# A comparison of different choices of clocks in a reduced phase space quantization in loop quantum cosmology with an inflationary potential using effective techniques

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We review recent results that apply a reduced phase space quantization of loop quantum cosmology (LQC) for a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with reference fields and an inflaton field in a Starobinsky inflationary potential. All three models that we consider are two-fluid models and they differ by their choice of global clock which are chosen to be either Gaussian dust, Brown-Kuchař dust or a massless Klein-Gordon scalar field. Although two-fluid models are more complicated than models involving the inflaton only, it turns out that some of the technical hurdles in conventional quantum cosmological models can be bypassed in these models. Using the effective dynamics resulting from the reduced phase space quantization we discuss some phenomenological implications of these models including the resolution of the big bang singularity via a quantum bounce and in addition address the question whether different choices of clocks can leave an imprint on the inflationary dynamics.

*Keywords*: Loop quantum cosmology, reduced phase space quantization, effective techniques.

## 1. Introduction

Within the last decade in the framework of loop quantum gravity new models have been introduced that apply the technique of reduced phase space quantization to construct the physical Hilbert space and thus the physical sector of the theory.[1–9] This requires to construct Dirac observables at the classical level and derive their corresponding algebra. For this purpose all models have in common that they couple some kind of additional matter to gravity. In the context of the relational formalism, introduced in Refs. 10, 11 and further developed and applied in Refs. 12–17, these additional degrees of freedom are used as reference matter to construct Dirac observables and their corresponding dynamics. The latter is encoded in a so called physical Hamiltonian that becomes a non-vanishing Hamiltonian operator in the physical Hilbert space involved either in a Schrödinger-like or Heisenberg-like equations. For the reason that these reference fields are dynamically coupled to general relativity an interesting question in this context is how a given choice of reference fields influences the physical properties of the model. In full loop quantum gravity these dynamical equations have a complicated structure and an analysis for individual models as well as a comparison between different models is a non-trivial task. Therefore, the work in Ref. 18 focus on a simplified setting in the framework of loop quantum cosmology where techniques are already available to analyze such questions. The relational formalism has been applied in the context of loop quantum cosmology for instance in Refs. 18–33. The main difference to former models in loop quantum cosmology with reference matter, often also called clocks in cosmology, is that the work in Ref. 18 considers two-fluid models because the clock is coupled in addition to an inflaton. One of the main questions that one is interested in is how the inflationary scenario is affected by the presence of the additional clock degree of freedom and how the imprint of the clock compares for different models. Such questions will be investigated using effective techniques that in LQC have in former work mainly be applied to one-fluid models.

### 1.1. *Dust and scalar field clock models in loop quantum cosmology*

All three models in the analysis in Ref. 18 are based on models in general relativity coupled to an inflaton field and an additional coupling of 8 and 7 respectively additional dust and scalar fields respectively yielding to a system with second class constraints. After the reduction with respect to the second class constraints one ends up with first class systems with four additional reference fields, that can be used as reference matter for the Hamiltonian and spatial diffeomorphism constraint. The details about the Brown-Kuchař dust model can be found in Ref. 34 and its quantization using LQG techniques has been performed in Ref. 1. For the classical Gaussian dust model we refer the reader to Ref. 35 and its LQG implementation has been discussed in Ref. 36. The four scalar field model in Refs. 5, 6 can be understood as a modification of the model in Ref. 37, that, as shown in Ref. 5 cannot be quantized in the framework of LQG. For flat FLRW spacetimes where the spatial diffeomorphism constraint vanishes identically, the corresponding symmetry reduced models involve one temporal reference field, the clock, only. A reduced phase space quantization for all three symmetry reduced models has been derived in Ref. 18. There it is shown that in all three models the quantum dynamics is encoded in a Schrödinger-like equation in the physical Hilbert space. The explicit form of the physical Hamiltonian differs for the dust and scalar field models where the latter involves a square root. Although the model involve an inflaton with a generic potential due to the fact that clock is coupled additionally and one does not use the inflaton as the clock, as it has for instance be done in the APS-model in Ref. 21, all models possess physical Hamiltonians that are time-independent. This is of advantage for the construction of the physical inner product of the individual models. In this review we will focus on the effective dynamics of these models that were used in Ref. 18 to investigate the above mentioned questions. The quantum dynamics is formulated in the volume representation in Ref. 18 and thus the set of elementary variables in the dynamical equations are $\mathcal{O}_b, \mathcal{O}_V, \mathcal{O}_\varphi, \mathcal{O}_{\pi_\varphi}$, where $\mathcal{O}_f$

