DETERMINATION OF MAGNETIC MULTIPOLES USING A HALL PROBE

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Abstract

In this work we present a method that allows determining the harmonic content of the magnetic field generated by an accelerator magnet using a Hall probe bench. The method is based on measuring the three components of the magnetic field on a cylindrical surface parallel to the longitudinal axis of the magnet. The Fourier decomposition of the magnetic field along a circle at a given longitudinal position yields the harmonic terms of the field at a reference radius equal to the circle's radius. As a result the method provides the longitudinal dependence of the harmonic terms, and in particular it allows analyzing their behavior in the fringe field region. We present an example of the application of this method to the measurement of a reference quadrupole magnet. A comparison with the integrated results provided by a rotating coil bench is also shown.

INTRODUCTION

Magnetic multipoles of accelerators' magnets are customarily measured using rotating coil techniques. Such techniques provide information based on the integration of the magnetic field along the length of the measuring coil, and hence they have a very limited longitudinal resolution.

At the magnetic measurements laboratory at ALBA, apart from a conventional rotating coil bench, we have a Hall probe bench equipped with a 3-axis Hall sensor [1]. The Hall probe bench allows to measure, in on-the-fly mode, the magnetic field along a longitudinal line with an arbitrary transversal coordinate within a square of 500 mm horizontal times 250 mm vertical. This bench has been typically used to measure the magnetic field in the mid plane of bending magnets and insertion devices, but we have taken profit of its features in order to determine the harmonic content of multipolar magnets as quadrupoles and sextupoles. Given the local nature of the measurements with a Hall probe, the obtained data contains information regarding the dependence of the field along the longitudinal direction.

DESCRIPTION OF THE METHOD

Consider an accelerator magnet with the coordinate system defined in Fig. 1. In principle the harmonic content of a magnet can be determined from a measurement of the magnetic field within its mid plane ($y = 0$) by means of a polynomial fitting of the magnetic field dependence along the horizontal (x) direction, i.e.

$$B_y(x,z) + i\, B_x(x,z) = \sum_{n=1}^{\infty} C_n(z) \left(\frac{x}{R_{\text{ref}}}\right)^{n-1}, \quad (1)$$

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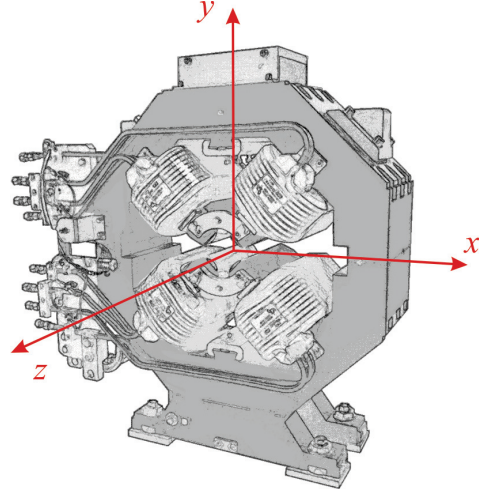
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Figure 1: Sketch of a quadrupole magnet showing the coordinate system used throughout this paper.

where $C_n(z)$ is the $2n$-polar complex harmonic coefficient at the longitudinal position z

$$C_n(z) = B_n(z) + i\, A_n(z) \quad (2)$$

calculated at a reference radius R_{ref}, and B_n and A_n are its normal and skew components, respectively. The problem of such a Cartesian expansion of the magnetic field is that the used basis functions (the x^n monomials) are not orthogonal, and hence the fitted values of the harmonic coefficients depend on both the degree N at which the fit is truncated and the horizontal range which is taken into account [2].

