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THE RUDIMENTS OF LIGHT-CONE PHYSICS*

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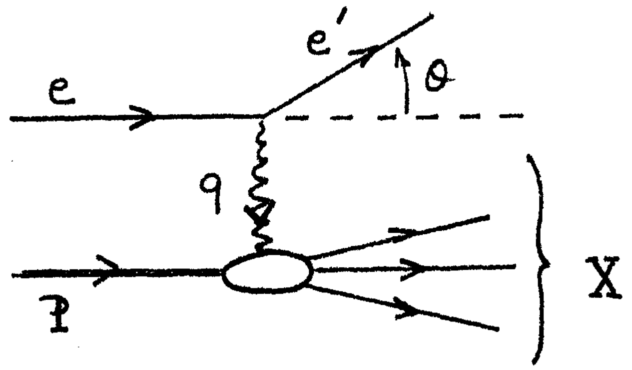
These notes are intended as an elementary exposition of the concepts and mathematical techniques of light-cone physics. They are directed toward experimentalists or theorists not working directly in the field and therefore contain little that is unknown to the initiate. I apologize at the outset for the obvious lack of rigor: Whenever rigor and simplicity were at odds, I have chosen simplicity.

Fortunately it is not difficult to present the fundamentals of light-cone physics, at least as abstracted from deep inelastic scattering, in simple terms. Mathematically, little more than a knowledge of the Fourier transform is required. Conceptually, the coordinate space structure of the parton model will provide a guide to the light-cone. It is impossible in these brief notes to describe many of the interesting applications of light-cone physics—for these, I refer to the literature and to the lectures of Professors Brandt and Preparata at this School.

The outline is as follows: first, some kinematic preliminaries so we will all agree on what we are talking about; second, a geometrical argument that effects near the light-cone dominate the cross section for inelastic lepton scattering in the Bjorken limit; third, mathematical techniques including discussions of light-cone expansions, more formal arguments for light-cone dominance and "measurement" of the light-cone singularities of operator products in deep inelastic scattering experiments; and lastly, some intuition for the origin of light-cone singularities in the framework of the parton model.

I. PRELIMINARIES

Consider the scattering of some lepton (for concreteness an electron, although the discussion applies as well to muons, neutrinos, or antineutrinos) off of a nucleon (practically, a proton or deuteron) with large transfer of both energy and momentum. Assuming one photon exchange, we have, diagrammatically,



where the final hadron state $|X\rangle$ is not observed. The mass and laboratory energy of the virtual photon are fixed by the laboratory energy and scattering angle of the lepton:

$$q^2 \equiv -Q^2 = -4ee' \sin^2\theta/2$$

$$P\cdot q \equiv \nu = (e - e')M$$

The deep inelastic, or Bjorken,[1] limit is that in which ν and Q^2 approach infinity with the ratio $x \equiv \frac{Q^2}{2\nu} \equiv 1/\omega$ fixed (caution—what I call x some call ω, with much ensuing confusion).

The cross section may be written in terms of the matrix element for the virtual photon to excite the nucleon to the state $|X\rangle$:

$$\frac{d\sigma}{dQ^2 d\nu} \propto \langle j^\mu j^\nu \rangle \frac{1}{Q^4} \left\{ \sum_X \langle P|J_\mu(0)|X\rangle \langle X|J_\nu(0)|P\rangle \otimes (2\pi)^4 \delta^4\left(P+q-P_X\right) \right\} \qquad \text{(I. 1)}$$

where $J_\mu(0)$ is the hadronic electromagnetic current operator. The lepton current, j^μ, is presumed known from QED or from some phenomenological weak interaction theory, so our ignorance is isolated in the quantity in curly brackets, conventionally defined as $4\pi W_{\mu\nu}$. In (I. 1) $\langle\ \rangle$ denotes an average over the lepton spins, and

$W_{\mu\nu}$ is understood to be averaged over the proton spin. Our states are covariantly normalized[2]:

$$\langle P|P'\rangle = (2\pi)^3\, 2E\, \delta^3(\vec{P}-\vec{P}')$$

which, we note for later use, implies that $|P\rangle$ carries with it the dimension $(\text{mass})^{-1}$.[3] The appearance of a local operator, $J_\mu(0)$, in (I.1) is an essential simplification which does not occur, for example, in purely hadronic reactions. In inelastic lepton scattering, we at least study hadronic structure with a well-defined probe.

The four-momentum-conserving δ-function prevents us from carrying out the sum over states in Eq. (I.1). Translational invariance removes the problem since

$$\langle P|J_\mu(0)|X\rangle (2\pi)^4\, \delta^4(P+q-P_X) \equiv \int e^{iq\cdot y} d^4y\, \langle P|J_\mu(y)|X\rangle \quad .$$

Using this, the label X occurs only in the state vector, and completeness $\left(\sum_X |X\rangle\langle X| = 1\right)$ may be used to obtain:

$$W_{\mu\nu} = \frac{1}{4\pi}\int e^{iq\cdot y} d^4y\, \langle P|J_\mu(y)\, J_\nu(0)|P\rangle \tag{I.2}$$

It is slightly more convenient to deal with operator commutators than with operator products. Notice that

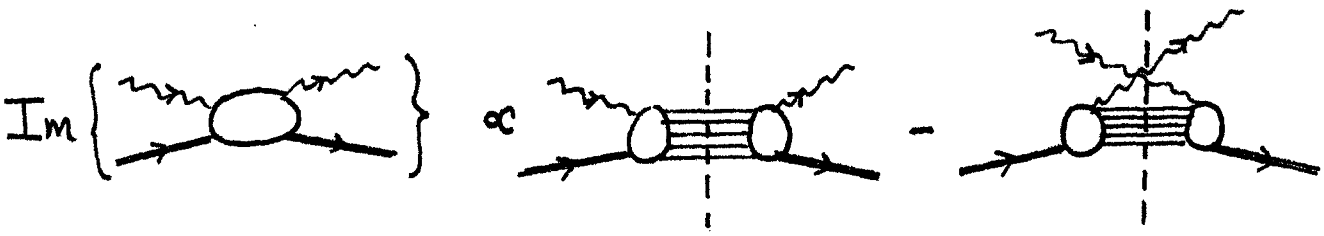
$$\int e^{iq\cdot y}\langle X|J_\mu(y)|P\rangle = (2\pi)^4\,\delta^4(P_X+q-P)\,\langle X|J_\mu(0)|P\rangle$$

$$= 0$$

because there is no state $|X\rangle$ for which the δ-function can be satisfied (i.e., a nucleon cannot decay by emission of a positive energy photon to another physical state). This allows us to subtract from (I.2) the same expression with the currents in the opposite order, since it is zero:

$$W_{\mu\nu} = \frac{1}{4\pi} \int e^{iq\cdot y} d^4y \langle P|\left[J_\mu(y), J_\nu(0)\right]|P\rangle \tag{I.3}$$

Notice that (I.3) is the imaginary part of the virtual forward Compton scattering amplitude (this is just the optical theorem).[4] Graphically:



where the dashed line indicates that the intermediate states are on-mass-shell, physical states. We shall have much to say about the two space time points (0 and y) in (I.3). While they have no direct interpretation in electroproduction, it is obvious that in the Compton amplitude, they are the points of absorption and emission of the virtual photon on the nucleon. We shall see that the behavior of $\langle P|\left[J_\mu(y), J_\nu(0)\right]|P\rangle$ at nearly light-like points ($y^2 \approx 0$) governs inelastic electron scattering in the Bjorken limit.

