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MEASUREMENT OF BETATRON OSCILLATION PARAMETERS β AND α IN THE MAIN RING LONG STRAIGHT SECTION

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I. Motivation

Characteristics of the beam in a synchrotron are determined by betatron oscillation parameters β and α (and dispersion parameters η and η'). It is a standard practice to compute them around the ring with a computer program such as SYNCH and these parameters, together with the betatron phase angle which is simply an integral of $1/\beta$, are used for many purposes. However, when one needs precise values of β and α , the standard table for the ideal focusing properties of ring magnets may not be adequate and measurements of these parameters may be of practical importance.

There are many factors that contribute to the deviation of β and α from their ideal values in the main ring.

1) There are forty old-type quadrupoles which are ~1% lower in gradient compared to the standard ones. In principle, one can prepare a new table taking this difference into account.

2) One of the acceptance criteria[1] for quadrupoles specifies that the integrated field gradient at 200 GeV level (B' ~120 kG/m) be within ±0.25%. With this value, one can predict from a simple formula[2] that the expected deviation of β is ~ ±6%. The deviation may be larger at injection due to the remanent field gradient, and at 400 GeV or higher due to the saturation effect.

3) Nonlinear fields coupled with the closed-orbit distortion can change the focusing properties of the ring. Even with the ideal closed orbit, the sagitta in dipoles must be taken into account.[3]

One of several projects of the main ring group which have been planned but never undertaken is to measure β and α in the long straight section of A sector. It was felt that a good matching of the injected beam in the transverse phase space requires a precise knowledge of β and α at the injection point. Although the present matching seems to be adequate judging from the overall transmission of better than 90%, the higher intensity beam with its larger emittance may demand a further improvement of the matching. More recently, the Colliding Beams Department has been proposing a very low value of β at the crossing point in the main ring in order to increase the luminosity. It is indeed difficult to achieve a usable luminosity for most of the planned experiments unless β at the crossing point is reduced from the normal value of 70 m to less than ~5 m.[4] At energies higher than ~100 GeV, the beam size will be less than a mm for $\beta \lesssim 5$ m and its absolute measurement becomes very difficult. It is therefore highly desirable that one should be able to measure β with a reasonable accuracy (10% or better).

II. Method

A standard way of measuring β is to introduce a localized gradient perturbation and find the resulting change in the tune of the ring. This has been used in the booster[5] where there are forty-eight trim quadrupoles which can be excited independently. For a localized perturbation of $\Delta\,(B'\,\ell/B\rho)$, the tune changes from ν_o to ν where[6]

$$\cos(2\pi\nu) - \cos(2\pi\nu_o)$$
$$= -\frac{1}{2}\sin(2\pi\nu_o)\cdot\Delta\,(B'\,\ell/B\rho)\cdot\beta \qquad (1)$$

It is clear from this relation that, when β is small, one must introduce a large perturbation in order to get a reasonable separation of ν from ν_o which is necessary for reducing the error in β. In the booster, the maximum available gradient of trim quadrupoles was not large enough to get a reliable value of $\beta_V(\cong 5.3\ \text{m})$ in the short straight sections.

In the main ring long straights, β varies from 50 m to 120 m for the normal mode of operation. One can probably use a standard trim quadrupole ($B'\ell = 0.244$ kG/amp) to measure β at 8 GeV. Five to ten amp will change the tune from 19.4 to 19.44. If β is measured at both upstream and downstream ends of long straights, one can calculate α at any point inbetween although its accuracy will in general be poor. It is easy to see that, at the center of the long straight section,

$$\alpha^* = (\beta_u - \beta_d)/(2s) \tag{2}$$

and $$\beta^* = (\beta_u + \beta_d)/4 \pm A/2 \tag{3}$$

where β_u and β_d are, respectively, upstream and downstream values of β, s is the drift distance between two points of measurements, and $A = \sqrt{\beta_u\beta_d - s^2}$. Two values of β^* are possible but it is obvious in most cases which one to take. The disadvantage of this method is the necessity of replacing a portion of pipe in order to install a trim quadrupole. For the low-beta insertion of colliding beam experiments, one is interested in β^* only. It is possible to install a quadrupole at the crossing point and to measure the change in tune. However, with $\beta^* = 2.5$ m (the ultimate goal for the colliding beams), it is necessary to have $B'\ \ell \simeq \pm 300$ kG at 100 GeV and, even with a normal main ring quadrupole ($\ell = 2.1336$ m), the required current is more than ~2,000 amp.