In order to avoid such a difficulty, the natural choice is to carry out an expansion of the magnetic field in polar coordinates:

$$B_\varphi(r,\varphi,z) + i\, B_r(r,\varphi,z) = \sum_{n=1}^{\infty} C_n(z) e^{i\,n\,\varphi} \left(\frac{r}{R_{\text{ref}}}\right)^{n-1} \quad (3)$$

If the magnetic field is measured along a circle of radius R_{meas}, the harmonic coefficients can be calculated from the azimuthal component of the field:

$$C_n(z) = \frac{1}{\pi} \left(\frac{R_{\text{ref}}}{R_{\text{meas}}}\right)^{n-1} \int_0^{2\pi} B_\varphi(\varphi,z)\, e^{-i\,n\,\varphi}\, d\varphi \quad (4a)$$

or alternatively from its radial component:

$$C_n(z) = \frac{i}{\pi} \left(\frac{R_{\text{ref}}}{R_{\text{meas}}}\right)^{n-1} \int_0^{2\pi} B_r(\varphi,z)\, e^{-i\,n\,\varphi}\, d\varphi. \quad (4b)$$

Assuming that the field is measured at P equally-spaced points around the circle, previous expressions can be rewrit-

ten as

$$C_n(z) = \frac{2}{P}\left(\frac{R_{\text{ref}}}{R_{\text{meas}}}\right)^{n-1}\sum_{p=0}^{P-1}B_\varphi(\varphi_p,z)\,e^{-\frac{i\,2\pi\,n\,p}{P}}$$

$$= \frac{2}{P}\left(\frac{R_{\text{ref}}}{R_{\text{meas}}}\right)^{n-1}DFT_n[B_\varphi(\varphi_p,z)] \tag{5a}$$

and

$$C_n(z) = i\frac{2}{P}\left(\frac{R_{\text{ref}}}{R_{\text{meas}}}\right)^{n-1}DFT_n[B_r(\varphi_p,z)] \tag{5b}$$

where $DFT_n[x_p]$ is the n-th term of the Discrete Fourier Transform of the data set x_p with $p = 0\ldots(P-1)$.

PRACTICAL APPLICATION

The described method has been applied to measuring the harmonic content of a reference quadrupole magnet with an iron yoke length of 140 mm and an aperture radius of 27.5 mm. In order to carry out the measurements we have selected the longest Hall probe available at our laboratory, an L-shaped probe with an usable length of 230 mm that has allowed us to measure the full length of the magnet plus a portion of the fringe field region (see Fig. 2).
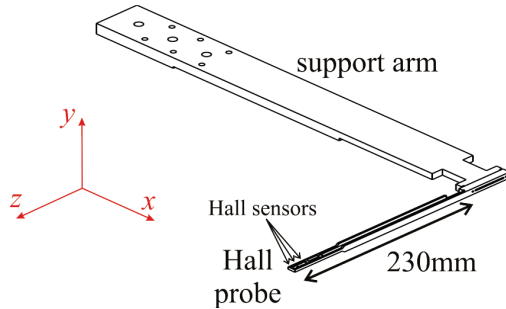

Figure 2: Hall probe used to carry out the determination of magnetic multipoles.

Regarding the measuring radius, R_{meas}, from Eq. (5) it is clear that it is convenient to maximize it within the constraints imposed by the transversal dimensions of the probe and the physical aperture of the magnet to be measured. In our case, a value of $R_{\text{meas}} = 17.5$ mm has been used.

Measurements consisted on a series of longitudinal scans starting at $z_{\text{ini}} = -200$ mm and finishing at $z_{\text{fin}} = +120$ mm from the center of the magnet. For each scan data has been acquired with the probe moving in forward and backward directions, and both sets of data have been averaged. A total of $P = 128$ of these scans with their transversal positions uniformly distributed around a circle of radius R_{meas} have been carried out, leading to a duration of 3 hours for the complete measurement.

