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Bounce Cosmology in a Locally Scale Invariant Physics with a U(1) Symmetry

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Abstract: An asymmetric non-singular bouncing cosmological model is proposed in the framework of a locally scale-invariant scalar-tensor version of the standard model of particle physics and gravitation. The scalar field ϕ is complex. In addition to local scale invariance, the theory is U(1)-symmetric and has a conserved global charge associated with time variations of the phase of ϕ. An interplay between the positive energy density contributions of relativistic and non-relativistic matter and that of the negative kinetic energy associated with the phase of ϕ results in a classical non-singular stable bouncing dynamics deep in the radiation-dominated era. This encompasses the observed redshifting era, which is preceded by a blueshifting era. The proposed model potentially avoids the flatness and horizon problems, as well as allowing for the generation of a scale-invariant spectrum of metric perturbations of the scalar type during a matter-dominated-like pre-bounce phase, with no recourse to an inflationary era.

Keywords: cosmology; nonsingular bounce; scalar-tensor gravity

1. Introduction

General relativity (GR), the backbone of the standard cosmological model, has successfully passed numerous tests within our solar system. However, it is not comparably successful on larger, galactic and supergalactic scales, unless cold dark matter (CDM) and dark energy (DE) are included in the cosmic energy budget. The latter are foreign to the standard model (SM) of particle physics and only appear in our cosmological model as nearly perfect non-interacting fluids with their respective characteristic equations of state (EOSs).

In addition, GR—a classical field theory of gravitation—is genuinely plagued by singularities. A few singularity theorems imply that curvature is inevitably singular unless certain plausible 'energy conditions' are violated. For the latter to take place, some form of exotic matter is needed. Curvature or energy density singularities are encountered either at the centers of black holes or at the Big Bang, even in the presence of a very early inflationary phase in the latter case [1]. It is widely hoped that a would-be quantum theory of gravity will ameliorate these unwelcome singularities, thereby constituting a major thrust behind the quest for such a theory.

The largest physical scales, over which the short-range nuclear interactions and the highly screened electromagnetic interaction are irrelevant, are an ideal testbed for alternative theories of gravitation, and indeed, a few persistent anomalies of the concordance ΛCDM cosmological model may possibly indicate that GR requires modifications on cosmological scales.

The GR-based ΛCDM, with an early inflationary phase and a dominant dark sector [the latter contains CDM and DE components that determine the background evolution,

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large-scale structure (LSS) formation history, and gravitational potential wells on galactic and supergalactic scales], has clearly proved to be a very successful paradigm that provides a compelling interpretation of essentially all currently available cosmic microwave background (CMB) and LSS observational data, as well as of light element abundances based on Big Bang nucleosynthesis (BBN). It is truly remarkable that ΛCDM provides a very good fit to these extensive observational data, that sample a wide range of phenomena over a vast dynamical range, using only half a dozen free parameters.

However, ΛCDM also lacks in a few ways. A major drawback is that the essence of DE and CDM remains elusive. Another problem is that, the currently leading inflationary scenario, 'eternal inflation', seems to lack predictive power as it is most naturally realized in the multiverse. Importantly, as a paradigm, inflation—which has been advanced as a possible solution to a few fine-tuning problems in the Hot Big Bang model—seems to suffer its own fine-tuning and conceptual problems, e.g., [2–12]. In addition, conceptually, the existence of the Big Bang, which essentially signals the breakdown of the underlying theory of gravitation, is also a major problem of the standard cosmological model.

Moreover, a few mild to strong inconsistencies between various datasets in comparison to ΛCDM have been found, e.g., [13]. These include the existence of a persistent relative deficit in power of density perturbations on superhorizon scales, e.g., [14–16]; an anomalously large weak lensing of the CMB anisotropy by the intervening LSS between the present and the last scattering surface [17]; a statistically significant 'Hubble tension' between local and high-z inferences of the local expansion rate, e.g., [18–21]; and others. In addition, the claimed 5σ evidence [22] against statistical isotropy—a principal pillar on which the model rests—could potentially undermine the standard cosmological model.

A very early and brief epoch of inflation very efficiently addresses the 'flatness' and 'horizon' problems, and in addition explains the observed slightly red-tilted power spectrum of scalar metric perturbations with adiabatic initial conditions. However, the nearly flat power spectrum is not a prediction of inflation but was rather postulated and explicated a decade before the inflationary scenario was proposed [23–25]. Whereas inflation does provide a concrete mechanism to seed such scalar metric perturbations, e.g., [26], alternative mechanisms have been proposed as well [27–29] within non-inflationary expanding Universe scenarios, which could potentially (if not in their original form) explain the observed near flatness of the spectrum. Although current observational limits on the spatial curvature could be interpreted as evidence for a 'flatness' (fine-tuning) problem if not for inflation that 'naturalizes' the observed spatial flatness of the Universe, there is also the alternative view, according to which there is no problem to begin with, e.g., [30–32]. A non-inflationary solution to the horizon problem within the expanding Universe scenario has been proposed in, e.g., [33]. Nevertheless, a phase of cosmic inflation (or a very similar scenario, e.g., [34]), seems to be the leading candidate to successfully address these (otherwise 'naturalness') problems in an expanding Universe scenario. However, a phase of contraction that precedes expansion is currently neither theoretically nor observationally ruled out.

A possibly viable alternative to inflationary cosmology is the bouncing Universe scenario [35–37]. Bouncing cosmological models avoid a few of the classical problems of the Hot Big Bang model with no recourse to an inflationary era, e.g., [35,38,39]. However, bouncing models have their own generic problems, e.g., [40,41], that need to be addressed in a model-specific fashion.

The main objective of the present work is to demonstrate the viability of an alternative, classical, asymmetric non-singular 'bouncing' cosmological model within a physical framework based on a globally U(1)-symmetric locally scale-invariant version of GR and the SM of particle physics. Embedding GR within a locally scale-invariant scalar-tensor formulation (and to a lesser extent the SM of particle physics as well) have been discussed in, e.g., [42–56]. Unlike in quantum-gravity-inspired bouncing models, classical bounces may take place at energies that do not require significant extrapolations of the SM of particle physics all the way to GUT or quantum gravity energy scales. This is a clear advantage, given that neither consistent theories of quantum gravity nor any observational/experimental evidence in favor of quantum gravity is currently available.

The terminology that is adopted in the main part of this work is somewhat different from the one familiar from either the standard, bouncing or cyclic cosmological models that are conventionally formulated in the 'Einstein frame' (EF), where, e.g., mass scales are fixed and only spacetime is dynamical. In this picture, the observed redshift on cosmological scales is solely due to space expansion, and in bouncing cosmological models, it is actually meant that space contraction itself momentarily halts at the 'bounce' followed by the space expansion that is what we observe. Since the proposed model could be viewed as presented in a specific 'Jordan frame' (JF), where space is static and cosmic evolution is regulated by the temporal evolution of mass, e.g., [57–60], we adopt the more appropriate notions of 'turning point', 'blueshifting' and 'redshifting-phase' instead of the commonly used parlance of 'bounce', 'contraction' and 'expansion', respectively.

Throughout, a mostly positive signature for the spacetime metric $(-1,1,1,1)$ is adopted. Our units convention is $\hbar = c = 1$. The theoretical approach underlying the proposed model is outlined in Section 2, followed by a description of the cosmological model in Section 3. The main results are summarized in Section 4. A few (mostly unwelcome) consequences of the symmetric bounce scenario are discussed in Appendix A.

2. Theoretical Framework

As proposed in [61], the SM of particle physics, as well as GR, can be promoted to Weyl-invariant (WI) theories with the following action

$$\begin{aligned}\mathcal{I}_{WI} &= \int \Big(\frac{1}{12}(\phi^2 - 2H^\dagger H)R + \frac{1}{2}\phi^\mu\phi_\mu - D^\mu H^\dagger D_\mu H - \frac{\lambda}{4}(H^\dagger H - \alpha^2\phi^2)^2 \\ &- \frac{\lambda'}{4}\phi^4 + \mathcal{L}_{SM}\Big)\sqrt{-g}d^4x, \end{aligned} \tag{1}$$

where D_μ is the gauge-covariant derivative, H is the Higgs field, ϕ is a newly introduced scalar field, R is the Ricci curvature scalar obtained from the metric field in the usual way, and α is defined via $H_0^\dagger H_0 = \alpha^2\phi_0^2 \equiv \frac{v^2}{2}$, where ϕ_0 is the expectation value of ϕ and $v \approx 246$ GeV, and $\mathcal{L}_{SM}$ is the Lagrangian density associated with the SM of particle physics [except for the Higgs kinetic and potential terms that explicitly appear in Equation (1)]. Here and throughout, Greek indices run through spacetime coordinates, $f_\mu \equiv \frac{\partial f}{\partial x^\mu}$ for any scalar function f, and summation convention applies. This action is invariant under local field rescaling inversely proportional to their corresponding mass dimensions, e.g., $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $g^{\mu\nu} \to \Omega^{-2}g^{\mu\nu}$, $\phi \to \phi/\Omega$, $H \to H/\Omega$, $A_\mu \to A_\mu$ and $\psi \to \Omega^{-3/2}\psi$, where A_μ and ψ are vector and spinor fields, respectively, and $\Omega(x)$ is an arbitrary function of spacetime.

