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Structure of Exotic Mg Isotopes and Temperature Dependence of the Symmetry Energy of Finite Nuclei

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Abstract. We study various ground-state properties of neutron-rich and neutron-deficient Mg isotopes with $A=20-36$ in the framework of the self-consistent deformed Skyrme-Hartree-Fock plus BCS method. The nuclear symmetry energy is investigated for the same isotopic chain following the theoretical approach based on the coherent density fluctuation model. The results of the calculations show that the behavior of the nuclear charge radii and the nuclear matter properties in the Mg isotopic chain is closely related to the nuclear deformation. The temperature dependence of the symmetry energy for isotopic chains of even-even Ni ($A=58-82$), Sn ($A=124-152$), and Pb ($A=202-214$) nuclei is investigated in the framework of the local density approximation. The results for the thermal evolution of the symmetry energy coefficient show that for all isotopic chains considered and for both Skyrme forces used in the calculations the symmetry energy coefficient decreases with the increase of the mass number in the temperature interval $T=0-5$ MeV.

1. Introduction

The study of exotic nuclei is one of the main topics of modern nuclear structure physics. The recent developments of radioactive beams enable us to study such unstable nuclei toward the neutron and proton drip lines. Close to them, a large variety of formerly unknown nuclear configurations has been observed. The magic numbers in such exotic systems can be a quite intriguing issue. New magic numbers appear and some others disappear in moving from stable to exotic nuclei in a rather novel manner due to specific components of the nucleon-nucleon interaction (see, for example, Ref. [1]).

Low-lying states of neutron-rich nuclei around the neutron number $N=20$ attract a great interest, as the spherical configurations associated with the magic number disappear in the ground states. For ^{32}Mg , from the observed population of the excited 0_2^+ state (found at 1.058 MeV) in the (t, p) reaction on ^{30}Mg , it is suggested [2] that the 0_2^+ state is a spherical one coexisting with the deformed ground state and that their relative energies are inverted at $N=20$. Very recently, a new signature of an existence of "island of inversion" [3] has been experimentally tested by measuring the charge radii of all magnesium isotopes in the sd shell at ISOLDE-CERN [4] showing that the borderline of this island lies between ^{30}Mg and ^{31}Mg .



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In the light of the new precise spectroscopic measurements of the neutron-rich ^{32}Mg nucleus that lies in the much explored "island of inversion" at $N=20$, in the present work (see also Ref. [5]) we aim to perform a systematic study of the nuclear ground-state properties of neutron-rich and neutron-deficient Mg isotopes with $A=20\text{--}36$, such as charge and matter rms radii, two-neutron separation energies, neutron, proton, and charge density distributions, neutron (proton) rms radii and related with them thickness of the neutron (proton) skins. The new data for the charge rms radii [4] is a challenging issue to test the applicability of the mean-field description to light nuclei, thus expecting to understand in more details the nuclear structure revealed by them. Following our recent works [6, 7], we analyze the correlation between the skin thickness and the characteristics related to the density dependence of the nuclear symmetry energy for the same Mg isotopic chain. Such an analysis may probe the accurate account for the effects of interactions in our method within the considered Mg chain, where the breakdown of the shell model could be revealed also by the nuclear symmetry energy changes. A special attention is paid to the neutron-rich ^{32}Mg nucleus by performing additional calculations modifying the spin-orbit strength of the effective interaction, to check theoretically the possible appearance of the "island of inversion" at $N=20$.

As known, the nuclear symmetry energy is a measure of the energy gain in converting isospin asymmetric nuclear matter (ANM) to a symmetric system. Its value depends on the density and temperature. The need of information for the symmetry energy in finite nuclei, even theoretically obtained, is a major issue because it allows one to constrain the bulk and surface properties of the nuclear energy-density functionals (EDFs) quite effectively. Therefore, in the present paper the thermal behavior of the symmetry energy is investigated for Ni, Sn, and Pb isotopic chains using different model local density distributions for these nuclei and the symmetry energy coefficient for a specific nucleus is calculated in the local density approximation (LDA) [8].

2. Theoretical framework

The results for the nuclear properties of Mg isotopes have been obtained from self-consistent deformed Hartree-Fock calculations with density dependent Skyrme interactions [9] and pairing correlations. Pairing between like nucleons has been included by solving the BCS equations at each iteration with a fixed pairing strength that reproduces the odd-even experimental mass differences [10]. We consider the Skyrme forces SLy4, Sk3, and SGII because they are among the most extensively used and are considered as standard references.

