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Investigating three body charm hadron decays with missing energy within non-minimal SU(5)

Gaber Faisel[a]

Department of Physics, Faculty of Engineering and Natural Sciences, Süleyman Demirel University, Isparta 32260, Turkey

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Abstract In the Standard Model, the $c \to u\nu\bar{\nu}$ transition arises only at the one-loop level, leading to extremely suppressed branching ratios for the corresponding three-body decays of charmed hadrons. This suppression motivates the search for possible enhancements from physics beyond the Standard Model. In this work, we study the contributions to the $c \to u\nu\bar{\nu}$ transition from the R_2^{a2} scalar leptoquark predicted in the Non-Minimal SU(5) GUT framework. We derive constraints on the model parameters using the recent BESIII results on $D^0 \to \pi^0\nu\bar{\nu}$ and available data on D^0–$\bar{D}^0$ mixing. Additional bounds from direct LHC searches for scalar leptoquarks, lepton flavor violation processes, and the LHCb upper limit on $\mathcal{B}(D^0 \to \mu^+\mu^-)$ are also taken into account. After imposing all constraints, we show that the branching ratios of the three-body charmed hadron decays with missing energy can be enhanced up to $\mathcal{O}(10^{-6})$.

1 Introduction

Hadron decays originating from Flavor-Changing Neutral Current (FCNC) transitions in the up-quark sector can serve as a probe for New Physics (NP) beyond the Standard Model (SM). In particular, hadron decays generated from the transitions altering the charm quantum number by one unit ($|\Delta C| = 1$) with missing Energy ($\not{E}$) in the final states [1–28]. In the SM these decays can be generated from the quark transition $c \to u\nu\bar{\nu}$ at loop-level. The undetected neutrino ($\nu\bar{\nu}$) pair in the final state serves as the missing energy and the predicted Branching Ratios ($\mathcal{BR}$) of a number of hadron decays arising from this transition are predicted to be highly suppressed due to the highly efficient Glashow–Iliopoulos–Maiani cancellation mechanism as reported in Ref. [1]:

$$
\begin{aligned}
&\mathcal{BR}(D^0 \to \nu\bar{\nu})_{\text{SM}} = 0\,,\\
&\mathcal{BR}(D^0 \to \gamma\nu\bar{\nu})_{\text{SM}} = 1.8 \times 10^{-19}\,,\\
&\mathcal{BR}(D^0 \to \pi^0\nu\bar{\nu})_{\text{SM}} = 2.5 \times 10^{-17}\,,\\
&\mathcal{BR}(D^0 \to \rho^0\nu\bar{\nu})_{\text{SM}} = 1.1 \times 10^{-17}\,,\\
&\mathcal{BR}(D^+ \to \pi^+\nu\bar{\nu})_{\text{SM}} = 1.3 \times 10^{-16}\,,\\
&\mathcal{BR}(D^+ \to \rho^+\nu\bar{\nu})_{\text{SM}} = 5.9 \times 10^{-17}\,,\\
&\mathcal{BR}(D_s^+ \to K^+\nu\bar{\nu})_{\text{SM}} = 4.5 \times 10^{-17}\,,\\
&\mathcal{BR}(D_s^+ \to K^{*+}\nu\bar{\nu})_{\text{SM}} = 3.3 \times 10^{-17}\,,\\
&\mathcal{BR}(\Lambda_c^+ \to p\nu\bar{\nu})_{\text{SM}} = 7.3 \times 10^{-17}\,,\\
&\mathcal{BR}(\Xi_c^+ \to \Sigma^+\nu\bar{\nu})_{\text{SM}} = 1.1 \times 10^{-16}\,,\\
&\mathcal{BR}(\Xi_c^0 \to \Sigma^0\nu\bar{\nu})_{\text{SM}} = 1.8 \times 10^{-17}\,,\\
&\mathcal{BR}(\Xi_c^0 \to \Lambda\nu\bar{\nu})_{\text{SM}} = 6.5 \times 10^{-18}\,,
\end{aligned}
\qquad (1)
$$

It should be noted that long-distance contributions to these decays are also tiny as estimated in Refs. [7,23,27].

Experimentally, hadron decays carried out through FCNC $|\Delta C| = 1$ transitions with missing energy in the final states have not been observed up to date. BESIII and Belle Collaborations searched for the decays of the hadrons D^0 and Λ_c^+. With the null result of the search, upper bounds on their branching ratios have been obtained [29–32]. Specifically, Belle Collaboration has set the upper bound $\mathcal{B}(D^0 \to \text{invisibles}) < 9.4 \times 10^{-5}$ [32], while BESIII Collaboration has reported the upper bounds $\mathcal{B}(D^0 \to \pi^0\nu\bar{\nu}) < 2.1 \times 10^{-4}$ [29] and $\mathcal{B}(\Lambda_c^+ \to p\gamma') < 8.0 \times 10^{-5}$ [31], all at 90% confidence level, with γ' referring to an unobservable massless dark photon.

The experimentally obtained upper bounds reported above by BESIII and Belle Collaborations can be used to constraint the couplings of the NP contributing to the amplitudes

[a] e-mail: gaberfaisel@sdu.edu.tr (corresponding author)

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of the searched hadron decays. Consequently, the resultant restraints on the NP couplings can affect the predictions of other hadron decays originating from the same $c \to u\nu\bar{\nu}$ transition. In some scenarios of NP, the missing energy can be carried away through non standard particles which can be SM singlet spin-1/2 fermions [1,8,9,14,19,22] or a pair of SM-gauge-singlet spin-0 bosons [1,16,17,22]. In other NP scenarios, the missing energy can be carried away either by a SM-gauge singlet spinless boson [3,15,16] or by a SM-gauge singlet vector boson such as a massless dark photon [12,13,20].

Grand Unified Theory (GUT) has been proposed as a possible candidate for NP beyond the SM [33–38]. The theory can be regarded as a fundamental high-energy theory. GUT is motivated by the remark that it can elucidate several major issues that the SM failed to address. For instances, the gauge couplings of the SM can be unified in the framework of GUT at a high scale [36]. Another issue that GUT can address is the charge quantization of the SM fermions. The quantization can be achieved upon unifying the SM gauge groups $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ to a GUT gauge symmetry group. In turn, this unification demand that the leptons and quarks have to be unified in a single or a few representations of the GUT group. The absence of FCNC couplings at tree-level in the SM can be resolved in some versions of GUT theories such as a non-minimal SU(5) theory as we will discuss in the next section.

Motivated by the aforementioned recent results of BESIII Collaborations, and the so tiny SM predictions of the branching ratios of the set of the decays listed in Eq. (1), we investigate in this work the $c \to u\nu\bar{\nu}$ transition in the framework of the non minimal SU(5) GUT theory. We apply the upper bounds obtained by BESIII and other constraints on the coupling constants of the theory affecting this transition and give the predictions for the set of the decays listed in Eq. (1). Such predictions are important to discriminate among different NP scenarios given that Belle II, BESIII can improve their search results in the near future and also the planned Future Colliders such as the Circular e^+e^- Collider (FCC-ee), the Circular Electron Positron Collider (CEPC) and the Super Tau-Charm Facility (STCF) can probe these decays as discussed in details in Ref. [1].

The remaining part of the paper is organized as follows. In Sect. 2, we briefly discuss the Non-Minimal SU(5) GUT theory adopted in this work. In Sect. 3, we derive the effective Lagrangian accounting for the $c \to u\bar{\ell}_{L_i}\ell_{L_j}$ transition in the Non-Minimal SU(5) GUT theory and study the constraints on the model parameters related to our study. We then move to Sect. 4 in which we derive the expressions of the amplitudes for the charm hadron decays of our interest and the corresponding differential decay rates. This will allow us to estimate the branching ratios and perform the numerical analysis which is devoted to Sect. 5. Finally, we give our conclusion in Sect. 6.

2 The non-minimal SU(5) GUT theory

Various gauge groups including SU(5), SO(10), E_6, etc., can serve as GUT gauge symmetry groups [39]. The minimal version of the GUT theories is referred to as the minimal SU(5) GUT theory. It is the minimal group containing the SM gauge groups $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ that was originally introduced by Georgi and Glashow [35]. In the theory, the left-handed quark doublets, the right-handed charged leptons and the right-handed up quarks are embedded in **10** representations of the SU(5) GUT gauge symmetry group. On the other hand, the left-handed lepton doublets and the right-handed down type quarks are embedded in $\bar{\mathbf{5}}$ representations of the SU(5) group. Regarding the SM Higgs doublet, it is embedded in a **5** representation of scalars. The theory additionally has a **24** representation of scalars that breaks the SU(5) gauge symmetry to the SM ones.

Although being minimal is favored, the minimal SU(5) GUT theory cannot be a realistic model. One of the reasons behind that is attributed to the remark that the absence of new contributions to the RG runnings between the EW and the GUT scales does not allow the unification of the three gauge couplings at a high-energy scale using only the RG runnings in the SM [40]. Another reason is that the measured values of the masses of the down-type quarks and the charged leptons cannot be accommodated within the theory set-up [40]. For all these reasons, it is mandatory to consider a non-minimal version of SU(5) GUT theory in which an additional Higgs multiplet with a 45-dimensional representation has been introduced to allow for gauge couplings unification and to accommodate the measured fermion masses [41,42]. Adding 45-dimensional representation results in new sources of flavor violation in the up quark sector relevant to our study and can have impact on the FCNC processes [40,43–45].