Figure 3 illustrates the cylindrical measurement mesh resulting from the combination of longitudinal scans. The magnetic field vector has been measured at every analyzed position, and its horizontal and vertical components have
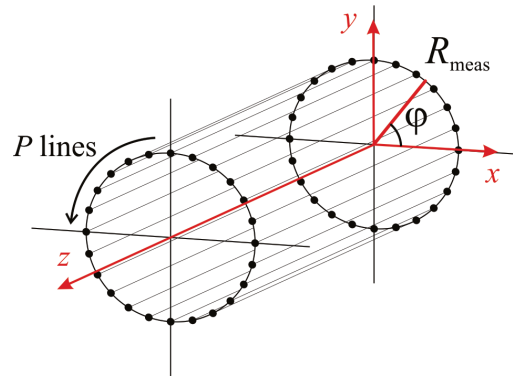
Figure 3: Diagram illustrating the cylindrical measurement grid used to determine the harmonic content of the field generated by an accelerator magnet.

been combined in order to determine the corresponding azimuthal and radial ones:

$$\varphi = \arctan(y/x)$$
$$B_r(R_{\text{meas}},\varphi,z) = +B_x\,\cos\varphi + B_y\,\sin\varphi \tag{6}$$
$$B_\varphi(R_{\text{meas}},\varphi,z) = -B_x\,\sin\varphi + B_y\,\cos\varphi$$

Afterwards, at each longitudinal position z we have applied Eq. (5) to the set of P field values for obtaining the harmonic coefficients $C_n(z)$.

The obtained field harmonics up to $n = 7$ calculated at a reference radius of $R_{\text{ref}} = 21$ mm are shown in Fig. 4. This figure illustrates that the difference between taking the azimuthal or the radial component of the magnetic field in order to determine the harmonic content is small. Figure 4 also displays the behavior of the field harmonics in the edge region of the magnet, a feature which is not easily accessible when measuring with a rotating coil system.

COMPARISON WITH ROTATING COIL

It is interesting to compare the obtained field harmonics with those measured using a conventional rotating coil system. In order to carry out the measurements, an in-house developed coil based on printed circuit boards has been used [3]. The measuring shaft has a main radial coil of $N_{\text{coil}} = 342$ turns with an effective width of $\Delta r = 2.3$ mm and a length $L_{\text{coil}} = 500$ mm at a radius $R_{\text{coil}} = 9.2$ mm, plus three identical auxiliary radial coils at smaller radii for quadrupole bucking. The total radius of the measuring shaft is 12 mm.

The integrated multipoles measured with the rotating coil are shown in Fig. 5 together with the result of integrating the harmonic coefficients determined from Hall probe data along the longitudinal direction. Data in Fig. 5 is shown, as is customary, in normalized form, relative to the magnitude of the main harmonic

$$c_n\,[\text{units}] = \frac{C_n}{|C_2|}\times 10^4 \tag{7}$$

where $C_n = \int_{-\infty}^{+\infty}C_n(z)\,dz$ are the integrated values of the corresponding coefficients.

Figure 5: Comparison between the integrated field harmonics calculated from local Hall probe measurements and directly measured using a rotating coil.