The dimensionless action (i.e., action in $\hbar$ units) describing the classical motion of a massive point particle m is $\mathcal{I}_{pp} = \int mds = \int m\frac{ds}{d\tau}d\tau =$ where $ds^2 \equiv g_{\mu\nu}(x)dx^\mu dx^\nu$ is the infinitesimal line interval on the spacetime described by the metric field $g_{\mu\nu}$, and τ is an affine parameter. It readily follows from $m \to \Omega^{-1}(x)m$ and $g_{\mu\nu} \to \Omega^2(x)g_{\mu\nu}$ that $\mathcal{I}_{pp}$ is Weyl-invariant, and so are the timelike geodesics derived from it. Massless particles follow null-geodesics, and it is straightforward to show that null geodesics derived from $ds \to \tilde{ds} = \Omega(x)ds$ are equivalent to those derived from ds, irrespective of Weyl-invariance, provided that the affine parameter is re-parameterized, $\tau \to \Omega^{-2}(x)\tau$, e.g., [62].

The kinetic term associated with ϕ appears in Equation (1) with the 'wrong sign'. Whereas ϕ is formally classified as a ghost field, the theory described by

$$\mathcal{I} = \int \left(\frac{1}{12}\phi^2 R + \frac{1}{2}\phi^\mu \phi_\mu\right)\sqrt{-g}d^4x, \tag{2}$$

is countably renormalizable and has other appealing properties as well [63]. In addition, it has been compellingly argued about the Weyl-invariance of this action that it is a 'sham symmetry' or that Equation (2) is 'GR in a guise', i.e., that it contains no degrees of freedom beyond those already existing in GR, e.g., [63–67]. In other words, the Weyl current associated with the Weyl symmetry identically vanishes [67]. Surely, in that case, the ghost field could always be gauged away by simply making the choice $\phi = constant$ because unlike in the case of scalar-tensor theories in general, the system of field equations governing the dynamics of $g_{\mu\nu}$ and ϕ are under-determined, as we see below. The fact that we can endow ϕ with spacetime dependence by itself does not render it dynamical as the Weyl transormation $\phi \to \phi/\Omega(x)$ is not governed by any equation—this fact is manifested precisely by the arbitrariness of the spacetime dependence of ϕ. However, endowing ϕ with a phase, i.e., promoting it to a complex field, does add a new degree of freedom that genuinely corresponds to new physics, well beyond GR. In the case at hand, it is a global U(1) current that does not generally vanish. This latter point is further elaborated upon in the cosmological context in Section 3.3 below. All this implies that the theories described by Equations (1) or (2)—in spite of their appearance—do not bear with them the pathologies that quantum ghost fields are usually infamous for.

Specifically in the cosmological context, the FRW action for the scale factor $a(\eta)$ itself has exactly the same structure as that of Equation (2), as is explicitly shown in Section 3.3. Thus, any 'pathology' that could be associated with ϕ due to its ghostly nature would be equally well associated with the scale factor $a(\eta)$ of the FRW spacetime. Normally, the latter is not considered a problem because it is understood that GR is a classical theory, and the scale factor is correspondingly considered as a classical function of time. In the same vein, $\phi(\eta)$ is considered a classical function; the metric field $g_{\mu\nu}$ and the scalar field ϕ are both treated as classical fields. There is no clear justification for considering a quantum Planck mass, $\phi(\eta)$ (i.e., a quantum Planck length), while rendering the metric field, i.e., $a(\eta)$, classical, other than merely that we have no consistent quantum theory of gravity at our disposal yet.

Here, we put forward the proposal that ϕ is promoted to a complex scalar field, and consequently, Equation (1) is replaced by

$$\begin{aligned}\mathcal{I}_{WI} &= \int \Big(\frac{1}{6}(|\phi|^2 - H^\dagger H)R + \phi^\mu \phi^*_\mu - D^\mu H^\dagger D_\mu H - \frac{\lambda}{4}(H^\dagger H - \alpha^2|\phi|^2)^2 \\ &- \frac{\lambda'}{4}|\phi|^4 + \mathcal{L}_{SM}\Big)\sqrt{-g}d^4x, \end{aligned} \tag{3}$$

where $|\phi|^2 \equiv \phi\phi^*$, and in going from Equation (1) to (3), the parameter rescaling $\alpha^2 \to \alpha^2/2$ has been made. Matching with GR implies that $\frac{1}{6}|\phi_0|^2 \approx \frac{1}{6}(|\phi_0|^2 - \frac{v^2}{2}) = (16\pi G)^{-1}$, where G is the universal gravitational constant and ϕ_0 is of order the Planck mass, M_p, which is much larger than v, the electroweak scale. The action is globally U(1)-symmetric [56]; i.e., non-derivative terms include only $|\phi|$ and not its phase. As usual, the matter Lagrangian $\mathcal{L}_{SM}$ depends on all the SM fields, including H. We emphasize that for the action described by Equation (3) to be WI, the term $\int \mathcal{L}_{SM}\sqrt{-g}d^4x$ has to be WI, i.e., $\mathcal{L}_{SM} \to \Omega^{-4}\mathcal{L}_{SM}$ under local rescaling of the fields according to their mass dimension, as described above.

From Equation (3), it follows that the gravitational sector of the fundamental interactions is described by the following action:

$$\mathcal{I} = \mathcal{I}_{gr} + \mathcal{I}_m, \tag{4}$$

where

$$\mathcal{I}_{gr} \equiv \int \left(\xi|\phi|^2 R + \phi^*_\mu \phi^\mu\right) \sqrt{-g} d^4x \tag{5}$$

$$\mathcal{I}_m \equiv \int \mathcal{L}_m(|\phi|; \{\psi\}) \sqrt{-g} d^4x, \tag{6}$$

are the 'free gravitational' and 'source' actions, respectively. Here, we replaced the prefactor 1/6 by a dimensionless parameter ξ (that will be fixed by the requirement of local scale invariance below), and $\phi \equiv \chi e^{i\Psi}$. Our purpose in doing that is to crisply illustrate in passing what WI really implies before specializing to the case $\xi = 1/6$ in latter sections.

The matter Lagrangian density, $\mathcal{L}_m(\chi; \{\psi\})$, depends on both the modulus χ of the scalar field and $\{\psi\}$. The latter collectively denotes all fields other than ϕ, including $g_{\mu\nu}$. Aside from being non-negative, $\mathcal{L}_m$ is only required to satisfy the relevant field equations. Since $\mathcal{L}_m$ is independent of the phase Ψ, the latter is a 'cyclical coordinate' in field space that appears in Equation (3) only via its derivative coupling. Consequently, Equation (3) has a global U(1) symmetry and a conserved global charge, which lies at the heart of the proposed bouncing model, as will be discussed in the next section. By construction, Equations (4)–(6) are (Weyl-) invariant under $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$, $\phi \to \phi\Omega^{-1}(x)$ and $\mathcal{L}_m \to \mathcal{L}_m \Omega^{-4}(x)$ if $\xi = 1/6$. Specifically, for the kinetic term in Equation (5) to have the canonical form $\phi^*_\mu \phi^\mu$ (up to a sign difference) while maintaining Weyl-invariance, ξ must be fixed to 1/6.

If the modulus of the scalar field is fixed $\chi = \sqrt{\frac{3}{8\pi G}}$ (and Ψ is a fixed constant), then Equations (4)–(6) reduce to the Einstein–Hilbert (EH) action. The specific constant G that appears in the EH action guarantees that the resulting gravitational field equations reduce to the Poisson equation in the weak field limit within, e.g., the solar system, where the 'universality' of G has been reasonably established. Notably, even if we favor the idea that dark matter (DM) exists in the form of some exotic, beyond-the-SM particles, we still lack Cavendish-like *experimental* evidence that they 'couple' gravitationally via the same 'Universal' strength G either to each other or to baryons. More generally, since CDM is still no more than an effective description, we have no direct evidence that the Equivalence Principle applies to DM particles as it does to ordinary matter.

Clearly, the kinetic term of the scalar field appears in Equation (5) with the 'wrong' sign. Normally, a wrong sign of the kinetic term is considered a pathological property of the theory, which leads to tachyonic/'ghost' instabilities, e.g., [68,69]. However, in the special case of the WI action, Equations (1), (3) or (4)–(6), this is not an issue classically as the scalar fields ϕ and ϕ^* are *non-dynamical* due to the very nature of this WI theory. In other words, classically, there is no dynamical equation that determines the evolution of ϕ that could potentially drive the kinetic term to ever negative values. This will be explicitly discussed in Section 3.3 below in the context of the cosmological model. In particular, unlike in the generic tachyonic instability case, perturbations of the scalar fields are not *dynamically* driven towards unbounded growth. More generally, with the underlying theory being a WI version of GR, Equations (1), (3) or (4)–(6) do not possess instabilities that are not already present in GR. It should be perhaps stressed here that the entire discussion is limited to the framework of a low-energy effective *classical* theory, as in, e.g., [70]; in the same fashion that GR is a classical theory, its generalization considered here is assumed to be classical as well, and so a runaway decay of fields with negative kinetic energy to an unbounded number of new particles is not a scenario that is relevant to the present work. The latter catastrophic scenario and its implications are already considered and discussed in, e.g., [71].

