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Status of the anomalous magnetic moment of the muon in spring 2019

Pere Masjuan
Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona
Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology (BIST)
Campus UAB, E-08193 Bellaterra (Barcelona), Spain

Abstract

In this talk we review the recent progress on the numerical determination of the Hadronic contributions to the anomalous magnetic moment of the muon. Special emphasis on the role of experimental data on the light-by-light scattering is made.

1 Introduction

The anomalous magnetic moment of the muon $(g-2)_\mu$ is one of the most accurately measured quantities in particle physics, and as such is a very promising signal of new physics if a deviation from its prediction in the Standard Model is found. The present experimental value for $a_\mu = (g-2)_\mu/2$, is given by $a_\mu^{\rm EXP} = 11659209.1(6.3)\times 10^{-10}$, as an average of $a_{\mu^+} = 11659204(7.8)\times 10^{-10}$ and $a_{\mu^-} = 11659215(8.5)\times 10^{-10}$ [1, 2]. Since statistical errors are the largest source of uncertainties, a new measurement with a precision of 1.6×10^{-10} is been pursuit at FNAL [3] nowadays and at JPARC in a near future [4]. In particular, the new experiment at FNAL [3] pretends to reduce the overall error by storing 20 times more muons, by producing a more stable magnetic field (better and more carefully measured), and by improving the precision of the frequency measurement (via high-fidelity recording of muon decay).

At the level of the experimental accuracy, the QED contributions has been completed up to the fifth order $\mathcal{O}(\alpha_{em}^5)$, giving the QED contribution $11658471.885(4)\times 10^{-10}$ [5], using the Rydberg constant and the ratio m_{Rb}/m_e as inputs [2]. Also electroweak contributions are necessary, since they reach $15.4\pm 0.1\times 10^{-10}$ [6]. Hadronic contributions in terms of the hadronic vacuum polarization (HVP) and the hadronic light-by-light scattering (HLBL) represent the main uncertainty in the Standard Model. In this talk, we will attempt to update the hadronic contributions to a_μ.

A white paper by the newly created *The Muon g-2 Theory Initiative* trying to get a *consensus* is on its way, accompanying the aforementioned experimental effort, and paving the path to new physics [7]. The results here described update Refs. [8] reaching spring 2019 only. We propose a particular procedure to combine the different results in the literature concerning hadronic contributions to the muon $(g-2)$.

2 The Hadronic Vacuum Polarization (HVP)

The hadronic vacuum polarization contributions are calculated utilising dispersion integrals and the experimentally measured cross section $\sigma^0_{\rm had,\gamma}(s) \equiv \sigma^0(e^+e^- \to \gamma^* \to {\rm hadrons} + \gamma)$, where the superscript 0 denotes the bare cross section and the subscript γ indicates the inclusion of effects from final state photon radiation. At leading order, the dispersion relation reads

$$a_\mu^{\rm had,\,LO\,VP} = \frac{\alpha^2}{3\pi^2}\int_{s_{\rm th}}^{\infty} \frac{{\rm d}s}{s} R(s) K(s)\,, \tag{1}$$

where $\alpha = \alpha(0)$, $s_{\rm th} = m_\pi^2$, $R(s)$ is the hadronic R-ratio given by

$$R(s) = \frac{\sigma^0_{\rm had,\gamma}(s)}{\sigma_{\rm pt}(s)} \equiv \frac{\sigma^0_{\rm had,\gamma}(s)}{4\pi\alpha^2/(3s)}\,, \tag{2}$$

