Reconstructing the equation of state and density parameter for dark energy from combined analysis of recent SNe Ia, OHD and BAO data

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Abstract. We reconstruct the dark energy equation of state by analyzing 5 sets of SNe Iadata along with Baryon Acoustic Oscillation (BAO) and Observational Hubble Data (OHD). A closed form parametrization of the luminosity distance in terms of redshift is assumed for the χ^2 analysis of the observational data. A strong dependence of dark energy equation of state on the matter density in the present and earlier epoch is obtained and the lower limit of matter density parameter at an earlier epoch for a flat FRW universe has been predicted. The variation of dark energy density parameter and the matter density parameter are also obtained.

1. Introduction

Observations on type Ia supernovae (SNe Ia) [1, 2], Baryon Acoustic Oscillation (BAO) [3], Hubble data based on differential ages of the galaxies (OHD) [4, 5] reveal that the universe is undergoing accelerated expansion in the present epoch. This can be explained by invoking the existence of dark energy - a hypothetical energy component with a negative pressure. In this paper we have made an attempt to reconstruct the equation of state of dark energy from the analysis of observational data from SNe Ia, BAO and OHD. Taking a parametric form of the luminosity distance $d_L(z)$ in terms of redshift z and considering the matter density at the present epoch (Ω_m^0) a free parameter, we perform a χ^2 analysis of these combined data sets to obtain the best-fit values and allowed ranges of the parameters - Ω_m^0 and those appearing in the parametrization of $d_L(z)$ - from the observational data. Assuming present universe to be spatially flat and containing only matter and dark energy, we further estimate the variation of $\omega_X(z)$ as a function of z for the best-fit values of the parameters and their 1σ limits as well for different data sets considered. The results of the analysis show that knowledge of the matter density of the universe at some earlier epoch is instrumental in providing observational evidences in favour of varying dark energy or cosmological constant solutions. We have also shown the simultaneous variation of matter density parameter $\Omega_m(z)$ and dark energy density parameter $\Omega_X(z)$ with z for the best-fit values of the parameters (obtained from χ^2 fitting) and their 1σ range. We also found the epoch at which the dark energy started dominating over the matter component of the universe.

2. Reconstructing the dark energy equation of state

In standard FRW cosmology, for a spatially flat universe, the luminosity distance $d_L(z)$ of an object at a redshift z is related to the Hubble parameter $H(z)$ as $H(z) = c[(d/dz)\{d_L(z)/1+z\}]^{-1}$ (c is the speed of light). Assuming an effective equation of state for the dark energy $w_X(z) = \rho_X(z)/p_X(z)$, the Hubble parameter can be expressed in terms of $w_X(z)$ which leads to an expression for $w_X(z)$ as given below.

$$\frac{H^2(z)}{H_0^2} = \Omega_m^0(1+z)^3 + \Omega_X^0 e^{\left(\int_0^z 3(1+w_X(z'))\frac{dz'}{1+z'}\right)}, \quad w_X(z) = -1 + \left[\frac{\frac{2}{3}\frac{(1+z)}{H(z)}\frac{dH(z)}{dz} - \Omega_m^0\frac{(1+z)^3}{H^2(z)/H_0^2}}{1-\Omega_m^0\frac{(1+z)^3}{H^2(z)/H_0^2}}\right]. \quad (1)$$

In the above H_0 is the value of the Hubble parameter at the present epoch. Ω_m^0 and Ω_X^0 respectively denote values of matter density ($\Omega_m(z)$) and dark energy density ($\Omega_X(z)$) parameters at the present epoch. The observational Hubble Data (OHD) based on the differential ages of the galaxies also provide values of the Hubble parameter $H(z)$ at some redshift z values. The Baryon Acoustic Oscillations (BAO) data is expressed in terms of the quantity $A(z_1)$ [3, 6] given as $A(z_1) = \frac{\sqrt{\Omega_m^0}}{[H(z_1)/H_0]^{1/3}}\left[\frac{1}{z_1}\int_0^{z_1}\frac{dz}{H(z)/H_0}\right]^{2/3}$ with $z_1 = 0.35$. A joint analysis of the SNe Ia, OHD and BAO data simultaneously constrain the parameters $H(z)$ and Ω_m^0.