The low-beta insertion in the main ring proposed by T. Collins[7] is shown schematically in Fig. 1. Three independent power supplies (a), (b) and (c) are connected to three groups of quadrupoles around the colliding point in an antisymmetric arrangement. For the measurement of β^* (and α^* if needed), one can connect another power supply to (dD) and to (Ff) quadrupoles and measure the change in tune of the ring. The perturbation current in these quadrupoles are of the order of ±50 amp for $\beta^* = 2.5$ m at 100 GeV. One complication here is that the value of β changes substantially in the two quadrupoles and the perturbation cannot be treated as a local one. For example, with $\beta^* = 2.5$ m, β_h changes from 555 m to 267 m in (dD). In Eq. (1), the quantity $\Delta(B'\ \ell/B\rho)\cdot\beta$ must be replaced by

$$\Delta(B'/B\rho) \int \beta \, ds$$

where the integral is over two quadrupoles. Since the total current in these quadrupoles is ~2,500 amp, it is probably still safe to assume that the amp factor is unity, that is, $dB'/dI = 57.8$ G/m/amp. With the measurement of the tune change first with (dD) perturbation and then with (Ff) perturbation, one can write down two linear relations for(β^*, α^*, γ^*) which, combined with the realtion $\gamma^*\ \beta^* = 1 + \alpha^{*2}$, give two sets of solutions for (β^*, α^*). Again the difference in two possible values of β^* is large enough to make the proper choice easy. A simple computer program for PDP-10 has been written for this purpose. Input quantities are the unperturbed tune ν_o and the unperturbed field gradient, which is the same in magnitude for (dD) and (Ff), the fractional changes in gradients, which may be different, and two perturbed tunes ν_1(dD) and ν_2(Ff). Also specified is the expected maximum error common to ν_o, ν_1 and ν_2. For up to 500 (uniform) random samples of ν's, the program calculates the average values of (β^*, α^*), their rms deviations and their maximum and minimum values.

III. Examples

In the following examples, tunes are always for the horizontal betatron oscillation.

A <u>Normal main ring</u>

$(B'/B\rho) = \pm 0.01802/\text{m}^2$, $\nu_o = 19.39536$

$\beta^* = 72.88$ m, $\alpha^* = -0.7259$

fractional change of the gradient

in (dD): - 15%, $\nu_1 = 19.44538$

in (Ff): + 6.5%, $\nu_2 = 19.44338$

results from the computer program:

1. maximum error in $\nu = 0.005$

   $\beta^* = (72.37 \pm 9.1)$ m, 85% of 500 cases

   $\alpha^* = (-0.7265 \pm 0.103)$

2. maximum error in $\nu = 0.0025$

   $\beta^* = (72.49 \pm 4.5)$ m

   $\alpha^* = (-0.7264 \pm 0.052)$

B. <u>Low-beta insertion</u>

$(B'/B\rho) = \pm 0.017527/m^2$ for main quadrupoles,

$= -0.043368/m^2$ for (a), see Fig. 1.

$= 0.048808/m^2$ for (b),

$= -0.041746/m^2$ for (c).