where $K(s)$ is a well known kernel function [9] and behaves as $1/s$, thus causing the HVP contributions to be dominated by the low-energy domain. At next-to-leading order, the data input is identical, but modified kernel function. Notice that final state radiative corrections and vacuum polarization corrections are taken into account. At spring 2019, two different results confront each other: DHMZ17 yields $693.1 \pm 3.4 \times 10^{-10}$ [10] and KNT18 yields $693.3 \pm 2.5 \times 10^{-10}$ [11]. Both using the same data sets, do not find yet perfect agreement, even though the distance among them have been reduced since their last respective publications (DHMZ10 yielding $692.3 \pm 4.2 \times 10^{-10}$ [12], and HLMNT11 $694.9 \pm 4.3 \times 10^{-10}$ [13]). The central value is in agreement, the error is still slightly different, but most important, reduced by $30-40\%$ in both cases. Nonetheless, the central values is a coincidence. Such value is obtained after summing up the different exclusive channels accounting for the inclusive measurement. When exploring in detail each individual contribution, the agreement is not that good. The largest difference occurs in the $\pi\pi$ channel, where the mean value in KNT18 is lower by almost 1σ of the DHMZ17 analysis. Notice that the KNT18 is also lower to the previous HLMNT11 result due to the new, precise and highly correlated radiative return data from KLOE and BESIII and the capability of the new data combination method to utilise the correlations to their full capacity. The difference between KNT18 and DHMZ17 in this $\pi\pi$ channel is larger than the global error of the sum of all channels. This difference which comes from how choices with regard to data combination affect results is a systematic error that should be taken into account to get a final HVP result. We propose to consider the individual differences in each channel as a systematic error. For example, the $\pi\pi$ from KNT18 reads $503.74 \pm 1.96 \times 10^{-10}$, while from DHMZ17 reads $507.14 \pm 2.58 \times 10^{-10}$, a difference of -3.40×10^{-10}, a 1.05σ effect. The systematic difference -3.40×10^{-10} translates into a systematic error that should be add as well. We opt to combine statistical errors uncorrelated (they are correlated though in a way we cannot guess) and the -3.40×10^{-10} as a 100% correlated systematic error (accounting for the unknown statistical correlation) to yield in the $\pi\pi$ case $504.98 \pm 3.74 \times 10^{-10}$. Doing so for all channels returns the final value $a_\mu^{\rm had,\,LO\,VP} = +693.47 \pm 4.36 \times 10^{-10}$. Using the same set of data and procedure, we would obtain $a_\mu^{\rm had,\,NLO\,VP} = -9.82 \pm 0.04 \times 10^{-10}$ but also $a_\mu^{\rm had,\,NNLO\,VP} = +1.24 \pm 0.01 \times 10^{-10}$ [14], in agreement with KNT18. The final result is then

$$a_\mu^{\rm had,\,Total\,VP} = +684.89 \pm 4.33 \times 10^{-10}\,. \tag{3}$$

3 The Hadronic Light-by-Light contribution (HLBL)

For the HLBL, two reference numbers can be found in the literature: $a_\mu^{\rm HLBL} = (11.6\pm4.0)\times10^{-10}$ [15] but also $(10.5\pm2.5)\times10^{-10}$ [16]. The overall HLBL contribution is twice the order of the present experimental error, thus we really need to reduce the errors down to 10%. The progress on the field is captured in [17] and the aforementioned theory initiative (https://indico.fnal.gov/conferenceDisplay.py?confId=13795).

The HLBL cannot be directly related to any measurable cross section and requires knowledge of QCD at all energy scales. Since this is not known yet, one needs to rely on hadronic models to compute it. Such models introduce systematic errors which are difficult to quantify. Using the large-N_c and the chiral counting, de Rafael proposed [18] to split the HLBL into a set of different contributions: pseudo-scalar exchange (PS, dominant [15, 16]), charged pion and kaon loops, quark loop, and higher-spin exchanges. The large-N_c approach however has at least two shortcomings: firstly, it is difficult to use experimental data in a large-N_c world. Secondly, calculations carried out in the large-N_c limit demand an infinite set of resonances. As such sum is not known, one truncates the spectral function in a resonance saturation scheme, the Minimal Hadronic Approximation (MHA) [31]. The resonance masses used in each calculation are then taken as the physical ones from PDG [2] instead of the corresponding masses in the large-N_c limit. Both problems might lead to large systematic errors not included so far [22, 24, 25, 28].

Actually, most of the results in the literature follow de Rafael's proposal finding values for $a_\mu^{\rm HLbL}$ between 6×10^{-10} and up to 14×10^{-10} (see [19, 20, 21, 15, 16, 32, 22, 23, 25, 27, 24, 26, 28, 29], including full and partial contributions to $a_\mu^{\rm HLbL}$). Such range almost reaches ballpark estimates based on the Laporta and Remiddi (LR) [33] analytical result for the heavy quark contribution to the LBL. The idea in such ballparks is to extend the perturbative result to hadronic scales low enough for accounting at once the whole HLBL. The free parameter is the quark mass m_q. The recent estimates using such methodology [34, 23] found $m_q\sim0.150-0.250$ GeV after comparing the particular model with the HVP. The value for the HLBL is around $a_\mu^{\rm HLBL} = 12-17\times10^{-10}$, which seems to indicate that subleading pieces of the standard calculations are not negligible.

Jegerlehner and Nyffeler's review [15] together with the *Glasgow consensus* written by Prades, de Rafael, and Vainshtein [16] represent, in our opinion, the two reference numbers. They agree well since they only differ by few subtleties. For the main contribution, the pseudoscalar, one needs a model for the pseudoscalar Transition Form Factor (TFF). They both used the model from Knecht and Nyffeler [20] based on MHA, but differ on how to implement the high-energy QCD constrains coming from the VVA Green's function. In practice, this translates on whether the piece contains a pion pole or a pion exchange. The former would imply that the exchange of heavier pseudoscalar resonances is effectively included in PS [21], while the latter demands its inclusion. The treatment of errors, summed linearly [15] or in quadrature [16], is also a difference. All in all, even though the QCD features for the HLbL are well understood [15, 16], the details of the particular calculations are important to get the numerical result to the final required precision. We think we need more calculations, closer to experimental data if possible.

