A STUDY OF THE BEAM STACKING PROCESS

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I. INTRODUCTION

A statistical study has been made of a particularly simple beam-stacking process with the aid of the CERN IBM 709 computer. A method was developed for evaluating the effect of this stacking process on a stacked beam of any given energy spectrum. Given this method, and a means of adding the particles which are stacked with each repetition of the stacking process, a wide variety of beam-stacking "experiments" may be simulated.

The beam-stacking process can be described with the aid of diagrams drawn in the synchrotron phase space.[1,2] The canonical coordinates of this space are ϕ and W, where ϕ is the relative phase of the particle with respect to the accelerating voltage, and where W is an action variable defined by

$$W(E) = \int_{E_0}^{E} dE/f(E) ,$$

where $f(E)$ is the frequency of revolution of a particle of energy E around the accelerator.

In the situation where the fractional change in $f(E)$ over the range of energies of interest is small, and where $f(E)$ is approximately a linear function of E, one sees that the ordinates of such diagrams, when fitted with the proper scale, can be taken to represent either the particle energy E, or the particle revolution frequency $f(E)$, or the proper canonical variable W. We have chosen to label Fig. 1 in terms of an energy scale.

Let us consider that the rectangular region of phase space, lying between E_s and $E_s - \Delta E_s$, is available for the stacked beam. And let us assume that the injector and the inflector place the injected particles in a band of phase space of width ΔE_i, located near the injector energy E_i.

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II. THE BEAM STACKING PROCESS

For the purpose of this study we have defined a beam-stacking process as a process by which a portion of the injected particles may be transferred into, and accumulated in, the region of phase space available for the stacked beam.

We have chosen to study a particularly simple beam-stacking process. The process may be divided into four phases, as follows:

1. *Capture.* A portion of the injected particles is captured by applying an rf voltage to an insulated gap in the vacuum chamber, at a constant frequency harmonically related to the frequency of revolution of particles of energy E_i. This creates, in the vicinity of the injected beam, a region of stable phase oscillations known as a "bucket."
2. *Alteration.* A slow (or adiabatic) alteration of the bucket parameters is necessary in order to convert the "stationary bucket" into a "moving bucket" capable of changing the mean energy of the particles trapped inside.

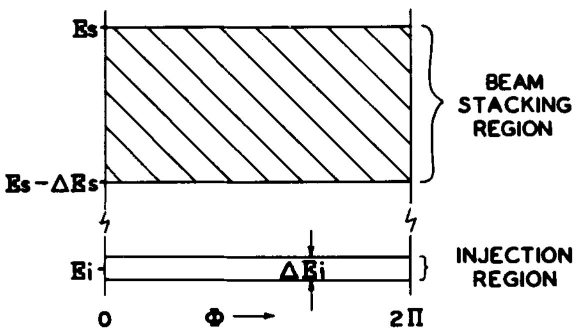

Fig. 1 The beam-stacking region.

3. *Acceleration.* A period of acceleration follows, employing a moving bucket of *constant area* and of *constant stable phase angle* ϕ_s.
4. *Stacking.* When the mean energy of the particles in the bucket reaches some given value, the particles are released from the bucket by an abrupt turn-off of the rf voltage.

III. THE METHOD

An important measure of the quality of a beam-stacking process is its stacking efficiency η. We define the stacking efficiency as the ratio of the phase-space density in the stacked beam to the phase-space density in the injected beam. We know from rather fundamental arguments[1,2] that we cannot increase the density of particles in the stacked beam over the density available in the injected region. Hence, the stacking efficiency can be no larger than 1, and is in practice less than 1.

Since the stacking efficiency for a beam-stacking process will most likely be a function of energy, it is desirable to obtain this function, rather than some average value of the stacking efficiency. This function $\eta(E)$ is proportional to the number of particles per energy interval as a function of E, and will henceforth be referred to as the energy spectrum of the stacked beam.

In order to find the energy spectrum, resulting after one cycle of the stacking process is superimposed on a stacked beam of a given initial energy spectrum, it is necessary to know:
1. The effect of the moving bucket on the initial energy spectrum.
2. The energy spectrum of the particles added to the stack by the moving bucket.

In answer to the first requirement, a method was developed for evaluating the effect of the moving bucket on the initial energy spectrum. The development of this method constitutes the major problem encountered during the study. The method is described in detail.