Following Ref. [40], the SU(5)-symmetric renormalizable Lagrangian, after adding the Higgs multiplet with a 45-dimensional representation, can be expressed as

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}(V^{\mu\nu})^B{}_A (V_{\mu\nu})^A{}_B + i(\bar{\Psi}_{10i})_{AB}\gamma^\mu D_\mu(\Psi_{10i})^{AB} \\ & + i(\bar{\Psi}_{\bar{5}i})^A\gamma^\mu D_\mu(\Psi_{\bar{5}i})_A + \left[D^\mu\Sigma^B{}_A\right]\left[D_\mu\Sigma^A{}_B\right] \\ & + \left[D^\mu(\Phi_5^\dagger)_A\right]\left[D_\mu(\Phi_5)^A\right] + \left[D^\mu(\Phi_{45}^\dagger)^C_{AB}\right] \\ & \times\left[D_\mu(\Phi_{45})^{AB}_C\right] + \mathcal{L}_Y - V(\Sigma, \Phi_5, \Phi_{45}),\end{aligned} \tag{2}$$

here $V_{\mu\nu}$ symbolizes the field strength tensor of the gauge bosons associated with the SU(5) gauge group, $A, B, C = 1, \ldots, 5$ are SU(5) indices, $\Psi_{\bar{5}i}$ and Ψ_{10i}, with $i = 1, 2, 3$

as a generation index, denote the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of the SU(5) gauge group respectively. In the preceding equation, Φ_5, Σ, and Φ_{45} stand for one **5**, one **24**, and one **45**-dimensional scalar representation, respectively. Additionally, in the same equation, $V(\Sigma, \Phi_5, \Phi_{45})$ and $\mathcal{L}_Y$ represent the scalar potential and the Yukawa interactions, respectively. The explicit expression for the scalar potential $V(\Sigma, \Phi_5, \Phi_{45})$ can be found in the Appendix of Ref. [40] while $\mathcal{L}_Y$ is given as [40]:

$$-\mathcal{L}_Y = \frac{1}{8}(Y_5^U)_{ij}\epsilon_{ABCDE}(\Psi_{10i})^{AB}(\Phi_5)^C(\Psi_{10j})^{DE} + (Y_5^D)_{ij}(\Psi_{10i})^{AB}(\Phi_5^\dagger)_A(\Psi_{\bar{5}j})_B + \frac{1}{4}(Y_{45}^U)_{ij}\epsilon_{ABCDE}(\Psi_{10i})^{AB}(\Phi_{45})_F^{CD}(\Psi_{10j})^{EF} + \frac{1}{2}(Y_{45}^D)_{ij}(\Psi_{10i})^{AB}(\Phi_{45}^\dagger)_{AB}^C(\Psi_{\bar{5}j})_C + \text{h.c.}, \quad (3)$$

In the previous equation, the tensor ϵ_{ABCDE} is totally antisymmetric and is defined as $\epsilon_{12345} = 1$. Moreover, the matrices Y_{45}^U and Y_5^U are antisymmetric and symmetric in the generation space, respectively. The known quarks and leptons q_{Li}, u_{Ri}^c, d_{Ri}^c, ℓ_{Li}, and e_{Ri}^c reside in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations as [40]

$$(\Psi_{10i})^{AB} = \frac{1}{\sqrt{2}}\begin{pmatrix} \epsilon^{\hat{a}\hat{b}\hat{c}}(V_{QU})_i{}^k u_{Rk\hat{c}}^c & q_{Li}^{\hat{a}\beta} \\ -q_{Li}^{\hat{b}\alpha} & \epsilon^{\alpha\beta}(V_{QE})_i{}^k e_{Rk}^c \end{pmatrix},$$
$$(\Psi_{\bar{5}i})_A = \left(d_{Ri\hat{a}}^c \;\; \epsilon_{\alpha\beta}(V_{DL})_i{}^k \ell_{Lk}^\beta\right), \quad (4)$$

where the indices i, k run over the three generations of the quarks and leptons, the totally antisymmetric tensors ϵ^{ijk} and ϵ^{ij} are defined as $\epsilon^{123} = \epsilon_{123} = 1$ and $\epsilon^{12} = \epsilon_{12} = 1$ and finally V_{QU}, V_{QE}, and V_{DL} are unitary matrices. The basis of $\Psi_{\bar{5}}$ and Ψ_{10} can be rotated with the help of arbitrary unitary matrices in the generation space namely, U_5 and U_{10} as

$$\Psi_{\bar{5}} \to U_5\Psi_{\bar{5}}, \quad \Psi_{10} \to U_{10}\Psi_{10}, \quad (5)$$

Doing so, one can adopt the basis where the charged leptons and up-type quarks are in their mass eigenstates [40]:

$$q_{Li} = \begin{pmatrix} \hat{u}_{Li} \\ (V_{\text{CKM}})_i{}^j \hat{d}_{Lj} \end{pmatrix}, \quad u_{Ri} = \hat{u}_{Ri}, \quad d_{Ri} = \hat{d}_{Ri},$$
$$\ell_{Li} = \begin{pmatrix} \hat{\nu}_{Li} \\ \hat{e}_{Li} \end{pmatrix}, \quad e_{Ri} = \hat{e}_{Ri}, \quad (6)$$

where a hat over a field indicates that the field is a mass eigenstate, and V_{CKM} is the familiar Cabibbo–Kobayashi–Maskawa (CKM) matrix in the Particle Data Group (PDG) convention.

Of particular interest to our study is the scalar Φ_{45} which consists of the scalars $\tilde{S}_1$, R_2^*, S_3^*, S_6^*, S_8, $H^{(45)}$, and $S_1^{(45)*}$ as [40]

$$(\Phi_{45})_{\hat{c}}^{\hat{a}\hat{b}} = \frac{1}{\sqrt{2}}\epsilon^{\hat{a}\hat{b}\hat{d}}\left[(\eta_a)_{\hat{c}\hat{d}} S_6^{*a} - \frac{1}{2}\epsilon_{\hat{c}\hat{d}\hat{e}} S_1^{(45)*\hat{e}}\right],$$
$$(\Phi_{45})_\gamma^{\hat{a}\hat{b}} = \frac{1}{\sqrt{2}}\epsilon^{\hat{a}\hat{b}\hat{d}} R_{2\hat{d}\gamma}^*,$$
$$(\Phi_{45})_{\hat{c}}^{\hat{a}\beta} = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\lambda_a)^{\hat{a}}{}_{\hat{c}} S_8^{a\beta} + \frac{1}{2\sqrt{3}}\delta_{\hat{c}}^{\hat{a}} H^{(45)\beta}\right],$$
$$(\Phi_{45})_{\hat{c}}^{\alpha\beta} = \frac{1}{\sqrt{2}}\epsilon^{\alpha\beta}\tilde{S}_{1\hat{c}},$$
$$(\Phi_{45})_\gamma^{\alpha\hat{b}} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(\sigma_a)^\alpha{}_\gamma S_3^{*\hat{b}} - \frac{1}{2}\delta_\gamma^\alpha S_1^{(45)*\hat{b}}\right),$$
$$(\Phi_{45})_\gamma^{\alpha\beta} = -\frac{\sqrt{3}}{2\sqrt{2}}\epsilon^{\alpha\beta}\epsilon_{\gamma\delta}H^{(45)\delta}, \quad (7)$$

After $SU(5)$ symmetry breaking, the interaction Lagrangian of the scalar R_2 with the SM fermions can be obtained from Eq. (3) as [40]

$$-\mathcal{L}_{R_2} = (Y_2^{UL})_{ij}\epsilon_{\alpha\beta}\bar{u}_{Rai}R_2^{a\alpha}\ell_{Lj}^\beta + (Y_2^{EQ})_{ij}\bar{e}_{Ri} R_{2a\alpha}^* q_{Lj}^{a\alpha} + \text{h.c.}, \quad (8)$$

where $\alpha, \beta = 1, 2$ are SU(2) indices and $a, b = 1, 2, 3$ are SU(3) indices. Upon using the expressions in Eq. (6), the Lagrangian $\mathcal{L}_{R_2}$ in Eq. (8) can be expressed in terms of the component fermion fields in the mass eigenstate as

$$-\mathcal{L}_{R_2} = (Y_2^{UL})_{ij}\bar{\hat{u}}_{Rai}R_2^{a1}\hat{e}_{Lj} - (Y_2^{UL})_{ij}\bar{\hat{u}}_{Rai}R_2^{a2}\hat{\nu}_{Lj} + (Y_2^{EQ})_{ij}\bar{\hat{e}}_{Ri}R_{2a1}^*\hat{u}_{Lj}^a + (Y_2^{EQ})_{ij}\bar{\hat{e}}_{Ri}R_{2a2}^*(V_{\text{CKM}})_j{}^k\hat{d}_{Lk}^a + \text{h.c.} \quad (9)$$

Clearly, from the Lagrangian in the preceding equation, the interactions of the scalar R_2 with the SM fermions can lead to FCNC at tree level in both quark sectors. Aside from the terms containing the down type quarks, the other terms in the Lagrangian $\mathcal{L}_{R_2}$ are important to the FCNC transitions in the up-type quark sector and hadron decays generated from these transitions. In the upcoming sections, we will focus on the FCNC $\Delta C = 1$ transition and explore the resultant effects on the set of decay modes mentioned in the introduction section.