Choosing a parametric form for $d_L(z)$ as [7], $d_L(a,b;z) = \frac{c}{H_0}\left[\frac{z(1+az)}{1+bz}\right]$, the quantities $H(z)$, $A(z_1)$ and $w_X(z)$ can be expressed analytically in terms of the parameters a, b and Ω_m^0. With these the expression for $w_X(z)$ and $\Omega_m(z)$ are now given by

$$\begin{aligned} w_X(a,b,\Omega_m^0;z) &= \frac{\frac{4}{3(1+bz)}\left[1+b+2bz-\frac{(1+z)(1+bz)(a+(ab+a-b)z)}{1+2az+(ab+a-b)z^2}\right]-1}{1-\Omega_m^0\frac{[1+2az+(ab+a-b)z^2]^2}{(1+z)(1+bz)^4}} \\ \Omega_m(a,b,\Omega_m^0;z) &= \Omega_m^0\frac{(1+z)^3(1+2az+(ab+a-b)z^2)^2}{[(1+z)(1+bz)]^4} \end{aligned} \quad (2)$$

3. Analysis of data

The parameters a, b and Ω_m^0 are obtained by performing a χ^2 analysis which involves minimization of suitably chosen χ^2 function. In the present analysis we have used the data from SNe Ia, BAO and OHD and define the χ^2 function as $\chi^2(a,b,\Omega_m^0) = \chi^2_{\rm SN}(a,b) + \chi^2_{\rm BAO}(a,b,\Omega_m^0) + \chi^2_{\rm OHD}(a,b)$. Here $\chi^2_{\rm SN}(a,b) = \sum_{i=1}^N(\mu_{obs}(z_i) - \mu_{th}(a,b,z))^2/\sigma_i^2$ is the χ^2 function for SNe Ia data, where $\mu_{\rm th}(a,b;z) = 5\log[H_0 d_L(z)/c] + \mu_0 = 5\log[z(1+az)/(1+bz)] + \mu_0$ is the distance modulus expressed in terms of redshift z and parameters a and b with $\mu_0 = 42.38 - 5\log_{10}h$, $H_0 = 100h$ km s^{-1} is the value of the Hubble parameter at the present epoch. $\mu_{\rm obs}(z)$ is its observed value. The χ^2 function for analysis of BAO data is given as $\chi^2_{\rm BAO}(\Omega_m^0,a,b) = [A(\Omega_m^0,a,b) - A_{\rm obs}]^2/(\Delta A)^2$ with $A_{\rm obs} = 0.469$, $\Delta A = 0.017$ and $A(\Omega_m^0,a,b)$ is the quantity A, expressed in terms of a and b as discussed earlier. The χ^2 function for the analysis of this observational Hubble data can be defined as $\chi^2_{\rm OHD}(a,b) = \sum_{i=1}^{15}[H(a,b;z_i) - H_{\rm obs}(z_i)]^2/\Sigma_i^2$ where $H_{\rm obs}$ is the observed Hubble parameter value at z_i with uncertainty Σ_i. In this paper we have considered different compilations of SNe Ia observations including the recent UNION2 data [8]. The other SNe Ia data sets considered here are [9, 10, 11, 12, 13]. Compilation of the observational data based on measurement of differential ages of the galaxies by Gemini Deep Deep Survey GDDS [14], SPICES and VDSS surveys provide the values of the Hubble parameter at 15 different redshift values [15, 16, 17, 18]. The values of the parameters a, b and Ω_m^0 at which minimum of χ^2 is obtained are the best-fit values of these parameters for the combined analysis of the observational data from SNe Ia, BAO and OHD. With these values of the parameters we find the variation of the dark energy equation of state $w_X(z)$ and the dark energy density parameter

($\Omega_X(z) = 1 - \Omega_m(z)$) respectively. We also find the 1σ ranges of the parameters a, b and Ω_m^0 from the analysis of the observational data discussed above and consequently the 1σ ranges of the quantities $w_X(z)$, $\Omega_X(z)$ and $\Omega_m(z)$. The computation of $\Omega_m(z)$ with the parameters a, b and Ω_m^0 as inputs from their 1σ ranges obtained in a way described above does not directly ensure that the condition $\Omega_m(z) \le 1$, which follows from the definition of $\Omega_m(z)$, is always respected. To circumvent this, $\Omega_m(z)$ at some particular value of z corresponding to an earlier epoch (beyond the range of measured redshifts $z \lesssim 1.76$ of SNe Ia events) is not allowed to exceed some chosen benchmark value (say, α) below 1. To take into account this constraint we find the domain of the (a, b, Ω_m) parameter space for which $\Omega_m(a, b, \Omega_m^0; z) \le \alpha$ is satisfied within the 1σ range. The 1σ range of the parameters thus obtained are, therefore, dependent on the initial condition of matter density at some earlier epoch which we choose here as $z = 2$.