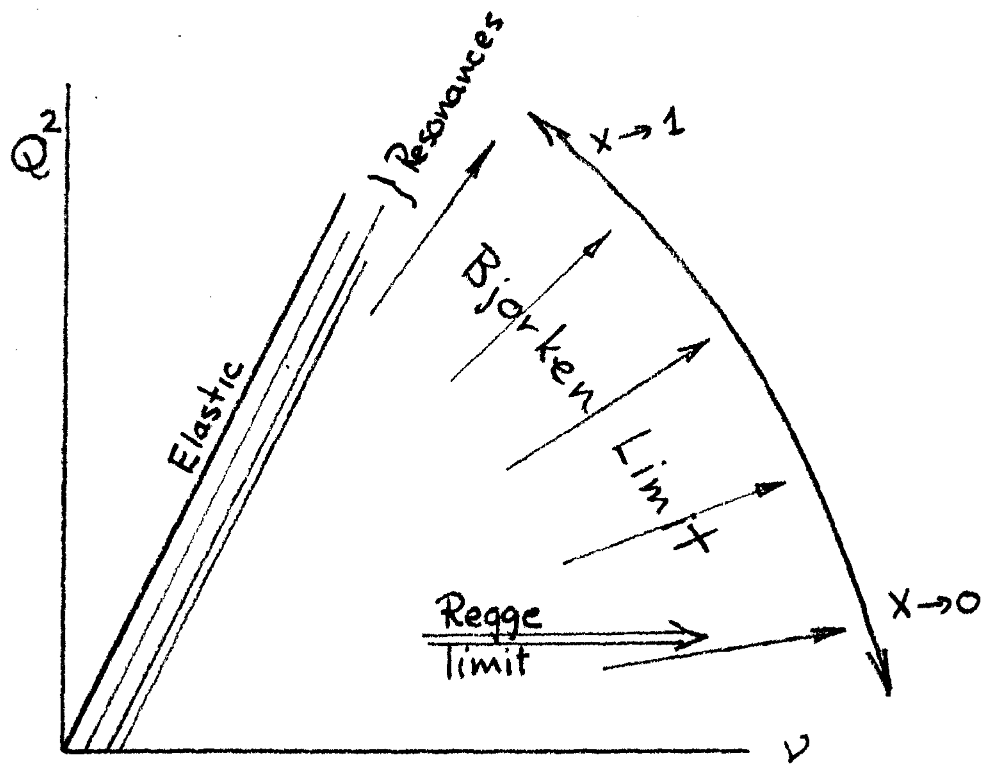
Since $W_{\mu\nu}$ is a Lorentz tensor and (for the case of electron scattering) must satisfy current conservation, $q^\mu W_{\mu\nu} = W_{\mu\nu} q^\nu = 0$, it has a particularly simple decomposition into invariant "structure functions"[5]:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) + \frac{1}{M^2}\left(P_\mu - \frac{\nu}{q^2} q_\mu\right)\left(P_\nu - \frac{\nu}{q^2} q_\nu\right) W_2(q^2,\nu)$$

In terms of W_1 and W_2 the cross section is given by

$$\frac{d\sigma}{d\nu\, dQ^2} = \frac{4\pi\alpha^2}{M^2 Q^4}\left(\frac{e'}{e}\right)\left[W_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

Before confronting the light cone, it is useful to look at the ν, Q^2 plane over which the functions W_1 and W_2 are defined.



$W_{\mu\nu}$ must vanish whenever it is kinematically impossible to scatter to a real state $|X\rangle$ whose mass is given by

$$M_X^2 = (P+q)^2 = M^2 + 2\nu - Q^2$$

Since $M_X^2 \geq M^2$, $W_{\mu\nu}$ must vanish for $2\nu < Q^2$. The line $2\nu = Q^2$ corresponds to elastic scattering where the state $|X\rangle$ is just the original nucleon. A resonance of mass M_R lies on the line $2\nu - Q^2 = M_R^2 - M^2$ parallel to the elastic line. The conventional Regge limit lies along lines parallel to the ν axis, i.e., masses (including the virtual photon's) are held fixed, while the energy of the virtual photon becomes infinite. The Bjorken limit, $\nu, Q^2 \to \infty$ with the ratio $x = Q^2/2\nu$ fixed, is the realm of light-cone physics. It is reached in the Q^2, ν plane by going out along rays through the origin, the slope corresponding to the value of x.

Although the Bjorken limit is only achieved formally at infinite ν and Q^2, it has become commonplace to set infinity equal to 1 or 2 GeV^2 in confronting data. The reason for this is of course hindsight: predictions which are derived for infinite ν and Q^2 seem borne out by experiment at rather small values. Experimentally it is found that for $Q^2, \nu > 1$ or 2 GeV, the structure functions $W_1(Q^2, \nu)$ and $\nu W_2(Q^2, \nu)$ reduce to functions of x alone[6]:

$$W_1(Q^2, \nu) \longrightarrow F_1(x)$$
$$\frac{\nu}{M^2} W_2(Q^2, \nu) \longrightarrow F_2(x)$$

This was predicted by Bjorken[1] and is known as Bjorken scaling.

At this time, neither the light-cone nor any other approach to the problem has succeeded in providing a satisfactory explanation for the early onset of seemingly asymptotic behavior. Of course, it is possible that the present experiments only give the illusion of asymptopia and that surprises await us at higher values of Q^2 and ν. Light-cone techniques are formally valid only at infinite ν and Q^2. If Bjorken scaling is found to break down at larger values of Q^2 and ν, it does not invalidate light-cone physics. It would mean, however, that we have been mistaken in applying it to the present data.