3 $c \to u\bar{\ell}_{L_i}\ell_{L_j}$ in the non-minimal SU(5) GUT theory

In this section, we discuss the transitions $c \to u\bar{\ell}_{L_i}\ell_{L_j}$ where i and j are family indices and ℓ_{L_i} defined in Eq. (6). The transitions can be generated from the tree-level Feynman diagrams constructed from the Lagrangian $\mathcal{L}_{R_2}$, given in in Eq. (9), upon integrating the leptoquarks R_{2a1} and R_{2a2}. Doing so, we get the effective Lagrangian accounting for the

$c \to u\bar{\ell}_{L_i}\ell_{L_j}$ transition as

$$\mathcal{L}_{|\Delta C|=1} = -\sqrt{2}\,G_F\,\bar{u}\gamma_\beta P_R c \left(\mathcal{Y}^\nu_{ij}\overline{\nu_{e_i}}\gamma^\beta P_L \nu_{e_j} + \mathcal{Y}^e_{ij}\overline{e_i}\gamma^\beta P_L e_j\right) - \sqrt{2}\,G_F\,\tilde{\mathcal{Y}}^e_{ij}\,\bar{u}\gamma_\beta P_L c\,\overline{e_i}\gamma^\beta P_R e_j + \text{h.c.}, \tag{10}$$

where G_F is the Fermi constant and a summation over the family indices $i, j = 1, 2, 3$ is understood and the dimensionless coefficients $\mathcal{Y}^{\nu,e}_{ij}$ are given as

$$\mathcal{Y}^\nu_{ij} = \frac{(Y_2^{UL})^*_{2i}(Y_2^{UL})_{1j}}{2\sqrt{2}\,G_F m^2_{R_2^{a2}}}$$
$$\mathcal{Y}^e_{ij} = \frac{(Y_2^{UL})^*_{2i}(Y_2^{UL})_{1j}}{2\sqrt{2}\,G_F m^2_{R_2^{a1}}}$$
$$\tilde{\mathcal{Y}}^e_{ij} = \frac{(Y_2^{EQ})^*_{j1}(Y_2^{EQ})_{i2}}{2\sqrt{2}\,G_F m^2_{R_2^{a1}}} \tag{11}$$

where $m_{R_2^{a1}}$ and $m_{R_2^{a2}}$ are the mass of the scalar leptoquarks R_2^{a1} and R_2^{a2} respectively. Clearly, non-vanishing values of the coefficients $\mathcal{Y}^\nu_{ij}$ and $\mathcal{Y}^e_{ij}$ induce FCNC charmed-hadron decays with missing energy and with charged leptons in the final states respectively. Before treating the former processes, we look at some potentially important constraints on the LQ parameters in Eq. (11). As can be seen from Eq. (10) the couplings $\mathcal{Y}^\ell_{ij}$ and $\tilde{\mathcal{Y}}^e_{ij}$ do not contribute to the transition $c \to u\nu\bar{\nu}$ relevant to the processes under concern in this work. However, the coupling $\mathcal{Y}^e_{ij}$ can be related to the coupling $\mathcal{Y}^\nu_{ij}$ as can be seen from Eq. (11). Accordingly, we need to investigate the possible constraints that can be imposed on the couplings $\mathcal{Y}^\nu_{ij}$ and $\mathcal{Y}^e_{ij}$.

The leptoquarks R_2^{a1} and R_2^{a2} have electric charges $+5/3$ and $+2/3$ respectively as can be inferred from in Eq. (9). From the same equation, we can see that R_2^{a1} (R_2^{a2}) together with charged leptons (neutrinos) can mediate box diagrams contributing to the $D-\bar{D}$ mixing. Upon integrating the heavy leptoquarks R_2^{a1} and R_2^{a2} we find that the new contributions to the effective Hamiltonian describing the $D-\bar{D}$ mixing can be expressed as

$$\mathcal{H}^{R_2}_{|\Delta C|=2} = \tilde{C}^{R_2}_1\,\left(\bar{u}\gamma^\mu P_R c\right)\left(\bar{u}\gamma_\mu P_R c\right) + \text{h.c.} \tag{12}$$

where $\tilde{C}^{R_2}_1(\mu_{R_2})$ is the Wilson coefficient corresponding to the four-quark operator listed in the above equation. Neglecting lepton masses compared to the heavy leptoquark masses and dropping the terms proportional to the couplings $(Y_2^{EQ})_{ij}$ as they are not relevant to our analysis, we find that

$$\tilde{C}^{R_2}_1(\mu_{R_2}) = \frac{1}{128\pi^2}\left(\frac{1}{m^2_{R_{2a1}}} + \frac{1}{m^2_{R_{2a2}}}\right)\sum_{j,k=1}^{3}(Y_2^{UL})^*_{2j}(Y_2^{UL})^*_{2k}(Y_2^{UL})_{1j}(Y_2^{UL})_{1k}, \tag{13}$$

Experimentally, the mixing observable reads $\Delta m_D^{\text{exp}} = \left(99.7 \pm 11.6\right) \times 10^8/\text{s}$ [46]. With the large hadronic uncertainty associated with the SM prediction for the mixing observable [47], we assume that only the contributions from the effective Hamiltonian $\mathcal{H}^{R_2}_{|\Delta C|=2}$ can saturate the experimental result. Consequently, we can write $\Delta m_D = |\langle \bar{D}^0|\mathcal{H}_{|\Delta C|=2}|D^0\rangle|\tilde{r}/m_{D^0}$, where the factor $\tilde{r}$ arises from the renormalization-group running of $\tilde{C}^{R_2}_1(\mu_{R_2})$ from the leptoquark masses scale $m_{R_{2a1}}$ and $m_{R_{2a2}}$ down to the low energy scale μ at which the hadronic matrix element of the four-quark operator in Eq. (12) is computed [47]. Using $\langle \bar{D}^0|\left(\bar{u}\gamma^\mu P_R c\,\bar{u}\gamma_\mu P_R c\right)|D^0\rangle = 0.0805(57)\,\text{GeV}^4$ at the scale of 3 GeV from a lattice QCD calculations [48], we obtain the 2σ upper-limit

$$\left(\frac{\tilde{r}}{m^2_{R_{2a1}}} + \frac{\tilde{r}}{m^2_{R_{2a2}}}\right)\left|\sum_{j,k=1}^{3}(Y_2^{UL})^*_{2j}(Y_2^{UL})_{1j}(Y_2^{UL})^*_{2k}(Y_2^{UL})_{1k}\right| < \frac{3.20 \times 10^{-4}}{\text{TeV}^2}. \tag{14}$$

With the presence of the scalar leptoquatrks R_2^{a1} and R_2^{a2} and the up-type quarks running in the loops, the Yukawa coupling Y_2^{UL} can induce lepton flavor violation (LFV) processes $e_j \to e_i\gamma$ at one-loop level. Upon integrating the scalar leptoquarks, the amplitudes of these processes will receive new contributions that are proportional to $(Y_2^{UL})^*_{ki}(Y_2^{UL})_{kj}$ where $k = u, c, t$ in addition to other contributions proportional to the coupling Y_2^{EQ} which are not relevant to our study. Consequently, the products of the couplings $(Y_2^{UL})^*_{ki}(Y_2^{UL})_{kj}$ can be constrained using the experimentally reported upper bounds on $\mathcal{BR}(\ell_j \to \ell_i\gamma)$ which currently read $\mathcal{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [49], $\mathcal{BR}(\tau \to e\gamma) < 3.3 \times 10^{-8}$ [50], and $\mathcal{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ [50].

The effective Lagrangian $\mathcal{L}_{|\Delta C|=1}$, in Eq. (10), yields new contribution proportional to $|\mathcal{Y}^e_{22}|^2$ to the branching ratio of the process $D^0 \to \mu^+\mu^-$. On the other hand, the LHCb collaborations obtained the upper bound on the branching ratio of the process $D^0 \to \mu^+\mu^-$ that reads $\mathcal{BR}(D^0 \to \mu^+\mu^-) < 3.1 \times 10^{-9}$ at a 90% CL [51]. Thus, we can use this upper bound to set constraint on the quantity $\mathcal{Y}^e_{22} \propto (Y_2^{UL})^*_{22}(Y_2^{UL})_{12}$.

