CONSTANT PHASE VELOCITY ACCELERATION SECTIONS IN A PROTON LINAC*

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For an accelerator with constant phase velocity acceleration sections, it is not possible to define a synchronous particle. One can however define a reference particle such that the motion of all particles might be related to this reference particle (1).

1 - Motion in one acceleration section

The equations of the motion in one acceleration section are:

$$\frac{dE}{ds} = e \xi \sin \varphi$$

$$\frac{d\varphi}{ds} = \frac{2\pi}{\lambda} \left(\frac{1}{\beta} - \frac{1}{\beta_r} \right)$$

where s is the direction of the motion, E the energy, ξ the electric field, φ the phase with respect to the accelerating field, λ the wave-length, β the relative velocity and β_r the phase velocity of the accelerating field in the section.

By writing $E - E_r = \Delta E$, the hamiltonian of the motion may be written

$$H = - \frac{\pi}{\lambda} \frac{E_o^2}{\left[E_r^2 - E_o^2\right]^{3/2}} \Delta E^2 + e \xi \cos \varphi$$

E_o is the rest energy of the particle.

* This work was started at the CERN.

A new variable energy may be defined by:

$$W = \frac{\Delta E}{e} A_o \quad \text{with} \quad A_o = \left[\frac{2\pi}{\xi \lambda \frac{Eo}{e} \beta_r^3 \gamma_r^3} \right]^{1/2}$$

The trajectories in the (W, φ) space will be the curves having for equation

$$\left(\frac{W}{2}\right)^2 + \sin^2 \frac{\varphi}{2} = ct$$

The length of an acceleration section is

$$L = \pm \frac{1}{\Omega_o} \int_{\alpha_o}^{\alpha_e} \frac{d\theta}{(1 - k^2 \sin^2 \theta)^{1/2}}$$

with $\Omega_o = \left[\frac{2\pi\xi}{\lambda \frac{Eo}{e} \beta_r^3 \gamma_r^3} \right]^{1/2}$ $\quad k^2 = \left(\frac{W}{2}\right)^2 + \sin^2 \frac{\varphi}{2}, \sin\alpha = \frac{\sin \frac{\varphi}{2}}{k}$

This equation may be solved by introducing the $F(\alpha, k)$ elliptic integral of first kind. If $\phi = \Omega_o L$, one gets

$$\phi = \pm \left[F(\alpha_e, k) - F(\alpha_o, k) \right]$$

2 - The reference particle

The reference particle has for velocity the phase velocity in the middle of the section. At the beginning of the section, the initial conditions for this reference particle are $(\varphi_r, -W_r)$, the final conditions will be $(\varphi_r, +W_r)$. W will then increase by $2 W_r$. From W, φ diagram, two points may have this property, one of them will be stable, the other one unstable.

The phase of the reference particle is variable all the way through the section; in the middle of the section, the phase will have its maximum value φ_m,

$$\text{with} \quad \sin \frac{\varphi_m}{2} = \left[\sin^2 \frac{\varphi_r}{2} + \left(\frac{W_r}{2} \right)^2 \right]^{1/2} .$$

One can compute the length of the acceleration section corresponding to the motion of the reference particle.

$$\phi = 2 \left[F \left(\frac{\pi}{2}, k_r \right) - F \left(\alpha_r, k_r \right) \right]$$

Figure No. 1 gives the curve ϕ = ct in the (W, φ) space. If the length of the section is small, the phase does not vary much, and the motion will be the same as in an accelerator with a synchronous particle, $\varphi_r \simeq \varphi_s$.