The spin-independent proton and neutron densities are given by [11, 12]

$$\rho(\vec{R}) = \rho(r, z) = \sum_i 2v_i^2 \rho_i(r, z), \quad (1)$$

where r and z are the cylindrical coordinates of \vec{R} , v_i^2 are the occupation probabilities resulting from the BCS equations and ρ_i are the single-particle densities. The mean square radii for protons and neutrons are defined as

$$\langle r_{p,n}^2 \rangle = \frac{\int R^2 \rho_{p,n}(\vec{R}) d\vec{R}}{\int \rho_{p,n}(\vec{R}) d\vec{R}}, \quad (2)$$

and the rms radii for protons and neutrons are given by

$$r_{p,n} = \langle r_{p,n}^2 \rangle^{1/2}. \quad (3)$$

Having the neutron and proton rms radii [Eq. (3)], the neutron skin thickness is usually estimated as their difference:

$$\Delta R = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}. \quad (4)$$

The mean square radius of the charge distribution in a nucleus can be expressed as

$$\langle r_{ch}^2 \rangle = \langle r_p^2 \rangle + \langle r_{ch}^2 \rangle_p + (N/Z) \langle r_{ch}^2 \rangle_n + r_{CM}^2 + r_{SO}^2, \quad (5)$$

where $\langle r_p^2 \rangle$ is the mean square radius of the point proton distribution in the nucleus (2), $\langle r_{ch}^2 \rangle_p$ and $\langle r_{ch}^2 \rangle_n$ are the mean square charge radii of the charge distributions in a proton and a neutron, respectively, r_{CM}^2 is a small correction due to the center of mass motion, and the last term r_{SO}^2 is a tiny spin-orbit contribution to the charge density. Correspondingly, we define the charge rms radius $r_c = \langle r_{ch}^2 \rangle^{1/2}$.

We calculate the symmetry energy s of Mg isotopes using the coherent density fluctuation model (CDFM) [13, 14]. The CDFM allows us to make a transition from the properties of nuclear matter to those of finite nuclei. The analysis of the infinite nuclear matter has been carried out on the basis of the Brueckner EDF [15, 16]. It can be shown in the CDFM that under some approximation the properties of finite nuclei can be calculated using the corresponding ones for ANM, folding them by the weight function $|f(x)|^2$. The latter can be obtained by using a known density distribution $\rho(r)$ for a given nucleus (see, e.g. [5])

$$|f(x)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho(r)}{dr} \right|_{r=x}, \quad (6)$$

where $\rho_0(x) = 3A/4\pi x^3$ and with the normalization $\int_0^\infty dx |f(x)|^2 = 1$. In the CDFM the symmetry energy for finite nuclei is expressed by that of ANM $s^{ANM}(x)$:

$$s = \int_0^\infty dx |f(x)|^2 s^{ANM}(x). \quad (7)$$

For finite systems, different definitions of the symmetry energy coefficient are adopted in the literature yielding different values. In our case we calculate the symmetry energy coefficient for a specific nucleus within the LDA and it is given as

$$e_{sym}(A, T) = \frac{1}{I^2 A} \int \rho(r) e_{sym}[\rho(r), T] \delta_l^2(r) d^3r. \quad (8)$$

In Eq. (8) $e_{sym}[\rho(r), T]$ is the symmetry energy coefficient at temperature T of infinite matter at the value of the local density $\rho(r)$, $\delta_l(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$ is the isospin asymmetry of the local density, where $\rho_n(r)$ and $\rho_p(r)$ are the neutron and proton densities, $\rho(r) = \rho_n(r) + \rho_p(r)$ and $I = (N - Z)/A$. The symmetry energy coefficient can be approximated by

$$e_{sym}(\rho, T) = \frac{e(\rho, \delta, T) - e(\rho, \delta = 0, T)}{\delta_l^2}, \quad (9)$$

where $e_{sym}(\rho, \delta, T)$ is the energy per nucleon in an asymmetric infinite matter, while $e(\rho, \delta = 0, T)$ is that one of symmetric nuclear matter. For an infinite system the energy per nucleon is calculated as $e = \varepsilon(r)/\rho$. We use for the total energy density of the system $\varepsilon(r)$ the Skyrme energy density functional with two Skyrme-class effective interactions, SkM* and SLy4. The symmetry coefficient $e_{sym}(\rho, T)$ can then be computed from Eq. (9). In contrast to the methodology employed in Ref. [8], where the kinetic energy density $\tau_q(r)$ (with $q=(n, p)$ referring to neutrons or protons) entering the expression for $\varepsilon(r)$ is considered in the Thomas-Fermi approximation at finite temperature, we use in our theoretical scheme $\tau_q(r)$ [17]