The information pertinent to the second requirement, listed above, is obtainable through studies of the capture, alteration and acceleration phases of the stacking process. These studies may be based on certain assumptions concerning the details of the injection process, the magnitude and the rate of change of the bucket parameters, the duration of the acceleration phase, the noise content of the various parameters, etc. These studies involve only those particles inside the bucket, or, in certain cases, those particles in the immediate vicinity of the bucket. Although some of these studies have been completed, their results are not included in this report. When such studies are done, their results may readily be utilized along with the method mentioned in the previous paragraph to evaluate the energy spectra produced by any number of repetitions of this particular stacking process.

Let us return to the problem of determining the effect of the stacking process on the particles in a stacked beam of some given energy spectrum.

A computer program, similar to a program which has existed at the MURA laboratory for a number of years, was employed for this study. Given sufficient information about the beam-storage device, including details of the rf system, and the revolution frequency of particles as a function of energy, this program is prepared to simulate the passage of a particle around and around the accelerator, keeping track of the particle energy, the time, and the phase of the particle with respect to the rf accelerating voltage. With such a program, one could study the effect of a moving bucket on each of an array of particles representing a stacked beam of some given initial energy spectrum.

One finds it necessary to use a very large number of particles (10,000 particles) to represent a stacked beam with a typical energy spectrum. Thus, a considerable amount of computer time (1,000 hours) is consumed just to evaluate the effect of the stacking process on *one* particular energy spectrum.

The problem can be greatly simplified now at the expense of two assumptions, neither of which seems unreasonable. First, we assume that there is no azimuthal structure to the density of particles in the stacked beam at the beginning of the stacking process. Hence, all the information concerning the distribution of particles at the beginning of the stacking process is contained in the initial energy spectrum. We now divide the beam-stacking region into 100 energy channels, and our second assumption is that any initial energy spectrum can be adequately represented by a histogram constructed on these channels.

We proceed by studying the effect of the stacking process on a uniform initial distribution of particles in each of the 100 channels. From the results of the studies on each channel, we abstract a histogram, based on the same energy channels, which represents the final energy spectrum resulting from a uniform initial distribution of particles in that channel. The information contained in these 100 histograms, each based on 100 energy channels, can be displayed in a square table having 100 rows and 100 columns.

If we choose to consider this array of coefficients as a 100×100 matrix, (A), and the energy spectra as 100×1 vectors, (V), we notice that the process of obtaining the final energy spectrum (or vector, V_f) resulting from any initial energy spectrum (or vector, V_i) is simply a matter of matrix multiplication. And the process of including the energy spectrum of the particles added to the stacked beam by the stacking process (V_s) is simply a matter of vector addition.

Hence, the energy spectrum of the stacked beam after the Nth cycle of the beam stacking process is given by

$$V_{f,N}=AV_{i,N}+V_s$$

where $V_{i,N}$ is the initial energy spectrum for the Nth cycle. By letting $V_{i,N+1}=V_{f,N}$ the stacking process may be carried through any desired number of cycles.

The matter of determining the matrix A is still a very lengthy calculation (1,000 hours). For certain beam-stacking processes, including the process we have chosen to study, the computing time may be reduced by a factor of 100 (10 hours). This reduction results from the possibility of studying a sample placed in one channel while the bucket stops at 100 predetermined energies, instead of studying samples placed in each of 100 channels while the bucket stops at one energy. For the sake of clarity, we shall ignore this aspect of the study in the discussion of the results.

IV. RESULTS

This program and its data-handling programs were first used to study the simple beam-stacking process described above for the following set of parameters.

The stable phase angle of the moving bucket, ϕ_s, was chosen so that $\sin\phi_s=0.3$ (i.e $\phi_s\cong17°$). The region available for the stacked beam was divided into 100 channels and these were numbered 1–100 from top to bottom. The area of the bucket was chosen to be equal to the area of five of these channels. Hence, the region available for the stacked beam has an area 20 times larger than the area of the moving bucket. The capture and alteration phases of the stacking process are assumed to be so far away in energy that they have no effect on the energy spectrum of the particles in the stacked beam.

The computation gives results that are equivalent to the effect, on a sample of particles, of a bucket that moves from far below the stacked-beam region up to channel 20, where the voltage is abruptly turned off. If the sample of 100 particles were placed initially in each of the 100 channels, 100 histograms would result. It is interesting to look at a few of such histograms as shown in Fig. 2. The upper two histograms are typical of those for channels 1–10. Nothing is very typical of the histograms for the channels lying near channel 20. The lower two histograms are typical of those for channels 30–100.

