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SOLENOIDAL DETECTOR NOTES

SYSTEMATIC ERRORS AND ALIGNMENT FOR BARREL DETECTORS

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1. Introduction

We discuss below the effect of systematic alignment errors in the tracking detectors. To describe these, we consider for every point in the detector an ideal and real location. Note, the ideal position is gotten following a calibration procedure and is not meant to be just the theoretical position assumed prior to construction. The difference between the two positions is a misalignment vector. This vector is most conveniently discussed in terms of its components in cylindrical coordinates, described by ϕ, r, and z values. We note that angle measurements on tracks used in invariant mass reconstruction are sufficiently precise even with small misalignments; therefore the primary quantity we are concerned with is the momentum measurement. This will be true for angle measurements with errors $\lesssim$ few $\times 10^{-3}$. In the case of precision vertexing better angle measurements are needed. The goals discussed below should be good enough for vertexing in the bend plane. The tracking system as presently envisioned is not good enough for very high vertexing accuracy along the beam direction.

We use a somewhat simplified picture of the measurement process for the sake of clarity. We treat the track measurements as approximately 8 measurements of ϕ in the strip system and another 8 in the straw system. The 8 straw measurements represent the average for each superlayer. We assume statistical errors for the circumferential distance of $\sigma = 12\ \mu m$ for a double-sided silicon measurement and 80 μm for a superlayer of straws. We look at the beam constrained momentum which is the most accurate quantity we want to measure.

The radii assumed for the devices are given in table 1.

For a high momentum track from the origin, the azimuth and radius (in cylindrical coordinates) are related by

$$\phi = \phi_0 + Kr.$$

ϕ_0 and K are the constants describing the track. The tracking detectors are taken as providing measurements of the circumferential distances d_i at radii r_i. The track fit then corresponds to minimizing the squared deviations

$$\chi^2 = \sum_{i=1}^{16} \frac{(r_i\phi_i - d_i)^2}{\sigma_i^2}$$

with $\phi_i = \phi_0 + Kr_i$ and $\sigma_i = 12\ \mu$m for $i \leq 8$, $\sigma_i = 80\ \mu$m for $i \geq 9$.

Table 1. Radii of Tracking Elements

Layer # for Silicon	Radius(m)	Layer # for Straws	Radius(m)
1	0.18	1	0.686
2	0.21	2	0.825
3	0.24	3	0.954
4	0.27	4	1.093
5	0.30	5	1.223
6	0.33	6	1.360
7	0.36	7	1.492
8	0.39	8	1.627

Differentiating χ^2 with respect to ϕ_0 or K and setting to zero leads to the equations:

$$\begin{pmatrix} \sum \frac{r_i^2}{\sigma_i^2} & \sum \frac{r_i^3}{\sigma_i^2} \\ \sum \frac{r_i^3}{\sigma_i^2} & \sum \frac{r_i^4}{\sigma_i^2} \end{pmatrix} \begin{pmatrix} \phi_0 \\ K \end{pmatrix} = \begin{pmatrix} \sum \frac{r_i d_i}{\sigma_i^2} \\ \sum \frac{r_i^2 d_i}{\sigma_i^2} \end{pmatrix}$$

and a solution

$$\begin{pmatrix} \phi_0 \\ K \end{pmatrix} = \underbrace{\left[\frac{1}{\sum \frac{r_i^2}{\sigma_i^2} \sum \frac{r_i^4}{\sigma_i^2} - \left(\sum \frac{r_i^3}{\sigma_i^2} \right)^2} \right] \begin{pmatrix} \sum \frac{r_i^4}{\sigma_i^2} & -\sum \frac{r_i^3}{\sigma_i^2} \\ -\sum \frac{r_i^3}{\sigma_i^2} & \sum \frac{r_i^2}{\sigma_i^2} \end{pmatrix}}_{M} \begin{pmatrix} \sum \frac{r_i d_i}{\sigma_i^2} \\ \sum \frac{r_i^2 d_i}{\sigma_i^2} \end{pmatrix}.$$

This gives for the matrix M, given the detector dimensions and errors in table 1,

$$\begin{pmatrix} 391.7 & -407.5\ m^{-1} \\ -407.5\ m^{-1} & 693.6\ m^{-2} \end{pmatrix} \times 10^{-12}.$$

The dimensions (in meters) are indicated for each term. The error estimate for K is then

$$\sigma_K = \sqrt{693.6 \times 10^{-12} (m^{-2})} = 26.3 \times 10^{-6}\ m^{-1}.$$

The relation between this and the measured transverse momentum error is

$$\frac{\sigma_p}{p^2} = \frac{2\sigma_K}{0.3B}\ \mathrm{GeV}^{-1},$$

which gives 0.09 at $p = 1$ TeV, for $B = 2$ Tesla. This is the beam constrained momentum error for the dimensions and errors assumed.

