Stefano and Marcello @ Work

A personal selection of results obtained by Stefano Catani and Marcello Ciafaloni

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Abstract. To honor the memory of Stefano Catani and Marcello Ciafaloni, I describe part of their scientific results in the field of Quantum ChromoDynamics during their long and fruitful careers.

1 Introduction

Stefano Catani and Marcello Ciafaloni (Stefano and Marcello henceforth) have given important contributions to many aspects of quantum field theory and its applications to high energy physics. Reviewing all of them in a short talk is an impossible task. I will limit myself to a selection of topics, largely influenced by the subject of this Workshop (Quantum Chromodynamics) and by my personal interests.

2 Soft gluon resummation

Stefano's and Marcello's results I am most familiar with are those obtained in the context of the all-order resummation of large logarithmic contributions in the perturbative expansion of cross sections close to the threshold energy for the production of heavy systems. I summarize here the basic ideas of this subject following a simple and beautiful review presented by Stefano at a workshop in Montpellier in 1996. [1]

Physical cross sections are always inclusive over arbitrarily soft particles in the final state, because of finite detector resolution. On the other hand, inclusiveness plays a crucial role in QCD calculations, because infrared divergences from virtual corrections are cancelled by radiation of undetected real gluons. The finite left-over of these cancellations give large contributions if the tagged final state is forced to take most of the available energy.

Let us consider a generic process, e.g. the production of some heavy system accompanied by unobserved radiation, which carries a fraction $1 - z$ of the available energy $\sqrt{s}$. Loop corrections and real emission contributions to the cross section at order α_s are separately infrared divergent, and must be regularized by an infrared cut-off ϵ:

$$\frac{dw_{\text{virtual}}}{dz} = -2C\,\alpha_s\,\delta(1-z)\int_0^{1-\epsilon}\frac{dy}{1-y}\log\frac{1}{1-y} + \text{finite terms} \tag{1}$$

$$\frac{dw_{\text{real}}}{dz} = +2C\,\alpha_s\,\frac{1}{1-z}\log\frac{1}{1-z}\,\theta(1-\epsilon-z) + \text{finite terms} \tag{2}$$

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because of the bremsstrahlung energy spectrum $d\omega/\omega$ and the spectrum for collinear emission, dk_T^2/k_T^2. The sum of the two contributions is finite as $\epsilon \to 0$, and can be written

$$\frac{dw(z)}{dz} = \frac{dw_{\text{virtual}}}{dz} + \frac{dw_{\text{real}}}{dz} = 2\,C\,\alpha_{\text{s}}\left[\frac{1}{1-z}\,\log\frac{1}{1-z}\right]_+ \tag{3}$$

with the usual definition of the + distributions

$$\int_0^1 dz\,[D(z)]_+ f(z) = \int_0^1 dz\,D(z)\,[f(z) - f(1)]\,. \tag{4}$$

The contribution of virtual corrections is concentrated at $z = 1$, while the real emission contribution is spread over the range $x < z < 1$, where x is the fraction of total energy carried by the observed final state. Hence, the contribution of soft emission to the cross section is proportional to

$$\int_x^1 dz\,\frac{dw}{dz} = -C\,\alpha_{\text{s}}\,\log^2(1-x), \tag{5}$$

a finite left-over of the cancellation of infrared divergences. When observations are restricted to the region of x close to 1, the phase space for real emission is suppressed, and the finite left-over becomes large. The same mechanism is replicated at higher orders; at order n, at most two powers of $\log(1-x)$ for each power of α_{s} appear in the perturbative coefficients:

$$C_n(x)\alpha_{\text{s}}^n = \alpha_{\text{s}}^n \sum_{m=1}^{2n} c_{nm}\,\log^m(1-x) + \text{ non singular terms.} \tag{6}$$

The perturbative expansion becomes unreliable in the limit $x \to 1$, and therefore logarithmically enhanced contributions must be resummed to all orders. Typical examples of processes where resummation of soft gluons plays a role are lepton-nucleon scattering in the quasi-elastic limit ($x = x_{\text{Bj}} \to 1$), the production of systems with a large invariant mas Q^2 (Drell-Yan pairs, Higgs) close to threshold ($x = \frac{Q^2}{s}$ for $s \gtrsim Q^2$), transverse momentum q_T spectra in the small-q_T region ($x = 1 - \frac{q_{\text{T}}^2}{Q^2}, q_{\text{T}}^2 \ll Q^2$)).