Dispersive approaches [35, 29] relies on the splitting of the former tensor into several pieces according to low-energy QCD, which most relevant intermediates states are selected according to their masses [18, 36]; see Refs. [29] for recent advances. An advantage we see in this approach is that by decomposing the HLBL tensor in partial waves, a single contribution may incorporate pieces that were separated so far, avoiding potential double counting. The example is the $\gamma\gamma\to\pi\pi$ which includes the two-pion channel, the pion loop, and scalar and tensor contributions.

Finally, there have been different proposals to perform a first principles evaluation by using lattice

QCD [37]. They studied a non-perturbative treatment of QED which later on was checked against the perturbative simulation. With that spirit, they considered that a QCD+QED simulation could deal with the non-perturbative effects of QCD for the HLBL. Whereas yet incomplete and with some progress still required, promising advances have been reported already [37].

The lack of experimental data, specially on the doubly virtual TFF, is an obstacle for calculations. Fortunately, data on the TFF when one of the photons is real is available, from different collaborations, for π^0, η and η'. It is common to factorize the TFF, and describe it based on a rational function. One includes a modification of its numerator to fulfil high-energy QCD constraints. Although the high-energy region of the model is not very important, it still contributes around 20%. More important is the double virtuality, especially if one uses the same TFF model (as it should) for predicting the $\pi^0 \to e^+e^-$ decay. Current models cannot accommodate its experimental value (see [38]). The worrisome fact is that modifying the model parameters to match such decay and going back to the HLBL, would result in a dramatic decrease of the HLBL value [38].

While the HLBL requires knowledge at all energies, it is condensed in the Q^2 region from 0 to 2 GeV^2, in particular above around 0.5 GeV^2. Therefore a good description of TFF in such region is very important. Such data are not yet available, but any model should reproduce the available one. That is why the authors of [22, 23, 24, 28], in contrast to other previous approaches, did not used data directly but the low-energy parameters (LEP) of the Taylor expansion of the TFF and reconstructed it *via* the use of Padé approximants (PA) and Canterbury approximants (CA) for the two dimensional case [38]. As demonstrated in Ref. [28], the pseudoscalar TFF is Stieltjes functions for which the convergence of PA's sequence is guaranteed in advanced. As such, a comparison between two consecutive elements in this sequence estimates the systematic error yield by the method. In other words, PA for the TFFs take full advantage of analyticity and unitary of these functions to correctly extrapolate low- and high-energy regions. The LEPs were obtained in [22] for the π^0, in [39] for the η-TFF and in [40] for the η'-TFF, taking into account the $\eta - \eta'$ mixing [40, 41] and the determinations of the double virtual π^0 [38] and η, η' [42] TFFs. Ref. [28] collects the most updated results for the space- and time-like TFF together with $\gamma\gamma$ decays from 13 different collaborations, to yield a most precise PS contribution to the HLBL.

The aforementioned White Paper pretends a consensus for the HLBL with the following criteria: *i*) the TFF normalisation should be given by real-photon decay and should follow high-energy constraints. *ii*) At least space-like experimental data for the single-virtual TFF must be reproduced. *iii*) Systematic uncertainties must be assessed with reasonable procedure. Among all the aforementioned calculations only two of them satisfy the criteria for the π^0 and only one for the $\eta(')$. For the π^0, Ref [28] yields $a_\mu^{\text{HLBL},\pi^0} = 6.36 \pm 0.36 \times 10^{-10}$ which was later on corroborated by [30] yielding $a_\mu^{\text{HLBL},\pi^0} = 6.26^{+0.30}_{-0.25} \times 10^{-10}$. Taken the difference among them as a purely systematic 100% correlated error, we can combine them to obtain $a_\mu^{\text{HLBL},\pi^0} = 6.30 \pm 0.24 \times 10^{-10}$. Adding the $a_\mu^{\text{HLBL},\eta} = 1.63 \pm 0.19 \times 10^{-10}$ and $a_\mu^{\text{HLBL},\eta'} = 1.45 \pm 0.17 \times 10^{-10}$ from Ref. [28] the final PS contribution would result in:

$$a_\mu^{\text{HLBL,PS}} = 9.38 \pm 0.67 \times 10^{-10}\,. \tag{4}$$

In conclusion, new experimental data used to update the PS in Ref. [28] seem to reveal larger contributions from pseudsocalar mesons, and that the TFF is more important than expected. Also, systematic errors are important and difficult to evaluate, but PAs can help. Lattice QCD seems promising but only in the long run. Dispersion relations are useful at low energies but a consensus will be needed in order to combine with other contributions. On top, ballpark predictions coincide on drawing larger values, indicating the need to better understand the whole HLBL.

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