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A RELATIVISTIC "POTENTIAL MODEL" FOR N-PARTICLE SYSTEMS*

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ABSTRACT

Neither quantum field theory nor S-Matrix theory have a well defined procedure for going over to an approximation that can be reliably used in non-relativistic models for nuclear physics. We meet the problem here by constructing a finite particle number relativistic scattering theory for (scalar) particles and mesons using integral equations of the Faddeev-Yakubovsky type. Restricted to N particles and one meson, we can go from the relativistic theory to a "potential theory" in the integral equation formulation by using boundary states which do not contain the meson asymptotically. The meson-particle input amplitudes contain a pole at the particle mass, and the particle-particle input amplitudes are null. This gives unique definition (numerically calculable) to the particle-particle off-shell amplitude, and hence to the covariant "scattering potential" (but *not* to the non-invariant concept of "potential energy"). As we have commented before, if we take these scattering amplitudes as input for relativistic Faddeev equations, the results are identical to those obtained from the same model starting from three particles and one meson. In this paper we explore how far we can extend this relativistic "potential model" to higher numbers of particles and mesons.

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In a paper presented this year[1] I noted that "In effect we have discovered a 'relativistic potential model' which does not generate 'three body forces'." In fact all that I showed in that paper was that the "confined meson model" led to the same dynamical equations whether one first computed a "potential" in the two particle-one meson system and then added a third particle or started from three particles and one meson and then confined the meson – a conjecture that Sawicki had stated but did not prove. I assumed that the extension of the theorem to N particles and one meson was obvious. Perhaps so. I will try to convince you below that starting from a relativistic N-particle plus one meson system and confining the meson leads to a relativistic N-particle theory which is well defined, unitary and relativistically invariant as to predictions. I will then indicate what problems lie in wait when one goes on (a) to include two distinct mesons in the N-particle system or (b) to include "anti-particles" or "crossing symmetry"; conjoining (a) and (b) lies beyond even the speculative horizon of this paper.

The "confined quantum" approach[2] used in this paper started from the relativistic three-particle theory for three distinguishable scalar particles developed by Lindesay[3] . We specialize this theory by making a distinction between particles and mesons (of finite mass, which we also can call "quanta" if we wish to include the zero mass case). Particles are not allowed to scatter directly, while quanta scatter by forming an intermediate state with the same mass and quantum numbers as the particle. For an "effective Lagrangian" perturbation theory these restrictions define a "Yukawa vertex" and, by extension, an "Abelian gauge theory". Our scattering is driven by an s-channel "bound state" or "resonance" kinematically defined by requiring this input scattering amplitude to have a pole in the invariant four momentum of this subsystem at this mass. The energy for this "off-shell" state comes from the uncertainty principle rather than from some "Hamiltonian"; 3-momentum is conserved in any coordinate system. These principles give us an explicit model for the Wick[4] - Yukawa[5] description of the origin of nuclear forces. As has been shown already, this leads to a well defined relativistic theory of particle scattering and meson production due to single meson exchange[6] .

We trust this discussion specifies clearly how the two technical terms "particle" and "meson" enter into what follows. Particles and mesons are assumed to have unique masses, or decay widths so small for the problems of interest that the mass uncertainty can be ignored. In our theory "particles" do not scatter directly from each other, and the number of "particles" is conserved.

For convenience we will use the "zero momentum frame" for the N particle plus 1 meson system and leave discussions of "manifest covariance" to more detailed presentations of this class of models. The number of particles, N, is conserved. The number of mesons would be conserved if we could specify the *external* measurement procedure which would discriminate between a particle and a "particle plus any finite number of quanta bound state"; of course we have ruled this possibility out by our construction. The Faddeev theory conserves the number of mesons as well as particles, and then (at the 2 particle plus one meson level) implicitly defines the "coupling constant" through unitarity. In the confined quantum theory presented here the number of mesons flips back and forth between 0 and 1 depending on whether we are looking at a system of $N - 1$ particles and one "bound state" – which is indistinguishable from a particle –or N particles and one meson. All of this fits comfortably into the framework of a relativistic, unitary Faddeev-type scattering theory, as my students and I have taken some pains to establish.

The concept of "zero momentum frame" is invariant for any finite system. The velocity of any massive system can be observationally specified by its finite velocity with respect to the system in which the $2.7^o K$ background radiation is isotropic by calculating its invariant mass and the implied velocity that would bring it to rest in that system. This clearly allows us to calculate the velocity of any terrestrial laboratory with respect to that (locally and empirically) specified system.