2. Errors in Measurement

We write the matrix M as

$$\begin{pmatrix} A & -B \\ -B & C \end{pmatrix}.$$

For the curvature measurement, since

$$K = C\sum \frac{r_i^2 d_i}{\sigma_i^2} - B\sum \frac{r_i d_i}{\sigma_i^2},$$

we can write

$$K = \sum_{i=1}^{16}\left(C\frac{r_i^2}{\sigma_i^2} - B\frac{r_i}{\sigma_i^2}\right) d_i = \sum_{i=1}^{16}\alpha_i d_i$$

which will have an error

$$\delta K = \sum_{i=1}^{16}\alpha_i \delta d_i$$

for errors δd_i in evaluating the circumferential distances. The coefficient α_i gives the contribution to the momentum measurement for each layer as well as the sensitivity to alignment errors in a given layer. We tabulate in table 2 the coefficients α_i. They satisfy $\Sigma\alpha_i r_i = 0$ which is required in order that K is independent of the choice of direction from which we measure azimuthal angles.

From the table we see that:

(*a*) The silicon layers each have about the same effect on the momentum.

(*b*) The inner few straw layers have almost no effect on the momentum. They are important mainly for pattern recognition.

(*c*) The outer few straw layers have a large effect on the momentum.

Table 2. Coefficients of α_i

Layer # for Silicon	α_i (m^{-2})	Layer # for Straws	α_i (m^{-2})
1	-0.35	1	0.007
2	-0.38	2	0.021
3	-0.40	3	0.038
4	-0.41	4	0.060
5	-0.42	5	0.084
6	-0.41	6	0.114
7	-0.39	7	0.146
8	-0.37	8	0.183

We can now phrase various alignment questions by specifying the set of δd_i along a track corresponding to the real detector. We emphasize again that these misalignments correspond to the deviations relative to a database established after an attempt at a final alignment. We categorize the errors as circumferential, radial, and longitudinal specified by deviations in the detector location by $\delta\phi$, δr, δz for each measurement.

For a circumferential error δd is given by $r\delta\phi$ directly. We will see below that these have the tightest tolerance.

We next look at radial displacements. Since the track trajectory is given by

$$\phi = \phi_0 + Kr,$$

a radial displacement leads to an apparent ϕ displacement, that is,

$$\delta\phi = K\delta r$$

due to a δr. The effect is seen to be momentum dependent, vanishing as the momentum gets very large. In looking at this, multiple scattering has to be considered, otherwise unreasonable requirements will be set for low momentum tracks. For the present detector, the multiple scattering momentum error is about 1/2%, thus the measuring and multiple scattering errors are about equal at 50 GeV. We choose below a value of K corresponding to about 50 GeV as the value for which we require the systematics to be sufficiently small. This gives

$$K = \frac{0.3B}{2p} \simeq 0.006\ m^{-1}$$

for $p = 50$ GeV. For this case $\delta\phi = 0.006\ \delta r$, and we will choose below $\delta d = 0.006\ r\delta r$ for a radial error.

Finally, a longitudinal error is important only for the stereo layers. In this case a displacement along z causes a rotation along ϕ, governed by the stereo ratio for the layer. We can re-express this in terms of the statistical errors in the circumferential and longitudinal direction expected for a given layer i as

$$\delta d = \left(\frac{\sigma_i}{\sigma_z}\right)\delta z.$$

For $\sigma_z = 1.5$ mm, which is assumed below, this gives

$$\delta d = \left(\frac{\sigma_i}{1.5\ \text{mm}}\right)\delta z.$$

Since σ_i is either 12 μm or 80 μm, the error in z is multiplied by a small number.

Finally, we need to specify a limit on how much worsening we will allow in the curvature measurement. We choose a worsening of $\lesssim 10\%$ due to random placement errors and $\lesssim 33\%$ for correlated errors. We will have to apportion the error between the circumferential, radial, and longitudinal contributions. It is desirable to leave most of the error budget for circumferential errors, thus we choose the other two small enough to have little impact on the overall momentum error. The choice below is rather conservative in this respect and a factor of two worse radial or longitudinal errors could be tolerated if it turns out to be difficult or expensive to achieve the goals below.

3. Individual Detector Placement Goals

In positioning the individual detectors and maintaining the alignment, the detector locations contain systematic uncertainties. This leads to errors which we simplify as uncorrelated. In this case, one gets a contribution to the curvature error, after averaging over detector locations, which is

$$(\delta K)^2 = \sum \alpha_i^2 (\delta d_i)^2.$$

Note, for $\delta d_i = \sigma_i$, $(\delta K)^2 = \sigma_K^2$. Thus we need to ensure that δd_i is sufficiently small compared to σ_i. For the silicon a reasonable goal is $\delta d_i < 5$ μm, for the straws $\delta d_i < 35\ \mu$m. These will act as errors in quadrature with the statistical errors increasing the individual position errors to $\sqrt{5^2+12^2}\mu$m $= 13\ \mu$m for silicon and $\sqrt{35^2+80^2}\mu$m $= 87\ \mu$m for the straws. The positioning errors include the contributions from circumferential, radial, and longitudinal errors in the stereo case. The radial error contributes to $\delta d_i = 0.006\ r_i \delta r_i$. For the outermost silicon and straw detectors, these are $0.003\ \delta r$ and $0.01\ \delta r$, respectively. Keeping the contribution less than about 2 μm for the silicon and 12 μm for the straws gives

$\delta r < 700\ \mu$m for silicon, and

$\delta r < 1200\ \mu$m for the straws.