II. LIGHT-CONE DOMINANCE

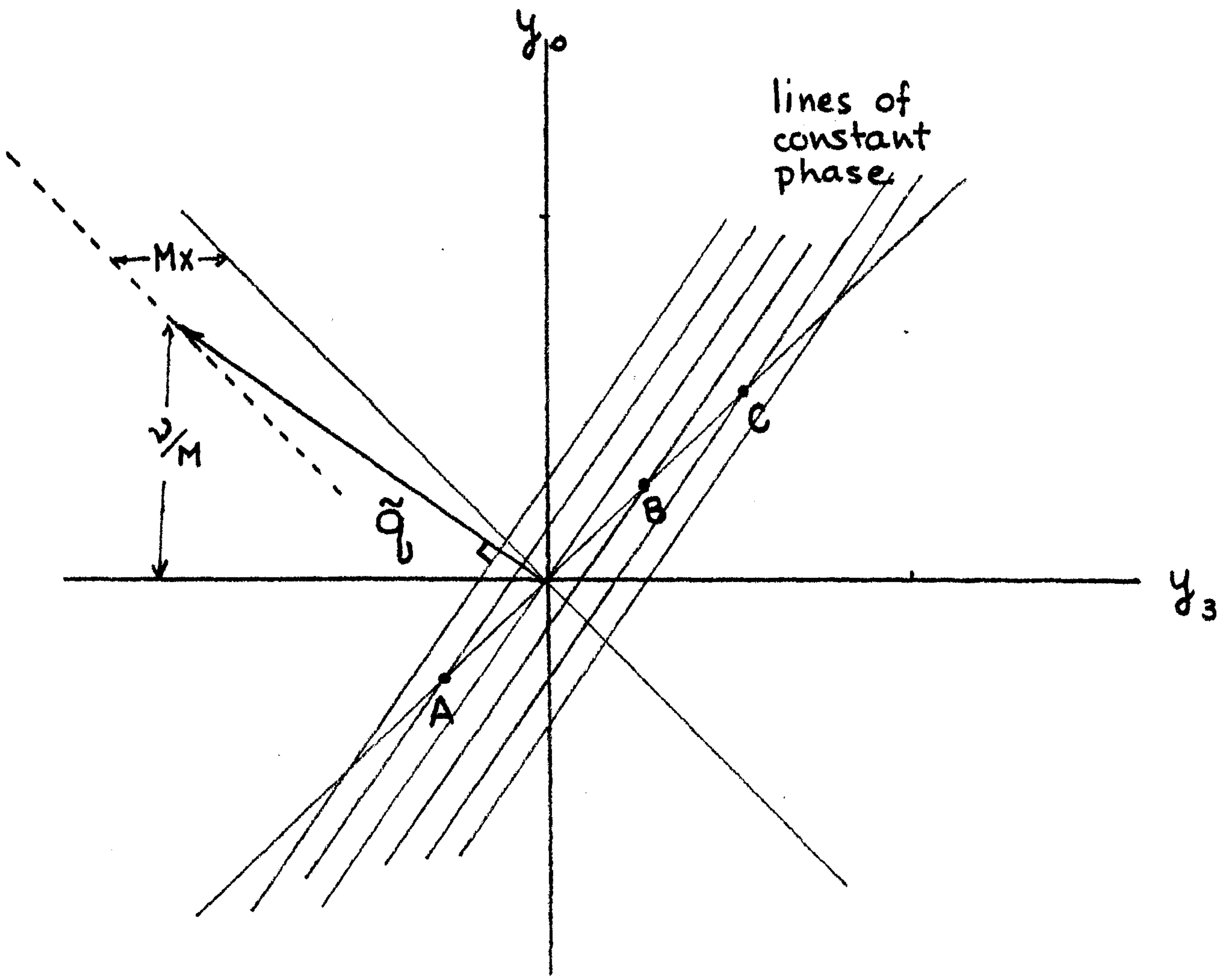
By light-cone dominance, we shall mean two things: first, that in the Bjorken limit, the dominant contribution to the Fourier transform of (I.3) comes from the region $0 \leq y^2 \lesssim 1/Q^2$; and, second, that it arises from the term in $\langle P | [J_\mu(y), J_\nu(0)] | P \rangle$ (referred to hereafter as the current correlation function) which is most singular on the light cone. If true, this provides great simplification—most of the complexity of the current correlation function may be ignored, only the most violent behavior near $y^2 = 0$ need be studied. It may seem that these two requirements are identical—in fact, for lepton scattering they are—however, in other processes, we shall see that the first is satisfied while in general the second is not. Unless the leading singularity can be shown to dominate, light-cone techniques lose most of their predictive power. Here we present a geometrical argument in support of the first requirement, and return to the second in a later section.

The requirement that $y^2 \geq 0$ follows from causality. The current correlation function vanishes unless y^2 is timelike. The conventional argument for $y^2 \lesssim 1/Q^2$ is based on the oscillation of the phase $e^{iq\cdot y}$ in (I.3).[7] We attempt to interpret this argument geometrically.[8] Consider the laboratory frame:

$$P = (M, 0, 0, 0)$$

$$q = \left(\frac{\nu}{M}, 0, 0, \sqrt{v^2/M^2 + Q^2}\right)$$

A plane of constant phase obeys the equation $q \cdot y = \Phi$ or $q_0 y_0 - q_3 y_3 = \Phi$. In Euclidean geometry, this is the equation of a family of planes perpendicular to the vector $\tilde{q} \equiv (q_0, 0, 0, -q_3)$. If we suppress the other two directions (which are inessential for the argument), we may plot the planes (now lines) of constant phase and simply see what happens to them in the Bj-limit.



In the figure, these lines are plotted for some phase interval, say $\Delta\Phi = 2\pi$, and are explicitly perpendicular to $\tilde{q}$. Now, for large Q^2 and ν, the vector $\tilde{q}$ approaches $\left(\frac{\nu}{M}, 0, 0, -\frac{\nu}{M} - Mx\right)$, it lies a distance $\frac{\nu}{M}$ up the zero-axis and a distance Mx outside the light cone. As the Bjorken limit is taken, two things happen: first, the vector $\tilde{q}$ runs out along the dashed line parallel to, but a distance Mx away from, the light cone. Consequently the lines of constant phase tilt over and become progressively more nearly parallel to the light cone. Second, the distance between successive lines of constant phase, which is easily seen to be $\sim M/\nu$, goes to zero. <u>Right on the light cone, these effects cancel,</u>

that is, the point of intersection of a phase line with the light cone (e.g., the points A, B, or C in the figure) remains fixed in the limit. These points are the solutions of the equation

$$q_0 y_0 - q_3 y_3 = \Phi$$

when

$$y_0 = y_3$$

and are given by

$$y_0 (q_0 - q_3) = \Phi$$

$$y_0 = y_3 = \Phi/Mx ,$$

which are clearly fixed in the limit. In fact, the distance between the successive points of intersection (e.g., between B and C) is given by