We next derive the field equations. Variation of the action, Equation (4), with respect to $g^{\mu\nu}$ and ϕ^*, results in the following field equations

$$2\xi|\phi|^2 G_{\mu\nu} = T_{\mu\nu} + \Theta_{\mu\nu} \tag{7}$$

$$\xi\phi R - \Box\phi + \frac{\partial\mathcal{L}_m}{\partial\phi^*} = 0, \tag{8}$$

respectively, where $G_{\mu\nu}$ is the Einstein tensor and

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} \tag{9}$$

$$\Theta_{\mu\nu} \equiv 2\xi\phi^*\phi_{;\mu;\nu} + (2\xi-1)\phi^*_\mu\phi_\nu - 2\xi g_{\mu\nu}\left[\phi^*\Box\phi - \left(\frac{1}{4\xi}-1\right)\phi^*_\alpha\phi^\alpha\right] + c.c. \tag{10}$$

The complex conjugate of Equation (8) is similarly obtained by varying Equation (4) with respect to ϕ. Multiplying Equation (8) by ϕ^*, adding the result to its complex conjugate and to the trace of Equation (7) results in

$$\phi\frac{\partial\mathcal{L}_m}{\partial\phi} + \phi^*\frac{\partial\mathcal{L}_m}{\partial\phi^*} = T + (1-6\xi)(\phi^*\Box\phi + \phi^*_\alpha\phi^\alpha + c.c.). \tag{11}$$

Together, Equations (7), (8) and (11) provide a system of dynamic equations for the metric components and the complex scalar field. However, in the special case, $\xi = 1/6$, Equation (11) reduces from being a *dynamical* equation to being a *constraint* on the functional dependence of $\mathcal{L}_m(g_{\mu\nu}, \chi)$. This is expected, for in WI theories, $\phi(x) = \phi_0/\Omega(x)$, and $\Omega(x)$ is an *arbitrary* function of spacetime—there can simply be no equation that determines its 'dynamics'. In other words, $\phi(x) = \phi_0$ in the EF, and any arbitrary choice of $\Omega(x)$ amounts to a specific JF, one of infinitely many such possible JFs.

In addition, a straightforward calculation using the field equations and their contractions leads to the non-conservation of $T_{\mu\nu}$,

$$T^{;\nu}_{\mu\nu} = \phi_{,\mu}\frac{\partial\mathcal{L}_m}{\partial\phi} + \phi^*_{,\mu}\frac{\partial\mathcal{L}_m}{\partial\phi^*}, \tag{12}$$

a well-known result (in more general scalar-tensor theories, e.g., [72]). Indeed, energy–momentum conservation is broken once masses, e.g., the Planck mass, are allowed to vary in space and time.

Equation (8) is conveniently replaced by an equation governing the evolution of the phase Ψ as follows. The imaginary part of the product of Equation (8) with ϕ^* results in an equation for the scalar field phase, Ψ,

$$\chi\Box\Psi + 2\chi_\mu\Psi^\mu = 0, \tag{13}$$

merely a statement about a conserved global U(1) 'current'. In the EF, this reduces to

$$j^{\mu}_{;\mu} = 0 \tag{14}$$

where $j^\mu \equiv \sqrt{-g}g^{\mu\nu}\Psi_\nu$ is a conserved 'current'. Plugging this back into Equation (5), the kinetic term becomes

$$\mathcal{I}_{kin} = \int \chi_\mu \chi^\mu \sqrt{-g} d^4x + \int \chi^2 \Psi_\mu j^\mu d^4x. \quad (15)$$

The last term is a total derivative that becomes a surface term upon integration $j^0\chi^2 \int \Delta\Psi_0 d^3x$, where $\Delta\Psi_0 = \Psi_{0,f} - \Psi_{0,i}$, and we made use of the fact that χ is constant in the EF. Whereas for a constant χ the EH action is recovered, Ψ has no analog in GR, and its variation due to the conserved current j^μ modifies the dynamics. In particular, in the case at hand, it is responsible for the bounce.

Since in the case $\xi = 1/6$, Equation (11) reduces to a constraint, it then follows that only Equations (7) and (13) are dynamical. This implies that we have only eleven dynamical equations for twelve degrees of freedom: $g_{\mu\nu}$, χ and Ψ. This under-determined system reflects the underlying Weyl invariance in the case $\xi = 1/6$.

According to Equation (4), spacetime curvature is sourced by the kinetic and matter Lagrangian terms that appear in Equations (5) and (6), respectively. This is not a radically different situation than in the GR-based ΛCDM model where gravitation is not exclusively determined by $\mathcal{L}_{SM}$; in order for the standard cosmological model to provide a reasonably good fit to observational data, $\mathcal{L}_{SM}$ is amended by CDM and DE, which together comprise up to $\sim$95% of the cosmic energy budget at present. However, a notable conceptual difference is that in the framework described by Equations (4)–(6), a single scalar field, χ, is responsible for the evolution of the Planck mass, M_p, particle masses, and potentially also to the existence of what is normally interpreted as CDM and DE on cosmological scales. The latter, as we see below, may be viewed as different terms in the potential for the field χ. The fact that χ determines the Planck mass follows directly from the term $\propto \chi^2 R$ in Equation (5) that replaces the term $\frac{1}{2}M_p^2 R$ in the standard EH action. In addition, χ also regulates the evolution of particle masses as is evident from its presence in $\mathcal{L}_m$. In comparison, in ΛCDM, the Planck mass is fixed, and cosmic evolution is determined by space expansion, which in turn is driven by the energy budget of which CDM and DE are key building blocks. The former is thought to be in the form of some exotic beyond-the-SM particles, and the latter is believed to be the manifestation of some slow-rolling quintessence field (or some generalization thereof).

3. Cosmological Model

We begin this section with an outline of the assumed symmetries underlying the proposed model in Section 3.1. The proposed bouncing model is described at the background (Section 3.2) and linear perturbation (Section 3.4) levels. In Section 3.3, the standard cosmological model description (in the EF) is compared to the proposed model (formulated in the JF) at the background level. This is essentially a comparison between EF and JF descriptions, up to the bounce that is absent from the standard cosmological model. We end this section with a stability analysis of a toy bounce model in Section 3.5.

3.1. Underlying Symmetries of the Model

In arriving at the cosmological model that will be described in Sections 3.2 and 3.4, we make two assumptions that are not foreign to the standard cosmological model. First, the Universe is homogeneous on cosmological scales. In the standard model, this Cosmological Principle results in the FRW spacetime. For the present model, we adopt a static metric conformally related to FRW, $g_{\mu\nu} = diag(-1, \frac{1}{1-Kr^2}, r^2, r^2\sin^2\theta)$, where the scale-factor has been scaled out [i.e., $a(\eta) \equiv 1$], the time coordinate is conformal η and K is the spatial curvature parameter. As a result, the entire cosmic evolution is accounted for by the Planck mass and particle masses' evolution, as described below. This is a JF version of the standard

EF picture. The second assumption is that $\mathcal{L}_m(\chi)$ is an analytic polynomial in χ with no negative powers thereof (much like, e.g., how only non-negative integer powers of the Higgs field appear in the fundamental Lagrangian of the SM of particle physics). As we see below, this latter 'assumption' is actually consistent with the fact that the major contributions to the cosmic energy budget, according to ΛCDM, are characterized by equations of state which are integer multiples of 1/3, i.e., radiation ($w = 1/3$), NR matter ($w = 0$), the effective EOS of the spatial curvature term ($w = -1/3$) and DE ($w = -1$).

We assume that the energy–momentum tensor is isotropic, $T^{\nu}_{\mu} = -\rho \cdot diag(1, -w, -w, -w)$, and is characterized by an energy density ρ and an EOS w. The isotropy of $T_{\mu\nu}$, which induces the isotropy of the spacetime metric, follows directly from the assumed underlying U(1) symmetry of $\mathcal{L}_m$ in the proposed framework (i.e., that the redshift depends on a single function, χ, and is therefore isotropic). In other words, the observational fact that there is no preferred direction in the Universe from our vantage point, which by induction is applied to any observer— the Cosmological Principle—is here manifested by the assumed U(1) symmetry of the underlying WI theory. In fact, introducing additional field degrees of freedom to the WI theory by assuming larger symmetry groups, which contain U(1) as a subgroup, will result in an isotropic cosmological model. The model proposed here is 'minimal' in the sense that it is based on the simplest such symmetry group that allows a bounce to take place.

Since $T^0_0 = \mathcal{L}_m$, it immediately follows from Equation (11) in the case $\xi = 1/6$ that

$$\mathcal{L}_m \propto (\sqrt{-g})^{-3(1+w)/4}\chi^{1-3w}, \tag{16}$$

where an effective single fluid with a constant EOS w has been assumed is used in the relation $\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}$. Consequently, $\mathcal{L}_m$ is quartic in χ in the case that $w = -1$, is independent of χ, i.e., of masses, in the case that $w = 1/3$, and is linear in masses in the case of NR matter, i.e., the case of $w = 0$. The standard, EF, scaling of the Lagrangian/energy density of matter with the scale factor is recovered from Equation (16) once χ is fixed and $\sqrt{-g} = a^4$ is employed (the conformal time coordinate is used), in which case it follows that $\mathcal{L}_m \propto a^{-3(1+w)}$. Under Weyl transformations $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ and $\chi \to \chi/\Omega$, Equation (16) implies that the Lagrangian density transforms $\mathcal{L}_m \to \Omega^{-4}(x)\mathcal{L}_m$, as it should. As mentioned above (our third assumption), although there is no fundamental principle that requires $\mathcal{L}_m$ to be an analytic polynomial in χ we do impose this restriction. This by itself corresponds to a constraint on the EOS, $w \leq 1/3$, in the case that $\mathcal{L}_m$ was a monomial. This assumption is actually very reasonable and needs no special justification. Potentials with negative powers of scalar fields have been considered in the literature, but these are considered 'non-canonical' and have the undesired property that the potential diverges when the field obtains very small values. More generally, as is typically the case, there is no prescription for choosing the form of the matter Lagrangian. The latter is designed, subject to certain symmetry requirements, to recover (along with the kinetic terms) the required dynamics of the fields, as determined by experiments/observations (e.g., invoking CDM and DE within the ΛCDM model so as to render GR consistent with observations on cosmological scales). Therefore, and following the foregoing discussion, the 'coefficients' $f_i(\psi)$ in the matter Lagrangian

$$\mathcal{L}_m(|\phi|; \{\psi\}) = \sum_{i=0}^{i_{max}} f_i(\{\psi\})\chi^i, \tag{17}$$