Note that the choice of value of α is arbitrary. Such a choice has to be made as we see from the present analysis that the condition $\Omega_m(z) \le 1$ is not satisfied for all values of a, b and Ω_m^0 within their 1σ domain. But by such a choice this can be clearly demonstrated (as is done in the present work) that $\Omega_m(z)$ at an epoch (z) as evaluated from observational data requires the knowledge of Ω_m at some earlier epoch.

4. Results and discussions

The SNe Ia data sets considered here are HST+SNLS+ESSENCE [9, 10, 11], SALT2 data and MLCS data [13], UNION data [12] and UNION2 data [8]. In the analysis, we have taken each one of these five sets of SNe Ia data at a time with OHD and BAO data to compute $\chi^2 = \chi^2_{\rm SN} + \chi^2_{\rm OHD} + \chi^2_{\rm BAO}$ for different sets of values of the parameters a, b and Ω_m^0.

SNe Ia data sets + BAO + OHD	No of data points	best-fit values of $(a,\ b,\ \Omega_m^0)$	Minimum value of χ^2
(Set: I) HST+SNLS+ESSENCE + BAO + OHD	192+1+15	(1.437, 0.550, 0.268)	199.267
(Set: II) SALT2 + BAO + OHD	288+1+15	(1.401, 0.542, 0.272)	560.083
(Set: III) MCLS + BAO + OHD	288+1+15	(1.401, 0.653, 0.296)	783.078
(Set: IV) UNION + BAO + OHD	307+1+15	(1.635, 0.699, 0.268)	311.615
(Set: V) UNION2 + BAO + OHD	557+1+15	(1.289, 0.458, 0.272)	544.074

Table 1. Best-fit values of parameters and minimum values of χ^2 for each of the 5 data sets considered.

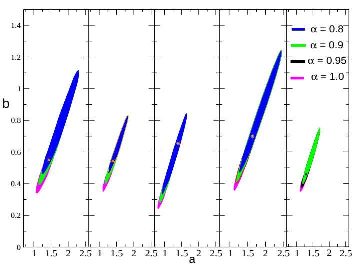

Figure 1. 1σ contour in the a - b parameter space (with marginalization over $\Omega_m^{(0)}$).

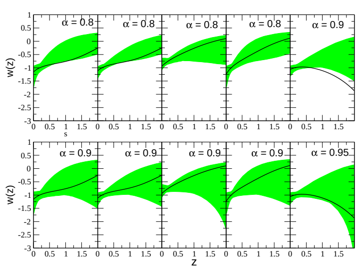

Figure 2. Plots of $w_X(z)$ vs z for different data sets and their 1σ limits for different values of α

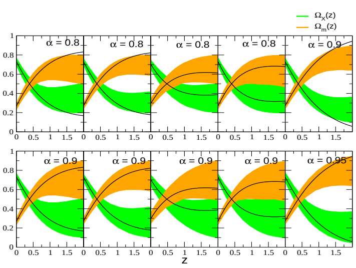

Figure 3. Plots of $\Omega_X(z)$ and $\Omega_m(z)$ vs z for different data sets

In Table 1 we present the best-fit values of the parameters a, b and Ω_m^0 obtained from analysis of different data sets. We then find the 1σ ranges of the parameters a, b and Ω_m^0 for each of the data sets (I-V). To ensure the boundedness: $\Omega_m(z) < 1$, we obtain the allowed ranges of the parameters for different values of α. The 1σ allowed region of the parameters obtained from the combined analysis of SNe Ia, OHD and BAO data are presented in the planes of any two parameters of set $\{a, b, \Omega_m^0\}$ by marginalizing over the third one. In Figure 1 we show the 1σ contours in parameter planes $a - b$ for $\alpha = 0.8, 0.9, 1.0$ ($\Omega_m(z = 2) \leq \alpha$). For data set V the same is plotted for $\alpha = 0.9, 0.95$ and 1. The values of α below 0.9 are not chosen for data set V (UNION2+OHD+BAO) because the value of $\Omega_m(z = 2)$ exceeds 0.9 even when calculated at the best-fit values of parameters a, b and Ω_m^0 obtained from the analysis of data set V. The UNION2 data (along with OHD and BAO) thus restricts the matter density parameter value at an epoch $z = 2$ to lie slightly below 0.95 (at 1σ level).