We now proceed to simulate several beam-stacking "experiments." For the first experiment, we begin with zero stacked beam (i.e. $V_{i_1}=0$), and with a vector V_s representing a uniformly filled bucket of unit density. This experiment simulates the accumulation of stacked beam. The resulting energy spectra are shown in Fig. 3.

One can define an average stacking efficiency $\bar{\eta}$ as the average density of stacked beam in a rectangular region of phase space having an area equal to the sum of the areas of the buckets used to stack the beam. This quantity was evaluated for each of the previous energy spectra, and the results are shown in Fig. 4.

For the next experiment, we begin with a stacked beam having a unit density in channels 13–27, and with a vector V_s representing an empty bucket (i.e. $V_s=0$). This experiment demonstrates the effect of stacking empty buckets within a stacked beam of uniform density. These energy spectra are shown in Fig. 5.

In the last experiment, we begin with a stacked beam having a unit density in channels 43–57, and with a vector V_s representing an empty bucket. This experiment demonstrates the effect of passing through a stacked beam with an empty bucket. The resulting energy spectra are shown in Fig. 6.

V. CONCLUSION

One sees that the stacking process studied, in spite of its simplicity, is a reasonably efficient beam-stacking process, provided one stacks a sufficiently large number of buckets. The number of buckets required for a reasonable efficiency is shown in Fig. 4 to depend on the stable phase angle ϕ_s.