$$\Delta y_0 = \Delta y_3 = \frac{\Delta \Phi}{Mx} .$$

The significance of this is as follows: The phase oscillations, becoming infinitely closely spaced, will tend to wipe out any smooth contribution from the current correlation function inside the light cone. If there is some discontinuous behavior in the current correlation function across the light cone, this contribution to $W_{\mu\nu}$ will persist in the Bjorken limit because the phase lines do not become asymptotically close along the light cone. Indeed, we can guess (correctly) that, since the wavelength along the light cone is of the order 1/Mx, the Bjorken limit will sample behavior along the light cone with a frequency proportional to Mx. Some discontinuous behavior is expected at the light cone on the basis of causality. $\left[J_\mu(y), J_\nu(0)\right]$ should be zero outside the light cone, but is presumably not zero for $y^2 \geq 0$. Unless the correlation function vanishes like e^{-1/y^2} as $y^2 \longrightarrow 0$, it or its derivatives will be discontinuous or singular near $y^2 = 0$.

Several of the assumptions made in this argument can be shown to be unnecessary. First, the current correlation function may have certain singular behavior inside the light cone, yet the light cone still dominates.[9] Second, even for current products—rather than commutators—which do not necessarily vanish for $y^2 < 0$, the region $|y^2| \lesssim 1/Q^2$ can be shown to dominate in the Bjorken limit.[10] This may seem surprising since a discontinuity across $y^2 = 0$ figured heavily in our discussion. A simple demonstration that this is not necessary follows from arguments given in Appendix C of the second paper of Ref. 27.

In the following section, we return to the question of light-cone dominance, especially to the requirement that the leading singularity dominate, in a more mathematical context.

III. LIGHT-CONE EXPANSIONS AND THE MEASUREMENT OF LIGHT-CONE SINGULARITIES

If we are convinced that the region $y^2 \lesssim 1/Q^2$ dominates the Fourier transform, the next step is to develop a formalism for studying objects like the current correlation function in this region. Such a formalism—operator product expansions about $y^2 = 0$—was developed by Frishman[11] and by Brandt and Preparata.[10] A fine review of this subject, which is too extensive to cover here, is given by Frishman in his Schladming Lectures.[12]

From now on, we shall deal only with a hypothetical structure function, $V(Q^2, \nu)$, arising from the scattering of some scalar current J(y) off of the nucleon.

$$V(Q^2, \nu) \equiv \frac{1}{4\pi} \int e^{iq\cdot y} d^4y \, \langle P | [J(y), J(0)] | P \rangle \qquad \text{(III. 1)}$$

with

$$C(y^2, y \cdot P) \equiv \langle P | [J(y), J(0)] | P \rangle$$

The complications of spin are not essential for the discussion.

We wish to study $C(y^2, y \cdot P)$ near $y^2 = 0$. Simple examples such as free field theory lead one to expect a product of operators to satisfy an expansion of the following form:

$$A(x)\,B(y) = \sum_{[\alpha]} C^{[\alpha]}(x-y)\,F_{[\alpha]}(x,y)$$

where the $C^{[\alpha]}$ are not operators but may be singular as $(x-y)^2 \longrightarrow 0$, while the $F_{[\alpha]}$ are bilocal operators (i.e., they are operator-valued at both x and y)[13] regular near $(x-y)^2 \sim 0$. The indices $[\alpha]$ are Lorentz indices and any other labelling (e.g., internal symmetry, strength of singularity) which may occur. An example is in order: Consider $J(y) = :\phi^+(y)\,\phi(y):$ where $\phi(y)$ is a free scalar field. It is easy to show then[14] $[J(x), J(y)] = i\,\Delta(x-y)\,\Delta_1(x-y) + i\,\Delta(x-y) \otimes [:\phi^+(x)\,\phi(y): + :\phi^+(y)\,\phi(x):]$ where Δ and Δ_1 are certain functions (singular near the light cone) about which we shall have more to say. Although the example we have chosen is a commutator, a similar expansion exists for the product.

The basic assumption is that the <u>singularities</u>, if any, may be isolated in the C-number function $C^{[\alpha]}$. At least three courses of investigation are suggested by these expansions: 1. attempting to justify such expansions rigorously, for example in model field theories; 2. studying the way various terms in the expansion contribute to structure functions like $V(Q^2, \nu)$; and, 3, trying to calculate these expansions by approximate methods in more "realistic" model field theories. About 1, we have little to say except to give a reference.[15]

The second subject is very fertile. We shall see that it is possible to "measure" the strength of the most singular term in the expansion by doing deep inelastic scattering experiments,[16] and that the results are surprising. First, take the one-nucleon matrix element of the light-cone expansion of a current commutator:

$$C(y^2, y\cdot P) \equiv \langle P|[J(y), J(0)]|P\rangle$$

$$= \sum_{[\alpha]} E_{[\alpha]}(y^2)\, F^{[\alpha]}(y\cdot P, y^2)$$

In the Bjorken limit, we need only the behavior of C near $y^2 = 0$; since $F^{[\alpha]}$ is assumed to be regular near the light cone, we may set $y^2 = 0$ in $F^{[\alpha]}$. We shall now suppose that the most singular term in the expansion has a certain form and compute its contribution to $V(Q^2, \nu)$ in the Bjorken limit. This will illustrate the sort of mathematics involved in light-cone calculations. We then return to the expansion and show that terms less singular on the light cone are less important in the Bjorken limit.

Certain properties of $C(y^2, y\cdot P)$ constrain the allowed form of $E_{[\alpha]}$ and $F^{[\alpha]}$. In particular

1. $C(y^2, y\cdot P) = 0$ for $y^2 < 0$ (causality)
2. $C(y^2, y\cdot P) = -C(y^2, -y\cdot P)$ (crossing)
3. $V(Q^2, \nu) = 0$ for $|2\nu| < Q^2$,[17] puts constraints on $F_{[\alpha]}(y\cdot P)$ which we will formulate more carefully later. (spectral condition)

Suppose the most singular term in $\sum_{[\alpha]}$ were

$$\frac{1}{\pi i}\,\delta(y^2)\,\epsilon(y\cdot P)\,F(y\cdot P) \qquad \left(\epsilon(y\cdot P) = \begin{cases} 1 & y\cdot P > 0 \\ -1 & y\cdot P < 0 \end{cases}\right) \tag{III.2}$$

If $F(y\cdot P) = +F(-y\cdot P)$, conditions 1 and 2 are satisfied.