$$\tau_q(r) = \frac{2m}{\hbar^2} \varepsilon_{K_q} = \frac{3}{5} (3\pi^2)^{2/3} \left[\rho_q^{5/3} + \frac{5\pi^2 m_q^2}{3\hbar^4} \frac{1}{(3\pi^2)^{4/3}} \rho_q^{1/3} T^2 \right], \quad (10)$$

which is valid at low T . The temperature-dependent densities are calculated through the HFBTHO code that solves the nuclear Skyrme-Hartree-Fock-Bogoliubov (HFB) problem by using the cylindrical transformed deformed harmonic-oscillator basis [18]. In addition, two other density distributions of ^{208}Pb [19], namely the Fermi-type density determined within the extended Thomas-Fermi (ETF) method [20] and the symmetrized-Fermi local density obtained within the rigorous density functional approach (RDFA) [21], are used.

3. Results and discussion

The evolution of the quadrupole parameter β of the ground states as a function of the mass number A of neutron-rich and neutron-deficient Mg isotopes is presented in Fig. 1. As expected, the semi-magic ^{20}Mg isotope ($N=8$) is spherical. As the number of neutrons increases we start populating first the $d_{5/2}$ orbital between ^{22}Mg and ^{26}Mg , leading to prolate and oblate deformations in the energy profiles. Prolate shapes with $\beta \sim 0.4$ are ground states in ^{22}Mg and ^{24}Mg , whereas an oblate shape at around $\beta \sim -0.25$ is developed in ^{26}Mg in competition with a prolate shape at around $\beta \sim 0.35$. Due to the conjunction of the $N=Z=12$ deformed shell effects, the nucleus ^{24}Mg is the most deformed of the isotopic chain. We obtain rather flat profiles of the corresponding potential-energy curves around sphericity in $^{28,30}\text{Mg}$, whose correct description would need beyond mean-field techniques involving configuration mixing of the shape fluctuations. ^{32}Mg becomes also spherical, thus showing that the magic number $N=20$ exists for the ground-state of ^{32}Mg in the mean-field theories [22]. For heavier isotopes the prolate deformation grows with the increase of the neutron number for $^{32-36}\text{Mg}$. In general, we find almost identical values of the quadrupole parameter β with the three Skyrme parametrizations.

The results for the symmetry energy s [Eq. (7)] as a function of the mass number A for the whole Mg isotopic chain ($A=20-36$) are presented in Fig. 2. It is seen that the SGII and Sk3 forces yield values of s comparable with each other that lie above the corresponding symmetry energy values when using SLy4 set. Although the values of s slightly vary within the Mg isotopic chain (23–26 MeV) when using different Skyrme forces, the curves presented in Fig. 2 exhibit the same trend. It is useful to search for possible indications of an "island of inversion" around $N=20$ revealed also by the symmetry energy. We would like to note that the modification of the spin-orbit strength of the SLy4 effective interaction by increasing it by 20% corresponding to prolate deformed ground-state of ^{32}Mg leads to a smaller value of $s=23.67$ MeV compared with the one for the spherical case $s=24.75$ MeV. Indeed, the results shown in Fig. 2 are related to the evolution of the quadrupole parameter β as a function of the mass number A that is presented in Fig. 1. Although the considered Mg chain does not contain a double-magic isotope, it is worth mentioning that we find maximum values within a plateau around the semi-magic isotope $A=32$ that resembles the sharp peak observed in previous works including double-magic nuclei [6, 7].

The charge radius is related to the deformation and the isotope shifts of charge radii can be used to investigate the deformations in the isotopic chains. Our results for the squared charge radii differences $\delta < r_c^2 >_{26,A}^2 = < r_c^2 >^A - < r_c^2 >^{26}$ taking the radius of ^{26}Mg as the reference are compared in Fig. 3 with the experimental data [4]. In general, different Skyrme forces do not differ much in their predictions of charge rms radii of magnesium spanning the complete *sd* shell. The trend of the behavior of the experimental points and theoretical values strongly corresponds to the neutron shell structure. For $^{21-26}\text{Mg}$ isotopes the charge distribution is compressed due to the filling of the $d_{5/2}$ orbital and the charge radii do not fluctuate too much. The addition of more neutrons on either $s_{1/2}$ or $d_{3/2}$ in the range $^{28-30}\text{Mg}$ results in a fast increase of the radius. Finally, for isotopes beyond ^{30}Mg , where the "island of inversion" does exist in terms of the rms charge radius [4], the theoretical results underestimate clearly the experimental points. Obviously, an additional treatment is needed to understand in more details this specific region. We note the intermediate position of ^{27}Mg , where a minimum is observed in Fig. 3, since one of