We restrict our degrees of freedom to mass shell particles specified by the 3-vectors $\underset{\sim}{k}_i, i \in 1, 2, ..., N$ and the momentum (when present) of the meson $\underset{\sim}{q}$. Since the "bound state" is indistinguishable either kinematically or by quantum numbers from one of the particles, we can define $\underset{\sim}{K} = \Sigma_{i=i}^{N} \underset{\sim}{k}_i$ whether the quantum is present or absent. In the first case $\underset{\sim}{K} + \underset{\sim}{q} = 0$, and in the second $\underset{\sim}{K} = 0$. We generalize the starting point of our earlier three-body treatments (two particles plus 1 meson) to N particles plus 1 meson (following roughly our "zero range" treatment of the non-relativistic four body problem[7]) by writing the initial equations as

$$\mathbf{F}_{i\mu,j\mu} = \mathbf{t}_{i\mu}\left[\delta_{ij} - \Sigma_{k=1}^{N}\bar{\delta}_{ik}\mathbf{R}_0\mathbf{F}_{k\mu,j\mu}\right]$$

Since the Faddeev spectator indices (a, b, c) do not generalize to N-particle systems, we use instead pair indices $(i\mu, j\mu, k\mu)$ for the input two-body amplitudes $\mathbf{t}$. The form of the equations then follows immediately from our assumption that any particle can only scatter from the meson, and not from another particle. Our other basic assumptions then fix the input amplitudes as $t_{i\mu} = \frac{g_i^2\Pi_{j=1}^{N}\bar{\delta}_{ij}\delta^3(\underset{\sim}{k}_j-\underset{\sim}{k}_j^0)}{D_{i\mu}}$ where $D_{i\mu}$ is required to vanish when the invariant four momentum of the quantum-particle system is equal to m_i, the mass of the particle. Using mass shell states with the usual invariant completeness relation $\int \frac{d^3k}{\epsilon_m}|\underset{\sim}{k}><\underset{\sim}{k}| = 1$, $\epsilon_m = \sqrt{k^2+m^2}$, and removing the dependence on $\underset{\sim}{q}$ by overall 3-momentum conservation will then leave us N 3-vector (particulate) degrees of freedom and a factor of $[\epsilon_\mu(-\underset{\sim}{K})]^{-1}$ when we reduce the operator equations to integral equations.

Rather than following through this explicit treatment here, we argue instead that our model already specifies the structure we must end up with. As we develop the multiple scattering series for the amplitudes T_{ij}, which are defined by $g_i \underset{s_{i\mu}\to m_i^2}{lim}(s_{i\mu}-m_i^2)g_j \underset{s_{j\mu}\to m_j^2}{lim}(s_{j\mu}-m_j^2)F_{i\mu,j\mu}$, we find that we intersperse N-particle propagators with N particle plus 1 meson propagators conserving 3-momentum but (thanks to the Wick-Yukawa mechanism we have formalized) not the energy. However the energy with which the 1 meson states appear differs from that of the states by which it is clothed only by the meson energy itself. Further, since the momentum conservation relates the states with which we start and end, we conclude that the integral equations must be of the form

$$T_{ij}(\underset{\sim}{K};\underset{\sim}{K}_0) - V_{ij}(|\underset{\sim}{k}_i - \underset{\sim}{k}_j^0|)$$

$$\Sigma_{k=1}^{N} = \int \frac{d^3k'}{\epsilon'_{m_k}} V_{ik}(|\underset{\sim}{k}_i - \underset{\sim}{k}'_k|)\frac{1}{E'-E_0-i0^+}F_{kj}(\underset{\sim}{k}_1,..,\underset{\sim}{k}'_k,...,\underset{\sim}{k}_N;\underset{\sim}{K}_0)$$

where

$$V_{ij} = -\frac{g_i g_j \bar{\delta}_{ij}}{(\underset{\sim}{k}_i - \underset{\sim}{k}_j^0)^2 + \mu^2}$$

and $E = \Sigma_{k=1}^{N} \epsilon_{m_k}$. Since $V_{ij} = V_{ji}$, a large number of proofs exist which establish that the solution of these equations, if unique, is unitary.