Soft gluon resummations are typically performed after a suitable integral tranform. In the case of threshold production, one takes Mellin moments of the cross section with respect to x:

$$\hat{F}(N) = \int_0^1 dx\,x^{N-1}F(x); \qquad F(N) = \frac{1}{2\pi i}\int_{\bar{N}-i\infty}^{\bar{N}+i\infty} dN\,x^{-N}\hat{F}(N). \tag{7}$$

(in the case of transverse momentum the appropriate transform is two-dimensional Fourier transform with respect to $\vec{q}_T$.) In the space of the Mellin variable N, the region $x \to 1$ is mapped in the large N region; in this limit, the perturbative expansion of a coefficient function takes the form

$$C(N) = \sum_{n=0}^{\infty} \alpha_{\text{s}}^n \sum_{k=1}^{2n} c_{nk}\log^k N + \text{ non singular terms} \tag{8}$$

with at most two powers of $\log N$ for each power of α_{s}. Terms with $n+1 \le k \le 2n$ are leading logs, while terms with $k = n$ are next-to-leading logs.

Resummation of soft emission contributions at the leading logarithmic approximation can be performed easily in QED on the basis of the eikonal approximation. However, its generalization to QCD, and to higher logarithmic accuracy, is highly non trivial. Both Stefano and Marcello have given central contribution in this direction. I summarize them below.

1. Eikonal emission exponentiates in QED because soft photons are emitted independently of each other. This is not the case in QCD, because of the presence of three-gluon and four-gluon vertices. However, gluon correlations can be shown to cancel in the soft limit, therefore leading to the exponentiation of the single-emission cross section in the leading log approximation. A proof of the exponentiation at leading log in QCD was given by Stefano and Marcello in ref. [2].

2. In QCD, running coupling effects are especially relevant. It was shown by Marcello and collaborators (see e.g. refs. [3], [4]) that the appropriate scle choice for the running coupling is the transverse momentum k_T of emitted gluons with respect to the direction of the emitting parton. On the basis of kinematics, k_T^2 is bounded from above by the hard scale of the process Q^2 times a power of $1-x$. Hence, the upper bound on the transverse momentum can be very different from Q^2 in the threshold limit. This is an important point, because by this choice leading log terms of order $\alpha_s^k \log^{k+1} N$ are resummed.

3. Also next-to-leading logarithmic terms can be shown to take the form of an exponential of a function of the conjugated variable N. A proof was given by Stefano and Luca Trentadue in ref. [5].

In the case of deep-inelastic scattering and Drell-Yan pair production, the resummed coefficient functions in the conjugate space take the form

$$\frac{C_N(Q^2)}{C_N^{\rm LO}(Q^2)} = g_0(Q^2)\exp G_N(Q^2) + O\left(\frac{\log^k N}{N}\right) \tag{9}$$

where $C_N^{\rm LO}(Q^2)$ is the leading-order result, and

$$\begin{aligned} G_N^{\rm DIS} Q^2) &= \log\Delta_q(Q^2,\mu^2) + \log J_q(Q^2) + \log\Delta_{\rm int}^{\rm DIS}(Q^2) \\ G_N^{\rm DY} Q^2) &= 2\log\Delta_q(Q^2,\mu^2) + \log\Delta_{\rm int}^{\rm DY}(Q^2) \end{aligned}$$

with

$$\begin{aligned} \Delta_q(Q^2,\mu^2) &= \int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{Q^2(1-z)^2}\frac{dq^2}{q^2}A(\alpha_s(q^2)) \\ J_q(Q^2,\mu^2) &= \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dq^2}{q^2}A(\alpha_s(q^2)) + B(\alpha_s(Q^2(1-z)))\right] \\ \Delta_{\rm int}(Q^2,\mu^2) &= \int_0^1 dz \frac{z^{N-1}-1}{1-z} D(\alpha_s(Q^2(1-z)^2)) \end{aligned}$$

and A, B, D analytic functions of their argument. After performing the integrals, the exponent takes the form of an expansion in powers of α_s with fixed $\alpha_s \log N$.

