Effect of free parametrized TOV on properties of neutron stars

D Purnamasari and A Sulaksono[1, *]

[1] Physics Department, Faculty of Mathematics and Natural Sciences, University of Indonesia, Depok 16424, Indonesia

*anto.sulaksono@sci.ui.ac.id

Abstract. Recently, much progress has been reported about the properties of neutron stars (NS). However, in some parts such radius and moment of Inertia of NSs are still uncertain. We are interested in studying the properties of neutron stars also by using free parametric constraints. In this work, we use the modified Tolman Oppenheimer Volkoff (TOV) equation known as the parameterization of TOV (PTOV). Relativistic Mean Field (RMF) model is used to calculate the equation of state (EoS) of an NS. NS EoS is used as input to calculate some properties of NSs. The properties of neutron stars that are the focus of this research is mass, radius, and moment of inertia of slowly rotating NSs. We observe the effect of each free parameter added to the TOV equation, which results in a shift from the mass-radius diagram. We obtain the results which are consistent with the constraint of maximum mass $M < 2.35\ M_\odot$ and constraint of radius $11.38 < R_{1.4M\odot}$ (km) < 13.77. We observe that the contribution of each free parameter can increase mass and decrease the radii as well as provide an impact on the corresponding moment of inertia.

1. Introduction

Uncertainty in the equation of state (EoS) of neutron stars (NS) is one of the current problems for astrophysicists [1]. Constructing of EoS, the TOV equation, and macroscopic properties are the main steps for observing compact objects by astrophysicists. EoS is used as input to the TOV equation, which results in the properties of NSs. Many non-relativistic models have been developed up to now, and many equation states (EoS) are tested and confronted with experimental data and stellar observations. However, the theoretical prediction of NS radii is still poor [2].

The equation of state (EoS) is an essential component when observing neutron stars. In the previous studies have been observed that the PSR J1614 + 2230 and PSR J0348 + 0432 with maximum mass $(1.97 \pm 0.04)\ M_\odot$ and $(2.01 \pm 0.04)\ M_\odot$. The self-interaction of repulsive vector mesons in the RMF model plays a role in making EoS stiffer, which gives a larger maximum mass [3]. The radius constraint of the NS GW170817 is in a range of $12.00 < R_{1.4M\odot}$(km) < 13.45 in Ref. [4], but in Ref. [5] lies between $9.8 < R_{1.4M\odot}$(km) < 13.2 with a representative mass of $1.4M_\odot$. It seems that NS requires a relatively large maximum mass and a relatively short radius. These requirements are theoretically challenging.

We present the parameterization of the TOV (PTOV) model, where the model is characterized by the existence of free parameters where each parameter contributes to the TOV equation [2]. So in this work, the EoS is used as an input to calculate the properties of neutron stars based on the modified TOV equation in Ref. [1]. We also see the effect of varying the free parameters. The parameter variation aims

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to see which terms in the standard TOV equation influence to increase or decrease the mass and radius of the star which produce the maximum mass of neutron stars that will be indicated by the mass-radius diagram then we calculate the moment of inertia. The later has not been studied before.

2. Formalism

In this section, describe the state of an NS. The GR theory is used. EoS from an NS is known as the relationship between energy density and pressure in an NS that must be known in advance to solve the TOV equation. NSs are assumed to be neutral (not electrically charged). The particles in the neutron star core are assumed to be homogeneously distributed so that within the RMF approach, the system is in thermodynamic equilibrium.

2.1. Equation of State (EoS)

We use EoS which is calculated by using the RMF model. This RMF model uses effective Lagrangian density with the isovector scalar nucleon coupling (see Ref. [7-10]). In this work, we also construct the equation of state (EoS) with the RMF approach where the exchange of σ, ω and ρ mesons explains the interaction among the nucleons [11].

2.2. Standard Tolman Oppenheimer Volkoff (TOV) Equations

The Tolman Oppenheimer Volkoff (TOV) equation is usually used to describe the structure of a static, non-rotating and spherical compact star in general relativity. The TOV equations are given as,

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{c^2r^2}\left[1+\frac{P(r)}{\rho(r)}\right]\left[1+\frac{4\pi r^3P(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{c^2r}\right]^{-1} \tag{1}$$

$$\frac{dM(r)}{dr} = 4\pi\rho(r)r^2 \tag{2}$$

where G is gravitational constant and c speed of light propagation where we use c = 1 unit [6].

2.3. Parametrized TOV (PTOV) Equations

The parametrized TOV equations can be written as,

$$\frac{dP(r)}{dr} = -\frac{G(1+\alpha)\rho(r)\tilde{M}(r)}{c^2r^2}\left[1+\frac{\beta P(r)}{\rho(r)}\right]\left[1+\frac{\chi 4\pi r^3P(r)}{\tilde{M}(r)}\right]\left[1-\frac{\gamma 2G\tilde{M}(r)}{r}\right]^{-1} \tag{3}$$

$$\frac{d\tilde{M}(r)}{dr} = 4\pi r^2(\rho(r)+\sigma P) \tag{4}$$

where there are five free parameters namely $\alpha, \beta, \chi, \gamma$ and σ which have contribution for mass and radii of the compact object like an NS [7].