are determined on cosmological scales by the observed cosmic evolution. The matter Lagrangian, $\mathcal{L}_m(|\phi|; \{\psi\})$, serves as a potential for the scalar field χ. Other fields in $\mathcal{L}_m$, e.g., those that determine the number densities of relativistic as well as NR particles, are non-evolving on the homogeneous, isotropic and *static* spacetime. The effective EOS associated with the i-th term of the potential is

$$w_i \equiv \frac{1-i}{3}. \tag{18}$$

Equation (17), with (18), trivially satisfies Equation (11) with $\xi = 1/6$. The only terms allowed in Equation (17) are characterized by parameters w_i that are integer multiples of $1/3$ subject to the constraint that $w_i \leq 1/3$ for all i, $\mathcal{L}_m$ is a polynomial in χ that contains only non-negative powers of χ, and there is no 'anisotropy problem'. The latter is a well-known problem [73] that is generic to bouncing models [74]. In practice, the constraint that $w_i \leq 1/3$ is implicitly assumed to hold in the standard ΛCDM model as well. The 'mixmaster' model of [73] typically arises from allowing for different evolutions along three principal axes. This would require, in particular, that these functional degrees of freedom are manifested in $\mathcal{L}_m$. In the limit of small anisotropic evolution, this is represented by an effective stiff matter contribution to the energy budget that dominates the cosmic energy budget prior to radiation. This dynamic does not have an analog in the present model because $\mathcal{L}_m$ is 'protected' against this 'shearing' dynamic by the assumed underlying U(1) symmetry and analyticity, as will be shown in the next section.

There are arguments supporting the idea that global symmetries are not allowed in a would-be quantum theory of gravity. The proposed model should be viewed as a low-energy effective approximation, and so even if the assumed underlying $U(1)$ symmetry is broken at the Planck (or even lower) scale, it is our assumption that the bounce safely takes place at lower energies (insofar as typical energies at the bounce are sufficiently high to allow for standard BBN to take place).

3.2. Evolution of the Cosmological Background

Sufficiently far from the bounce, the proposed model is equivalent to the standard cosmological model described in the comoving frame, where spacetime is static when the conformal rather than cosmic-time coordinate is used. For this latter fact, this description of the model naturally lends itself to a recently proposed resolution of the 'cosmic coincidence' problem [75], i.e., the puzzling fact that we happen to find ourselves observing the Universe in the unique era when DE and NR matter comparably contribute to the cosmic energy budget in spite of their very different evolution histories. Two other classical problems that plagued the Hot Big Bang model before cosmic inflation was proposed as a possible resolution—the 'flatness' and 'horizon' problems—are discussed below; the proposed model does not invoke cosmic inflation. In the following paragraphs, we analyze the field equations that govern the background evolution and their consequences.

The infinitesimal line element describing the background spacetime is $ds^2 = -d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$, where the time coordinate is η. Making contact with the FRW spacetime of the standard cosmological model is possible if η is identified with conformal time η. The latter is related to cosmic time t via $d\eta \equiv dt/a(t)$. The nonvanishing components of the Einstein tensor G_μ^ν, associated with the metric $g_{\mu\nu} = diag(-1, \frac{1}{1-Kr^2}, r^2, r^2\sin^2\theta)$ are

$$\begin{aligned} G_\eta^\eta &= -3K \\ G_i^j &= -K\delta_i^j. \end{aligned} \tag{19}$$

Here, 'i, j' indices stand for the spatial coordinates.

Defining $\mathcal{Q} \equiv \chi'/\chi$, the non-trivial field equations associated with metric variations (Equation (7)) read

$$\mathcal{Q}^2 + K = \frac{2\rho}{\chi^2} - \Psi'^2 \tag{20}$$

$$\mathcal{Q}' - \mathcal{Q}^2 - K = -\frac{3(1+w)\rho}{\chi^2} + 3\Psi'^2, \tag{21}$$

analogous to the Friedmann equations. Equation (21) can be replaced with a certain combination of Equations (20) and (21)

$$\mathcal{Q}' + \mathcal{Q}^2 + K = \frac{\rho(1-3w)}{\chi^2} + \Psi'^2. \tag{22}$$

To close our system of equations, the evolution equation of the phase, obtained by taking the imaginary part of the field equation associated with the variation of the scalar field, is

$$\Psi'' + 2\mathcal{Q}\Psi' = 0, \tag{23}$$

which can be readily obtained from Equation (13).

Equations (20) and (21) have the form of the Friedmann and Raychaudhuri equations of the standard cosmological model (with the scale factor replaced by χ) augmented with $\propto \Psi'^2$ terms that effectively represent a negative energy source with a 'stiff' EOS ($w_\Psi = 1$), with an effective contribution $-c_\Psi^2\chi^{-2}$ to the effective matter Lagrangian, where c_Ψ is an integration constant. Assuming this contribution is negligible at present, as well as at any observationally accessible cosmological era, then at sufficiently small χ, this term competes with the radiation term, and at the 'turning point' (which is referred to as 'bounce' in the EF), χ_b, the rate $\mathcal{Q}$ momentarily vanishes and a transition from $\mathcal{Q} < 0$ to $\mathcal{Q} > 0$ ensues.

For reference, Equation (20) is the analog of the standard Friedmann equation (with the $\propto \Psi'^2$ term omitted) $a'^2 + K = 8\pi G a^2 \rho(a)/3$. The kinetic term associated with a appears with the 'wrong' sign, but this is never considered a problem for the FRW model, although a is a ghost scalar field per se (as will be more explicitly shown in Section 3.3). It is understood that a, being part of the the definition of the FRW metric, is a *classical* field, and in this context, there is no problem with it having a negative 'kinetic energy'. In the same vein, there is no problem with χ and Ψ having kinetic terms with the 'wrong' sign in Equation (20); certain parts of the metric can be gauged away by allowing χ to vary, as we effectively allow here in providing a particular JF description of the proposed model. This point is further elucidated in Section 3.3.

Light element abundances set a limit on the redshift at the turning point $z_b > 10^9$. Since the 'stiff' energy drops faster than that of radiation by a factor $(1+z)^2$, it then follows that if $z_b > 10^9$, then already by recombination the (effective) 'stiff' energy density dropped to minuscule levels, a trillionth of the radiation energy density at most, and therefore has virtually no impact on the standard ΛCDM-based descriptions of either structure formation history or cosmological distances.

Other bouncing models with similar behavior near the bounce that contain a negative-energy effective stiff matter component (which are sourced by other mechanisms) have been considered in [70,76,77]. We emphasize that in the present model, the energy condition is not violated; i.e., $\mathcal{L}_m$ still includes only positive contributions, and it is only the contribution of the U(1)-symmetric kinetic term that makes an *effective* negative contribution, $-c_\Psi^2\chi^{-2}$ to the source term in Equation (20), much like a spatially closed space ($K > 0$) contributes negatively to the effective energy density. We emphasize once again that the model is purely classical and is based on a classical theory, much like the standard cosmological model is based on classical GR.

In the proposed scenario, cosmic history is clearly asymmetric around the bounce; we assume that an overwhelmingly large fraction of the CMB photons ia generated via dissipative processes at the bounce and shortly after. The reason for this assumption is constraints set up by current cosmological observations (which, in our understanding rule out symmetric bouncing models), and the following argument in general applies to other bouncing models as well. Specifically, assuming that recombination did take place, i.e., the bounce redshift, z_b, is sufficiently larger than the redshift at recombination, z_{rec}, so that the observed temperature anisotropy and polarization of the CMB do indeed provide a snapshot of the Universe at z_{rec}, it is straightforward to see that the observed horizon scale from temperature anisotropy and polarization at angular degree scales is only consistent within a perfectly symmetric bouncing scenario when $z_b \sim 1800$ as discussed in Appendix A, for the reason that in such a scenario, the acoustic horizon at recombination results from acoustic oscillations between $-\eta_{rec}$ and η_{rec}, rather than between 0 and η_{rec}, as in the standard cosmological model. Since hydrogen recombination takes place at $z_{rec} \sim 1100$, it seems that this part of standard recombination is largely unaffected. However, standard helium recombination takes place along two channels at $z \sim 6000$ and $z \sim 2000$, so this important component of standard recombination will not take place within a symmetric bounce scenario, with clear implications for temperature anisotropy and polarization. Perhaps more significant, BBN will never take place in such a symmetric scenario, and we are left in this case with the undesired situation that the observed light element abundance should be imposed by the initial conditions rather than be generated dynamically by BBN.

To get around this possibility, we have to assume that the model is generally asymmetric around the bounce. Specifically, to avoid a situation where $z_b < z_{BBN} \sim O(10^9)$, the entropy is required to grow sufficiently fast around the bounce such that $\rho_\gamma(-\eta) \ll \rho_\gamma(\eta)$ for any time sufficiently close to $\eta_b = 0$. This guarantees that the speed of sound at which acoustic oscillations propagate is typically much lower before the bounce than after the bounce, and this, as detailed in Appendix A, typically results in very large z_b. We assume that the blueshifting phase was dominated by a dust-like component preceded possibly by DE, with a very low entropy. Sufficiently close to the bounce, dissipation processes damp entropy from matter into the CMB. Depending on the typical energies at the bounce, these processes could include quantum particle production, as well as bulk viscosity, radiative viscosity, the damping of acoustic waves, etc. [78]. For these photons to fully thermalize, this whole process must end by $z \sim 10^7$, e.g., [79]. BBN itself already imposes a restriction $z_b \gtrsim 10^9$. It is our assumption that the model is sufficiently asymmetric around the bounce (in the sense that the temporal gradient of entropy generation is sufficiently large) that the speed of sound in the plasma is negligible during the blueshifting epoch. Again, according to this scenario, the bounce takes place at sufficiently large redshifts that standard BBN could be safely started and concluded, consistent with the observed light element abundance. The horizons at the asymptotic past ($t \to -\infty$) and asymptotic future ($t \to \infty$) do not exactly mirror each other because the proposed scenario is asymmetric (at least) near the bounce.