$\nu_o = 19.37704$, $\beta^* = 2.50$ m, $\alpha^* = 0.0868$

fractional change of the gradient

in (dD): -1.25%, $\nu_1 = 19.44853$

in (Ff): +1.75%, $\nu_2 = 19.44188$

results from the computer program:

1. maximum error in $\nu = 0.005$

   $\beta^* = (2.54 \pm 0.30)$ m, 85% of 500 cases

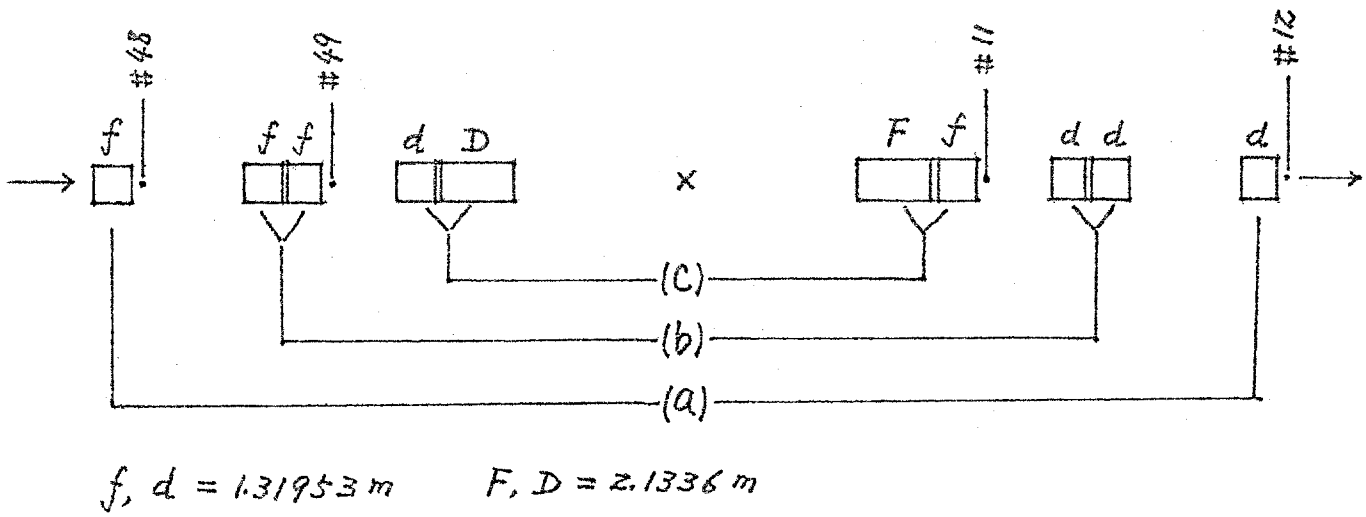
   $\alpha^* = (0.0869 \pm 0.18)$

2. maximum error in $\nu = 0.0025$

   $\beta^* = (2.51 \pm 0.097)$ m

   $\alpha^* = (0.0863 \pm 0.079)$.

Fig. 1 Low-beta insertion in the main ring proposed by T. Collins (ref. 7). Three independent excitations are used for quadrupoles around the long straight section. There are three dipoles between (f) and (ff), four between (dd) and (d).



References

1. Memo from E. Malamud to R. Juhala, February 4, 1972.
2. E. D. Courant and H. S. Snyder, Annals of Physics, 3, 1(1958); Eqs. (4.49) and (4.50), p. 26. There are misprints in these equations. There should be a minus sign in front of the right hand side expressions. The quantity $\phi_2 - \phi_1$ should be replaced by $\phi_1 - \phi_2$ in Eq. (4.49) and $\phi - \phi_1$ by $\phi_1 - \phi$ in Eq. (4.50).
3. S. Ohnuma, TM-437, August 10, 1973.
4. R. Diebold, Colliding Beams Seminar.
5. A series of measurements were made by R. E. Peters in October-December, 1974 but the results were never made 'public'. There are two volumes of data in the Booster Annex, Cross Gallery West.
6. E. D. Courant and H. S. Snyder, loc cit, Eq. (4.37).
7. T. L. Collins, TM-649, March 11, 1976. Examples given in this report are for ν_h= 20.28, the original design value.