{p} - (e/c)\boldsymbol{A}$ into the above, and assuming normal fields, thereby neglecting A^2, we have,

$$E_2 = ecG\left[\frac{mc^2}{E^2} + \frac{\langle A_o\rangle}{E^2}\right]\langle\boldsymbol{A}\cdot\boldsymbol{p}\rangle \tag{4.5}$$

which, for $\boldsymbol{A} = \frac{1}{2}\boldsymbol{B}\times\boldsymbol{r}$, reduces to,

$$E_2 = (mc^2)G\left[\frac{mc^2}{E^2} + \frac{\langle A_o\rangle}{E^2}\right]\frac{e}{2mc}\langle\boldsymbol{L}\rangle\cdot\boldsymbol{B} \tag{4.6}$$

Hence,the orbital g-factor is,

$$g_L = \frac{(mc^2)^2}{E^2}\left[1 + \frac{\langle A_o\rangle}{mc^2}\right]G \tag{4.7}$$

In the above equation (4.7), we will define $\langle A_o\rangle/mc^2 \equiv \overline{A_o^r}$. Thus, by substituting for the eigenvalue G from equation (3.8b), we observe that equation (4.7) can be written as,

$$g_L = (m^2c^4/E^2)[1 + \overline{A_o^r}][1 + \Delta_L] \tag{4.8}$$

where, Δ_L is expressed by equation (3.22). We note that in the low-energy limit $(m^2c^4)/E^2 \approx 1$, based on the Sommerfeld-Dirac equation, and also $\overline{A_o^r} \to \overline{A_o^{nr}}$. Thus, the orbital g-factor in the non- relativistic limit is

$$g_L \approx 1 + \Delta_L + \overline{A_o^{nr}} + \overline{A_o^{nr}}\Delta_L \text{ (4.9)} \tag{4.9}$$

It is observed that the correction in the presence of a potential is,

$$\Delta_L' = \Delta_L + \overline{A_o^{nr}} + \overline{A_o^{nr}}\Delta_L \tag{4.10}$$

where $\overline{A_o^{nr}}$ in (4.10) is proportional to the expectation value of the atomic potential A_o, in the low-energy limit.

According to Suslov [26], the expectation value $1/r$ for a relativistic electron in a Coulomb field, where r is the radial operator, is

$$\langle r^{-1}\rangle = \frac{\beta'}{\mu'\upsilon'}(1 - \varepsilon^2)(\varepsilon\upsilon' + \mu'\sqrt{(1-\varepsilon^2)}) \tag{4.11a}$$

where,

$$\varepsilon = E/mc^2 \; \beta' = mc/\hbar \; a_o = \hbar^2/me^2 \tag{4.11b}$$

$$\kappa = \pm\left(j + \frac{1}{2}\right), \;\; \upsilon' = \sqrt{\kappa^2 - \mu'^2} \; \mu' = \alpha Z = Ze^2/hc \tag{4.11c}$$

also, $n = n_r$, the radial quantum number and,

$$E_n = \frac{mc^2}{\sqrt{1 + (\alpha Z)^2/(n + \upsilon')^2}} = \frac{mc^2}{Q} \tag{12}$$

is the Sommerfeld-Dirac equation, where $Q = \sqrt{1 + (\alpha Z)^2/(n + \upsilon')^2}$.

Substituting (4.11b), (4.11c) into (4.11a), we obtain

$$\frac{\overline{A_0^r}}{Ze^2} \equiv \left\langle \frac{r^{-1}}{mc^2} \right\rangle = \frac{m^2c^4 - E_{nl}^2}{(m^2c^4)(mc^2)} \left(\frac{E_{nl}}{Ze^2} + \sqrt{\frac{m^2c^4 - E_{nl}^2}{\hbar^2c^2\kappa^2 - Z^2e^4}} \right) \tag{4.13}$$

The above expression shows that the quantity $\overline{A_o^r}$ is dependent on n and l, owing to the energy E_n and $\kappa = j \pm \frac{1}{2} = l - 1$. In the non-relativistic limit, $c \to \infty$ and $\overline{A_o^r} \to \overline{A_o^{nr}}$, where

$$\overline{A_o^{nr}} = \frac{1}{Q};\ \ Q = \sqrt{1 + (\alpha Z)^2/(n + \upsilon')^2} \text{ where } \upsilon' = \sqrt{\kappa^2 - \mu^2} \tag{4.14}$$

Equation (4.14) also has a dependence on n and l. Thus, equations (4.10) and (4.14) together show that the anomaly in the electron orbital g-factor is dependent on the principal quantum number n, atomic number Z, and orbital angular momentum quantum number l. This explains in a reasonable way, the differences in the anomalies determined for atomic states which differ in orbital angular momentum l, and for the anomalies determined for different atoms which are in the same states.