denotes the Dirac observable of quantity $f$. As shown in Ref. 18 the effective physical Hamiltonians of the dust and scalar field model take the form

$$\mathbf{H}_{\text{eff}}^{\text{FLRW, dust}} = -\frac{3\mathcal{O}_V}{8\pi G\lambda^2\gamma^2}\sin^2(\lambda\mathcal{O}_b) + \frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V} + \mathcal{O}_V U(\mathcal{O}_\varphi) \tag{1}$$

$$\mathbf{H}_{\text{eff}}^{\text{FLRW, scalar}} = \sqrt{-2\mathcal{O}_V\left(-\frac{3\mathcal{O}_V}{8\pi G\lambda^2\gamma^2}\sin^2(\lambda\mathcal{O}_b) + \frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V} + \mathcal{O}_V U(\mathcal{O}_\varphi)\right)}, \tag{2}$$

where $\lambda = \sqrt{\Delta} = \sqrt{4\sqrt{3}\pi\gamma\ell_p^2}$ denotes the polymerization parameter. Taking into account that the elementary Dirac observables satisfy the following standard Poisson brackets $\{\mathcal{O}_b, \mathcal{O}_V\} = 4\pi G\gamma$ and $\{\mathcal{O}_\varphi, \mathcal{O}_{\pi_\varphi}\} = 1$ with $\gamma$ denoting the Immirzi parameter and where all remaining Poisson brackets vanish, the system of equations of motion for the Dirac observables in the individual models has been been derived in Ref. 18 and they provide the basis for the results discussed in the next section. Note that although in the case of flat FLRW spacetimes the physical Hamiltonian of the Brown-Kuchař model and the Gaussian dust model agree, these model are still not identical because within the Brown-Kuchař model the dust energy density can be chosen to be either positive or negative and the effect of these two different choices has also been analyzed in Ref. 18.

## 2. The effect of choosing different clocks on inflation

Similar to the one-fluid models that are obtained via Dirac quantization as for instance the APS-model in Ref. 21, the effective dynamics in the scalar field and dust models can be rewritten in terms of a modified Friedmann equations of the form

$$\mathcal{O}_H^2 = \frac{\dot{\mathcal{O}}_V^2}{9\mathcal{O}_V^2} = \frac{8\pi G}{3}\mathcal{O}_\rho\left(1 - \frac{\mathcal{O}_\rho}{\rho_{\max}}\right) \tag{3}$$

with $\rho_{\max} = \frac{3}{8\pi G\gamma^2\lambda^2}$. The maximal density $\rho_{\max}$ is the same as in Refs. 21–23 but here $\mathcal{O}_\rho$ does not only depend on the inflaton but also on the clock energy density. Furthermore, the temporal coordinate with respect to which the Hubble parameter is determined is given by either the dust and scalar field clock respectively.