The analogous longitudinal errors are about $\delta z < 250\ \mu$m for both the silicon and the straws. These allow for the circumferential error nearly the full 5 μm for silicon and 35 μm for the straws.

4. Correlated System Errors

For correlated errors we have to keep the full expression

$$\delta K = \sum_{i=1}^{16} \alpha_i \delta d_i$$

where the δd_i are related in some way. We look at a few examples. The numbers are meant to apply to any slice through the detector which passes through the origin.

(A) Suppose the outer detector is rotated relative to the inner detector by an angle $\delta\phi$. How large can we allow $\delta\phi$ to be? In this case $\delta d_i = r_i \delta\phi$ for either the inner or outer detectors. This gives $\delta K = \sum_{i=1}^{8} \alpha_i r_i \delta\phi$. Setting $\delta K \le \sigma_K/3$ gives $\delta\phi \le 10^{-5}$ radians. This corresponds to a circumferential displacement of 16 μm for the outer straw layer.

(B) Suppose all the radii of the inner or outer detectors are systematically incorrect by δr. In this case $\delta d_i = 0.006\ r_i \delta r$ for i between 1 and 8 or between 9 and 16. Again, taking $\delta K \le \sigma_K/3$ would give $\delta r \le 1500\ \mu$m. In order to allow most of the error to come from circumferential shifts, we will take as a goal

$$\delta r \le 600\ \mu\text{m}.$$

(C) Suppose the centroids of the silicon and straw detectors are displaced by a vector $\vec{\Delta}$. Along the direction of $\vec{\Delta}$ this corresponds to a radial error. Going around in circumference by 90°, the error is circumferential with $\delta d_i = \Delta$ for $i = 9$ through 16. Thus in this case

$$\delta K = \left(\sum_{i=9}^{16} \alpha_i \right) \Delta.$$

Taking again

$$\delta K \leq \sigma_K / 3$$

gives

$$\Delta \leq 15 \ \mu\text{m}.$$

We note, finally, that because the stereo angle alternates in sign, that an overall longitudinal displacement does not tend to give important correlated errors.

5. Further Considerations

The allowed values for the radial and longitudinal misalignments are much greater than the allowed circumferential values. Thus it is important to avoid situations where a radial error is turned into a circumferential error because of approximations made in the discussions above. We look at two examples.

(A) Tracks have been assumed to come from the origin. This is only approximate because of decaying particles as well as displacements from the origin of the colliding beams. For stiff tracks with small impact parameters we have a more general approximate equation

$$\phi = \phi_0 + Kr + b/r.$$

This gives an additional error in ϕ associated with a radial error, beyond our earlier discussion, of

$$\delta\phi = -b/r^2 \ \delta r, \quad \text{and}$$
$$\delta d = -b/r \ \delta r.$$

For the silicon, the numbers arrived at earlier gave $\delta d \leq 2 \ \mu$m for $\delta r \leq 700 \ \mu$m for tracks from the origin. Now, for the case of a b value of about 1 mm we want $\delta d \leq 2 \ \mu$m. Thus for $r > 15$ cm, the silicon alignment goal for δr is now $\delta r \leq 300 \ \mu$m, somewhat more stringent than our earlier number. For the straws the effect of a finite b is much smaller and is thus not a significant issue.

(B) Since individual detector elements have a finite size, a radial displacement of the whole detector implies a circumferential displacement of the edge of the detector element. This is not significant for the straws, so we consider the silicon only below. The azimuthal angle of the edge of a detector is approximately given by h/r, where h is the detector element half width. Thus a radial misalignment, assumed to be the same for the whole detector, generates an error in ϕ given by

$$\delta\phi = -h/r^2\, \delta r \quad \text{and}$$
$$\delta d = -h/r\, \delta r.$$

If we want an error of 2 μm at the mean silicon detector radius for an average location on the detector (i.e., halfway between the edge and center) we get an alignment goal

$$\delta r \leq \frac{(4\ \mu\text{m})30\text{cm}}{h}.$$

For $h = 1.5$ cm this gives

$$\delta r \leq 80\ \mu\text{m}.$$

This is actually more stringent than any other limit on δr for the silicon. It also indicates that there is an advantage to using detectors which are not too wide.

6. *Summary*

We now summarize the results from above.

Maximum uncorrelated positioning errors:

Silicon detector:	Circumferential	5 μm
	Radial	80 μm
	Longitudinal	250 μm
Straw superlayer:	Circumferential	35 μm
	Radial	1200 μm
	Longitudinal	250 μm

Maximum correlated system error, silicon relative to straws:

Rotational alignment:	10^{-5} radians
Radial alignment:	600 μm
Longitudinal coordinate:	250 μm
Displacement of detector centroids:	15 μm

Note, all the numbers above are meant to be standard deviations, not the edges of box shaped distributions, which span a range of ± 1.73 times the values tabulated above.