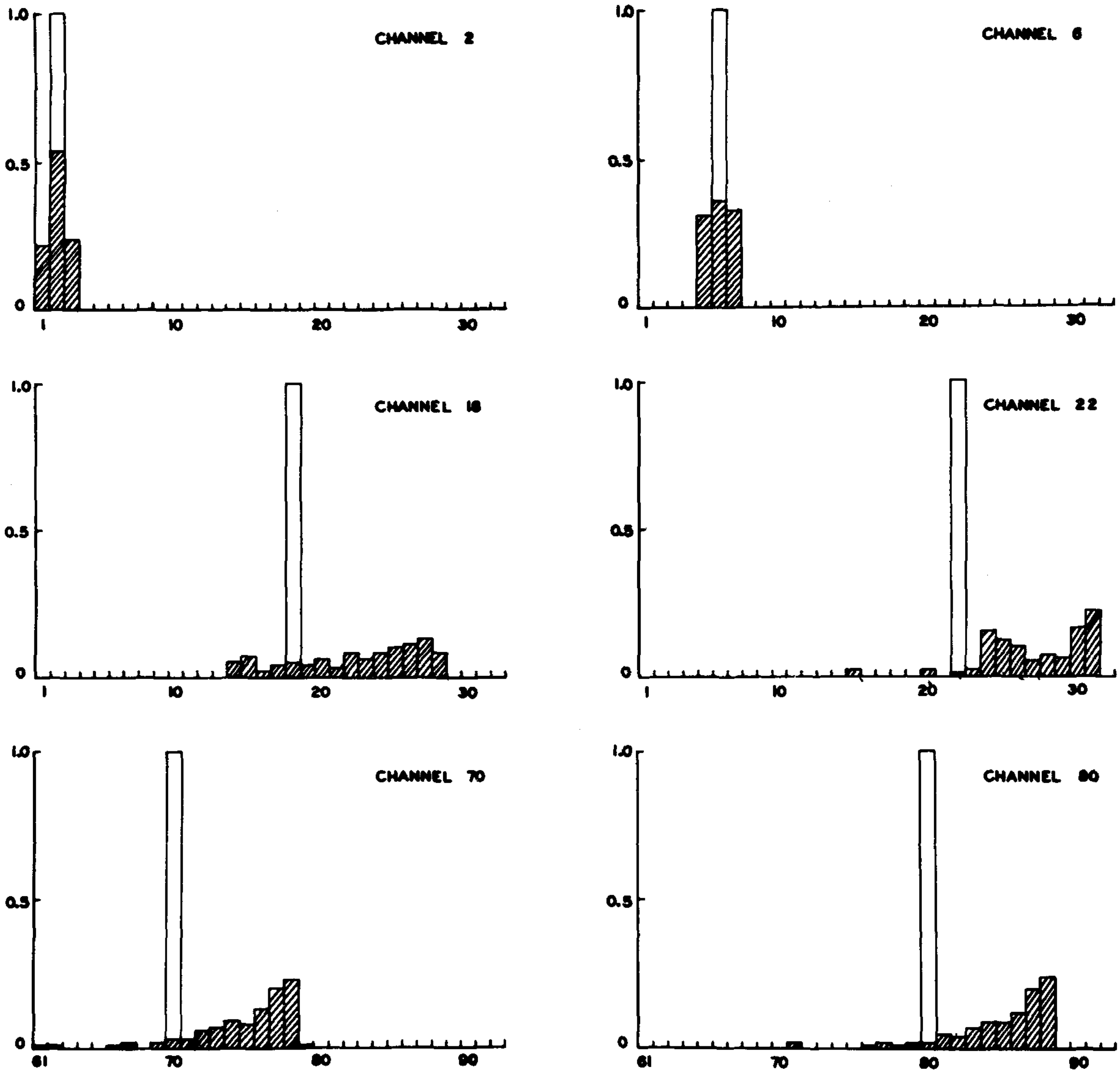

Fig. 2 The shaded portion of each histogram shows the energy spectrum resulting after one cycle of the beam-stacking process for a sample of particles placed initially in the open rectangle. The horizontal coordinate represents energy and is defined in terms of the energy channel number. The stacking process involves a moving rf bucket, which starts far to the right of the figure, and moves to the left until it arrives at channel 20, where it is abruptly turned off.

Many aspects of this statistical study simply verify once again the predictions of the analytical theory of synchrotron phase space.[1,2] In particular, a keen insight is provided into the problems of phase displacement, energy scattering, and the nature of this beam-stacking process.

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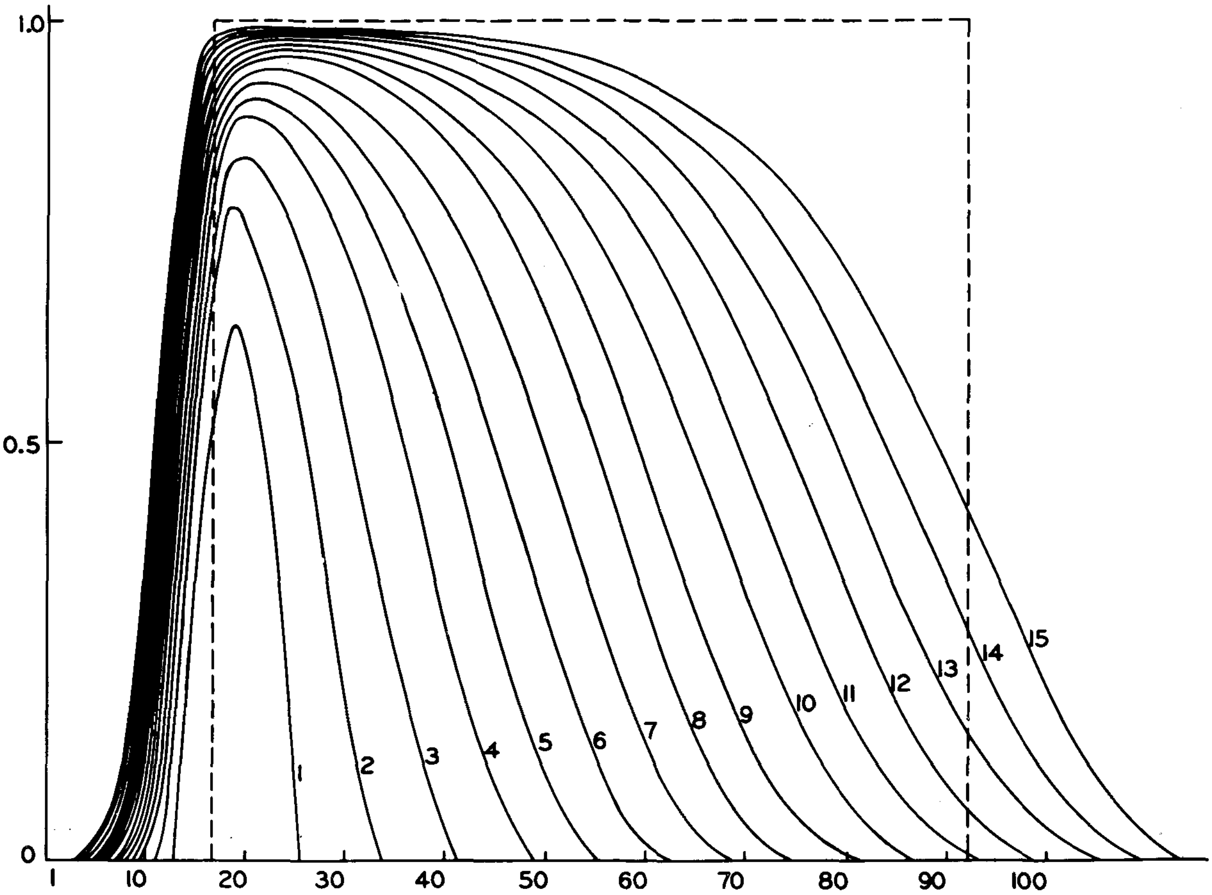

