Neutrino mass models with one vanishing minor under Z_8 cyclic group symmetry

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Some Type-I seesaw mass models [1] are revisited with the motivation to study their textures under Z_8 cyclic symmetry. These neutrino mass models are based on both diagonal charged lepton mass matrices and Dirac neutrino mass matrices with the non-diagonal right handed Majorana neutrino mass matrices M_R. We observe that the one zero texture of M_R propagate to the left-handed neutrino mass matrices M_ν as their vanishing minors and the symmetry of the textures corresponds to Z_8 cyclic group symmetry. It is done by extending the SM to include two or three Higgs doublet and some SU(2) singlet scalar.

1. Introduction

The series of neutrino experiments have confirmed without any doubt that the neutrinos are massive. To achieve massive neutrinos in theory, one has to move beyond the SM of particle physics which can accommodate massless neutrinos only. The basic SM extensions include the existence of right-handed Majorana neutrino. These RH Majorana neutrinos are often used to explain neutrino masses via the type-I seesaw mechanism.

In the Type-I seesaw mechanism, the effective neutrino mass matrix M_ν is given by

$$M_\nu = -M_D M_R^{-1} M_D^T, \tag{1}$$

where M_D and M_R are respectively the Dirac neutrino mass matrix and right-handed Majorana mass matrix. Zero textures [2, 3, 4], vanishing minors [5] and hybrid textures [6] are some of the interesting proposals that are being studied extensively so as to restrict the form of the neutrino mass matrix and thus reduce the number of free parameters. Zero textures in M_ν have been extensively studied because of their implications for the possible existence of family symmetries which require certain entries of the matrix, which are extremely small compared to the other elements of the matrix, to vanish. Since according to equation (1), M_ν is a combination of the Dirac mass matrix M_D and the heavy right-handed Majorana neutrino mass matrix M_R, so the zeros of M_D and M_R, propagate as zeros in M_ν. Thus the study of zero texture of M_D and M_R is more basic than the study of M_ν [7]. The zeros in M_D and M_R apart from propagating as zeros in M_ν may also reflect as vanishing minors in M_ν, provided M_D is diagonal. Zeros in any arbitrary entries of the mass matrices can be enforced effectively with the help of certain cyclic group symmetry [8]. In our paper, we have made use of the Z_8 cyclic group symmetry to enforce zeros in the mass matrices.

The paper is organised as follows: in section 2, we have presented a general form of texture zero and vanishing minor in the neutrino mass matrices. In section 3, we have studied the one zero textures of the symmetric RH Majorana mass matrix of the bimaximally mixed neutrino mass matrices [1], where M_D and M_l are chosen to be diagonal, also vanishing minors were observed in each case. We present symmetry realization of the mass matrices using an Abelian cyclic group Z_8 with suitable scalar singlets and Higg's doublets in section 4. And finally end up with conclusion in section 5.

2. General form of one zero textures of M_R and vanishing minors in M_ν

We work in the basis where the charged lepton mass matrix M_l is diagonal, and the Dirac mass matrix M_D [1] and the general form of the symmetric RH Majorana mass matrix respectively takes the form:

$$M_D = \tan\beta \begin{pmatrix} \lambda^m & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau, \quad M_R = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \tag{2}$$

where $\lambda = 0.22$ is the Wolfenstein parameter, $m = 6, n = 2$ and $\tan\beta m_\tau = C(say)$ is a constant. The effective neutrino mass matrix, in the context of Type-I seesaw mechanism becomes,

$$M_\nu = \frac{1}{|M_R|}\begin{pmatrix} (df-e^2)\lambda^{2m} & (bf-ce)\lambda^{m+n} & (be-cd)\lambda^m \\ (bf-ce)\lambda^{m+n} & (af-c^2)\lambda^{2n} & (ae-bc)\lambda^n \\ (be-cd)\lambda^m & (ae-bc)\lambda^n & (ad-b^2) \end{pmatrix} R_1, \tag{3}$$

where R_1 is a constant and $|M_R| = (adf - ae^2 - b^2f + 2bce - c^2d)$. All the possible one zero texture of the symmetric RH Majorana mass matrix are shown in the table below:

Table 1: All possible one zero texture of M_R.