A difficulty immediately arises. The ratio $\frac{C_N(Q^2)}{C_N^{\rm LO}(Q^2)}$ is a function of $\alpha_s(Q^2/N^a), a = 1,2$ which can be expanded in powers of $\alpha_s(Q^2)$. To next-to-leading log we have

$$\alpha_s\left(\frac{Q^2}{N^a}\right) = \frac{\alpha_s(Q^2)}{1+L}\left[1-\alpha_s(Q^2)\frac{\beta_1}{\beta_0}\frac{\log(1+L)}{1+L}\right]; \qquad L = \alpha_s(Q^2)\beta_0\log\frac{1}{N^a} \tag{10}$$

which has a simple pole at $L=-1$, or

$$N = N_L \equiv e^{\frac{1}{\bar\alpha}}; \qquad \bar\alpha = a\beta_0\alpha_s(Q^2) \tag{11}$$

(the so-called Landau singularity) and a branch cut on the real positive N axis for $\mathrm{Re}\,N > N_L$. Hence, the inverse Mellin transform of $C_N(Q^2)/C_N^{\mathrm{LO}}(Q^2)$ does not exist. This has to do with non-perturbative contributions, which are not under control in perturbation theory.

A solution, now widely adopted, is presented in a paper [6] by Stefano, in collaboration with M. Mangano, P. Nason and L. Trentadue. Their suggestion (named the minimal prescription) is in fact very simple: they suggest to define

$$\sigma(x, Q^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\, \mathcal{L}(N, Q^2)\, C(N, \alpha_s(Q^2)) \tag{12}$$

with the constant c taken to the right of all singularities of $C(N)$, but to the left of the Landau pole, as shown in fig. 1. Clearly, this is not a true inverse Mellin: the integrand is not analytical

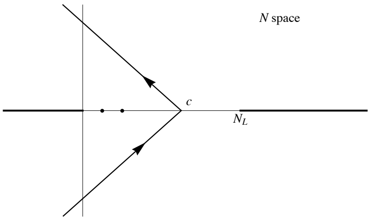

Figure 1. The inversion contour of the minimal prescription.

in any right half-plane, because of the branch cut due to the Landau pole. However, it is shown in ref. [6] that the minimal prescription, which provides a well-defined cross section for all values of x, is an asymptotic sum of the original (divergent) perturbative expansion. Furthermore, the difference between the original series, truncated at the best-approximation term, and the minimal prescription, is suppressed more strongly than any power of Λ^2/Q^2.

The minimal prescription allows one to obtain resummed predictions for physical observables, and since its formulation an impressive amount of phenomenological results have been produced by Stefano and collaborators. An important example is the calculation of the cross section for the production of a Higgs boson at hadron colliders, presented in ref. [7]. Figs. 2 and 3 are taken from that paper.

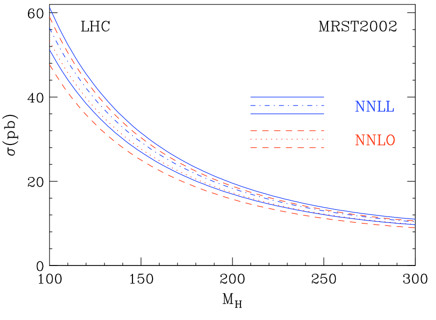

Figure 2. Higgs production at the LHC.

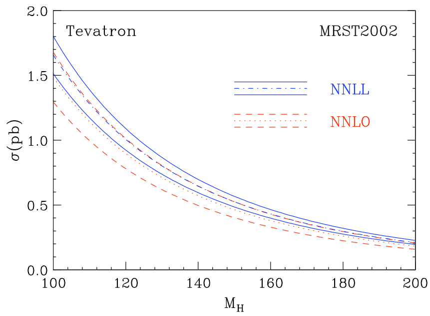

Figure 3. Higgs production at the Tevatron.