3 - Efficiency

For the reference particle, the energy increased in a section will be

$$2 \frac{\Delta E}{e} = \frac{2 W_r}{A_o}$$

If L is the length of the section, and $\mathcal{E}$ the electric field, the efficiency of the section may be written as

$$\eta = \frac{2 \frac{\Delta E}{e}}{\mathcal{E} L}$$

Using reduced variables $\eta = \dfrac{2 W_r}{\phi}$

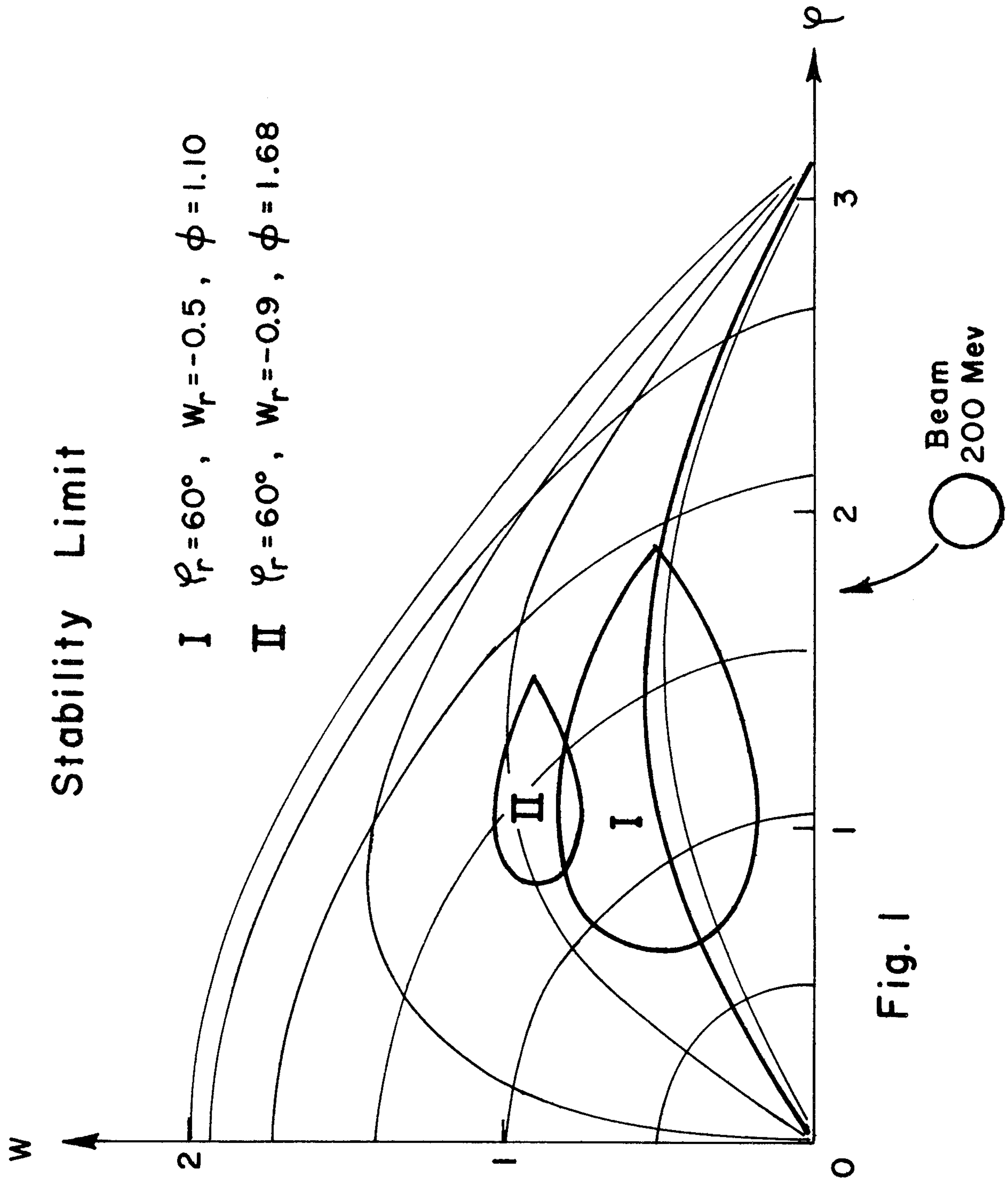
Stability Limit
I $\varphi_r = 60°$, $W_r = -0.5$, $\phi = 1.10$
II $\varphi_r = 60°$, $W_r = -0.9$, $\phi = 1.68$
W
2
1
0
1
2
3
φ
II
I
Beam
200 Mev

Fig. 1

Figure No. 2 gives the curves η = ct in the (W, φ) space. For each value of ϕ , a maximum value of efficiency may be found such that

$$\eta_{max} = \frac{2 \ (W_r) max}{\phi}$$

for smaller values of ϕ , η max = 1, for ϕ = 2, η max = 0. 99 but η_{max} decreases rapidly for $\phi > 2$.