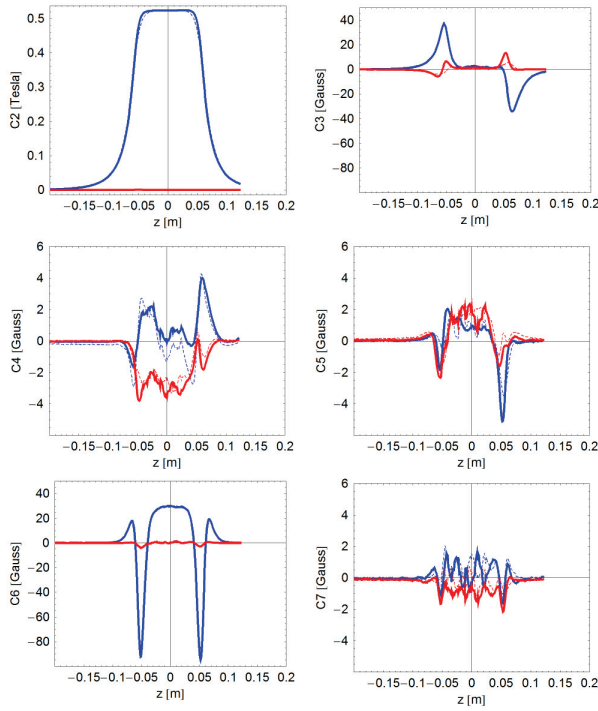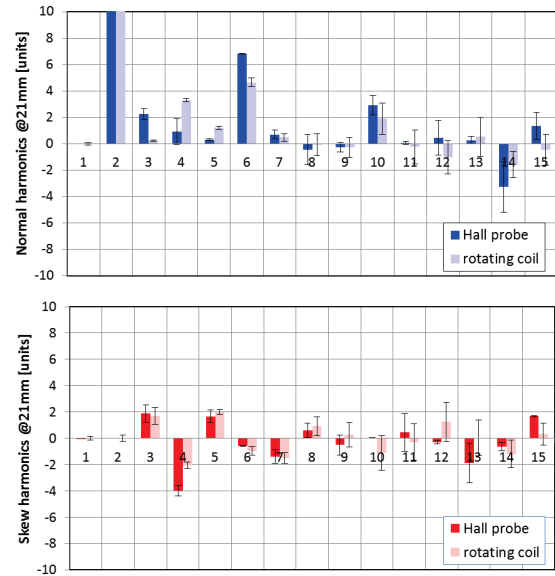Figure 4: Longitudinal dependence of the field harmonics calculated at R_{ref} = 21 mm from Hall probe measurements on a reference quadrupole. Blue curves correspond to normal coefficients (B_n) and red curves to skew ones (A_n). Continuous thick lines correspond to results obtained from the azimuthal component of the magnetic field, and dashed lines correspond to results obtained from the radial one.

Regarding the magnitude of the main harmonic, Hall probe data yield a value of $C_2^{Hall} = (72.47 \pm 0.07)$ Tesla·mm, whereas according to rotating coil measurements the integrated quadrupolar coefficient is $C_2^{coil} = (72.38 \pm 0.05)$ Tesla·mm, a reasonable agreement within 1×10^{-3}. It shall be pointed out that in the case of the Hall probe measurement the tail of the $C_2(z)$ curve has been fitted assuming an exponential decay dependence in order to extend the integration to the region beyond $z_{fin} = +120$ mm, not accessible with the Hall probe

On the other hand, as regards higher order harmonics ($n > 2$), Fig.5 shows that the two measurement techniques provide similar results within a range of ±2 units. In particular, in the case of the systematic harmonics ($n = 6, 10, 14 \ldots$), both techniques yield the same sign for the coefficients.

One possible reason for the discrepancy in the magnitude of the obtained higher order multipoles can be the presence of small inaccuracies in the calibrated value of the non-linear terms that characterize the response of the Hall sensors. As a consequence of such errors, the main harmonic would produce artifacts into the higher order ones, being the effect the most significant for the next-to-the-main harmonics. This hypothesis would explain the relatively high discrepancies displayed by harmonics C_3 and C_4 according to Fig. 5.

CONCLUSIONS

In this work we have analyzed the feasibility of using Hall probe measurements to determine the harmonic content of accelerator magnets. On the one hand, it has been proved that this technique provides relevant information regarding the longitudinal dependence of the field harmonics. This information can be useful to determine the optimum pole shimming during the development of a magnet prototype. In addition, the details of the harmonic content in the fringe field region can be included into particle tracking codes and lead to an improved modelization of accelerator magnets. On the other hand, and as far as accuracy is concerned, it has been shown that in terms of integrated multipoles the presented technique is inferior compared with standard ones based on rotating coils, but it still provides useful data with an accuracy of a few parts in 10^4.

For the future we foresee to manufacture a new Hall probe with a larger usable length (at least up to 400 mm) in order to enable the measurement of magnets with a longer iron yoke.

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