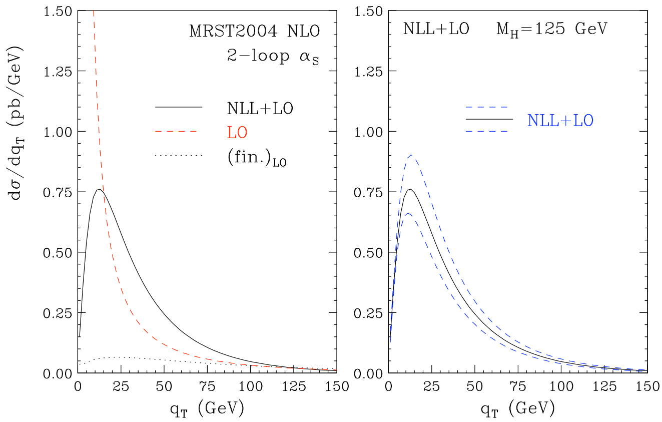

Figure 4. Left: NLL+LO compared with the LO spectrum. Right: uncertainty band from scale variations.

The uncertainty bands are obtained by varying the renormalization and factorization scales around a central value. We note that resummation has a sizable effect on central values, and considerably reduces the theoretical uncertainty.

In some cases, cross section are affected not only quantitatively, but also qualitatively, by the resummation of soft logarithms. This is the case for transverse momentum distributions, where the inclusion of large logs of q_T^2/Q^2 produces a suppression of the cross section in the low-q_T region. The q_T spectrum of Higgs production at the LHC was computed in ref. [8], from which figs. 4 and 5 are taken.

Many other applications of the resummation formalism outlined above were performed by Stefano and a long list of collaborators. [10]-[25].

The subject of soft gluon resummation was addressed later by Stefano with Pino Marchesini and Bryan Webber from a different point of view, which proves to be suitable for implementation in shower Monte Carlo Codes. [9]

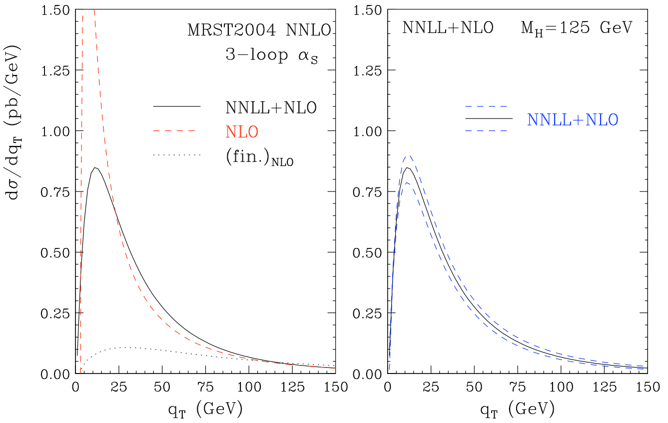

Figure 5. Left: NNLL+NLO compared with the LO spectrum. Right: uncertainty band from scale variations.

3 High-energy resummation

A relevant part of the collaboration between Stefano and Marcello is the study of perturbative QCD in the large energy regime. These subject was addressed in a series of papers in collaboration with Francesco Hautmann ([26]-[32]).

Perturbative QCD predictions for hadronic processes at squared energy s and large transverse momentum $p_T \sim s$ are remarkably accurate in the regime $p_T \gg \Lambda$, where Λ is the QCD energy scale, of order a few hundred MeV. Indeed, non-perturbative contributions suppressed by powers of Λ/p_T, while logarithmic corrections to the naive parton model are systematically computable as a power series in $\alpha_s(p_T^2) \sim (\beta_0 \log \frac{p_T^2}{\Lambda^2})^{-1} \ll 1$. However, in the regime $s \gg p_t^2 \gg \Lambda^2$, powers of $\log x = \log \frac{p_t^2}{s}$ (usually referred to as small-x, or high-energy logarithms) appear in the perturbative coefficients and spoil the accuracy of perturbative predictions. A similar situation arises whenever the process is characterized by two (or more) hard scales, sizably different from each other; an example is the production of heavy quarks.