During such a dust-like blueshifting ('contracting') phase, quantum fluctuations of χ are characterized by a scale-invariant spectrum, thereby providing an explanation for the primordial density perturbations with no recourse to an inflationary era, e.g., [80–91]. There are complications unique to this alternative mechanism that are related to mode-mixing during the bounce and causality issues, e.g., [92,93], that are absent from the inflation-generated scale-invariant spectrum. It is our assumption here that the observed nearly scale-invariant spectrum of density perturbations has been generated along these general lines.

Accounting for the vacuum-like, NR, radiation and effectively stiff energy densities, Equations (16) and (20) combine to

$$\mathcal{Q}^2 = f_4\chi^2 + f_1/\chi + f_0/\chi^2 - c_\Psi^2/\chi^4, \tag{24}$$

where f_0 appreciably changes during the asymmetric bounce, as well as possibly f_1 and even f_4 as well. The latter takes place at χ_b that satisfies $f_4\chi_\mathrm{b}^2 + f_1/\chi_\mathrm{b} + f_0/\chi_\mathrm{b}^2 - c_\Psi^2/\chi_\mathrm{b}^4 = 0$. Equation (24) integrates to

$$\eta_2 - \eta_1 = \int_{\chi_1}^{\chi_2} \frac{d\chi}{\sqrt{f_4\chi^4 + f_1\chi + f_0 - c_\Psi^2\chi^{-2}}}, \tag{25}$$

where, again, (at least) f_0 and f_1 may sharply change at around χ_b. Assuming spatial flatness, the future horizon size is given by $r_h = \eta_\infty$, where

$$\eta_\infty = \int_{\chi_\mathrm{b}}^{\infty} \frac{d\chi}{\sqrt{f_4\chi^4 + f_1\chi + f_0 - c_\Psi^2\chi^{-2}}}. \tag{26}$$

In the proposed scenario, f_1 and f_0 change (due to dissipation processes) near the transition from the blue- to the redshifting phase, and so the future and past horizons with respect to the bounce are not exactly equal. Specifically, whereas f_0 has little effect on η_∞ in both evolution epochs, f_1 may drop significantly during the bounce and is plausibly expected to impact η_∞.

Two interesting limits of this equation will suffice for our purposes. The first is obtained by neglecting the vacuum-like and NR terms near the turning point. In this case, Equation (24) (with f_1 and f_4 set to vanish) integrates to

$$\chi^2 = f_0\eta^2 + c_\Psi^2/f_0, \tag{27}$$

assuming that c_Ψ and f_0 are fixed, which we know is not the case in our asymmetric bounce. Under this oversimplified assumption, χ attains its minimum, $\chi_\mathrm{b} = c_\Psi/\sqrt{f_0}$, at $\eta = 0$. This represents a smooth transition between the radiation-dominated (RD) era $[a(\eta) \propto \eta]$ and the bounce $[a(\eta) = constant]$. The evolution rate, synonymous to the expansion rate in the EF, $\mathcal{Q} = f_0\eta/(f_0\eta^2 + c_\Psi^2/f_0)$, is anti-symmetric under $\eta \leftrightarrow -\eta$, as is appropriate for a symmetric bounce. To phenomenologically break this symmetry commensurate with an asymmetric bounce, we modify Equation (27) to

$$\chi^2 = f_0\eta^2 + c_\Psi^2/f_0 + C\eta, \tag{28}$$

where C is a constant. The bounce in this case takes place at $\eta_b = -C/(2f_0)$, where $\chi_b^2 = (c_\Psi^2 - C^2/4)/f_0$, provided that $C < 2c_\Psi$. The blue/redshifting rate now becomes $\mathcal{Q} = (f_0\eta + C/2)/(f_0\eta^2 + C\eta + c_\Psi^2/f_0)$, which is no longer antisymmetric under $\eta \leftrightarrow -\eta$. Physically, the parameter C represents the time-symmetry breaking processes that are responsible for entropy (photon) production. Additional terms can be added, and in general, χ^2 can be represented as a polynomial in η, but for our toy-model purposes, Equation (28) will suffice for the stability analysis near the bounce that is carried out in Section 3.5.

In the other extreme—sufficiently remote from the turning point, where the background dynamics is dominated by nonrelativistic (NR) matter or DE—Equation (24) integrates to either $\chi \propto \eta^2$ in the matter-dominated (MD) era, or $\chi = \left(\sqrt{\lambda}(\eta_c - \eta)\right)^{-1}$ in the DE-dominated era. The integration constant η_c represents the start/end of the (conformal) time coordinate in the proposed model that asymptotically approaches de Sitter spacetime in the distant past and future, i.e., $\eta \in (\eta_{c,-}, \eta_{c,+})$, where $\eta_{c,-} < 0$ and $\eta_{c,+} > 0$ and in general $|\eta_{c,-}| \neq \eta_{c,+}$. If η is bounded from below, this time due to the presence of the vacuum-like energy density component, then observed radial distances are bounded at $|\eta_{c,-}|$, where the scalar field diverges and the model breaks down [$\eta = \eta_{c,-}$ and $\eta = \eta_{c,+}$ correspond to $t \to -\infty$ and $t \to \infty$, i.e., past and future (cosmic) infinity, respectively]. In the case of MD contraction with no DE component, η is in principle unbounded from below, thereby trivially addressing the 'horizon problem'. In such a scenario where the Universe starts off DE-free at $\eta \to -\infty$, the 'flatness problem' goes away as well.

Regardless of whether the 'flatness problem' is a genuine fine-tuning problem of the standard cosmological model or not, e.g., [30–32], we lay out the problem as it is normally presented in the literature followed by its possible resolution by the proposed 'bouncing' model. The 'flatness problem' is often claimed to arise in the hot Big Bang model due to the monotonic expansion of space and the consequent faster dilution of the energy density of matter (either relativistic or NR) compared to the effective energy density dilution associated with curvature. It is thus hard to envisage how space could be nearly flat (as is indeed inferred from observations, e.g., [17]) if not for an enormous fine-tuning taking place at the very early Universe, or alternatively for an early violent inflationary era.

Specifically in the proposed model, if the initial conditions at $\eta \to -\infty$ are 'natural' in the sense that the initial energy density of NR matter, $\rho_m(a)$, is not much different from the (absolute value of the) effective energy density associated with spatial curvature, $\rho_k(a) = -3K/(8\pi G a^2)$; then, since $\rho_m/\rho_k \propto 1 + z$, and since the number of e-folds contraction is infinite in this scenario, whereas the number of e-folds expansion from the bounce to the current size of the Universe is finite, then it immediately follows that spatial curvature at present should be vanishingly small.

From the foregoing discussion, it becomes clear that the proposed model is identical to the standard cosmological model in the latter comoving frame presentation insofar as Ψ' is dynamically irrelevant, i.e., sufficiently remote from the bounce. As is clear from Equation (24), in this limit, $\mathcal{Q}$ is identical to the conformal Hubble function, $\mathcal{H} = a'/a$. All this is expected in light of the discussion in Section 3.1 and is further discussed in Section 3.3.

We stress that not only does this *classical* 'bouncing' cosmological model avoid the disturbing initial singularity problem, it also achieves this with no recourse to ideas from quantum gravity, a theory that does not currently exist (and in addition would likely require a hundred trillion times higher energy than is currently achievable in colliders to test).

3.3. Einstein vs. Jordan Frame

The discussion in the previous section that highlighted the role of static background and temporally evolving masses, which provide an alternative explanation for the observed cosmological redshift, as in e.g., [57–60], can be viewed as a JF description of our model. It is constructive to compare these results to the standard treatment in the EF, where masses are fixed and cosmological redshift is accounted for by space expansion. Assuming a single fluids for concreteness with EOS w the '00' and 'ii' components of the Einstein equations for the metric $g_{\mu\nu} = a^2(\eta) \cdot diag(-1, \frac{1}{1-Kr^2}, r^2, r^2 \sin^2\theta)$ are, respectively,

$$\mathcal{H}^2 + K = \frac{8\pi G a^2 \rho}{3} \tag{29}$$

$$2\mathcal{H}' + \mathcal{H}^2 + K = -8\pi G w a^2 \rho. \tag{30}$$

Combining them, we obtain

$$\mathcal{H}' - \mathcal{H}^2 - K \quad = \quad -4\pi G(1+w)a^2\rho. \tag{31}$$

Equations (29) and (31) can be recovered from (20) and (21) by making the replacements $\mathcal{Q} \to \mathcal{H}$, $\chi^{-2} \to \frac{4\pi G a^2}{3}$, and $\Psi' = 0$. As discussed in the previous section, the $\propto \Psi'^2$ terms, whose analogs are absent in Equations (29) and (31), represent a negative contribution to the cosmic energy budget with a 'stiff matter' EOS, $w_\Psi = 1$. To complete the analogy with the theory described by Equation (3), we explicitly write down the EH action specified to the FRW spacetime in the comoving frame and derive Equations (29) and (31) directly from the action. We start by applying the EH action

$$\mathcal{I}_{EH} = \int \left(\frac{1}{16\pi G} R + \mathcal{L}_m \right) \sqrt{-g} d^4x \tag{32}$$

to the FRW spacetime $ds^2 = a^2(\eta)\gamma_{\mu\nu}dx^\mu dx^\nu$, where $\gamma_{\mu\nu}dx^\mu dx^\nu = -d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$. The Ricci scalar calculated from this metric is $R = 6(a''/a + K)/a^2$. Fixing units such that $3/(8\pi G) \equiv 1$ and employing integration by parts, Equation (32) can be brought to the form

$$\mathcal{I}_{EH} = \int [-a'^2 + Ka^2 + a^4\mathcal{L}_m(a)]\sqrt{-\gamma}d^4x. \tag{33}$$