It is useful to check dimensions at each stage of these calculations. By dimension, we mean the physical dimension (see footnote 3) in units of mass (inverse length). $C(y^2, y \cdot P)$ has dimension +2 if we take the currents to have dimension 2 as suggested by free scalar field theory[18] (remember $|P\rangle$ has dimension -1). In the realistic case, the dimension of the electromagnetic current is fixed at 3 since $Q \equiv \int d^3x\, J_0(x)$ is dimensionless, but in our hypothetical case, we must fix the dimension by assumption. Equation (III.2) has the proper dimension $\left(\dim[\delta(y^2)] = +2\right)$ if $F(y \cdot P)$ is dimensionless.

The contribution of (III.2) to $V(Q^2, \nu)$ is given by

$$V_1(Q^2, \nu) \equiv \frac{1}{4\pi^2 i} \int e^{iq\cdot y}\, d^4y\, \delta(y^2)\, \epsilon(y_0)\, F(y \cdot P) \qquad \text{(III.3)}$$

We have replaced $\epsilon(y \cdot P)$ by $\epsilon(y_0)$ since sign $(y \cdot P)$ = sign(y_0) when multiplying a function which vanishes outside the light cone. We can guess some features of the Fourier transform by dimensional analysis: The dimension of the right-hand side of (III.3) is -2, and in the Bjorken limit, we expect this to be provided by a factor of $1/\nu$ ($1/\nu$ is equivalent to $1/Q^2$ in the limit). Hence we expect $V_1(Q^2, \nu) \rightarrow 0(1/\nu)$ in the limit.

Let us confirm this by direct calculation. First write $F(y \cdot P)$ in terms of its Fourier transform:

$$F(y \cdot P) = \int_{-\infty}^{\infty} d\alpha\, e^{i\alpha\, y\cdot P}\, f(\alpha) \quad ,$$

so (III.3) becomes

$$V_1(Q^2, \nu) = \frac{1}{4\pi^2 i} \int_{-\infty}^{\infty} d\alpha\, f(\alpha) \int d^4y\, e^{i(q+\alpha p)\cdot y}\, \delta(y^2)\, \epsilon(y_0) \qquad \text{(III.4)}$$

The function defined by the y integration in (III.4) is well known. A knowledge of a few of its properties is extremely useful. Consider

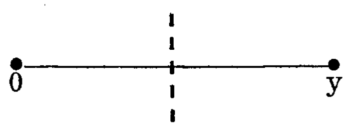
$$\Delta(y,m^2) \equiv \frac{1}{i(2\pi)^3}\int d^4k\, e^{ik\cdot y}\,\delta(k^2-m^2)\,\epsilon(k_0)$$

In field theory, $\Delta(y,m^2)$ is related to the imaginary part of the Feynman propagator for a free particle of mass m:

$$-\frac{1}{2}\epsilon(y_0)\Delta(y,m^2) = \mathrm{Im}\left\langle 0\middle|T(\phi(y)\phi(0))\middle|0\right\rangle$$

$$\Delta(y,m^2) = i\left\langle 0\middle|[\phi(y),\phi(0)]\middle|0\right\rangle$$

represented diagrammatically by



$\Delta(y,m^2)$ has the following useful properties[19]:

A. $$\Delta(y,m^2) = \frac{1}{2\pi}\epsilon(y_0)\delta(y^2) - \frac{m^2}{4\pi}\epsilon(y_0)\,\theta(y^2)\left[\frac{J_1\left(m\sqrt{y^2}\right)}{m\sqrt{y^2}}\right]$$

B. $$\Delta(y,0) = \frac{1}{2\pi}\epsilon(y_0)\delta(y^2)$$

C. $$\frac{d}{dm^2}\Delta(y,m^2)\Big|_{m^2=0} = -\frac{1}{8\pi}\epsilon(y_0)\,\theta(y^2)$$

where $J_1\left(m\sqrt{y^2}\right)$ is the regular Bessel function. Note the light-cone singularities in $\Delta(y,m^2)$ and the fact (property B) that the Fourier transform of $\delta(k^2)\epsilon(k_0)$ is, up to numerical factors, just $\delta(y^2)\,\epsilon(y_0)$.

We can now evaluate III. 4:

$$V_1(Q^2,\nu) = 2\pi \int_{-\infty}^{\infty} d\alpha\, f(\alpha)\,\Delta(\alpha\, P+q, 0)$$

$$= \int_{-\infty}^{\infty} d\alpha\, f(\alpha)\,\delta(\alpha^2 M^2 + 2\alpha\nu - Q^2)\,\epsilon(\alpha M + \nu)$$

Now there are two solutions to the δ-function:

$$\alpha_{\pm} = -\frac{\nu}{M^2} \pm \sqrt{\nu^2/M^4 + Q^2/M^2}$$

In the Bjorken limit, these reduce to

$$\alpha_+ \cong x$$

$$\alpha_- \cong -2\nu/M^2$$

and we find, doing the integral against the δ-function

$$\underset{Bj}{\text{Lim}}\; V_1(Q^2, \nu) = \frac{1}{2\nu}\left[f(x) - f\left(\frac{-2\nu}{M^2}\right)\right]$$

Returning briefly to the spectral condition (no. 3 listed above), we find we can make $V(Q^2, \nu)$ vanish for $|2\nu| < Q^2$ if $f(x) = 0$ for $|\alpha| > 1$. This translates into a restriction of $F(y \cdot P)$. It ensures that $f(-2\nu/M^2)$ vanishes in the Bjorken limit, leaving

$$\underset{Bj}{\text{Lim}}\; V_1(Q^2, \nu) = \frac{1}{2\nu}\, f(x).$$

Our conjecture based on the geometrical picture of the previous section is borne out: The structure function measures the Fourier transform of the smooth function multiplying the light-cone singularity with a frequency proportional to x. The scaling law is analogous to Bjorken's scaling laws for W_1 or W_2.

Suppose, now, that we look at a term in $C(y^2, y \cdot P)$ with a weaker singularity on the light cone, for example:

$$\frac{1}{\pi i} \mathcal{M}^2 \, \theta(y^2) \, \epsilon(y_0) \, G(y \cdot P)$$

where the constant $\mathcal{M}^2$ (with dimension $[\mathrm{mass}]^2$) is necessary to preserve the dimension of $C(y^2, y \cdot P)$. This term contributes to $V(Q^2, \nu)$ a term of the form

$$V_2(Q^2, \nu) \equiv \frac{1}{4\pi^2 i} \int e^{iq \cdot y} d^4y \, \mathcal{M}^2 \, \theta(y^2) \, \epsilon(y_0) \, G(y \cdot P)$$

Again we may guess the answer by dimensional analysis: We expect $V_2 \sim \mathcal{M}^2/\nu^2$ in the Bjorken limit. Property C of the function $\Delta(y, m^2)$ allows us to perform this integral—substitute for $\theta(y^2) \, \epsilon(y_0)$, do the integral as for V_1, differentiate by m^2 and set m^2 to zero—with the result

$$V_2(Q^2, \nu) \propto \frac{\mathcal{M}^2}{\nu^2} \, g(x)$$

where $g(x)$ is the Fourier transform of $G(y \cdot P)$.