The aforementioned constraints discussed above can be evaded once we choose the appropriate texture of the Yukawa coupling matrix Y_2^{UL} providing that the choice will not restrain all contributions to the coupling $\mathcal{Y}_{ij}^{\nu}$ generating the transition $c \to u\nu_i\bar{\nu}_j$, see Eq. (10), relevant to the processes under concern in this work. To do this, we see from the first line in Eq. (11) that $\mathcal{Y}_{ij}^{\nu}$ depends only on the elements of the first and second rows of the coupling Y_2^{UL}. Thus, we can choose a texture for Y_2^{UL} in which the elements of the third row are zeros. Concerning the $D - \bar{D}$ mixing bound in the inequality (14), we find that the mixing requirement is escapable if the contributing elements of the first and second rows of Y_2^{UL} belong to different columns. On the other hand, the constraints from the upper bounds on $\mathcal{BR}(\ell_j \to \ell_i\gamma)$ ($\mathcal{BR}(D^0 \to \mu^+\mu^-)$) can be satisfied if the product $(Y_2^{UL})^*_{ki}(Y_2^{UL})_{kj}$ ($(Y_2^{UL})^*_{22}(Y_2^{UL})_{12}$) vanishes. Bearing in mind all these requirements, we find that there is a possible texture of the Yukawa coupling Y_2^{UL} can be chosen

$$\mathrm{Y}_2^{UL} = \begin{pmatrix} 0 & \mathrm{Y}_{u2} & 0 \\ 0 & 0 & \mathrm{Y}_{c3} \\ 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

It should be noted that this choice of the Y_2^{UL} matrix allows us to avoid the restraints come from $\mathcal{BR}(D^+ \to \pi^+e^-\mu^+)_{\rm exp} < 3.6 \times 10^{-6}$ $\mathcal{BR}(D^+ \to \pi^+e^+\mu^-)_{\rm exp} < 2.9 \times 10^{-6}$, both at 90% CL [52] and the others implied by $D^0 \to e^\pm\mu^\mp$ data [52] and the constraints inferred from quests for the flavor-violating processes $pp \to e^\pm\mu^\mp$ and $pp \to e^\pm\tau^\mp$ at the LHC [53]. This can be explained as the amplitudes of these processes are proportional to $y_{ce}y_{u\mu} = y_{c\mu}y_{ue} = 0$ in the case of the final states have $e^\pm\mu^\mp$ and $y_{ce}y_{u\tau} = y_{c\tau}y_{ue} = 0$ in the case of the final states have $e^\pm\tau^\mp$ as can be seen from the second row in the matrix Y_2^{UL}. Moreover, with the texture of the matrix Y_2^{UL} given above, one can also evade the constraints imposed from data on the high-invariant-mass tails of the dilepton reactions $pp \to e^+e^-, pp \to \tau^+\tau^-$ at the LHC and $D \to \pi e^+e^-, \pi\mu^+\mu^-$ measurements [54]. The reason can be attributed to the fact that the amplitudes of these processes are proportional to $y_{ce}y_{ue} = 0$, $y_{c\mu}y_{u\mu} = 0$ and $y_{c\tau}y_{u\tau} = 0$ in case of the presence of $e^+e^-, \mu^+\mu^-$ and $\tau^+\tau^-$ in the final states respectively.

The non vanishing elements of the coupling matrix (Y_2^{UL}) can be constrained from search for the flavor-violating process $pp \to \mu^+\tau^-$ at the LHC [53] and from the upper bound $\mathcal{BR}\big(D^0 \to \pi^0\nu\bar{\nu}\big) < 2.1 \times 10^{-4}$ at 90% CL reported by the BESIII Collaboration [29]. From the result of the search for the flavor-violating process $pp \to \mu^+\tau^-$ at the LHC one can obtain the bound [9,53] $\big(|\mathcal{Y}^e_{\mu\tau}|^2 + |\mathcal{Y}^e_{\tau\mu}|^2\big)^{1/2} < 6.4 \times 10^{-3}$ at the 2σ level. Using the definitions in Eq. (11) the bound reads

$$\frac{1}{m^2_{R_{2a2}}}\left|(Y_2^{UL})_{u2}(Y_2^{UL})^*_{c3}\right| < 2.11 \times 10^{-7} \tag{16}$$

The interactions of the scalar leptoquarks described by the Lagrangian $\mathcal{L}_{|\Delta C|=1}$, shown in Eq. (10), bring about the decay process $\mathbb{D} \to \mathbb{P}\nu_i\bar{\nu}_j$ where $\mathbb{D} = D^0, D^+, D_s^+$ and the final state pseudoscalar meson $\mathbb{P}$ of our interest can be taken as $\mathbb{P} = \pi^0, \pi^+, K^+$. The amplitudes for these decay processes is given as

$$\mathcal{M}_{\mathbb{D}\to\mathbb{P}\nu_i\bar{\nu}_j} = \frac{\sqrt{2}}{4}G_{\rm F}\mathcal{Y}^{\nu}_{ij}\left\{\langle\mathbb{P}|\bar{u}\gamma^\mu c|\mathbb{D}\rangle\rangle + \langle\mathbb{P}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle\rangle\right\}$$
$$\overline{\nu_i}\gamma_\mu(1-\gamma_5)\nu_j\,, \tag{17}$$

the matrix elements of the four-quark operators in the above equation can be evaluated as

$$\langle\mathbb{P}|\bar{u}\gamma^\mu c|\mathbb{D}\rangle = \big(p^\mu_{\mathbb{D}} + p^\mu_{\mathbb{P}}\big)f_+ + \big(p^\mu_{\mathbb{D}} - p^\mu_{\mathbb{P}}\big)\big(f_0 - f_+\big)\frac{m^2_{\mathbb{D}} - m^2_{\mathbb{P}}}{q^2_{\mathbb{DP}}}$$
$$\langle\mathbb{P}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle = 0, \tag{18}$$

where $p^\mu_{\mathbb{D}}$ and $p^\mu_{\mathbb{P}}$ are the momenta of the $\mathbb{D}$ and $\mathbb{P}$ mesons respectively. The quantities f_+ and f_0 are the form factors which are functions of the squared momentum-transfer $\hat{q}^2$. After, calculating the amplitude square and performing the integral over one of the two kinematic variables, one finds that the differential decay rate in terms of other kinematic variable $\hat{s} = (p^\mu_{\mathbb{D}} - p^\mu_{\mathbb{P}})^2$ is given as

$$\frac{d\Gamma_{\mathbb{D}\to\mathbb{P}\nu_i\bar{\nu}_j}}{d\hat{s}} = \frac{\tilde{\lambda}^{3/2}_{\mathbb{DP}}\,G^2_{\rm F}\,f^2_+(\hat{s})\,|\mathcal{Y}^{\nu}_{ij}|^2}{768\pi^3 m^3_{\mathbb{D}}}, \tag{19}$$

with $\tilde{\lambda}_{\rm XY} = \lambda\big(m^2_{\rm X}, m^2_{\rm Y}, \hat{s}\big)$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. The decay rate can be obtained upon doing the integration with respect to $\hat{s}$ over the range $(m_{\nu_i} + m_{\nu_j})^2 \le \hat{s} \le (m_{\mathbb{D}} - m_{\mathbb{P}})^2$. In the calculations of the decay rate we adopt the lattice-QCD results for $D \to \pi$ and use [55]

$$f_+(\hat{s}) = \frac{1}{1 - \hat{s}/(2.00685\,{\rm GeV})^2}\sum_{n=0}^{3} a_n\left[z^n - \frac{n\,z^4}{4(-1)^{4-n}}\right],$$
$$z = \frac{\sqrt{(M_D + M_\pi)^2 - \hat{s}} - M_D - M_\pi}{\sqrt{(M_D + M_\pi)^2 - \hat{s}} + M_D + M_\pi},$$
$$M_D = 1864.83{\rm MeV}\,, \quad M_\pi = 134.9768{\rm MeV}, \tag{20}$$

the constants a_n can be found in Refs. [1,55]. It is direct now to apply the upper limit result due to BESIII $\mathcal{B}\big(D^0 \to \pi^0\nu\bar{\nu}\big) < 2.1 \times 10^{-4}$ at 90% CL [29] to constrain the non-vanishing couplings $\mathcal{Y}^{\nu}_{32}$ and $\mathcal{Y}^{\nu}_{12}$. The obtained upper limits can be then translated to the bound

$$\frac{1}{m^2_{R_{2a2}}}\left|(Y_2^{UL})_{u2}(Y_2^{UL})^*_{c3}\right| < 4.04\times 10^{-6} \quad (21)$$

Once, leptoquark mass is determined, one can obtain the constraints on the $\left|(Y_2^{UL})_{u2}(Y_2^{UL})^*_{ck}\right|$ using the bound in the above equation. Empirically, CMS collaborators set constraint on the scalar leptoquark masses from the search at LHC for a scalar leptoquark decaying to a quark and a neutrino [56]. The search results exclude scalar leptoquark masses up to 1.1 TeV at 95% CL. On the other hand, ATLAS collaboration set constraints on the masses of the Scalar leptoquarks pair-produced in $p\,p$ collisions at $\sqrt{s}=13$ TeV at the Large Hadron Collider. Based on the negative result of the direct search for the produced pair of the scalar leptoquarks decaying fully into a quark and an electron (muon), ATLAS collaboration excluded the scalar leptoquark mass region below 1.8 (1.7) TeV [57]. ATLAS collaboration also conducted a search for pair production of third-generation scalar leptoquarks decaying into a top quark and a τ-lepton [58]. The outcome of the search showed that Scalar leptoquarks decaying exclusively into a top quark and a τ-lepton are excluded up to masses of 1.43 TeV while, for a branching ratio of 50% into a top quark and a τ-lepton, the lower mass limit is 1.22 TeV [58]. Regarding third-generation leptoquark coupled exclusively to a τ lepton and a b quark, CMS collaborations excluded below masses of $1.22-1.88$ TeV for different leptoquark models and varying coupling strengths up to 2.5 [59]. Recently, ATLAS collaboration has presented statistical combination of various searches for pair-produced leptoquarks, using the full LHC Run 2 ($2015-2018$) data set of 139 fb^{-1}. The resulting lower limits on leptoquark masses exceed those from the individual analyses by up to 100 GeV, depending on the signal hypothesis [60].