Applying Equation (5) to the FRW spacetime and setting $\xi = 6$, we obtain

$$\mathcal{I}_{gr} = \int \left[-\chi'^2 - \chi^2\Psi'^2 + K\chi^2 + \mathcal{L}_m(\chi) \right] \sqrt{-\gamma}d^4x. \tag{34}$$

Making the replacements $a \rightleftarrows \chi$ and $a^4\mathcal{L}_m(a) \rightleftarrows \mathcal{L}_m(\chi)$, we can freely switch between the EF (Equation (33)) and JF (Equation (34)) descriptions up to the $-\chi^2\Psi'^2$ term, which is responsible for the bounce and is absent from the standard EH action. It is perhaps worth noting that the replacements $a \rightleftarrows \chi$ and $a^4\mathcal{L}_m(a) \rightleftarrows \mathcal{L}_m(\chi)$ are fully consistent with Equation (16). Friedmann Equations (29) and (30) can be directly derived from Equation (33) by introducing a Lagrange multiplier $\tilde{\lambda}$ (as in, e.g., [94])

$$\mathcal{I}_{EH} = \int [-\frac{a'^2}{\tilde{\lambda}} + \tilde{\lambda}\left(Ka^2 + a^4\mathcal{L}_m(a)\right)]\sqrt{-g}d^4x. \tag{35}$$

Except for the term that allows for a bounce, Equations (32) and (33) also differ in how the cosmic evolution is understood. Whereas in the EH action, Equation (33), $\mathcal{L}_m$ represents genuine matter, in the scalar-tensor action, Equation (34), the role of matter in shaping up the dynamics of cosmic evolution is taken over by a polynomial potential for the scalar field.

Notably, the scale factor $a(\eta)$ appearing in the FRW action, Equation (33), is a ghost field. Yet, Equation (33) and the Friedman equation derived from it are the backbone of the modern cosmology that describes the entire cosmic history at the background level, except at the singularity. The concern with quantum ghost fields is that their kinetic energies can go ever more negative, thereby allowing for a copious creation of particles. In reality, this 'nightmare scenario' is not what is observed with the scale factor $a(\eta)$. In fact, we do find ourselves in an expanding rather than contracting Universe where the kinetic term $-a'^2$ in Equation (33) increases over time rather than going ever more negative.

Whereas the kinetic term $-\chi'^2$ in Equation (34) is not a problem for the model, one could legitimately wonder how dangerous the $-\chi^2\psi'^2$ term is. In this regard, generically, bouncing models must violate the energy conditions by positing the existence of a negative energy contribution. If the latter is realized by some form of matter, specifically described by fields, it could in principle run unstably. However, it is an implicit assumption of bouncing models that the Universe never attains sufficiently high energy densities for these fluctuations to be significant at a level that would invalidate the model, which is the working assumption of the present work as well.

3.4. Linear Perturbation Theory

ΛCDM has successfully passed numerous tests and has proven to be remarkably effective in explaining the formation and linear growth of density perturbations, predicting the CMB acoustic peaks, polarization spectrum, and damping features on small scales. It also correctly describes the linear and nonlinear evolution phases of the LSS (on sufficiently large scales) as well as the abundance of galaxies and galaxy cluster halos. Therefore, it would seem essential to establish the equivalence of linear perturbation theory between the model proposed here and ΛCDM.

Consider linear perturbations over the FRW spacetime in the comoving frame. Metric perturbation variables include the scalars φ and ψ, vector mode v_i, and tensor modes h_{ij}, where the latter are subject to the constraint $\gamma^{ij}h_{ij} = 0$, and $\gamma_{ij} \equiv diag[1/(1-Kr^2), r^2, r^2\sin^2\theta]$. The weakly perturbed line element is $ds^2 = -(1+2\varphi)d\eta^2 + 2v_i d\eta dx^i + [(1-2\psi)\gamma_{ij} + 2h_{ij}]dx^i dx^j$. When stress is negligible, then $\varphi = \psi$. We define the fractional energy density and pressure perturbations (in energy density units) $\delta_{\rho_M} \equiv \delta\rho_M/\rho_M$ and $\delta_{P_M} \equiv \delta P_M/\rho_M$ ($= w\delta_{\rho_M}$), respectively. The matter velocity is v.

Transforming from the frame where $G = constant$ [in which case Equation (4) reduces to GR] to an arbitrary field frame with $\Omega(x) = 1 - \delta_\chi(x)$ and assuming $\delta_\chi \equiv \frac{\delta\chi}{\chi} \ll 1$ implies in particular that the scalar and metric field transform as $\chi \to \chi/\Omega \approx \chi(1+\delta_\chi)$ and $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, respectively, i.e., $\psi \to \psi + \delta_\chi$, and $\delta_{\rho_M} \to \delta_{\rho_M} + 4\delta_\chi$ to the leading order. Consequently, the new, 'shifted', perturbation variables, e.g., $\tilde{\varphi} \equiv \varphi + \delta_\chi$, $\tilde{\psi} \equiv \psi + \delta_\chi$, and $\tilde{\rho}_M \equiv \delta_{\rho_M} + 4\delta_\chi$, obey the same perturbation equations that are satisfied by the old perturbation quantities. The fact that the structure of the perturbation equations is unchanged under Weyl transformations is crucial in the context of stability near the bounce because perturbations of the scalar field in bouncing scenarios based on scalar-tensor theories of gravity are a potential cause for instability near the bounce, e.g., [91].

3.5. Stability Analysis for a Toy Bouncing Model

In this section, we analyze the dynamics of scalar perturbations at and near the bounce and show that all linear perturbation variables smoothly transform through the bounce. Clearly, this is a worry of any bouncing model, e.g., [95]. This is in contrast to the singular Big Bang model in which it is simply posited that singular perturbation modes, e.g., in the RD era, are vanishing by construction.

The following considerations start with a symmetric bounce, then generalized to a toy asymmetric bounce model. Near the bounce, the energy budget is dominated by radiation and the kinetic term associated with Ψ. In this case, $w_M = 1/3$ is a very good approximation. The Friedmann-like equation then integrates to Equation (27)

$$\chi^2 = A\eta^2 + B, \tag{36}$$

where $A \equiv f_0$ and $B \equiv c_\Psi^2/f_0$, and it is assumed that f_0 is fixed through the bounce. The analog of the conformal Hubble function in this case becomes $\mathcal{Q} = A\eta/(A\eta^2 + B)$.

Neglecting anisotropic stress, the linear perturbation equations (e.g., [72]) result in two coupled equations for $\delta\Psi$ and φ

$$\begin{aligned} \delta\Psi'' + 2\mathcal{Q}\delta\Psi' + k^2\delta\Psi &= -4\Psi'\varphi' \\ \varphi'' + 4\mathcal{Q}\varphi' + \frac{k^2}{3}\varphi &= 2\Psi'\delta\Psi', \end{aligned} \tag{37}$$

which, in the case $\chi^2 = A\eta^2 + B$, read

$$\delta\Psi'' + \frac{2A\eta}{A\eta^2+B}\delta\Psi' + k^2\delta\Psi = -\frac{4\sqrt{AB}}{A\eta^2+B}\varphi' \tag{38}$$

$$\varphi'' + \frac{4A\eta}{A\eta^2+B}\varphi' + \frac{k^2\varphi}{3} = \frac{2\sqrt{AB}}{A\eta^2+B}\delta\Psi'. \tag{39}$$

Equation (39) is a generalized Bardeen equation that is obtained from a combination of the Arnowitt–Deser–Misner (ADM) energy constraint and the Raychaudhuri equation. In the long-wavelength limit, $\delta\Psi'$ can be substituted from (39) into the derivative of (38) with respect to η. The resulting third-order equation for φ then reads

$$(A\eta^2+B)\varphi''' + 8A\eta\varphi'' + 12\left(\frac{A^2\eta^2+AB}{A\eta^2+B}\right)\varphi' = 0, \tag{40}$$

which is satisfied by

$$\varphi = c_1 + \frac{c_2\eta + c_3 B(A\eta^2 - B)}{(A\eta^2+B)^2} \tag{41}$$

where c_1, c_2 and c_3 are three integration constants. Substituting Equation (41) back into Equation (39) then results in

$$\frac{\delta\Psi'}{\Psi'} = -\frac{4c_2\eta + c_3(A^2\eta^4 + 6AB\eta^2 - 3B^2)}{(A\eta^2+B)^2}. \tag{42}$$

It is clear from Equations (41) and (42) that both φ and $\delta\Psi'/\Psi'$ are well behaved near $\eta = 0$ due to the non-vanishing B, i.e., the non-vanishing of Ψ', and perturbation theory does not break down there. Note that it is $\delta\Psi'/\Psi'$ rather than $\delta\Psi/\Psi$ that is the quantity of interest in this model.