The relation between the light-cone singularity of a term in $C(y^2, y \cdot P)$ and the power of ν in the ensuing scaling law is completely general. In deep inelastic scattering, only the most singular term on the light cone survives in the limit so the scaling law measures the strength of that singularity. By the way, this establishes the second "condition" for light-cone dominance mentioned at the beginning of Section II: The leading singularity indeed dominates in the Bj limit.

What is remarkable in the SLAC-MIT experiments is that scaling for νW_2 and W_1 in the Bjorken limit is what would be expected if the electromagnetic current were constructed from free fields: That is, construct electromagnetic currents from some hypothetical free fields and commute them as we did for the scalar case $:\phi^+(y)\phi(y):$ above. Certain singular functions will occur and give rise to the prediction that νW_2 and W_1 scale. Of course, this is not to say that the proton is described by free-field theory. If it were, only elastic scattering could occur. Only leading singularities are from free-field theory. The matrix elements of bilocal operators, $F_{[\alpha]}(y \cdot P)$, whose Fourier transforms produce the observed scaling functions involve interactions and should be rich sources of information about the structure of the nucleon. Just how much free-field theory is necessary in order to recover free-field singularities on the light cone will become apparent when we study the parton model in the next section.

We now turn to the third aspect of light-cone expansions: trying to calculate by approximate methods in more "realistic" models. Here we must be very brief. One's first instinct might be to try perturbation theory in some reasonable model like QED with massive photons. However, one finds that in every order scaling for νW_2 and W_1 is broken by logarithms of Q^2, or correspondingly the leading light-cone singularities of free-field theory are not preserved in perturbation theory.[20] Only very smooth (super-renormalizeable) theories scale order-by-order in perturbation theory—but these do not provide a very attractive model for hadronic currents. For example, there is no known super-renormalizeable theory in four dimensions which describes spin 1/2 particles—an unfortunate situation for quark enthusiasts.

Faced with this, one could either forget perturbation theory or try to sum all orders and show that scaling, and therefore free-field light-cone singularities, reappears. While there has been work on the latter,[21] the former has attracted the most attention recently. There are two popular ways to ignore perturbation theory: The first is to do free-field theory, the second is to introduce interactions through the canonical equations of motion and canonical commutation relations in some model field theory (knowing full well that the canonical equations of motion are modified by the divergences of perturbation theory, if we believed perturbation theory). The first way, free-field theory, was emphasized by Fritzsch and Gell-Mann[22] and proceeds as follows. To calculate the leading term in the light-cone expansion of the commutator of two currents, imagine the currents to be constructed of some free fields (e.g., for quarks: $J_\mu^{e.m.}(y) = \bar{\psi}(y)\gamma_\mu Q\psi(y)$, where Q is the 3x3 quark charge matrix) and compute $\left[J_\mu(y), J_\nu(0)\right]$ as we did for $:\phi^+(y)\phi(y):$ above. One finds a free-field light-cone singularity multiplying some bilocal operator. This singularity will give scaling as observed at SLAC. The matrix element of the bilocal operator determines the shape of the scaling functions $W_1(x)$ and $\nu W_2(x)$. One does not assume that free-field theory describes the bilocal operator, only the singularity. This leaves the shapes of $W_1(x)$ and $\nu W_2(x)$ unspecified. This approach is primarily useful for generating algebraic relations (e.g., sum rules) involving various deep inelastic processes. If the currents carry SU(3) labels, then the bilocal operators will carry other SU(3) quantum numbers by virtue of the f_{ijk} and d_{ijk} which appear when doing the commutators. How sum rules are constructed is discussed, for example, in Ref. 22. Not surprisingly, the sum rules derived in this manner can also be derived in the parton model—a fact which we will understand better in the next section.

The second approach: "Canonical manipulations" is more of a testing ground to tell us when we might trust free-field theory than a calculational scheme in its own right. That is: One verifies that the introduction of interactions, treated canonically, does not alter the singularities of free-field theory and then one proceeds with free-field theory. If one's results depend on the interaction, then they are thought less likely to be valid since we have little claim to know what are the correct interactions among the hypothetical fields which build up hadronic weak and electromagnetic currents. "Canonical manipulations" provide a guide which tells us how far free-field theory can be pushed. Using this approach, Brandt and Preparata[10] and Gross and Treiman[23] showed that introduction of a neutral vector interaction among hypothetical quarks does not alter the free-field light-cone singularity. Similar techniques were used by Llewellyn Smith to study scalar and pseudoscalar interactions with similar results.[24] A related approach, originated by Cornwall and Jackiw,[25] begins with a canonical field theory quantized on the plane $\tau = y^0 + y^3 = 0$ rather than at equal times. Light-cone, actually equal-τ, commutators of currents may be computed from the canonical equal-τ commutator of the fields just as equal-time current commutators are usually calculated from equal-time commutation relations of fields. This approach yields the same results as canonical manipulations at equal times but may be somewhat simpler from a computational standpoint.

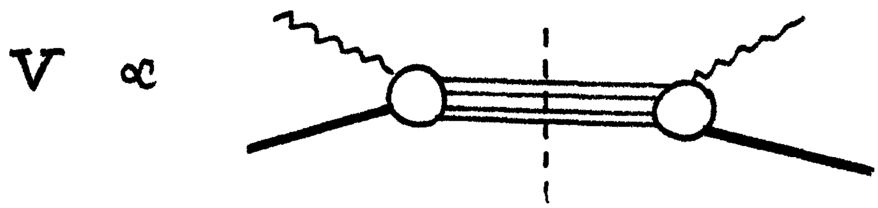
In any case, at the present time there does not seem to be any attractive way to calculate light-cone expansions beyond the leading term which is consistent with a leading singularity given by free-field theory.