The decay processes $K_{L/S}\to e_i^+e_j^-$ and $K_L\to\pi^0 e_i^+e_j^-$, where i and j are flavor indices, can be generated from the transition $s\to d\,e_i^+e_j^-$. This transition arises from integrating the scalar leptoquark R_{2a2} mediating the tree-level Feynman diagrams constructed from the interaction term in the second line in Eq. (9) and its hermitian conjugate. Hence, the empirical data related to the branching ratios of $K_{L/S}\to e_i^+e_j^-$ [30,61,62] and on the branching ratios of $K_L\to\pi^0 e_i^+e_j^-$ [63] can be used to constraint the products of the Yukawa couplings $|(Y_2^{EQ})_{j2}(Y_2^{EQ})^*_{i1}|^2$. However, these products of the Yukawa couplings do not contribute to the transition $c\to u\,\nu_i\bar{\nu}_j$ and hence the expected constraints are not important to our study. This conclusion is also valid regarding the constraints from $b\to s(d)\gamma$, $B_{s,d}-\bar{B}_{s,d}$ and $K-\bar{K}$ mixing. This can be explained as all these constraints are expected to be imposed on the elements of the matrix Y_2^{EQ} which are not relevant to the processes under concern in this study.

Finally, with the obtained constraints on the Yukawa couplings contributing to the transition $c\to u\nu\bar{\nu}$, discussed above, we proceed below to estimate the predictions for the branching ratios of the list of the decay modes presented in Eq. (1).

4 FCNC charm decay with missing Energy in Non-Minimal SU(5)

In this section we discuss a set of charmed hadron decays into final states with missing energy carried away by the neutrino pairs. Particularly, we will study the meson decays $\mathbb{D}\to\mathbb{V}\bar{\nu}\nu$ where $\mathbb{D}=D^0, D^+, D_s^+$, and $\mathbb{V}$ can be a vector meson ρ^0,ρ^+,K^{*+} or a photon γ. We also will study the baryonic decays $\mathcal{B}_c^{+,0}\to\mathcal{B}'^{+,0}\bar{\nu}\nu$ where $\mathcal{B}_c^{+,0}\in\left\{\Lambda_c^+,\Xi_c^+,\Xi_c^0\right\}$ and $\mathcal{B}'^{+,0}\in\left\{p,\Sigma^+,\Sigma^0,\Lambda\right\}$.

4.1 $\mathbb{D}\to\mathbb{V}\bar{\nu}\nu$ decays

The interaction Lagrangian $\mathcal{L}_{|\Delta C|=1}$ presented in Eq. (10) can induce the decay mode $\mathbb{D}\to\mathbb{V}\bar{\nu}\nu$. The amplitude of the process $\mathbb{D}(p_{D^0})\to\mathbb{V}(p_{\mathbb{V}})\bar{\nu}(\mathrm{k}')\nu(\mathrm{k})$ using $\mathcal{L}_{|\Delta C|=1}$ is given as

$$\mathcal{M}_{\mathbb{D}\to\mathbb{V}\nu_i\bar{\nu}_j} = \frac{\sqrt{2}}{4}\,G_{\rm F}\mathcal{Y}^{\nu}_{ij}\left\{\langle\mathbb{V}|\bar{u}\gamma_\mu c|\mathbb{D}\rangle - \langle\mathbb{V}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle\rangle\right\} \overline{\nu_i}\gamma_\mu(1-\gamma_5)\nu_j\,, \quad (22)$$

the matrix elements in the above equation for the case $\mathbb{V}=\rho^0,\rho^+,K^{*+}$ can be calculated as

$$\begin{aligned}\langle\mathbb{V}|\bar{u}\gamma_\mu c|\mathbb{D}\rangle &= \frac{2V}{m_{\mathbb{D}}+m_{\mathbb{V}}}\,\epsilon_{\mu\beta\eta\theta}\,\varepsilon_{\mathbb{V}}^{\beta*}\,p_{\mathbb{V}}^{\eta}\,p_{\mathbb{D}}^{\theta},\\ \langle\mathbb{V}|\bar{u}\gamma^\mu\gamma_5 c|\mathbb{D}\rangle &= i(m_{\mathbb{D}}+m_{\mathbb{V}})\varepsilon_{\mathbb{V}}^{\mu*}A_1 - \left[\frac{p_{\mathbb{D}}^\mu+p_{\mathbb{V}}^\mu}{m_{\mathbb{D}}+m_{\mathbb{V}}}A_2 \right.\\ &\left.\quad +\frac{p_{\mathbb{D}}^\mu-p_{\mathbb{V}}^\mu}{q^2_{\mathbb{DV}}}(A_3-A_0)2m_{\mathbb{V}}\right]i\varepsilon_{\mathbb{V}}^{*}\cdot p_{\mathbb{D}},\end{aligned} \quad (23)$$

where V and $A_{0,1,2,3}$ are form factors which are functions of the squared momentum-transfer $q^2_{\mathbb{DV}}=(p_{\mathbb{D}}-p_{\mathbb{V}})^2$ and $2A_3 m_{\mathbb{V}}=(m_{\mathbb{D}}+m_{\mathbb{V}})A_1-(m_{\mathbb{D}}-m_{\mathbb{V}})A_2$. On the other hand, for the case $\mathbb{V}=\gamma$, the matrix elements are given as

$$\begin{aligned}\langle\gamma|\bar{u}\gamma_\mu c|D^0\rangle &= \frac{eF_V}{m_{D^0}}\epsilon_{\mu\zeta\eta\theta}\,\varepsilon_\gamma^{\zeta*}\,p_{D^0}^{\eta}\,p_\gamma^{\theta}\,,\\ \langle\gamma|\bar{u}\gamma^\mu\gamma_5 c|D^0\rangle &= \frac{ieF_A}{m_{D^0}}\left(p_\gamma\cdot p_{D^0}\,\varepsilon_\gamma^{\mu*}-\varepsilon_\gamma^{*}\cdot p_{D^0}\,p_\gamma^{\mu}\right),\end{aligned} \quad (24)$$

with e denoting the proton charge, ε_X standing for the polarization vector of X, and F_V and F_A are form factors which are functions of the squared momentum-transfer $(p_{D^0} - p_\gamma)^2 = (\mathrm{k} + \mathrm{k}')^2 \equiv \hat{s}$.

After, calculating the amplitude square, multiplying by the phase space and performing the integral over one of the two kinematic variables, one finds that the differential decay rate in terms of other kinematic variable $\hat{s} = (p^\mu_{\mathbb{D}} - p^\mu_{\mathbb{V}})^2$ is given for $\mathbb{V} = \rho^0, \rho^+, K^{*+}$ as

$$\frac{d\Gamma_{\mathbb{D}\to\mathbb{V}\nu_i\bar{\nu}_j}}{d\hat{s}} = \frac{\tilde{\lambda}^{3/2}_{\mathbb{D}\mathbb{V}}\, G_F^2 |\mathcal{Y}^\nu_{ij}|^2}{256\pi^3 m^3_{\mathbb{D}}} \left\{ \frac{2\hat{s}V^2}{3\,\tilde{\mathrm{m}}^2_+} + \frac{A_1^2\,\tilde{\mathrm{m}}^2_+}{6m^2_{\mathbb{V}}} \left(\frac{1}{2} + \frac{6m^2_{\mathbb{V}}\hat{s}}{\tilde{\lambda}_{\mathbb{D}\mathbb{V}}}\right) + \frac{\tilde{\lambda}_{\mathbb{D}\mathbb{V}} A_2^2}{12\,\tilde{\mathrm{m}}^2_+ m^2_{\mathbb{V}}} + \frac{\hat{s} - \tilde{\mathrm{m}}_+\tilde{\mathrm{m}}_-}{6m^2_{\mathbb{V}}} A_1 A_2 \right\}, \tag{25}$$

while for $\mathbb{V} = \gamma$ we have

$$\frac{d\Gamma_{D^0\to\gamma\nu_i\bar{\nu}_j}}{d\hat{s}} = \frac{\alpha_e\left(m^2_{D^0} - \hat{s}\right)^3 \hat{s}\, G_F^2 |\mathcal{Y}^\nu_{ij}|^2 \left(F_V^2 + F_A^2\right)}{384\pi^2 m^5_{D^0}} \tag{26}$$

the decay rate can be obtained after performing the integration of the differential decay rate with respect to the kinematic variable $\hat{s}$ over the range $0 \le \hat{s} \le (m_{\mathbb{D}} - m_{\mathbb{V}})^2$. To perform the integration and thus to calculate the decay rates and the branching ratios, we need to have the expressions showing the dependency of the form factors on $\hat{s}$. Regarding the form factors $V, A_{1,2}$, one can adopt the so-called symmetry-preserving formulation of a vector-vector contact interaction, which takes the form [1,64]