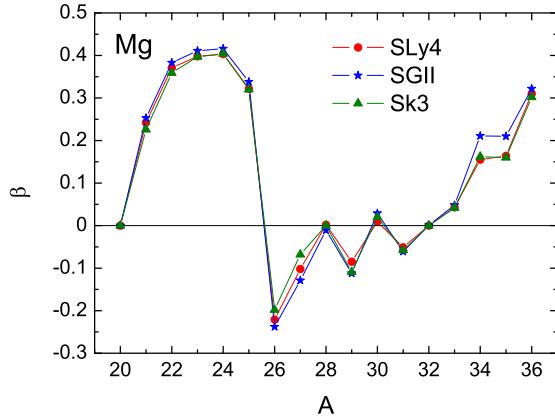


Figure 1. The quadrupole parameter β of the ground state as a function of the mass number A for Mg isotopes ($A=20-36$) in the cases of SLy4, SGII, and Sk3 forces.

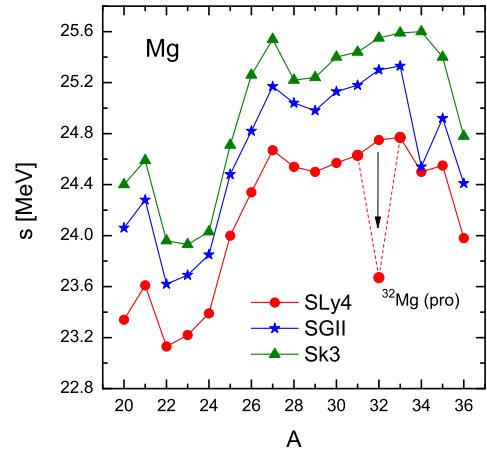


Figure 2. The symmetry energies s for Mg isotopes ($A=20-36$) calculated with SLy4, SGII, and Sk3 forces.

the neutrons added to ^{25}Mg fills the last $d_{5/2}$ hole and the other one populates the $s_{1/2}$ subshell.

We plot in Fig. 4 the differences between the rms radii of neutrons and protons $\Delta R = r_n - r_p$. The latter is a simple measure of a neutron (proton) skin emergence in Mg isotopes from the considered isotopic chain. We can see from Fig. 4 that ΔR increases monotonically with neutron excess in the chain of Mg isotopes. From the Skyrme HF analysis of the rms radii of proton and neutron density distributions Lenske *et al.* [23] have found a proton skin in the neutron-deficient Mg isotopes, while at the neutron dripline the Mg isotopes develop extended neutron skins. It can be also seen from Fig. 4 that, in particular when using SLy4 force, a rather pronounced proton skin in ^{20}Mg develops which exceeds the neutron density by 0.33 fm. The latter value almost coincides with the value of 0.34 fm that has been obtained in Ref. [23] using a standard Skyrme interaction. In the neutron-rich Mg isotopes our calculations predict quite massive neutron skins whose thickness for ^{36}Mg nucleus reaches 0.39 fm when the same SLy4 parametrization is used.

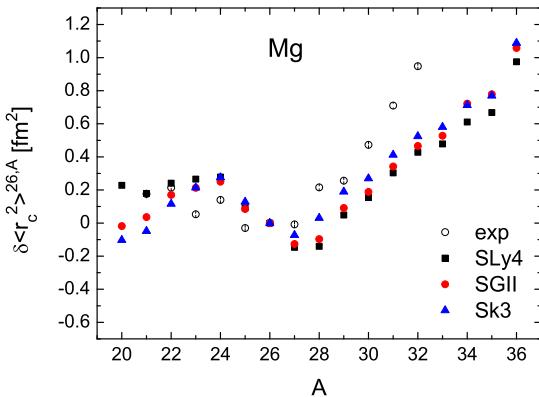


Figure 3. Theoretical (with different Skyrme forces) and experimental [4] isotope shifts $\delta < r_c^2 >^26A$ of magnesium isotopes relative to ^{26}Mg .