IV. FREE FIELD LIGHT-CONE SINGULARITIES— THE PARTON MODEL WITHOUT PARTONS

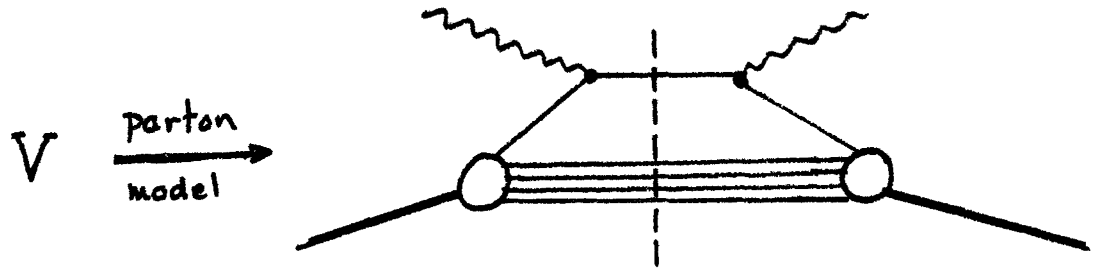
We now turn to the parton model to provide a simple picture of the origin of the free-field singularity which the SLAC-MIT electron scattering experiments[6] have found in the current correlation function.[26,27] This is an appropriate time to emphasize again that we are assuming that the experiments really are asymptotic. If scaling is violated at larger Q^2, then the important singularities are not those of free-field theory and this naive constituent picture is inapplicable.

Again we consider scalar currents and confine ourselves to scalar partons. To formulate the parton model, go to the target's infinite momentum frame[28] where its momentum $\vec{P}$ (in the z-direction) is larger than any other variable in the problem (e.g., $|\vec{P}| \gg Q^2/M,\ \nu/M$). At infinite momentum and in the Bj limit, it is argued that the interactions of the constituents in the target are slowed down and they appear instantaneously free when hit by the current. On the basis of this picture, supported by detailed calculations in models,[29] the constituents are taken to be scattered elastically by the current and not to interact with the spectator constituents after scattering. These assumptions of elasticity and incoherence are the heart of the model.

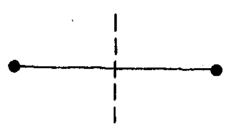
Diagrammatically, the model reduces $V(Q^2, \nu)$ (for positive energy photons, we need only the first ordering of currents in (III.1)):



in the Bjorken limit to the following:



Notice that in the parton model, the currents are connected by



which should produce a $\Delta(y, m^2)$ in coordinate space—let us see how this arises.

The parton model diagram reads as follows:

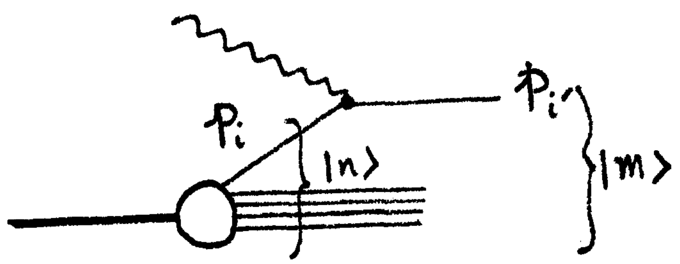
$$\lim_{Bj} V(Q^2,\nu) = \frac{1}{4\pi}\int d^4y\, e^{iq\cdot y} \sum_n |a_n|^2 \langle n|j(y)\,j(0)|n\rangle \qquad \text{(IV. 1)}$$

where $|n\rangle$ are parton states out of which the proton is composed with amplitude $a_n \left(|P\rangle = \sum_n a_n |n\rangle\right)$, and j is the free single particle current operator which can only scatter partons, not create pairs. Terms like $a_m^* a_n$ are all eliminated by the assumption of incoherence.

Elasticity and incoherence reduce J(y) to j(y): there are no form factors to diminish the vertex. Insert into (IV. 1) a complete set of states and identify the current correlation function:

$$\langle P|J(y)\,J(0)|P\rangle = \sum_{nm} |a_n|^2 \langle n|j(y)|m\rangle \langle m|j(0)|n\rangle$$

j(y) operates on each parton, p_i, in $|n\rangle$ individually so the sum on m reduces to the phase space for the single parton it scatters:



$$\langle P|J(y)\,J(0)|P\rangle = \sum_{ni} |a_n|^2 \int \frac{d^3p_{i'}}{2E_{i'}} \langle i|j(0)|i'\rangle \langle i'|j(0)|i\rangle$$

$$\otimes\ e^{i(p_i - p_{i'})\cdot y}$$

With our normalization $\langle i|j(0)|i'\rangle = \lambda_i$, the charge of the ith parton,

$$\langle P|J(y)\,J(0)|P\rangle = \int \frac{d^3 p_{i'}}{2E_{i'}}\, e^{-i p_{i'}\cdot y} \sum_{n\,i} \lambda_i^2 |a_n|^2\, e^{i p_i\cdot y} \tag{IV.2}$$

But

$$\int \frac{d^3 p_{i'}}{2E_{i'}}\, e^{-i p_{i'}\cdot y} \propto \int \frac{d^4 p_{i'}}{(2\pi)^3}\, \theta(p_{io})\, \delta\left(p_i^2 - m_i^2\right) e^{-i p_{i'}\cdot y} \equiv \Delta_-(y,m^2)$$

where the familiar $\Delta(y,m^2) = i\left(\Delta_+(y,m^2) - \Delta_-(y,m^2)\right)$ $\left(\Delta_+(y,m^2)\right.$ has the opposite sign of the exponential from $\left.\Delta_-(y,m^2)\right)$. Had we begun with the current commutator, rather than the product, we would have obtained $\Delta(y,m^2)$.

It remains to simplify the non-singular part of (IV.2). The sum over n includes a sum over the momenta of the ith particle—which is taken to be forward along the infinite momentum direction, so:

$$\sum_{n\,i} = \int_0^1 dx \sum_{n\,i} \delta(x - x_i)$$

where

$$p_i^\mu \cong x_i\, P^\mu \qquad\qquad 0 < x_i < 1$$

so that

$$\langle P|J(y)\,J(0)|P\rangle \propto \Delta_-(y,m^2) \int_0^1 dx\, e^{ixP\cdot y} \sum_{n\,i} |a_n|^2 \lambda_i^2 \quad \delta(x - x_i)$$

The last factor is the scaling structure function in the parton model:

$$\underline{\text{i.e.}}, \quad \lim_{Bj} \nu V(Q^2, \nu) \equiv f(x) = \sum_{n\,i} |a_n|^2 \lambda_i^2 \delta(x - x_i)$$

so we have:

$$\langle P|J(y)\,J(0)|P\rangle \propto \Delta_-(y, m^2) \int_0^1 dx\, e^{ixy\cdot P} f(x)$$

The analogous result for the commutator is

$$\langle P|[J(y), J(0)]|P\rangle \propto \Delta(y, m^2) \int_0^1 dx \cos(xy\cdot P)\, f(x)$$

Finally, taking the leading singularity of $\Delta(y, m^2)$:

$$\langle P|[J(y), J(0)]|P\rangle \propto \delta(y^2)\,\epsilon(y_0) \int_0^1 dx \cos xy\cdot P f(x) \qquad \text{(IV.3)}$$

+ less singular terms.