According to Ref. [1, 2], each free parameter namely $\alpha, \beta, \chi, \gamma$ and σ in the PTOV equations above have the following interpretation :

- In stellar objects such as neutron stars, the parameter α measures the degree of coupling of matter related to the possible effects of the gravity coupling G_{eff} = G (1 + α). The number α in GR is 0, and f (R) in the modified gravity theory, α is equal to 1/3 [13].
- The β parameter contributes to inertial pressure. β located in the term contribution $(\rho + P)$ arises from the hydrostatic equilibrium which plays the role of inert mass density derived from the conservation law $\nabla_\mu T^{\mu\nu} = 0$. The number of β in GR is 1.

- Parameter χ is related to pressure effects on self-gravity that investigated in GR so that the number of $\chi = 1$. If the readers are interested in learning this effect, see Ref. [14] for more details.
- γ contributes to intrinsic curvature for the relativistic case, not a non-relativistic physics. The number of parameter γ is equal to 0 in a non-relativistic physics case and equal to 1 in GR.
- The parameter σ affects the mass function in the PTOV equation. This parameter also measures the effect of the pressure contribution. The number of σ in GR is equal to 0.

This behavior has the potential to remedy the problem of standard TOV by assuming a realistic EoS and then varying these parameters [15].

3. Results and Discussions

We examine the contribution of the free parameter terms shown in the PTOV equation above, which are derived from star hydrostatic equilibrium. Starting from the EoS previously explained. We analyze the contribution of free parameters in the PTOV equation with the mass-radius diagram like Fig. 1 below. However, the values of each free parameter have been compared by looking at the effect on the mass-radius diagram.

The results obtained in Table. 1 below, we can see the effect from the new free parameter in the PTOV equation to mass-radius relation with fixed parameter γ =1 in the GR context. Here we want to know the effect of each parameter on the mass-radii diagram by varying the various values of each α, β, σ and χ parameter and getting the best plot, as shown in Fig. 1. We analyze that by varying α parameter, it will increase the mass of stars, which can be interpreted to be very influential on mass-radii relations, which can be seen in the plot between GR and set 1 in Fig.1 below.

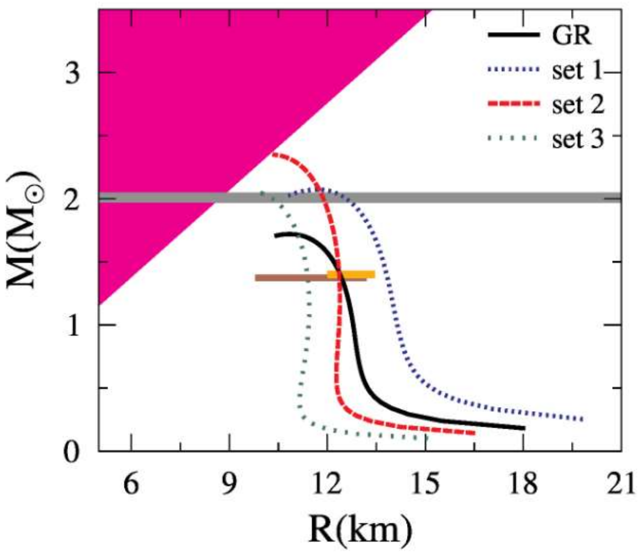

Figure 1. Mass-radii relation for neutron stars described by varying the free parameter.

Now we compare the plot set 1 and set 2 in Fig. 1. Here we vary the four parameters like Table. 1 with set γ equal to 1 and it can be seen that the results can increase the mass and decrease the radius. Comparing set 2 and set 3 can reduce the radius which contributes to the decrease in β and χ parameters and increases α parameter while the σ parameter is fixed.

Table 1. Values of the α , β , σ and χ parameter in Fig. 1 and Fig. 2.

		GR	Set 1	Set 2	*Set 3*
Parameters	α	0	-0.1	0.2	0.5
	β	1	1	0.3	0.2
	σ	0	0	- 0.8	- 0.8
	χ	1	1	0.2	0.1
Neutron stars	M_{max}	1.72 M⊙	2.07 M⊙	2.35 M⊙	2.05 M⊙
	R_{Mmax}	10.84 km	11.67 km	10.32 km	9.88 km
	$R_{1.4M\odot}$	12.38 km	13.77 km	12.36 km	11.38 km
	C_{Mmax}	0.16	0.18	0.23	0.21
	$C_{1.4M\odot}$	0.11	0.10	0.11	0.12

In this work, we check the implementation of the free parameters in Ref. [1,2]. The authors in Ref. [1] varies all free parameters where the value of α parameter equals to 0, 0.3 and -0.3. Whereas the authors in Ref. [2] maintain a fixed α parameter is equal to 0.3 which $\alpha = 0$ in GR case and fixed $\gamma = 1, \sigma = 1/6$, while the other parameters are varied, but there are additional correction terms introduced in Ref. [2].