For N=2 we see that our model is equivalent to ordinary Yukawa potential scattering except that the kinematics of the particles are relativistic. It is also convenient to note that $\underset{\sim}{k}_i + \underset{\sim}{k}_j = 0$, and hence that the dynamics involves only one vector variable. When we go from N=2 to N=3, we must use care, since the integral equations as they stand contain, on iteration, disconnected pieces and the summation becomes ambiguous. This is, of course, the same problem Faddeev faced in the non-relativistic potential theory, and can be met by the same method. We decompose the three body amplitude into a sum of pair plus spectator terms, where the driving term for the pair is the solution of the two body problem in the three body space, and then couple these in such a way as to produce a unique solution to the problem which was ambiguous as originally posed. The resulting equations are now in two vector variables rather than one. A check on this approach is provided, as already noted[1], by first formulating the 3 particle plus 1 meson problem as a four-body problem and noting that the confined quantum assumption reduces the 18 Faddeev-Yakubovsky amplitudes to three, which turn out to be identical to the Faddeev amplitudes obtained by the first approach.

For the four particle problem driven by single meson exchange, we start from the general pair equations, show that one iteration can be regrouped into (3,1) and (2,2) configurations, and then written again as a kernel which uses the solution of the 3-particle problems and the product of two two-particle problems as driving terms. The number of vector variables, and the number of non-cancelling terms in the energy propagator then goes from one to two. So far as we can see, this procedure generalizes from N to $N+1$ if we use the Faddeev-Yakubovsky combinatorics for any finite number of particles. Further, since the procedure starts from pairs coupled by a single meson, we can use a separate meson mass μ_{ij} for each pair, and need not use a common meson mass for the whole system.

The conclusion we have now reached is that if we consider a system of relativistic particles which interact only by the elementary exchange of a relativistic meson whose mass and coupling constants can be separately specified for each pair, the relativistic N-particle problem is equivalent to the corresponding Yukawa potential problem, *provided* that we use Faddeev-Yakubovsky integral equations *and* that we use relativistic kinematics for the particles. For some people this is not particularly startling. When discussing this with Brian Lynn[8] , he claimed that under the same assumptions the result could be obtained by starting from the Bethe-Salpeter equation. Perhaps so. What we claim here is that we have *proved* this proposition; I am eager to see other proofs.

Although, looked at superficially, the system we have just derived looks as if it contains $N(N-1)/2$ mesons "at the same time", this is an illusion. The "potential" is in fact a relativistic propagator in which only one mesonic degree of freedom appears. In order to obtain (if we can - which is at best speculative at this point) a genuine "two meson exchange potential", we would have to start from relativistic four particle equations containing two particles and two mesons, and apply the confined quantum restriction to that model. This is problem (a) as alluded to in our first remarks. At the N=2 level, we anticipate no difficulties in principle. If our method is correct, and applicable, we will be able to find out how to solve the problem of relating the "box", "crossed box" and iteration of the "single meson exchange" to each other in a way that leads to a unique specification of the "potential". Here we have an advantage over the discussions which started three decades ago. Since we start from integral equations which are unambiguously specified in terms of the multiple scattering series they generate, and are covariant, finite, unitary, "time reversal " invariant, ...most of the problems that arise when one starts from a "field theory" expansion in terms of powers of a "coupling constant" and then tries to make sense of this in terms of something that can be inserted in a non-relativistic Schroedinger equation[9] are missing. We trust that a "g^4 potential" which meets some of the needs of the relativistic

2-particle problem can be constructed this way. Whether a similar argument to that given above can be constructed to justify the use of this object in the N particle relativistic problem is more dubious.

The attack on problem (b) – the inclusion of antiparticles – has been undertaken by George Pastrana[10] . It is easy to start from Lindesay's three particle theory by calling the distinguishable elements "particle", "antiparticle" and "meson" with masses $m = \bar{m}$ and μ. Particle-antiparticle scattering then goes through state μ while particle (antiparticle) - meson scattering goes through state $m(\bar{m})$; we must now also require a conserved quantum number such that the number of particles minus the number of antiparticles is conserved. Starting from the $m\bar{m}, \mu\mu$ sector, the unitary theory which then results can be "crossed" to yield unique and unitary predictions for the $\mu m, \mu\bar{m}, mm, \bar{m}\bar{m}$ sectors. Putting these two approaches together should allow extensions to interesting quantitative applications in QED and QCD.

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On page 4, the first equation should read:

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