$$\mathbb{F}(\hat{\mathrm{F}}_0, \hat{\mathrm{A}}, \hat{\mathrm{B}}) = \frac{\hat{\mathrm{F}}_0}{1 - \hat{\mathrm{A}}\,\hat{s}/m_P^2 + \hat{\mathrm{B}}\left(\hat{s}/m_P^2\right)^2}, \tag{27}$$

where the values of $\hat{\mathrm{F}}_0$, $\hat{\mathrm{A}}$, $\hat{\mathrm{B}}$, and m_P can be found in Refs. [1,64]. For $D^{0,+} \to \rho^{0,+}$, the form factors $V, A_{1,2}$ can be calculated with the help of $\mathbb{F}(\hat{\mathrm{F}}_0, \hat{\mathrm{A}}, \hat{\mathrm{B}})$ as [1,64]

$$A_1 = \mathbb{F}(0.52, 0.15, -0.14), \quad A_2 = \mathbb{F}(0.36, 0.6, -0.042), \quad V = \mathbb{F}(0.83, 0.87, 0.0009), \quad m_P = 1.87\mathrm{GeV} \tag{28}$$

and for $D_s^+ \to K^{*+}$, the form factors $V, A_{1,2}$ are given as [1,64]

$$A_1 = \mathbb{F}(0.56, 0.22, -0.2), \quad A_2 = \mathbb{F}(0.4, 0.72, -0.047), \quad V = \mathbb{F}(0.94, 0.98, -0.0011), \quad m_P = 1.96\mathrm{GeV}. \tag{29}$$

Considering the $D^0 \to \gamma$ transition, the form factors F_V and F_A have been discussed in Refs. [22,28,65]. Here, we use the formulas given in Ref. [65] which are expressed as

$$F_V(\hat{s}) = \frac{(2/3)(-0.49)}{1 - \hat{s}/(2.0\,\mathrm{GeV})^2}, \quad F_A(\hat{s}) = \frac{(2/3)(-0.17)}{1 - \hat{s}/(2.3\,\mathrm{GeV})^2}, \tag{30}$$

4.2 $\mathcal{B}_c^{+,0} \to \mathcal{B}'^{+,0}\nu\bar{\nu}$ decays

The amplitude of the baryonic decay mode $\mathcal{B}_c^{+,0} \to \mathcal{B}'^{+,0}\,\nu_i\bar{\nu}_j$ can be evaluated using the effective Lagrangian $\mathcal{L}_{|\Delta C|=1}$, Eq. (10), as

$$\mathcal{M}_{\mathcal{B}_c^{+,0}\to\mathcal{B}'^{+,0}\,\nu_i\bar{\nu}_j} = \frac{\sqrt{2}}{4} G_F \mathcal{Y}^\nu_{ij} \Big\{ \langle\mathcal{B}'^{+,0}|\bar{u}\gamma^\mu c|\mathcal{B}_c^{+,0}\rangle - \langle\mathcal{B}'^{+,0}|\bar{u}\gamma^\mu\gamma_5 c|\mathcal{B}_c^{+,0}\rangle \Big\} \overline{\nu_i}\gamma_\mu(1-\gamma_5)\nu_j \tag{31}$$

the baryonic matrix elements given in the above equation can be estimated as [1]

$$\begin{aligned}
\langle\mathcal{B}'^{+,0}|\bar{u}\gamma^\mu c|\mathcal{B}_c^{+,0}\rangle &= \bar{u}_{\mathcal{B}'^{+,0}} \Bigg\{ \left[\gamma^\mu - \frac{\mathrm{M}_+\hat{p}^\mu - \mathrm{M}_-\hat{q}^\mu}{\mathrm{M}_+^2 - \hat{q}^2}\right]\mathrm{F}_\perp + \left[\hat{p}^\mu - \frac{\mathrm{M}_+\mathrm{M}_-\hat{q}^\mu}{\hat{q}^2}\right]\frac{\mathrm{M}_+\,\mathrm{F}_+}{\mathrm{M}_+^2 - \hat{q}^2} + \frac{\mathrm{M}_-\hat{q}^\mu}{\hat{q}^2}\mathrm{F}_0 \Bigg\} u_{\mathcal{B}_c^{+,0}}, \\
\langle\mathcal{B}'^{+,0}|\bar{u}\gamma^\mu\gamma_5 c|\mathcal{B}_c^{+,0}\rangle &= \bar{u}_{\mathcal{B}'^{+,0}} \Bigg\{ \left[\gamma^\mu + \frac{\mathrm{M}_-\hat{p}^\mu - \mathrm{M}_+\hat{q}^\mu}{\mathrm{M}_-^2 - \hat{q}^2}\right]\mathrm{G}_\perp - \left[\hat{p}^\mu - \frac{\mathrm{M}_+\mathrm{M}_-\hat{q}^\mu}{\hat{q}^2}\right]\frac{\mathrm{M}_-\,\mathrm{G}_+}{\mathrm{M}_-^2 - \hat{q}^2} - \frac{\mathrm{M}_+\hat{q}^\mu}{\hat{q}^2}\mathrm{G}_0 \Bigg\} \gamma_5 u_{\mathcal{B}_c^{+,0}},
\end{aligned} \tag{32}$$

where $u_{\mathcal{B}'^{+,0}}$ and $u_{\mathcal{B}_c^{+,0}}$ denote the Dirac spinors of the baryons, $\mathrm{G}_{\perp,+,0}$ and $\mathrm{F}_{\perp,+,0}$ represent the form factors which depend on the momentum transfer $\hat{s} = \left(p_{\mathcal{B}_c^{+,0}} - p_{\mathcal{B}'^{+,0}}\right)^2$, $\mathrm{M}_\pm = m_{\mathcal{B}_c^{+,0}} \pm m_{\mathcal{B}'^{+,0}}$, $\hat{p} = p_{\mathcal{B}_c^{+,0}} + p_{\mathcal{B}'^{+,0}}$. Upon performing the summation of the absolute square of the amplitude over the initial and final baryon polarizations and multiplying by the three-body phase space, one obtains the differential decay rate as

$$\frac{d\Gamma_{\mathcal{B}_c^{+,0}\to\mathcal{B}'^{+,0}\nu_i\bar{\nu}_j}}{d\hat{s}} = \frac{\tilde{\lambda}^{1/2}_{\mathcal{B}_c^{+,0}\mathcal{B}'^{+,0}}\, G_F^2 |\mathcal{Y}^\nu_{ij}|^2}{768\pi^3 m^3_{\mathcal{B}_c^{+,0}}} \left\{ \left(2f_\perp^2\hat{s} + f_+^2\mathrm{M}_+^2\right)\hat{\sigma}_- + \left(2g_\perp^2\hat{s} + g_+^2\,\mathrm{M}_-^2\right)\hat{\sigma}_+ \right\}, \tag{33}$$

where $\hat{\sigma}_\pm = \mathrm{M}_\pm^2 - \hat{s}$. and it has to be integrated over $0 \le \hat{s} \le (m_{\mathcal{B}_c^{+,0}} - m_{\mathcal{B}'^{+,0}})^2$. In order to calculate the decay rate and accordingly the branching ratio, we need to know the dependency of the form factors on the momentum transfer $\hat{s}$. For the matrix elements of the $\Lambda_c^+ \to p$, we use the lattice-QCD parametrization of Ref. [66] which can be written as [1]

$$\tilde{\mathbb{F}}(\mathrm{A}_0, \mathrm{A}_1, \mathrm{A}_2) = \frac{\mathrm{A}_0 + \mathrm{A}_1 \tilde{z} + \mathrm{A}_2 \tilde{z}^2}{1 - \hat{q}^2/m_{\text{pole}}^2},$$
$$\tilde{z} = \frac{\sqrt{\tilde{\mathrm{t}}_+ - \hat{q}^2} - \sqrt{\tilde{\mathrm{t}}_+ - (m_{\Lambda_c} - m_N)^2}}{\sqrt{\tilde{\mathrm{t}}_+ - \hat{q}^2} + \sqrt{\tilde{\mathrm{t}}_+ - (m_{\Lambda_c} - m_N)^2}}, \tag{34}$$

where $\tilde{\mathrm{t}}_+ = (1.87 + 0.135)^2\,\mathrm{GeV}^2$ and m_N is the average nucleon mass. The form factors can be obtained from $\tilde{\mathbb{F}}(\mathrm{A}_0, \mathrm{A}_1, \mathrm{A}_2)$ as [1,66]

$$\mathrm{F}_\perp = \tilde{\mathbb{F}}(1.36, -1.7, 0.71), \quad \mathrm{F}_+ = \tilde{\mathbb{F}}(0.83, -2.33, 8.41),$$
$$\mathrm{G}_\perp = \tilde{\mathbb{F}}(0.69, -0.68, 0.7), \quad \mathrm{G}_+ = \tilde{\mathbb{F}}(0.69, -0.9, 2.25), \tag{35}$$

with $m_{\text{pole}} = 2.01$ GeV for $\mathrm{F}_{\perp,+}$, 2.423 GeV for $\mathrm{G}_{\perp,+}$. Turning now to the form factors appearing in the matrix elements in the transition $\Xi_c \to \Sigma, \Lambda$, there is no lattice analysis on their form factors [1]. Accordingly, we follow Refs. [1,67] and use the light-front constituent quark model to estimate the form factors from the quantities $f_{1,2,3}$ and $g_{1,2,3}$ which for $\Xi_c^+ \to \Sigma^+$ are given as [1,67]

$$f_1 = \tilde{F}(0.73, 1.49, 2.35), \quad f_2 = \tilde{F}(0.99, 1.43, 2.38),$$
$$g_1 = \tilde{F}(0.63, 1.18, 1.79), \quad g_2 = \tilde{F}(0.11, 1.88, 2.88) \tag{36}$$

and for $\Xi_c^0 \to \Lambda$ they are given as [1,67]

$$f_1 = \tilde{F}(0.28, 1.5, 2.32), \quad f_2 = \tilde{F}(0.38, 1.35, 2.3),$$
$$g_1 = \tilde{F}(0.25, 1.18, 1.77), \quad g_2 = \tilde{F}(0.04, 1.71, 2.78), \tag{37}$$