The leading high-energy logarithmic contributions to total cross sections are powers of $\alpha_s \log x$, which is of order one in the small-x limit. Stefano, Marcello and F. Hautmann have shown in refs. [26]-[32] that these large logarithmic contributions can be summed to all orders for hard processes directly coupled to gluons. To this purpose, they prove a high-energy (or k_T) factorization property of the cross section, valid in the small x limit, where by a universal k_T depending parton density is convoluted with a partonic cross section with an off-shell gluon in the initial state.

4 All-order calculations and Monte Carlo codes

In a beautiful series of papers in collaboration with Mike Seymour [33]-[35], Stefano has introduced a generalization of the subtraction method for the computation of high-order perturbative contributions to QCD observables

The cross section for a process with m partons in the final state at leading order receives next-to-leading order corrections from emission of one extra parton, and from virtual corrections to the m parton process:

$$d\sigma^{\rm NLO} = d\sigma^{\rm R}_{\rm m+1 partons} + d\sigma^{\rm v}_{\rm m\, partons}. \tag{13}$$

Both contributions are divergent in the infrared region of the momentum of virtual or real gluons, and the divergence is cancelled in the sum.

The cancellation of infrared singularities is not easy to implement numerically, because it takes place between processes with different final states. In most calculations, the so-called subtraction method is employed, which amounts to add and subtract an arbitrary term with $m+1$ partons in the final state:

$$d\sigma^{\rm NLO} = \left[d\sigma^{\rm R}_{\rm m+1 partons} - d\sigma^{\rm A}_{\rm m+1 partons}\right] + d\sigma^{\rm A}_{\rm m+1 partons} + d\sigma^{\rm v}_{\rm m\, partons}. \tag{14}$$

The cross section $d\sigma^{\rm A}$ is chosen in such a a way that it cancels the singularities in $d\sigma^{\rm R}_{\rm m+1 partons}$ and can be integrated analytically in the singular region. Furthermore, it is convenient to choose it so that it is independent of the observable one wants to compute.

The Catani-Seymour formalism, named the dipole formalism by the authors, is a choice of $d\sigma^{\rm A}$ which is completely general, i.e. not only observable-independent for a given process, but also process independent. The dipole implementation of the subtraction method proves to be especially useful for the implementation of QCD corrections in a Monte Carlo event generator.

5 Non-QCD research

Although not directly related to QCD, some other achievements in Marcello's scientific work are worth mentioning. In the early stage of his scientific career Marcello gave significant contributions in the study of composite models and relativistic bound states, mainly in collaboration with Pietro Menotti at Scuola Normale Superiore in Pisa, where he was a student. [39]-[36]

More recently, Marcello has transferred his knowledge in QCD in an interesting series of papers about the role of large logarithmic corrections arising in the electroweak theory. Most of this work was performed in collaboration with his son Paolo. [40]-[47]

In the latest stage of his scientific activity, Marcello devoted his attention to gravitational scattering at transplanckian energies. [48]

6 Personal recollections

I would like to conclude with some personal reflections on my relationships with Marcello and Stefano. I had the opportunity to meet Marcello a few times between 1986 and 1988, when I was a student in Pisa. Later, in 2001, we were both visiting the CERN Theory Division for an extended period, which gave me many chances to spend time with him. I was always struck by Marcello's profound and vast knowledge of quantum field theory and advanced mathematics, as well as by his calm and pleasant way of discussing both physics and other topics.

Stefano, on the other hand, was a close friend. We first met at an International School organized by the Italian Physical Society in Varenna, on Lake Como, in 1984, and we stayed in close contact ever since. Our scientific interests overlapped significantly, yet for various reasons, we never had the chance to collaborate. However, I was fortunate to have many conversations with Stefano, which were often illuminating. In those moments, I always felt that Stefano's understanding of the subject went far deeper than mine, and that, in some way, I could not reach the level of insight he possessed. I cherish those conversations as a precious gift. For that, and for all the time we shared, I will miss Stefano deeply.

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