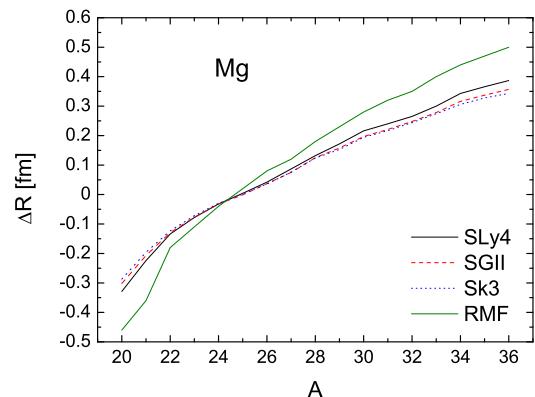


Figure 4. Difference between neutron and proton rms radii Δr_{np} of Mg isotopes calculated by using SLy4, SGII, and Sk3 forces. The RMF calculation results are from Ref. [22].

In Fig. 5 a comparison between the results for the symmetry energy coefficient for ^{208}Pb obtained by using three different densities, namely within the ETF, RDFA and HFB methods (with SkM* and SLy4 forces), is made. The results for the thermal evolution of the symmetry energy coefficient in the interval $T=0\text{--}5$ MeV show that its values decrease with temperature being larger in the case of symmetrized-Fermi density of ^{208}Pb obtained within the RDFA. Figure 6 illustrates the isotopic evolution of the symmetry energy coefficient on the example of the Pb chain ($A=202\text{--}214$) in the case of SLy4 Skyrme interaction. A smooth decrease of e_{sym} is observed with the increase of the mass number. We also would like to note the lack of kink for this Pb isotopic chain when looking at the values of e_{sym} at zero temperature. This result confirms our previous observations when studying the density dependence of the symmetry energy for Pb isotopes [6, 7].

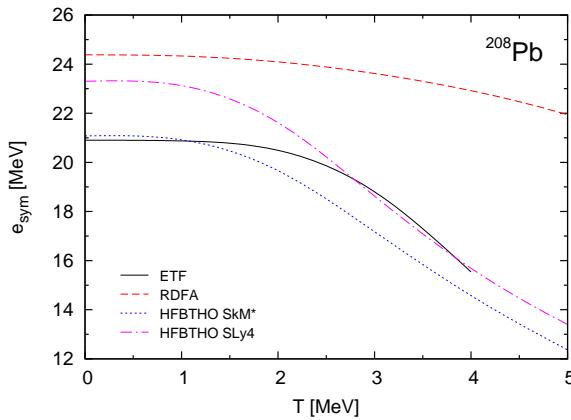


Figure 5. Comparison of the results for the symmetry energy coefficient e_{sym} for ^{208}Pb calculated with ETF, RDFA and HFB (with SkM* and SLy4 forces) densities.

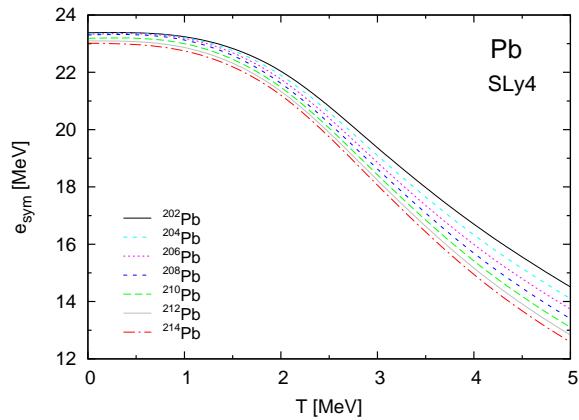


Figure 6. The temperature dependence of the symmetry energy coefficient e_{sym} obtained for several nuclei from Pb isotopic chain ($A=202\text{--}214$) in HFB method with SLy4 force.

4. Conclusions

The results of the present work can be summarized as follows:

- i) The charge and mass radii follow the trends observed in the experiment, with fluctuating values up to $A=26$, and smoothly increasing values with A , beyond $A=27$. These global properties are found to be rather similar with the three Skyrme forces.
- ii) The values of the symmetry energy s vary roughly between 23 and 26 MeV. More dramatic is the considerable change in the trend of symmetry energy evolution with the mass number, when we include the results for the prolate ($\beta=0.38$) ground-state of ^{32}Mg obtained with the spin-orbit modified SLy4 effective interaction.
- iii) The ETF and RDFA results for the density distributions demonstrate a smooth function of r at any temperature T , while the Skyrme HFB densities have a stronger T -dependence. In general, the density distributions decrease with the temperature.
- iv) It is observed that for all isotopic chains considered and for both Skyrme forces used in the calculations the symmetry energy coefficient decreases with the increase of the mass number in the same temperature interval.

To conclude, we would like to note that further study is necessary to prove theoretically the existence of an "island of inversion" probed by the REX-ISOLDE experiment. In particular, it is worth to perform calculations by including effects of tensor and three-body forces and exploring novel energy density functionals.

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