To solve for the short-wavelength limit, we differentiate Equation (38) with respect to η and then substitute for $\delta\Psi'$ from Equation (39). The resulting fourth-order equation for φ is

$$\begin{aligned} &\left(\frac{Ax^2}{k^2} + B\right)\varphi + \frac{18A}{k^2}x\varphi_x + \left(\frac{60A}{k^2} + \frac{4A}{k^4}x^2 + 4B\right)\varphi_{xx} \\ &+ \frac{30A}{k^2}x\varphi^{(3)} + 3\left(B + \frac{Ax^2}{k^2}\right)\varphi^{(4)} = 0, \end{aligned} \tag{43}$$

where, e.g., $\varphi_x \equiv \frac{\partial\varphi}{\partial x}$, in the large-$k$ limit, which has no closed-form solution. Here $x \equiv k\eta$, and, e.g., $\varphi^{(3)} \equiv \frac{d^3\varphi}{dx^3}$. In the limit $x \ll 1$, i.e., $\eta \to 0$, the potential φ decouples from both A and B, and the equation considerably simplifies to

$$3\varphi_{xxxx} + 4\varphi_{xx} + \varphi = 0, \tag{44}$$

with the general solution

$$\varphi = c_1 \cos x + c_2 \sin x + c_3 \cos\left(x/\sqrt{3}\right) + c_4 \sin\left(x/\sqrt{3}\right), \tag{45}$$

illustrating the fact that perturbations are manifestly stable on small scales as well.

Slightly off $\eta = 0$, $A\eta^2 \gtrsim B$ perturbations smoothly approach their short-wavelength-limit standard behavior in the RD era. In the latter, within ΛCDM, $\varphi = \frac{1}{x}\left[c_j j_1(x) + c_y y_1(x)\right]$ where j_1 and y_1 are the spherical Bessel and the modified spherical Bessel functions, respectively, of the first kind. In ΛCDM, $c_y = 0$ since $y_1(0)$ diverges, and the phase of the oscillating potential is 0, i.e., $\varphi \propto \sin x$. This needs not be the case in the proposed model, as is evident from this discussion insofar as $B \neq 0$. The implications for CMB observables of not neglecting the 'diverging' mode have been discussed by [96].

The case of asymmetric bounce is considered next. We consider here a mild asymmetry as in Equation (28). Here, then, Equation (36) is replaced by $\chi^2 = A\eta^2 + C\eta + B$, which incurs a change in $\mathcal{Q}$. It is convenient in this case to consider perturbations that take place at around $\eta = \eta_b$, the bounce time, rather than at $\eta = 0$. Transforming to the new time coordinate $\tau \equiv \eta + C/2A$, in addition to making the assumption that $B \gtrsim C^2/(4A)$ so as to avoid spoiling the bounce, it then readily follows that $\chi^2 = A\eta^2 + C\eta + B \approx A\tau^2 + B$. In terms of the new time coordinate, then, $\mathcal{Q}$ is nearly unchanged from the symmetric bounce, and Equations (38) and (39) maintain their forms but are parameterized by τ instead of η. It then follows that the long-wavelength solutions, Equations (41) and (42), and the short-wavelength solution, Equation (45), are unchanged with respect to the symmetric bounce case but with η replaced by τ and possibly the redefinition of integration constants. Once again, we see that scalar perturbations are well behaved in this more general case near the bounce.

Although the treatment in this section was based on several simplifying assumptions, the results give us confidence in the stability of the proposed model at and near the bounce. Adding this to the fact that the proposed model is entirely classical and that the bounce can take place at relatively low energies of a few MeV, which is many orders of magnitude lower than typical energies in bouncing models, we see no reason for a similar treatment to fail when applied to more realistic cases. Similar considerations likely apply to primordial gravitational waves as well if they are at all excited in the proposed (or similar) scenario.

4. Summary

While ΛCDM has clearly been remarkably successful in phenomenologically interpreting a wide range of observations, it still lacks a microphysical explanation of several key components, primarily the nature of CDM and DE. It also suffers from a few outstanding conceptual problems such as the initial singularity, in addition to a few 'coincidence' or 'naturality' problems. Inflation, which has long been part and parcel of our current understanding and acceptance of ΛCDM by compellingly explaining away the flatness and horizon problems, as well as the nearly scale-invariant spectrum of primordial scalar perturbations, has its own shortcomings and fine-tuning problems. In addition, a few persistent anomalies afflict ΛCDM, and so alternative models that address (at least) a few of these issues are of interest.

Direct spectral information on the CMB is unavailable (due to opacity) in the pre-recombination era ($z \gtrsim 1100$). From the observed cosmic abundance of light elements, BBN at redshifts $O(10^9)$ could be indirectly probed. Earlier on, at $z = O(10^{12})$ and $z = O(10^{15})$ [energy scales of $O(200)$ MeV and $O(100)$ GeV, respectively], the quantum chromodynamics (QCD) and electroweak phase transitions have presumably taken place, although their expected (indeed weak) signatures in, e.g., the CMB, have not been found yet.

In addition, inflation, a cornerstone of the standard cosmological model, is clearly beyond the realm of well-established physics; its ultimate detection via the B-mode polarization that it imprints on the CMB could be achieved only if inflation took place at energy scales $\sim$trillion times larger than currently achievable in colliders. Moreover, theoretical expectations for the amplitude of this B-mode as a function of the energy scale of inflation rely on the assumption that gravitation is genuinely quantized. The latter assumption lacks empirical basis at present. By itself, inflation is plagued by the η-problem, the trans-Planckian problem, and the 'measure problem' in the multiverse. The latter essentially implies a lack of predictive power.

Ideally, an alternative cosmological model that agrees well with ΛCDM at BBN energies and lower, i.e., $z < 10^{10}$, while still addressing the classical problems of the Hot Big Bang model that inflation was originally designed to undertake, as well as avoiding the initial curvature singularity, while never reaching Planck or even GUT scale energies, will be an appealing alternative. One conclusion of the present work is that this could be in principle achieved, at least in part, with a (*classical*) non-singular 'bounce' that also removes the technically and conceptually undesirable initial singularity problem of GR-based cosmological models. In order to achieve such a bounce within GR, or a conformally related theory, certain 'energy conditions' have to be effectively violated. One specific realization of this program has been the focus of the present work.

Symmetries play a key role in our current understanding of the inner workings of the fundamental interactions. For example, the SM of particle physics is based on a *local* $U(1) \times SU(2) \times SU(3)$ gauge group with quantized gauge fields. In addition, our favorite theory of gravitation, GR, is invariant under coordinate transformations. In the framework adopted in this work, GR and the SM of particle physics are endowed with local scale invariance, i.e., 'Weyl invariance', as well as an internal U(1) symmetry with a global charge, in addition to the standard coordinate-system covariance. Only the salient merits of the cosmological model based on this alternative theory of gravitation have been discussed in the present work.

In the proposed model, spacetime is described by the FRW metric in comoving frame, which in the absence of spatial curvature reduces to the Minkowski spacetime. Here, the role of the scale factor in ΛCDM as the regulator of cosmic evolution is played by the modulus, χ, of a complex scalar field ϕ that lives on a static background. The phase, Ψ, plays a crucial role near the turning point and is largely irrelevant elsewhere. There is no analog to Ψ in ΛCDM. Here, χ, which regulates the evolution of (dynamical) masses, starts infinitely large, first monotonically decreases until it 'bounces', then grows again without bound. Put alternatively, the Planck length starts out infinitely small, increases until it peaks at the turning point, then decreases again. Described in terms of these length 'units', the Universe is said to undergo a blueshifting phase of evolution, followed by a turnaround and redshifting. Cosmological redshift is then a manifestation of evolving Rydberg 'constant' rather than space expansion.

The modulus of the scalar field, χ, delineates essentially the same dynamics in the cosmological model that the scale factor $a(\eta)$ does in ΛCDM (insofar Ψ is dynamically irrelevant). However, unlike $a(\eta)$, which is part of the Friedmann–Robertson–Walker (FRW) metric, $\chi(\eta)$ is a scalar field living in a static space. If the time variation of Ψ is sufficiently slow, the entire observable cosmic evolution, from BBN (taking place at typical $\sim$1 MeV energies) onward, is essentially indistinguishable from that of ΛCDM, thereby retaining its merits. However, the very early Universe can be much different, e.g., there is no initial singularity in the proposed model and possibly also no primordial phase transitions that in ΛCDM are expected to have taken place at energy scales of O(100) MeV and O(100) GeV and possibly also at the GUT scale.

The alternative cosmological scenario explored in this work starts with a deflationary evolution, which culminates in a turning point when the (absolute value of the negative) energy density associated with the effective 'stiff matter' (provided by the kinetic term of Ψ) momentarily equals that of radiation. In the vacuum-like-dominated epoch, the energy density of the Universe is dominated by an $\propto \chi^4$ term in the matter Lagrangian that is genuinely classical with no quantum fluctuations. Therefore, DE, according to the present scenario, is not zero-point energy but rather a manifestation of the self-coupling of the scalar field, i.e., a term in the matter Lagrangian of the form $f_4\chi^4$, with f_4 being a dimensionless parameter. This DE-like contribution is characterized by a non-dynamical EOS with no recourse to, e.g., a new quintessence field; here, the same scalar field accounts for both 'G' and DE, and possibly also CDM, or any alternative that effectively manifests itself as CDM.

One of the most notable achievements of inflation was the realization that a slightly red-tilted primordial spectrum of Gaussian perturbations can be generated by quantum fluctuations in a vacuum-like expanding Universe. No specific mechanism has been adopted in the present work for the generation of primordial density perturbations (although a few known mechanisms capable of generating such a spectrum during an MD-like contracting phase have been briefly mentioned). Whatever this mechanism turns out to be, it does not necessarily involve quantum fluctuations of the metric field, unlike in inflation. Again, gravitation was treated in this work as a genuine classical interaction. However, admittedly, the only mechanism that is sufficiently well understood and can be readily integrated with the proposed model is that of primordial perturbations generated by quantum fluctuations of linearized gravity, as is the case with inflation.