A	B	C
$\begin{pmatrix} 0 & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$	$\begin{pmatrix} a & 0 & c \\ 0 & d & e \\ c & e & f \end{pmatrix}$	$\begin{pmatrix} a & b & 0 \\ b & d & e \\ 0 & e & f \end{pmatrix}$
D	E	F
$\begin{pmatrix} a & b & c \\ b & 0 & e \\ c & e & f \end{pmatrix}$	$\begin{pmatrix} a & b & c \\ b & d & 0 \\ c & 0 & f \end{pmatrix}$	$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & 0 \end{pmatrix}$

Here we have distinguished the different structures of texture zero in different classes. For example, class A: gives the mass matrix where the zero corresponds to the position 'a' of the matrix; class B: mass matrix where the zero corresponds to the position 'b' of the matrix and so on. As our M_D is diagonal, the zeros of M_R will show as a vanishing minor in M_ν. In the context of type-I seesaw, the effective neutrino mass matrix M_ν with M_D as diagonal and one zero texture of M_R (class A) becomes,

$$M_\nu = \frac{1}{|M_R|}\begin{pmatrix} (df-e^2)\lambda^{2m} & (bf-ce)\lambda^{m+n} & (be-cd)\lambda^m \\ (bf-ce)\lambda^{m+n} & -c^2\lambda^{2n} & -bc\lambda^n \\ (be-cd)\lambda^m & -bc\lambda^n & -b^2 \end{pmatrix}, \tag{4}$$

where $|M_R| = (-b^2f + 2bce - c^2d)$. Thereby giving rise to a vanishing minor in M_ν for a zero corresponding to class A in M_R. Similarly, for each class of M_R, a vanishing minor is obtained in M_ν with M_D as diagonal.

3. One zero texture of the RH Majorana mass matrix

When M_l = diag(m_e, m_μ, m_τ) and M_D = diag(λ^6, λ^2, 1) C (C being a constant) the neutrino mass matrix arises solely due to M_R. It is found that the texture study of the right handed Majorana mass matrices in each case, that is, normal, degenerate and inverted heirarchy from Ref. [1], gives different texture structure.

3.1 Normal heirarchy

I(A): The right-handed Majorana mass matrices [1] of the form,

$$M_R = \begin{pmatrix} \lambda^{11} & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^6 & 0 \\ \lambda^5 & 0 & 1 \end{pmatrix} v_R \tag{5}$$

with $v_R \approx 10^{14}$ gives rise to a structure with $e = 0$ (class E), thereby leading to the following form of effective neutrino mass matrix through seesaw mechanism:

$$M_\nu = \frac{1}{|M_R|}\begin{pmatrix} df\lambda^{12} & bf\lambda^8 & -cd\lambda^6 \\ bf\lambda^8 & (af-c^2)\lambda^4 & -bc\lambda^2 \\ -cd\lambda^6 & -bc\lambda^2 & (ad-b^2) \end{pmatrix} R_1, \tag{6}$$

where R_1 is a constant and $|M_R| = (adf - b^2f - c^2d)$. Thus a vanishing minor is obtained in M_ν for $e = 0$ in M_R.

3.2 Degenerate case

I(B): The right-handed Majorana mass matrix [1] is of the form,

$$M_R = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{12} & \delta_1\lambda^8 & \delta_1\lambda^6 \\ \delta_1\lambda^8 & (1+\delta_2)\lambda^4 & \delta_2\lambda^2 \\ \delta_1\lambda^6 & \delta_2\lambda^2 & (1+\delta_2) \end{pmatrix} v_R, \tag{7}$$

where $\delta_1 = 3.6\times10^{-5}, \delta_2 = 3.9\times10^{-3}, v_R \approx 10^{13}$ GeV, enforcing $(\delta_1+\delta_2)\lambda^{12} = 0, \delta_1\lambda^8 = 0$, M_R reduces to the following structure:

$$M_R = \begin{pmatrix} a & 0 & c \\ 0 & d & e \\ c & e & f \end{pmatrix} (\text{class B}). \tag{8}$$