Here $\tilde{F}(\kappa_0, \kappa_1, \kappa_2) = \kappa_0/(1 - \kappa_1 \hat{q}^2 + \kappa_2 \hat{q}^4)$, with $\kappa_{0,1,2}$ are constants that can be found in Refs. [1,67] and $f_3 = g_3 = 0$. The form factors $\mathrm{F}_{\perp,+}$ and $\mathrm{G}_{\perp,+}$ are related to the quantities $f_{1,2,3}$ and $g_{1,2,3}$ by [1]

$$\mathrm{F}_\perp = f_1 + \frac{\mathrm{M}_+ f_2}{m_{\Lambda_c}}, \qquad \mathrm{F}_+ = f_1 + \frac{\hat{q}^2 f_2}{m_{\Lambda_c} \mathrm{M}_+},$$
$$\mathrm{G}_\perp = g_1 - \frac{\mathrm{M}_- g_2}{m_{\Lambda_c}}, \qquad \mathrm{G}_+ = g_1 - \frac{\hat{q}^2 g_2}{m_{\Lambda_c} \mathrm{M}_-}, \tag{38}$$

With all this in hand, we can now proceed to give our predictions for the branching ratios of the decay processes mentioned above, This will be achieved in the next section.

5 Numerical results and analysis

In this section, we present our predictions for the branching ratios of the hadron decay processes discussed in the previous section. We start by showing the predictions for the D meson decays to final states having a pseudoscalar meson and neutrino–antineutrino pair. Then, we move to D meson decays to final states having a vector meson and neutrino–antineutrino pair. After that, we consider the decays of baryons to final states having a baryon and neutrino–antineutrino pair.

From the first line in Eq. (11) and from the matrix Y_2^{UL} in Eq. (15), we find that the non vanishing coupling $\mathcal{Y}_{32}^{\nu} \propto (Y_2^{UL})_{23}^*(Y_2^{UL})_{12}$ will contribute to the transition $c \to u\nu_3\bar{\nu}_2$ relevant to the processes under concern in this work. In the following discussion, we drop the flavor of the neutrinos.

In Fig. 1 we show our predictions of the branching ratios of the D meson decay processes $D^+ \to \pi^+\nu\bar{\nu}$ and $D_s^+ \to K^+\nu\bar{\nu}$ which include a pseudoscalar meson and missing energy in the final states. In the left (right) plot, the contour lines in red color represent the branching ratio of $D^+ \to \pi^+\nu\bar{\nu}$ ($D_s^+ \to K^+\nu\bar{\nu}$) decay in units of 10^{-7}. Moreover, in the same plots, the green region in the $m_{R_{2a2}} - |(Y_2^{UL})_{u2}(Y_2^{UL})_{c3}^*|$ plane is allowed by the constraints discussed in Sect. 3. With all these constraints, we find that the upper limits of the branching ratios of the decay processes $D^+ \to \pi^+\nu\bar{\nu}$ and $D_s^+ \to K^+\nu\bar{\nu}$ are 1.47×10^{-6} and 5.11×10^{-7} respectively. Comparing to the corresponding SM predictions, Eq. (1), we can deduce that sizable enhancements of the branching ratios are possible in the model under concern in this work.

We turn now to D meson decays into a final state with a vector boson and missing energy carried away by the neutrino antineutrino pair. Our predictions are displayed in Fig. 2. In the figure, we display the contour lines of the branching ratio of the decay process $D^0 \to \rho^0\nu\bar{\nu}$ and $D^+ \to \rho^+\nu\bar{\nu}$ in units of 10^{-7} in the top-left and top-right plots respectively. On the other hand, the contour lines of the branching ratio of $D_s^+ \to K^{*+}\nu\bar{\nu}$ ($D^0 \to \gamma\nu\bar{\nu}$) decay is depicted in the bottom-left (right) plot in units of 10^{-5}. It should be noted that, the yellow regions in the plots respect the constraints mentioned in Sect. 3. Bearing in mind all these constraints, we find that the upper bounds on the branching ratios of the decays $D^0 \to \rho^0\nu\bar{\nu}$, $D^+ \to \rho^+\nu\bar{\nu}$, $D_s^+ \to K^{*+}\nu\bar{\nu}$ and $D^0 \to \gamma\nu\bar{\nu}$ can be as large as 2.62×10^{-7}, 6.638×10^{-7}, 3.77×10^{-7} and 2.12×10^{-9} respectively which are sizable compared to the corresponding SM predictions listed in Eq. (1).

In Fig. 3 we show the contours of the branching ratios of the baryon decays $\Xi_c^0 \to \Sigma^0\nu\bar{\nu}$ (top-left), $\Xi_c^+ \to \Sigma^+\nu\bar{\nu}$ (top-right), $\Xi_c^0 \to \Lambda\nu\bar{\nu}$ (bottom-left) and $\Lambda_c^+ \to p\nu\bar{\nu}$ (bottom-right) in units of 10^{-7}. In the plots, the points in the orange region respect the aforementioned constraints stated

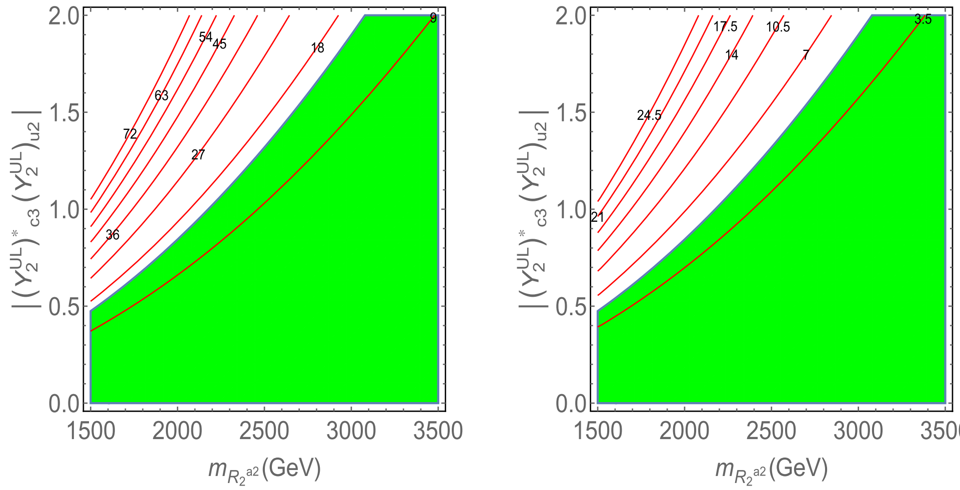

Fig. 1 Left (Right): Contour lines of the branching ratio of $D^+ \to \pi^+ \nu \bar{\nu}$ ($D_s^+ \to K^+ \nu \bar{\nu}$) decay in units of 10^{-7}. The points in the Green region satisfy the constraints discussed in Sect. 2