In addition, the 'anisotropy problem' that in general plagues bouncing scenarios does not exist in our construction. As discussed above, Weyl symmetry and the consequent absence of any dimensional parameter in the action, in addition to the postulated global U(1) symmetry, protect the model from running into a chaotic, anisotropy-dominated, evolution phase, unless we are willing to consider non-canonical terms in the matter Lagrangian of the form, e.g., $\mathcal{L}_{m,ani} \propto (\bar{\psi}\psi)^2\chi^{-2}$ and $C_{\alpha\beta}{}^{\gamma\delta}C_{\gamma\delta}{}^{\rho\sigma}C_{\rho\sigma}{}^{\alpha\beta}\chi^{-2}$, where ψ is, e.g., a Dirac field, and $C_{\rho\sigma}{}^{\alpha\beta}$ is the Weyl tensor.

Conformal time is both past- and future-bounded in this scenario, i.e., $\eta \in (\eta_{c,-}, \eta_{c,+})$, unless there is no component in the blueshifting phase that corresponds to $w \leq -1/3$. In principle, any 'horizon problem' could be avoided if $|\eta_{c,-}| \gg \eta_0$. Specifically, in this scenario, cosmic history starts with very large (and in principle infinite) particle masses, and therefore, the causal horizon is much larger than would be naively expected from monotonically growing masses (which to the redshifting era). Likewise, the 'flatness problem' afflicting the Hot Big Bang scenario stems from the slower decay of the energy density associated with curvature as compared to that of matter in a *monotonically* expanding Universe. In bouncing scenarios, the situation is reversed before the turning point; starting at infinitely large χ (masses), one typically expects to find that the energy density in the forms of NR matter and radiation largely exceeds that of curvature at any *finite* χ value in either the blueshifting or redshifting era. From this perspective, flatness is an attractor point rather than an unstable point that requires the fine-tuning of the initial conditions.

The model we considered is falsifiable in several ways: First, $w_{DE} = -1$ due to local scale invariance and any observationally inferred $w_{DE} \neq -1$ would either rule out this particular model or alternatively either imply the soft breakdown of WI at very early and late times or the existence of a non-canonical DE term in the matter Lagrangian of the form, e.g., $\mathcal{L}_{m,DE} \propto (\bar{\psi}\psi)^{-\varepsilon/3}\chi^{4+\varepsilon}$ (where $|\varepsilon| \ll 1$ is some dimensionless parameter) that involves non-integer, and possibly irrational, powers of the fields. Second, within the framework adopted here, if a scale-invariant B-mode polarization is ultimately measured, it would provide compelling evidence that gravity is quantized, in contradiction to the assumption made here that gravity is a genuinely classical interaction, which implies that its perturbations are not subject to the Bunch–Davies vacuum condition. Consequently, unlike the inflationary-induced B-mode polarization of ΛCDM, it does not follow from any fundamental principle that B-mode polarization has to be characterized by a flat spectrum. Third, signals from primordial phase transitions as well as leptogenesis or baryogenesis that ought to be imprinted in the CMB anisotropy and polarization (perhaps too weak to be detected) in the standard expanding model may not have taken place at all in the proposed model, depending on the typical temperature at the turning point.

We believe that, in addition to addressing the cosmological horizon and flatness problems, the framework proposed here provides important insight on the nature of DE, and initial singularity and stability near the turning point. Even so, the work presented here is by no means exhaustive, and indeed a few of its basic aspects will be further elucidated in future works.

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Appendix A. Consequences of a Symmetric Bounce Scenario

Here, we adopt an EF parlance and use notions such as 'contraction', 'bounce' and expansion since this Appendix is of relevance to symmetric bounce models in general (which are normally formulated in the EF). A symmetric bounce is only consistent with an adiabatic evolution, i.e., when entropy is not significantly generated throughout cosmic history, in particular not during the bounce. In a symmetric bounce scenario, the cosmic history of the post-bounce expanding phase exactly mirrors that of the pre-bounce contracting phase. This condition by itself sets certain stringent conditions, which, coupled with observationally inferred cosmological parameters from the post-recombination era, results in constraints on the bounce redshift, z_b, and the large- scale correlation of the CMB anisotropy and polarization on the sky.

In the standard cosmological model, the acoustic horizon at recombination is given by, e.g., [97].

$$\eta_{ac} = \int_0^{\eta_\star} \frac{d\eta'}{\sqrt{3[1 + R(\eta')]}}, \tag{A1}$$

where the integration starts at the Big Bang, $\eta = 0$, and ends at recombination, $\eta_\star$, and $R \equiv \frac{3\rho_b}{4\rho_\gamma} = 3 \times 10^4 (1+z)^{-1} \Omega_b h^2$ with ρ_b and ρ_γ as the baryon and radiation densities, respectively. The acoustic speed $1/\sqrt{3(1+R)}$ is lower than $1/\sqrt{3}$ due to the baryonic inertia. At recombination, the concordance model corresponds to $R_\star \approx 0.65$. Before and at around recombination, the energy budget consists of dust and radiation only. Consequently, $\mathcal{H}^2 = \frac{8\pi G\rho a^2}{3}$ can be integrated

$$d\eta \propto \frac{da}{\sqrt{1+(\frac{\rho_m}{\rho_\gamma})_\star a}}, \tag{A2}$$

and up to a multiplicative constant factor, the acoustic horizon at recombination is

$$\eta_{ac} \propto \int_0^1 \frac{da}{\sqrt{(1+\frac{4}{3}R_\star \mathcal{R} a)(1+R_\star a)}}, \tag{A3}$$

where $\mathcal{R} \equiv \frac{\Omega_m}{\Omega_b}$ ($\Omega_m = \Omega_c + \Omega_b$) and a is normalized to unity at recombination, i.e., $a_\star = 1$. This acoustic horizon scale is probed not only by CMB anisotropy and polarization but also with baryonic acoustic oscillations (BAOs).

Now, assume a symmetric bounce model instead of the Big Bang model. In this model, the horizon scale could be twice as large as that of the standard model, Equation (A1), if z_b is very large (but finite of course) because the boundaries of the integral in Equation (A1) will be replaced with integration over the range $[-\eta_\star, \eta_\star]$ or, equivalently, the integrand is symmetric around $a = 0$ (or any finite a_b) in Equation (A1), by assumption. Assuming that $a_b \ll a_\star$, the comoving distance to recombination, $\eta_0 - \eta_\star$, is unchanged, so the angular size on the sky of anisotropy/polarization correlations could be up to twice as large as in the SM. However, demanding that the observed angular scale of the acoustic horizon is consistent with that of the concordance model, then

$$\begin{aligned}&\int_0^1 \frac{da}{\sqrt{(1+\frac{4}{3}R_\star \mathcal{R} a)(1+R_\star a)}} \\ = \quad & 2\int_{a_b}^1 \frac{da}{\sqrt{(-\frac{c^2}{a^2}+1+\frac{4}{3}R_\star \mathcal{R} a)(1+R_\star a)}},\end{aligned} \tag{A4}$$

where $a_b = (1+z_b)^{-1}$. Here, c is the value of c_Ψ at recombination. The parameters c and a_b are not independent; they satisfy the constraint $-\frac{c^2}{a_b^2} + 1 + \frac{4}{3}R_\star \mathcal{R} a_b \equiv 0$. The observed value $\mathcal{R} = 6.2$ is consistent with Equation (A4) when $c = 1.25$ and $a_b \approx 0.606$, which corresponds to $z_b \approx 1800$. For comparison, the case $R_\star = 0.1$ corresponds to a bounce occurring at $z_b \approx 1390$. While the bounce redshift, $z_b \approx 1800$, guarantees that hydrogen recombination indeed takes place at $z_\star \approx 1100$, helium recombination that takes place at $z \sim 2000$ and 6000 in the SM does not take place in this scenario. This will have a significant impact on the CMB anisotropy and polarization. Since the width of recombination is still $\Delta z \sim 80$ and since $z_\star$ is sufficiently remote from z_b, the CMB, much like in the standard singular cosmological model, provides a snapshot of the Universe at recombination.

BBN is widely hailed as a great triumph of the singular Hot Big Bang model. BBN ostensibly took place at $z = O(1)$ MeV, when typical particle energies were ~ 1 MeV. Clearly, if the bounce took place at $z_b \approx 1800$, then BBN, electroweak and QCD phase transitions, lepto/baryogenesis, or inflationary era never took place in our Universe. This poses a significant challenge to such a symmetric bounce model.

A symmetric scenario must have started with a DE-dominated phase that would project over the entire sky very shortly after the bounce with an ever-shrinking correlation sky over the expanding Universe. The angular scale of the causal horizon at present should be the distance traveled by light from the start of the contracting phase until the bounce plus the relatively short distance travelled from the bounce until recombination in the expanding phase—all this divided by the distance travelled from recombination to here.

$$\theta_c \sim [r(-1, z_b) + r(z_\star, z_b)]/r(0, z_\star) \tag{A5}$$

where

$$r(z_1, z_2) \equiv \int_{1+z_1}^{1+z_2} \frac{dx}{\sqrt{\Omega_\Psi x^6 + \Omega_r x^4 + \Omega_m x^3 + \Omega_{DE}}}, \tag{A6}$$

and $\Omega_\Psi < 0$. Since $r(z_\star, z_b) \ll r(-1, z_b)$ and since $r(0, z_\star) \lesssim r(-1, z_b)$, it follows that $\theta_c \gtrsim 1$ radian. There is thus no reason why the CMB should be correlated over the entire sky at the *present* time, when $r(0, z_\star) \lesssim r(-1, z_b)$; observers at very high redshifts, z_x, that satisfy $r(z_x, z_\star) \ll r(-1, z_b)$ will see $\theta_c \sim 2\pi$ with no 'low multipole anomaly'.

Therefore, the observed low multipoles anomaly, e.g., [14–16], might be due to a symmetric bounce model (as opposed to a standard Big Bang one) coupled with our location along the cosmic timeline.

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