3.3 Inverted hierarchy

II(A): With $a = 0.5, \epsilon = 0.002, \eta = 0.0001, v_R \approx 10^{12}$ in

$$M_R = \begin{pmatrix} 2a\eta(1+2\epsilon)\lambda^{12} & \eta\epsilon\lambda^8 & \eta\epsilon\lambda^6 \\ \eta\epsilon\lambda^8 & a\lambda^4 & -(a-\eta)\lambda^2 \\ \eta\epsilon\lambda^6 & -(a-\eta)\lambda^2 & a \end{pmatrix} \frac{v_R}{2a\eta} \tag{9}$$

the terms $a\eta\epsilon\lambda^{12}$ and $\eta\epsilon\lambda^8$ can be effectively forced to zero.Thereby reducing M_R to the following form,

$$M_R = \begin{pmatrix} a & 0 & c \\ 0 & d & e \\ c & e & f \end{pmatrix} (\text{class B}). \tag{10}$$

Thus the structure of M_ν (I(B), II(A) with $b = 0$) becomes,

$$M_\nu = \frac{1}{|M_R|} \begin{pmatrix} (df-e^2)\lambda^{12} & -ce\lambda^8 & -cd\lambda^6 \\ -ce\lambda^8 & (af-c^2)\lambda^4 & ae\lambda^2 \\ -cd\lambda^6 & ae\lambda^2 & ad \end{pmatrix} m_0, \tag{11}$$

where m_0 is a constant and $|M_R| = (adf - ae^2 - c^2d)$. In each case a vanishing minor is obtained in M_ν for a corresponding zero in M_R. Similarly when a texture study is made on all the other models I(B), I(A), I(C), II(B) from Ref. [1] they reveals different texture structure as shown in the Table below:

Table 2: Texture structure of the mass models.

Mass Models ($M_R's$)		Zero Texture in M_R
I(B) (Normal)	$\begin{pmatrix} -\lambda^{10} & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda \\ \lambda^4 & \lambda & 1 \end{pmatrix} v_R$, where $v_R \approx 10^{13}$ GeV	$M_{R_{11}} = 0$ Class A
I(A) (Degenerate)	$\begin{pmatrix} -2\delta_2\lambda^{2m} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{\sqrt{2}}+\delta_1)\lambda^m \\ \frac{1}{\sqrt{2}}+\delta_1)\lambda^{m+n} & (\frac{1}{2}+\delta_1-\delta_2)\lambda^{2n} & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^n \\ (\frac{1}{\sqrt{2}}+\delta_1)\lambda^m & (-\frac{1}{2}+\delta_1-\delta_2)\lambda^n & (\frac{1}{2}+\delta_1-\delta_2) \end{pmatrix} v_R$, where $\delta_1 = 6.18\times10^{-3}$,$\delta_2 = 3.06\times10^{-3}$,$v_R \approx 10^{13}$ GeV	$M_{R_{11}} = 0$ Class A
I(C) (Degenerate)	$\begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{2m} & \delta_1\lambda^{m+n} & \delta_1\lambda^m \\ \delta_1\lambda^{m+n} & \delta_2\lambda^{2n} & (1+\delta_2)\lambda^n \\ \delta_1\lambda^m & (1+\delta_2)\lambda^n & \delta_2 \end{pmatrix} v_R$, where $\delta_1 = 3.6\times10^{-5}$, $\delta_2 = 3.9\times10^{-3}$, $v_R \approx 10^{13}$ GeV	$M_{R_{12}} = 0$ Class B
II(B) (Inverted)	$\begin{pmatrix} -\lambda^{15} & \lambda^8 & \lambda^6 \\ \lambda^8 & \lambda & \lambda^{-1} \\ \lambda^6 & \lambda^{-1} & \lambda^{-3} \end{pmatrix} v_R$, where $v_R \approx 10^{12}$ GeV	$M_{R_{11}} = 0$ Class A

4. Symmetry Realization

Texture zeros can be enforced in any arbitrary entries of the mass matrices with an extended scalar sector and Higg's doublet by means of Abelian symmetries. For symmetry realization of different textures of M_R we consider the Z_8 cyclic group symmetry. The leptonic field transformations are different for different texture mass matrices. Let the fields transform under Z_8 as