In order to analyze how a given choice of clock might affect inflation in Ref. 18 a Starobinsky potential

$$U = \frac{3m^2}{16\pi G}\left(1 - e^{-\sqrt{\frac{16\pi}{3}}\mathcal{O}_\varphi}\right)^2 \quad \text{with} \quad m = 2.44 \times 10^{-6} \tag{4}$$

was considered. The choice of initial conditions, set at the bounce, was guided by former results in LQC models[38] and chosen to be $\mathcal{O}_{b_i} = \pi/2\lambda$ and $\mathcal{O}_{V_i} = 10^3$ in Planck units. Fixing further values for $\mathcal{O}_{\varphi_i}, \mathcal{O}_{\pi_\varphi}$ determines $\mathbf{H}_{\text{eff}}^{\text{FLRW, dust}}$ from which one can obtain $\mathcal{O}_{\rho^{\text{clock}}}$. Therefore in Ref. 18 the initial values are parametrized by $(\mathcal{O}_\varphi, \mathcal{O}_{\rho^{\text{clock}}})$. In the case of the dust models a choice of initial conditions given by $\mathcal{O}_{\varphi_i} = -1.45$, $\mathcal{O}_{\rho^{\text{clock}}} = 10^{-8}$ yields to a quantum model with a pre-inflationary phase, often being present in LQC models after the bounce, and an inflationary phase where the number of inflationary e-foldings for the latter is 63.1. For a similar choice of initial conditions the one-fluid models without an additional clock degree of freedom yield 63.9 inflationary e-foldings, see Ref. 38. This shows that for these choice of initial conditions the dust clock plays only a subdominant role for inflation. This is exactly what one expects from a good clock because such a clock should not have a dominant imprint on the dynamics of the model. A similar analysis for the scalar field model leads to 63.8 inflationary e-foldings being closer to the value obtained in the one-fluid models and showing that the effect of the scalar field clock is weaker compared to the dust clock. However, this is also expected from the fact that the energy density of the scalar field comes with a higher inverse power of the scale factor than the dust contribution and thus decaying faster in the evolution. In a further investigation to better understand the influence of the clock energy density on inflation in Ref. 18 for the same set of initial conditions for the inflaton a varying dust energy density ranging from $10^{-8}$ up to $10^{-4}$ was considered. The results show that the number of inflationary e-foldings decrease with increasing dust energy density. The explanation given in Ref. 18 is that this results from the fact that $\mathcal{O}_{\varphi_{\text{on}}}$, the value of scalar field's Dirac observable at the onset of inflation, decreases due to a larger Hubble friction when the dust energy density is increased. For an initial dust energy of $\mathcal{O}_{\rho^{\text{clock}}} = 1.38 \times 10^{-4}$ in Planck units one reaches an upper bound for this set of initial conditions where inflation no longer occurs. The same analysis for the scalar field clock shows that also in this model the number of inflationary e-foldings decreases with higher clock energy density but the effect is less strong here. One sees a significant effect only if $\mathcal{O}_{\rho^{\text{clock}}} \geq 0.001$ in Planck units. As far as the number of pre-inflationary e-foldings is concerned the results from Ref. 18 demonstrate that the increase of the clock energy density has the opposite effect, namely that the number of pre-inflationary e-foldings increase.

## 3. Conclusions

As a first step towards an investigation how choices of different reference fields affect the physical properties of models the work in Ref. 18 considers different symmetry reduced reference matter models choosing either dust or a massless scalar field as the clock in addition to gravity and the inflaton leading to two-fluid LQC models. For the choice of a Starobinsky potential the fingerprint of the clock on the inflationary dynamics was analyzed in the framework of effective techniques. The analysis shows that initial conditions can be chosen such that the clock has no significant effect on the dynamics. Both models with dust and the scalar field clock respectively show a qualitatively similar behavior and can serve as good clocks. Some difference in the two models are found such as that the model with the scalar field clock has a larger number of inflationary e-foldings compared to the dust model. Furthermore, the scalar field model serves in larger parameter space as a good clock since it is less sensitive to the initial conditions of the clock energy density. Going beyond the first steps investigated in Ref. 18 and reviewed here requires a more detailed understanding on the physical solutions in these models at the level of the quantum theory. For this purpose the already existing numerical techniques[23, 39] need to be generalized to the two-fluid case. Next to the question of singularity resolution at the level of the physical Hilbert space, such kind of generalization will be important because it will also allow to test the validity of effective techniques for two-fluid models.



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