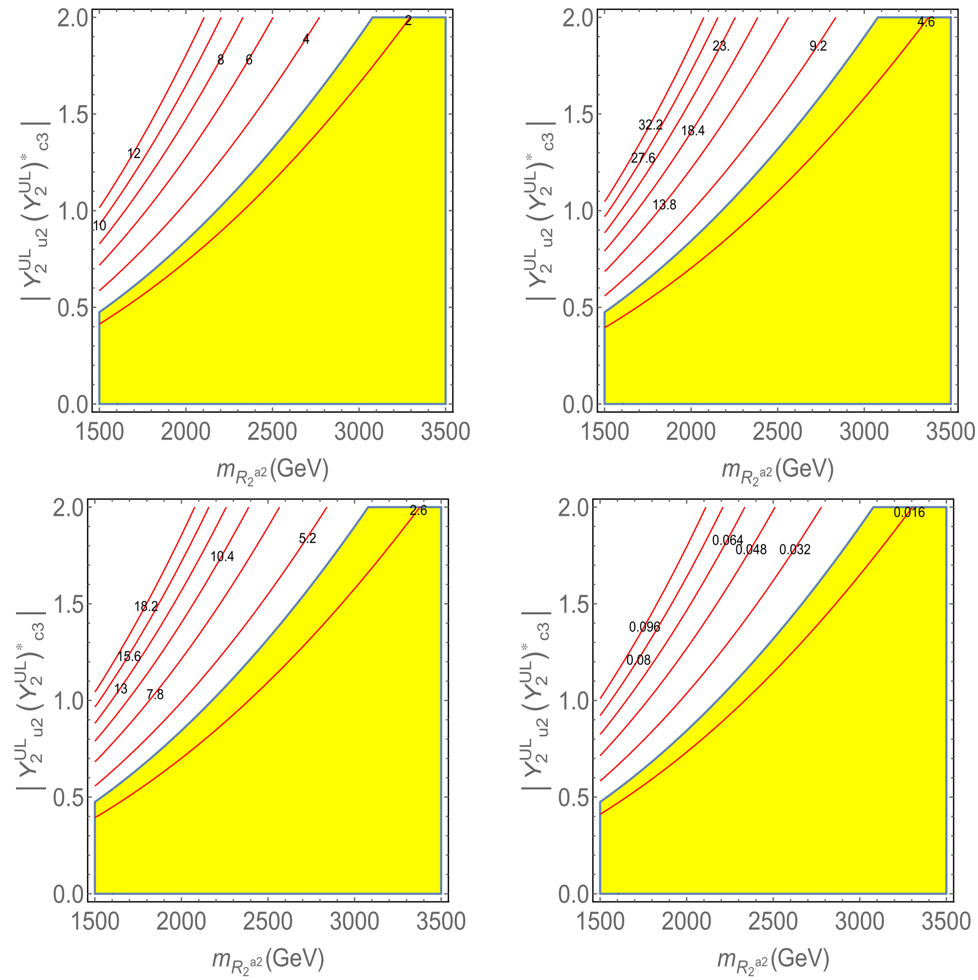

Fig. 2 Contour lines of the branching ratios of the decay processes $D^0 \to \rho^0 \nu \bar{\nu}$ (top-left), $D^+ \to \rho^+ \nu \bar{\nu}$ (top-right), $D_s^+ \to K^{*+} \nu \bar{\nu}$ (bottom-left) and $D^0 \to \gamma \nu \bar{\nu}$ (bottom-right) in units of 10^{-7}. The points in the yellow region satisfy the constraints discussed before in Sect. 3

in Sect. 3. For these points, we find that the branching ratios of the baryon decays $\Xi_c^0 \to \Sigma^0 \nu \bar{\nu}$, $\Xi_c^+ \to \Sigma^+ \nu \bar{\nu}$, $\Xi_c^0 \to \Lambda \nu \bar{\nu}$ and $\Lambda_c^+ \to p \nu \bar{\nu}$ can reach upper value 1.65×10^{-7}, 9.83×10^{-7}, 5.96×10^{-8} and 4.94×10^{-7} respectively. Clearly, the branching ratios of the aforementioned baryonic decay modes are enhanced in the model under concern compared to the corresponding SM predictions presented in Eq. (1).

It is worth stressing that the branching ratio predictions presented in this section are affected by theoretical uncertainties. The most important source comes from the hadronic form factors that parametrize the $D \to P$, $D \to V$, and baryonic transition matrix elements. For the pseudoscalar and vector meson channels, lattice QCD and light-cone sum rule inputs typically carry uncertainties of order 10–20%,

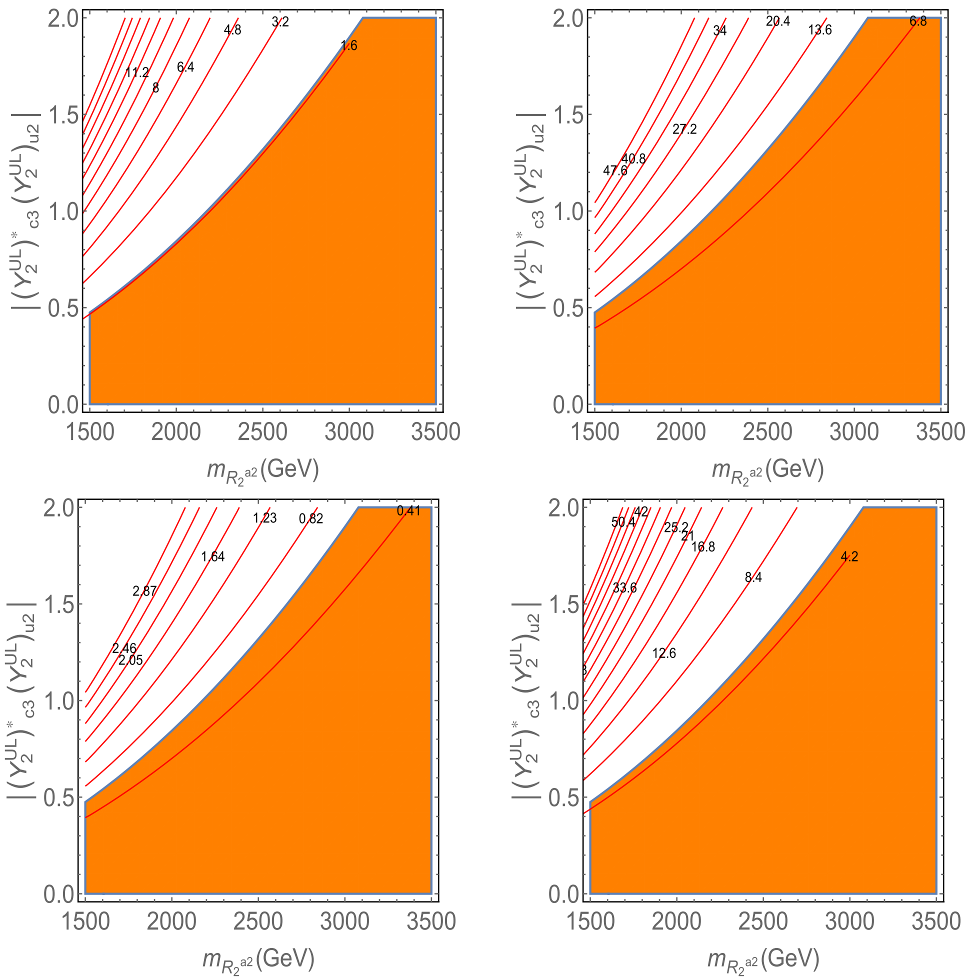

Fig. 3 Contours of the branching ratios of the decay modes $\Xi_c^0 \to \Sigma^0 \nu\bar{\nu}$ (top-left), $\Xi_c^+ \to \Sigma^+ \nu\bar{\nu}$ (top-right), $\Xi_c^0 \to \Lambda \nu\bar{\nu}$ (bottom-left) and $\Lambda_c^+ \to p \nu\bar{\nu}$ (bottom-right) in units of 10^{-7}. The points in the Orange region respect the constraints discussed above in Sect. 3

while for the baryonic decays, the form factors extracted from lattice QCD (for $\Lambda_c \to p$) and quark model estimates (for $\Xi_c \to \Sigma, \Lambda$) introduce larger uncertainties, potentially up to 30%. Subleading sources arise from input parameters such as CKM elements, decay constants, and hadron masses, which usually contribute below the 5% level. Finally, the overall normalization depends on the leptoquark mass and Yukawa couplings, which were consistently treated in accordance with current experimental bounds. Taking all these into account, we estimate the total theoretical uncertainty on the quoted branching ratios to be about 20–30%, with hadronic form factors being the dominant contribution. This level of uncertainty does not alter our conclusion that enhancements up to $\mathcal{O}(10^{-6})$ remain viable within the allowed parameter space.

6 Conclusion

The transition $c \to u\nu\bar{\nu}$ induces FCNC decays of charmed hadrons into three body final states with missing energy carried away by the neutrino–antineutrino pair. Within SM, these decays are predicted to have so tiny branching ratios. Based on this result, it turns to be important to go beyond SM physics with new interactions that can affect the $c \to u\nu\bar{\nu}$ transition and accordingly may lead to enhancements of the branching ratios.

In this study we have investigated the new contributions to the $c \to u\nu\bar{\nu}$ transition arising from the non-standard interactions of the R_2^{a2} scalar leptoquark in the Non-Minimal SU(5) GUT theory discussed in Sect. 2. We have derived the possible constraints that can be applied to the parameter space of the model using the results of the latest search for $D^o \to \pi^0\nu\bar{\nu}$ by BESIII collaboration. We also have estimated the bounds on the model parameters that can be obtained from the D meson mixing. Other constraints resulting from the direct search for the scalar leptoquark at the LHC and from lepton flavor violation processes have been discussed and were taken into account in the analysis.

After considering the relevant constraints on the parameter space of the model, we have shown the allowed parameter space permits large branching ratios of order $\mathcal{O}(10^{-6})$ are still possible associated with the decay $D^+ \to \pi^+\nu\bar{\nu}$, while other decay modes have smaller branching ratios.

Data Availability Statement This manuscript has no associated data. [Author's comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.]

Code Availability Statement This manuscript has no associated code/software. [Author's comment: Code/Software sharing not applicable to this article as no code/software was generated or analysed during the current study.]

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