$\bar{D}_{e_L} \to \omega \bar{D}_{e_L}, e_R \to \omega^6 e_R, \nu_{e_R} \to \nu_{e_R},$
$\bar{D}_{\mu_L} \to \omega^4 \bar{D}_{\mu_L}, \mu_R \to \omega^5 \mu_R, \nu_{\mu_R} \to \omega^4 \nu_{\mu_R},$
$\bar{D}_{\tau_L} \to \omega^6 \bar{D}_{\tau_L}, \tau_R \to \omega^2 \tau_R, \nu_{\tau_R} \to \omega \nu_{\tau_R},$

where $\omega = e^{i2\pi/8}$ is the generator of Z_8, $\bar{D}_{jL} (j = e, \mu, \tau)$ denotes $SU(2)_L$ doublets, $l_R(l = e, \mu, \tau)$ denotes the RH $SU(2)_L$ singlets and $\nu_{k_R}(k = e, \mu, \tau)$, the RH neutrino singlets.

The bilinears $\bar{D}_{j_L}\nu_{k_R}, \bar{D}_{j_L} l_R, \bar{\nu}^T_{k_R} C^{-1}\nu_{j_R}$ relevant for M_D ,M_l and M_R respectively transforms as

$$\bar{D}_{k_L}\nu_{j_R} = \begin{pmatrix} \omega & \omega^5 & \omega^2 \\ \omega^4 & 1 & \omega^5 \\ \omega^6 & \omega^2 & \omega^7 \end{pmatrix}, \quad \bar{D}_{k_L} e_{j_R} = \begin{pmatrix} \omega^7 & \omega^6 & \omega^3 \\ \omega^2 & \omega & \omega^6 \\ \omega^4 & \omega^3 & 1 \end{pmatrix}, \quad \bar{\nu}^T_{k_R} C^{-1}\nu_{j_R} = \begin{pmatrix} 1 & \omega^4 & \omega \\ \omega^4 & 1 & \omega^5 \\ \omega & \omega^5 & \omega^2 \end{pmatrix} \tag{12}$$

We introduce three $SU(2)_L$ doublet Higg's (ϕ_1, ϕ_2, ϕ_3) transforming under Z_8 as

$$\phi_1 \to \phi_1, \quad \phi_2 \to \omega\phi_2, \quad \phi_3 \to \omega^7\phi_3. \tag{13}$$

For the case i(a)(class E texture)we consider three scalar singlets $(\chi_{12}, \chi_{13}, \chi_{33})$ transforming under Z_8 as

$$\chi_{12} \to \omega^4\chi_{12}, \quad \chi_{13} \to \omega^7\chi_{13}, \quad \chi_{33} \to \omega^6\chi_{33}. \tag{14}$$

thereby leading to the following form of M_D, M_l and M_R:

$$M_D = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}, \; M_l = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}, \; M_R = \begin{pmatrix} X & X & X \\ X & X & 0 \\ X & 0 & X \end{pmatrix}. \tag{15}$$

The Z_8 invariant Yukawa Lagrangian becomes,

$$\begin{aligned} -\mathcal{L} = {} & Y^l_{11}\bar{D}_{e_L}\Phi_2 e_R + Y^l_{22}\bar{D}_{\mu_L}\Phi_3\mu_R + Y^l_{33}\bar{D}_{\tau_L}\Phi_1\tau_R + Y^D_{11}\bar{D}_{e_L}\tilde{\Phi}_3\nu_{e_R} + Y^D_{22}\bar{D}_{\mu_L}\tilde{\Phi}_1\nu_{\mu_R} \\ & + Y^D_{33}\bar{D}_{\tau_L}\tilde{\Phi}_2\nu_{\tau_R} + \frac{M^M_{11}}{2}\bar{\nu}^T_{e_R}C^{-1}\nu_{e_R} + \frac{Y^M_{12}}{2}\bar{\nu}^T_{e_R}C^{-1}\nu_{\mu_R}\chi_{12} + \frac{Y^M_{13}}{2}\bar{\nu}^T_{e_R}C^{-1}\nu_{\tau_R}\chi_{13} \\ & + \frac{M^M_{22}}{2}\bar{\nu}^T_{\mu_R}C^{-1}\nu_{\mu_R} + \frac{Y^M_{33}}{2}\bar{\nu}^T_{\tau_R}C^{-1}\nu_{\tau_R}\chi_{33} + h.c, \end{aligned} \tag{16}$$