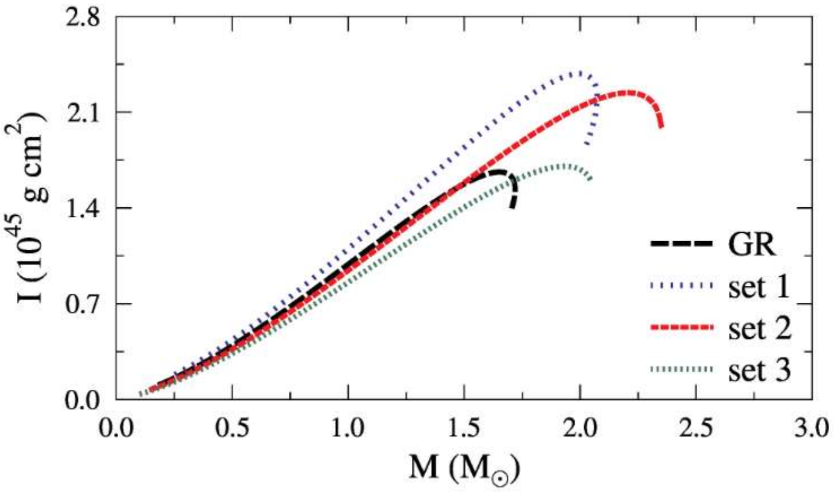

Figure 2. Momen of inertia as a function of NS mass.

By analyzing each new free parameter that exists in each term in the TOV equation, we can see the effect of each parameter. The reason we analyze each free parameter is that it can describe massive neutron (NS) stars but also marked with low radii and using parameterized TOV equations. It means this model can produce the appropriate maximum mass of neutron stars and also we can get proper canonical star radii. This analysis is crucial to obtain appropriate mass and radius of NS as well as the corresponding moment of inertia. The α parameter measures the level of matter coupling in NS objects which can be seen when we redefine the effective gravitational coupling of $G_{eff} \rightarrow G(1+\alpha)$ modified gravity theory. The β parameter measures the likelihood of contributions arising from inertia pressure which can be seen by the contribution of the term $(\rho + P)$ derived from conservation law. Parameter χ is related to pressure effects on self-gravity and the γ parameter contributes to intrinsic curvature. The parameter σ affects the mass function as well for the pressure contribution. We explore the effects of the new free parameters added to Eq. (3) and Eq. (4) using fixed configurations for the parameters α, β, γ, χ, and σ that are constructed to produce mass-radius diagrams. If we obtain mass and radius of NS then with a derivative we get the moment of inertia of the star which can be seen in Fig. 2.

The GW170817 binary system of combining two NSs gives the upper limit of the neutron star mass from 2.2 to 2.3 $M_\odot$ [16-18]. The compactness of the stars, C_{Mmax} and $C_{1.4M\odot}$, is a comparison between the corresponding masses and radii of the neutron stars in Table. 1. The compactness of neutron star in Ref. [19] is equal to 0.105 ± 0.002. Based on Table. 1 above, we see that neutron stars are more compact than their counterparts. In this work, we maintain maximum masses high and the radii can decrease according to Refs. [20-24]. We get the constraint of radii $11.38 < R_{1.4M\odot}(\text{km}) < 13.77$ almost the same with Ref. [4] and Ref. [5] that shown in Fig. 1 as yellow and brown lines, respectively. Then we also get the constraint of maximum mass $M < 2.35\ M_\odot$. The most influential parameter is the α parameter for radii, β and χ parameters for mass. The larger NS mass M and NS radius R affect the moment of inertia. We have shown this in Fig. 2. The moment of inertia of slowly rotating NSs affects the isovector scalar nucleon coupling in the RMF theory [25]. The changing of δ meson would soften the EOS and give the smallest stellar radius and maximum mass (see Ref.[25] for more details).

4. Conclusions
We can conclude that we have found appropriate parameterizations of the TOV equations (PTOV) which all astrophysical constraints proposed so far can be satisfied. As compared with the results presented in Refs. [1,2], our results can explain the effect of each free parameter in the parameterized TOV equation where we maintain maximum masses high, and the radii can decrease in such a way that all the values proposed can be attained that shown in the mass-radii diagram. This investigation not only studies the impact of |EoS on the properties of NS but also provides insight into the role of each term in the TOV equation. Finally, we obtain results that consistent with the constraint of maximum mass $M < 2.35\ M_\odot$ and constraint of radii $11.38 < R_{1.4M\odot}$ (km) < 13.77. For future work, we expand this study by using other EoS such as non-relativistic models by modifying the standard TOV equation to investigate white dwarfs.

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