To summarize:

I. The free-field light-cone singularity which corresponds to Bjorken scaling emerges from the free propagation of the elastically scattered parton in the final state.

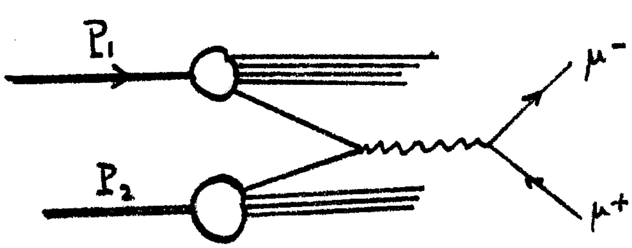
II. The smooth $y \cdot P$ dependence measures the Fourier transform of the parton probability distribution weighted by the squared charge.

Suppose we had begun with the light-cone result (IV.3) and tried to derive the parton model (IV.1). All of the algebraic results are reversible—however, at some point, it is necessary to assume that partons exist, i.e., that it makes sense to write $|P\rangle = \sum_n a_n |n\rangle$ for constituent states $|n\rangle$. Hence the title of this section: The light-cone formalism, including leading free-field singularities, does not seem to require the existence of real physical constituents. It is an open question how much can be abstracted from free-field theory without forcing real, physical constituents upon us. A more complete discussion of the relation between the two approaches is given in the second paper under ref. 27.

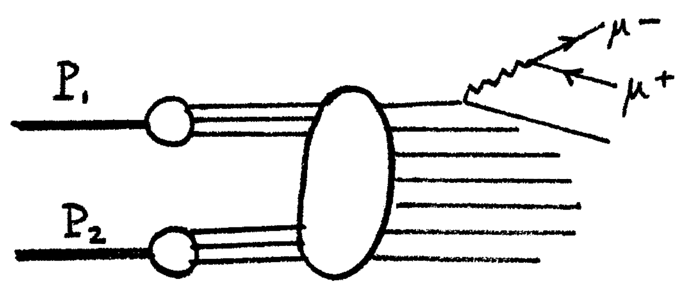
The parton model and light-cone analyses are not equivalent outside of the realm of highly inelastic lepton scattering. To illustrate the problems which arise when one tries to apply light-cone techniques to other processes, consider briefly massive muon pair production in the parton model. The experiment is:

$$P_1 + P_2 \to \mu^+ \mu^- + \text{anything}$$

at large $S \equiv (P_1 + P_2)^2$ and large $Q^2 \equiv (P_{\mu^+} + P_{\mu^-})^2$ with Q^2/s finite. A treatment of this process using light-cone techniques together with additional assumptions has been given by Professor Brandt[30] at this School. In the parton model[31] two diagrams are possible:



A. Annihilation



B. Bremsstrahlung

Drell and Yan use their parton model to show diagram A dominates for $Q^2, s \to \infty$; Q^2/s fixed. A free-field light-cone singularity is associated with the propagation of an elastically-scattered parton. Diagram A has no scattered parton, hence no light-cone singularity. One can show[27] that the matrix element corresponding to A is smooth across the light cone. Diagram B, however, has an elastically scattered parton and is singular on the light cone. In addition, it is not hard to see[27,30] that both diagrams receive contributions from the region $y^2 \lesssim 1/Q^2$ in coordinate space. Referring back to our formulation of light-cone dominance, we see that the first requirement ($y^2 \lesssim 1/Q^2$) is satisfied, but the second is violated—a non-leading singularity dominates. The resolution lies in the S-dependence of the diagrams—while diagram B is more singular at $y^2 = 0$, in the parton model it falls more quickly with S, leaving the non-singular diagram dominant.

In light-cone treatments of this process, additional assumptions must be made which re-establish the dominance of diagrams like B. The situation is certainly not so simple as it was in electron scattering.

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3. By dimension, we shall always mean the physical dimension—that is, the units in which a given physical quantity is expressed. This is not the same as, and should not be confused with, the scale dimension which determines the properties of some operator under scale transformations.

4. The term we have subtracted in (I.3) is non-vanishing for $q_0 < 0$. This has no particular significance for electroproduction. However, it corresponds to the u-channel cut in forward virtual Compton scattering and must be subtracted to give $W_{\mu\nu}$ the crossing properties appropriate to Compton scattering.

5. For neutrino scattering where the currents are not conserved, the analogous decomposition is

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1 + \frac{1}{M^2}\left(P_\mu - \frac{\nu}{q^2} q_\mu\right)\left(P_\nu - \frac{\nu}{q^2} q_\nu\right) W_2$$

$$+ \frac{i\,\epsilon_{\mu\nu\alpha\beta}}{2M^2} P^\alpha q^\beta W_3 + \frac{q_\mu q_\nu}{M^2} W_4 + \frac{(P_\mu q_\nu + P_\nu q_\mu)}{2M^2} W_5$$

$$+ i\,\frac{(P_\mu q_\nu - P_\nu q_\mu)}{2M^2} W_6$$

W_3 violates parity conservation, W_4 and W_5 violate current conservation, and W_6 violates both current conservation and time reversal invariance.

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13. A simple demonstration that $F_{[\alpha]}(x,y)$ must be an operator at both x and y is as follows. Let z be a point spacelike with respect to x, timelike with respect to y. In general, $\left[A(x)\,B(y),\ C(z)\right] \neq 0$ for an operator C(z). If we assume $F_{[\alpha]}(x,y)$ to involve only C-number functions of y, then $\left[F_{[\alpha]}(x,y),\ C(z)\right] = 0$ by causality for each $[\alpha]$. If the sum over $[\alpha]$ is finite (as expected), then a contradiction follows.

14. To verify this, write a momentum decomposition of the free field ϕ and use the canonical commutation relation of the creation and destruction operators together with the definitions of Δ and Δ_1 (cf ref. 19).

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17. Since we have defined $V(Q^2, \nu)$ in terms of a commutator, it receives contributions from both positive and negative energy photons. In both cases for $Q^2 > 0$, the threshold is $2|\nu| > Q^2$.

18. The dimension of $J(y) = :\phi^+(y)\,\phi(y):$ follows from the dimension of the field ϕ. Scalar and spinor fields have dimensions 1 and 3/2, respectively. Field dimensions are determined as follows: the Hamiltonian has dimensions of energy (+1) for scalars $H = \int d^3x \left[\ldots + m^2 \phi^+(x)\,\phi(x)\right]$ which implies dim $[\phi(y)] = 1$, the spinor result follows from $H = \int d^3x \left[\ldots + m\,\bar{\psi}\,\psi\right]$.

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