where $\tilde{\phi} = i\tau_2\phi*$. Similarly for the case I(B) (class B texture), we consider three scalar singlets:

$$\chi_{13} \to \omega^7\chi_{13}, \quad \chi_{23} \to \omega^3\chi_{23}, \quad \chi_{33} \to \omega^6\chi_{33}.$$

reducing M_R to the form,

$$M_R = \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}. \tag{17}$$

The Z_8 invariant Yukawa Lagrangian becomes,

$$\begin{aligned} -\mathcal{L} = {} & Y^l_{11}\bar{D}_{e_L}\Phi_2 e_R + Y^l_{22}\bar{D}_{\mu_L}\Phi_3\mu_R + Y^l_{33}\bar{D}_{\tau_L}\Phi_1\tau_R + Y^D_{11}\bar{D}_{e_L}\tilde{\Phi}_3\nu_{e_R} + Y^D_{22}\bar{D}_{\mu_L}\tilde{\Phi}_1\nu_{\mu_R} \\ & + Y^D_{33}\bar{D}_{\tau_L}\tilde{\Phi}_2\nu_{\tau_R} + \frac{M^M_{11}}{2}\bar{\nu}^T_{e_R}C^{-1}\nu_{e_R} + \frac{Y^M_{13}}{2}\bar{\nu}^T_{e_R}C^{-1}\nu_{\tau_R}\chi_{13} + \frac{M^M_{22}}{2}\bar{\nu}^T_{\mu_R}C^{-1}\nu_{\mu_R} \\ & + \frac{Y^M_{23}}{2}\bar{\nu}^T_{\mu_R}C^{-1}\nu_{\tau_R}\chi_{23} + \frac{Y^M_{33}}{2}\bar{\nu}^T_{\tau_R}C^{-1}\nu_{\tau_R}\chi_{33} + h.c. \end{aligned} \tag{18}$$

Here M_R contains two types of mass terms viz. the terms arising from Yukawa coupling to the scalar singlets

$\chi's$ and bare mass terms. The scale of the former depends upon the scale of the Abelian Group Z_8 breaking. The bare mass terms are the mass terms that arises without the involvement of the extra scalar singlets which were incorporated in our extended form of the SM. So there is no restriction on the scale of the bare mass terms, and hence can have a higher mass scale. Thus a large effective neutrino mass can arise in such a model.
For the case II(A), the texture structure and symmetry realization is same as I(B). The symmetry realization of all the other models are illustrated in the table below:

Table 3: Transformation properties of leptons and scalar fields under Z_8 cyclic group.

Models	$\bar{D}_{e_L}, \bar{D}_{\mu_L}, \bar{D}_{\tau_L}$	e_R, μ_R, τ_R	$\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R}$	ϕ's	χ's
I(B), I(A), II(B)	$\omega^7, \omega^5, \omega^4$	$\omega^5, \omega^3, \omega^4$	$\omega, \omega^3, 1$	$1, \omega^4$	$\omega^4, \omega^7, \omega^2, \omega^5$
I(C)	$\omega, \omega^4, \omega^6$	$\omega^6, \omega^5, \omega^2$	$1, \omega^4, \omega$	$\omega, 1, \omega^7$	$\omega^7, \omega^3, \omega^6$

5. Conclusion

We have explored one zero texture of the bimaximally mixed neutrino mass models [1] and achieved the symmetry realization of these textures under Z_8 abelian group. It is an interesting observation that the location of one zero of symmetric M_R determines the possible schemes of neutrino mass spectrum in certain cases. The $a = 0$ texture (class A) of M_R allows all the three schemes, viz., normal, degenerate and inverted schemes. The $b = 0$ texture (class B) favours the degenerate and inverted schemes, while the $e = 0$ texture (class E) allows only the normal hierarchical scheme. The $c = 0$ (class C), $d = 0$ (class D) and $f = 0$ (class F) textures of M_R have not been found to favour any models under consideration. For the a=0 texture models, two Higgs doublet and four scalar singlets need to be included to extend SM while for the $b = 0$ and $e = 0$ texture models, three Higg's doublet and three scalar singlets are required to extend the SM. It is the inherent problem of symmetry realization of one zero texture of neutrino mass matrices which requires a couple of Higg's doublets and scalar singlets.

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