4 - Motion of a particle around the reference particle

Let us now consider a particle with initial conditions (φ_1, W_1). Since this particle will travel the same length in the linac, one may compute the conditions at the end of the section.

$$F (\alpha_2, k) = 2 F (\frac{\pi}{2}, k) - F (\alpha_1, k) - \phi$$

$$\text{with } \sin \alpha_i = \frac{\sin \frac{\varphi_i}{2}}{k} \quad \text{and} \quad k^2 = \sin^2 \frac{\varphi_i}{2} + \left(\frac{W_i}{2}\right)^2$$

This expression gives φ_2 and W_2 , that is the position of the particle at the end of the section. When entering the next acceleration section, the particle has the same phase, but the phase velocity of the new section is different, so that

$$(\varphi_1)_{j+1} = (\varphi_2)_j \quad \text{and} \quad (W_1)_{j+1} = (W_2)_j - 2W_r .$$

The j indices are for the conditions in the j^{th} section. In all of this, no consideration has been given to the adiabatic evolution.

5 - Stability Area

If one computes the position of one particle at the end of each section by using the relation in Section 4, one can find that the consecutive positions of the particle are plotted on a curve in the (W, φ) space.

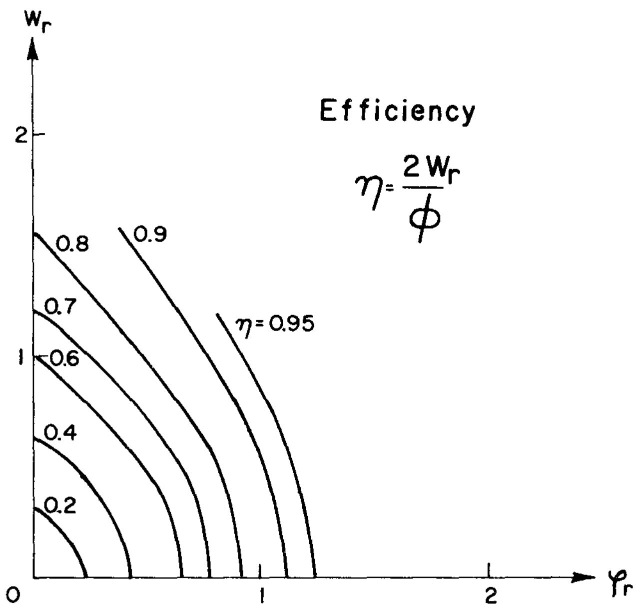
Efficiency
$\eta = \frac{2W_r}{\phi}$
W_r
2
1
0
0.8
0.9
0.7
0.6
0.4
0.2
η = 0.95
1
2
φ_r

Fig. 2

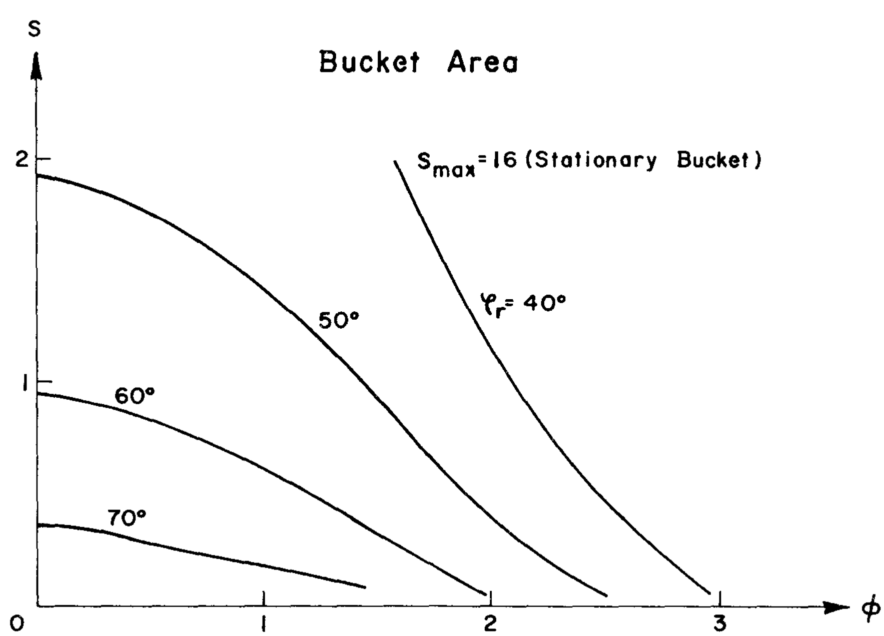
S
Bucket Area
2
1
0
S_{max} = 16 (Stationary Bucket)
φ_r = 40°
50°
60°
70°
1
2
3
ϕ