Fig. 3 A set of 15 energy spectra demonstrating the accumulation of stacked beam resulting from 15 repetitions of the beam-stacking process with full buckets. The dashed rectangle represents a stacked beam of unit density in a rectangular region of phase space having an area of 15 times larger than the area of the moving bucket. The horizontal coordinate represents energy defined in terms of channel number; the vertical coordinate is η.

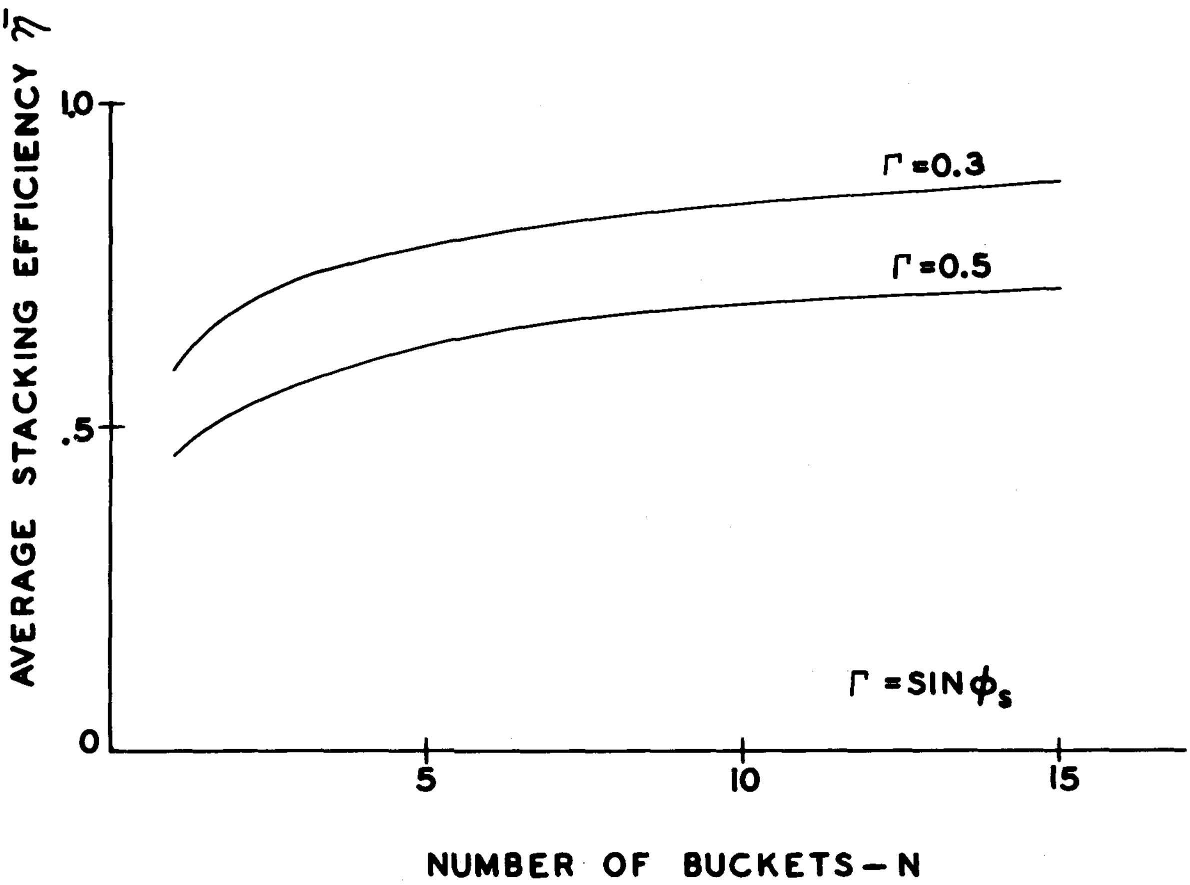

Fig. 4 The dependence of the average stacking efficiency on the number of repetitions of the stacking process for Γ = 0.3 and Γ = 0.5.