With the best-fit values of the parameters a, b and Ω_m^0 as obtained above (listed in Table 1 for different data sets) we compute the equation of state $w(z)$ of dark energy as a function of redshift z. The plots for $w(z)$ vs z are shown by solid curves in Figure 2 for all the 5 data sets. Plots from 1-5 (row-wise) correspond to data sets I-V. Using the 1σ range for the parameters a,b and Ω_m^0 as obtained above for different values of α ($\Omega_m(z) \leq \alpha$) we obtain the corresponding spread in $w(z)$. The shaded regions in Figure 2 show these 1σ bands of $w(z)$.

The same best-fit curves for $w_X(z)$ vs z are plotted both in the upper and lower panels of a given column. From Figure 2 we observe that, in some cases, the $w_X(z)$ vs z plots (solid curves) corresponding to the best-fit values of the parameters a, b and Ω_m^0 lie well within the respective 1σ regions. They barely remain within such regions in some other cases.

For a valid 1σ region the plot corresponding to the best-fit parameters should be fully contained within the 1σ spread. The obtained results given in Fig. 2 can thus be interpreted and summarized like this: The data sets I ((HST+SNLS+ESSENCE)+BAO+OHD) and II (SALT2+BAO+OHD) support the fact that matter density parameter Ω_m at an early stage of the universe at $z = 2$ was $\gtrsim 0.8$ whereas the data sets III (MLCS+BAO+OHD) and IV (UNION+BAO+OHD) can accommodate values of matter density parameter at the epoch $z = 2$ even a bit lower than 0.8. According to analysis of data set V (UNION2+BAO+OHD), the matter density parameter at $z = 2$ is only allowed to have values greater than 0.9. The 1σ ranges of the evolution of the parameters $\omega_m(z)$ and $\omega_X(z)$ for different choices of the value of α are also computed and are shown in Fig. 3. The analyses show that the nature of variations of the dark energy equation of state are similar for data sets I-IV and is different for the data set V. The results also indicate that the dark energy starts dominating the matter from the epoch $z \sim 0.4$ and the same from the analysis of data set V is found to be $z \sim 0.48$.

It is clear from the above discussion that in the present work we have performed an analysis without assuming any dark energy model. It may therefore be interesting to see how the parameters a and b in the present work behave for specific models. For example, in case of a flat $\Omega_m = 1$ model, the behaviour of a and b can be obtained from Eq. (1) by putting $\Omega_m(z) = \Omega_m^0 = 1$. This implies that the dark energy density $\Omega_X = 0$. The allowed values of a and b for such a situation are computed using Eq. (1) for non-zero z values upto $z = 2$ (around the SN Ia data limit) and the result is shown in Fig. 4 (left panel). The 1σ contour for $a - b$ for the representative case of UNION2 data is also shown in the same figure for comparison. It is evident that there is no overlap between the two. This is indeed expected since the dark energy density $\Omega_X = 0$. For Λ-CDM model however, the dark energy equation of state $w_X = -1$. The parameter space of $a - b$ for such a model can be obtained from Eq. (1) by demanding $w_X = -1$. The allowed parameter space of $a - b$ for such a model is obtained with $\Omega_m^0 = 0.27$. This is also shown in Fig. 4 (right panel) alongwith the allowed 1σ contour for $a - b$ obtained from UNION2 data, for comparison. The overlap of the two is expected as the 1σ spread for w_X obtained from the present analysis (and shown in Fig. 2) includes $w_X = -1$ for some values of z.

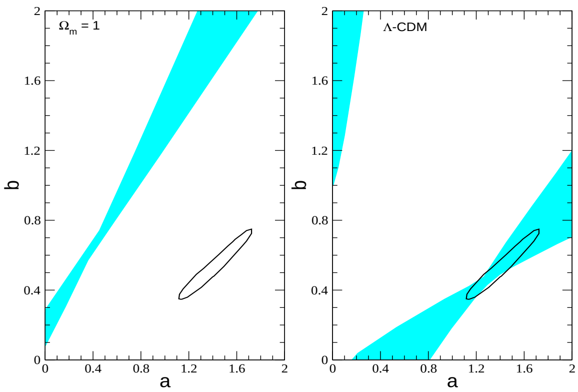

Figure 4. Left column: a-b parameter space that satisfy the model $\Omega_m = 1$. Right column: a-b parameter space for Λ-CDM model. The 1σ contour obtained from the analysis of Union2 data set is shown for comparison.

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