Fig. 3

For a small ϕ, one finds the curve of limit stability that is well known for an accelerator with synchronous particle. For greater ϕ, but ϕ still smaller than two, the curves have the same shape, the limit stability curves still go through the unstable point. Figure No. 1 shows the stability area for two values of ϕ.

For ϕ greater than two, that is for a number of synchrotron oscillations per section $\nu = \phi/2\pi$ of the order of 0.3 the motion is more complicated. For small amplitudes, the consecutive positions of the particle do not shape an ellipse centered on the stable point anymore. For larger amplitudes, the particle may become unstable after a few runs around the stable point. This case has not been thoroughly studied, since it is out of the limits of application.

The curves in Figure No. 3 give the surface of the limit stability curves versus ϕ for different values of φ_r. For ϕ smaller than two, it can be seen that the surface is the same as in accelerator with a synchronous particle the phase of which being such that $\eta = \sin\varphi_s$. For ϕ greater than two, the surfaces are smaller than it could be found from this approximation.

6 - Application

We shall now study a 200 MeV beam in a proton linac. For parameters defined in reference (2) that is $\xi = 2 \times 10^6$ V/m, $\lambda = 0.75$ m and an acceleration section length of 6 m, one finds $\phi = 1.4$. One still has to make choice of a reference phase such that all the particles might be contained inside the stability area. The 200 MeV beam has, in the (ΔE, φ) phase space, an area of 75.6×10^3 V.rd. In order to get a simpler image, one might admit that the adapted beam is a circle in the (W, φ) space, in that case

$$\Delta\varphi = \frac{A_0\, S(\Delta E, \Delta\varphi)}{\Delta W}. \quad \text{Hence} \quad \Delta\varphi \simeq 0.1 \text{ rd.}$$

The Figure No. 1 gives the cross section of the beam. For $\varphi_r = 60^\circ$, the stability limit area is still 10 times greater than the beam area. This might be necessary to take care of phase and amplitude errors of electric field. The value of ϕ decreases for increasing energies, it is then possible either to keep constant the length of the section and to increase the efficiency or to increase the section length by increasing the number of identical sections, for example. The following table gives the length L and its corresponding value of ϕ, as a function of energy.

Energy BeV	L m	ϕ
0.2	6	1.42
0.4	6	0.80
0.4	12	1.60
1.0	12	0.66
1.0	24	1.32
2.0	24	0.63
2.0	48	1.25
3.0	48	0.78

These lengths still fit the quadrupole doublet radial focusing requirements. One might think of starting with a 50 MeV beam and 3 m long sections, ξ being 2 10^6 V/m and ξ = 1.5 m, ϕ would then still be = 1.5.

7 - Conclusion

It is possible to use constant phase velocity acceleration sections in a linac if the number of synchrotron oscillations per section is smaller than 0.3. With this condition, it is possible to obtain a large enough area of stability and an efficiency of about 0.9 for a proton linac of energy between 200 MeV and 3 BeV.

References

(1) SMITH, L. - Stepped phase velocity linacs
Conference on linear accelerators for high energies.
Brookhaven August 1962

(2) SHERSBY-HARVIE, R. B. R. - Tentative proposal for a 3 Gev Linac injector - CERN AR/Int. SG/63-18 - 24th October 1963