5. Discussion and experimental considerations

The results described above provide yet another method for testing QED, as a quantum theory of the electron and photons. QED has been very successful in accounting for the anomaly in the spin g-factor g_S, both for the nearly free electron in a weak magnetic field and for the bound electron in the strong Coulomb fields of highly charged ions. Nevertheless, there has been a persistent need to test QED as a component part of the Standard Model in the quest to discover new physics. As shown above, QED is also in agreement with the empirical observation that there is a non-vanishing anomaly in the electron orbital g-factor. The calculated anomaly is of the correct order of magnitude and sign. However, further theoretical and experimental work are needed regarding the orbital anomaly (just as it was in the early days of QED concerning the spin-g anomaly), in order to bring closer together, the results of experiments and theory. High precision measurement of the orbital anomaly and its calculation from field theory provides yet another way to stringently test QED. A significant deviation between the precisely measured and calculated values of the orbital anomaly may indicate the occurrence of new physics. Well-developed experimental and theoretical techniques for bound state QED may be required for precise studies of the orbital anomalies in atoms [27, 28]. The discrepancy found with the spin-g anomaly of the muon is an indication that close testing of QED holds promise for the discovery of new physics [29]. Investigations of orbital anomalies will complement, very well, this search. The existence of an anomaly in the orbital g-factor also has implications for the structure of the electron [17] and the observation of zitterbewegung, the existence of which empirical evidence already suggests [19].

6. Conclusions

As a consequence of QED, it has been shown that the electron orbital *g*-factor (similar to the spin *g*-factor) has an anomalous part. The orbital *g*-factor g_L deviates significantly from 1, which is consistent with experimental observations. The calculation follows the standard QED procedures with some physical interpretations adapted from the quantum-relativistic dynamical fluctuation of the rest energy. The magnitude of the calculated Δ_L is in reasonable agreement with observation having the same sign and order of magnitude; it is satisfactory to note that it has been adequately shown that radiative interactions correct both the spin *g* and orbital *g*-factors as should, physically, be expected. It is hoped that further work will bring closer, agreement between refined calculations and high-precision experimental measurements.

The second-order radiative corrections described above have been simplified using some reasonable approximations. Unlike the spin g-factor anomaly, the electron orbital *g*-factor anomaly is not an intrinsic property but depends on the state of the electron in an atom. This was taken into considerations by including in the external electromagnetic field $A_\mu = (A_0,\ \boldsymbol{A})$, the screened Coulomb potential A_0. Notably, using standard QED, the anomaly in the orbital *g*-factor g_L has been shown to also have a radiative origin.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

M Awobode https://orcid.org/0000-0001-8660-569X

References

[1] Ngoko Djioka J M, Tetchou Ngana H M and Kwato Njock M G 2007 *Phys. Scr.* **75** 726
[2] Hanneke D, Fogwell S and Gabrielse G 2008 *Phys. Rev. Lett.* **100** 120801
[3] Aoyama T, Hayakawa M, Kinoshita T and Nio M 2012 *Phys. Rev. Lett.* **109** 111807
[4] Galison P and Pellegrino J 1987 *How Experiments End* (University of Chicago Press) 66
[5] Scott G G 1962 *Rev. Mod. Phys.* **34** 102
[6] Phipps T E and Taylor J B 1927 *Phys. Rev.* **29** 309
[7] Moore C E 1971 Atomic energy levels as determined by optical spectra US department of commerce *NBS, NSRDS-NBS* **35** 1 xiv
[8] Brandenberger J R 1979 *Optical Communication* **30** 181
[9] Scholl T Y, Rosner S D and Holt R A 1987 *Phys. Rev.* A **35** 1611
[10] Brandenberger J R, Dworshack W J and Kortenhoff T H J 2018 *Opt. Soc. Am.* B **35** 939
[11] Kusch P and Foley D 1947 *Phys. Rev.* **72** 1256
[12] Foley D and Kusch P 1948 *Phys. Rev.* **73** 250
[13] Louisell W H, Pidd R W and Crane H R 1954 *Phys. Rev.* **94** 7
[14] Van Dyck R S Jr, Dehmelt H and Schwinberg R 1987 *Phys. Rev. Lett.* **59** 26
[15] Gabrielse G, Hanneke D, Kinoshita T, Nio M and Odom B 2007 *Phys. Rev. Lett.* **99** 039902
[16] Awobode A M 2020 *Phys. Scr.* **95** 075401
[17] Awobode A M 2021 *Phys. Scr.* **96** 125411
[18] Awobode A M 2006 **121B** 563
[19] Awobode A M 2021 *Can. J. Phys.* **99** 8
[20] Greiner W and Reinhardt J 2009 *J Quantum Electrodynamics* 4th edn (Springer) 306–7
[21] Mandl F and Shaw G 2010 *Quantum Field Theory* 2nd edn (Wiley) 183–6
[22] Awobode A M 1994 *J. Math. Phys.* **35** 568
[23] Awobode A M 1990 *Found. Phys. Lett.* **2** 167
[24] Akhiezer A I and Berestetskii V B (ed) 1965 *Quantum Electrodynamics* 2/ed696–8
[25] Eisberg R and Resnick R 1985 Quantum physics of atoms *Molecules, Solids, Nuclei, and Particles* 2nd edn (Wiley) 217–9
[26] Suslov S K J 2009 Physics B: atomic *Molecular and Optical Physics* **42** 185003
[27] Sturm S, Werth G and Blaum K 2013 *Ann. Phys.* **525** 620
[28] Indelicato P and Mohr O 2016 Introduction to Bound-State Quantum Electrodynamics ed W Liu *Handbook of Relativistic Quantum Chemistry* (Springer)
[29] Abi A *et al* (Muon g—2 Collaboration) 2021 *Phys. Rev. Lett.* **126** 141801