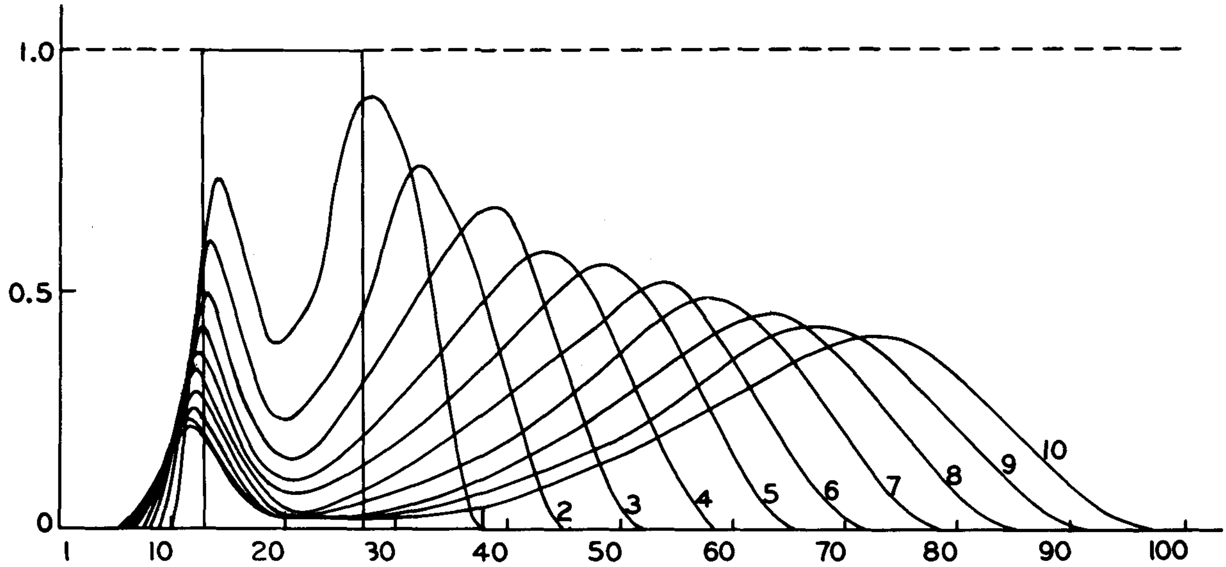

Fig. 5 A set of 10 energy spectra demonstrating the effect of the stacking process with empty buckets on a uniform distribution of stacked beam centered on channel 20.

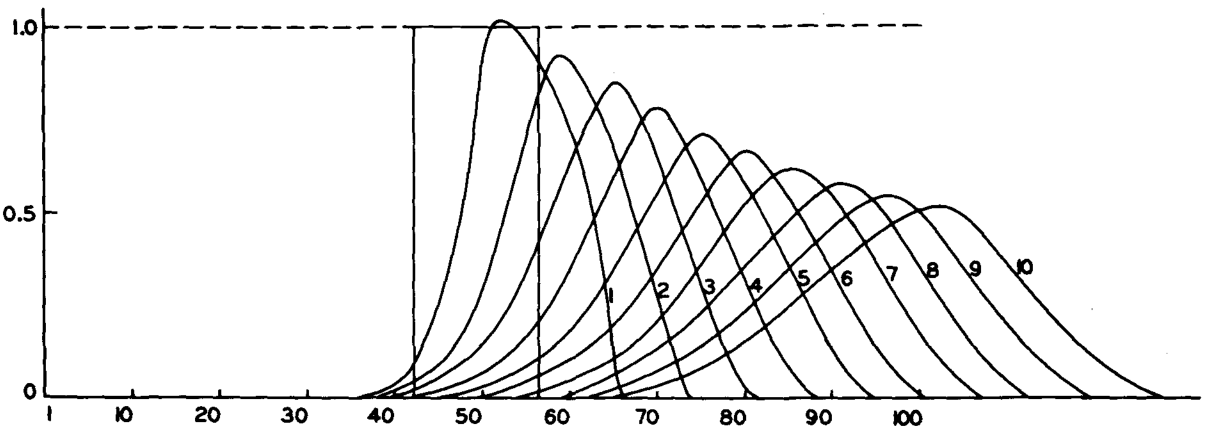

Fig. 6 A set of 10 energy spectra demonstrating the effect of the stacking process with empty buckets on a uniform distribution of stacked beam centered on channel 50.

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