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Addendum: M2-branes wrapped on a topological disk

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Abstract: We perform the flux quantization and, employing the results, obtain the explicit expression of Bekenstein-Hawking entropy for M2-branes wrapped on a topological disk, [1].

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1 Flux quantization

Note added. After completing this addendum, we noticed that just the result of the caluculation in this addendum was previously presented in (3.60)–(3.63) in section 3.2.2 of [2]. We present the calculation in detail in our conventions.

We perform the flux quantization and, employing the results, obtain the explicit expression of Bekenstein-Hawking entropy for M2-branes wrapped on a topological disk, [1].

We find the four-form flux to be[1]

$$
\begin{aligned}
G_{(4)} =& \frac{384\sqrt{2}gb^3\mathcal{C}y^4}{(1-y^2)^4}\left(y+2y^{-1/2}\Delta\right)dz\wedge dy\wedge \mathrm{vol}_{\mathrm{AdS}_2}\\
&+\frac{12\sqrt{2}b\mathcal{C}y^4h}{g\,(1-y^2)^2}\sin(2\xi)\,dz\wedge d\xi\wedge\mathrm{vol}_{\mathrm{AdS}_2}\\
&-\frac{4\sqrt{2}b}{g^2}\Big[\cos\xi\sin\varphi\,(\sin\xi\sin\varphi d\xi-\cos\xi\cos\varphi d\varphi)\wedge D\phi_1\wedge\mathrm{vol}_{\mathrm{AdS}_2}\\
&+\cos\xi\cos\varphi\sin\psi\Big(\sin\xi\cos\varphi\sin\psi d\xi+\cos\xi\,(\sin\varphi\sin\psi d\varphi-\cos\varphi\cos\psi d\psi)\Big)\wedge D\phi_2\wedge\mathrm{vol}_{\mathrm{AdS}_2}\\
&+\cos\xi\cos\varphi\cos\psi\Big(\sin\xi\cos\varphi\cos\psi d\xi+\cos\xi\,(\sin\varphi\cos\psi d\varphi+\cos\varphi\sin\psi d\psi)\Big)\wedge D\phi_3\wedge\mathrm{vol}_{\mathrm{AdS}_2}\Big]\,.
\end{aligned}
\tag{1}
$$

The Hodge dual of the four-form flux is

$$
\begin{aligned}
*G_{(4)} =& \frac{16}{g^6\Delta^2}\left(y+2y^{-1/2}\Delta\right)\cos^5\xi\sin\xi\,d\xi\wedge\mathrm{vol}_{S^5}\wedge d\phi_4\\
&+\frac{16}{g^6\Delta^2}\cos^6\xi\sin^2\xi\,dy\wedge\mathrm{vol}_{S^5}\wedge d\phi_4\\
&-\frac{48\mathcal{C}}{g^5y^3}\cos^3\xi\sin\xi\cos^2\varphi\sin(2\psi)\\
&\quad\times d\xi\wedge d\psi\wedge\Big(\cos^2\varphi\,d\phi_2\wedge d\phi_3+\sin^2\varphi\,d\phi_1\wedge(d\phi_2-d\phi_3)\Big)\wedge d\phi_4\wedge dy\wedge dz\\
&+\frac{24\mathcal{C}}{g^5y^3}\cos^3\xi\sin\xi\cos^3\varphi\csc\varphi\sin^2(2\psi)\,d\xi\wedge d\varphi\wedge d\phi_1\wedge(d\phi_2-d\phi_3)\wedge d\phi_4\wedge dy\wedge dz\\
&-\frac{48\mathcal{C}}{g^5y^{3/2}\Delta}\cos^4\xi\sin^2\xi\cos^3\varphi\sin\varphi\sin(2\psi)\\
&\quad\times d\varphi\wedge d\psi\wedge\Big(d\phi_1\wedge d\phi_2+d\phi_2\wedge d\phi_3+d\phi_3\wedge d\phi_1\Big)\wedge d\phi_4\wedge dy\wedge dz\,,
\end{aligned}
\tag{2}
$$

where we define the volume form of gauged five-sphere,

$$
\mathrm{vol}_{S^5} = \cos^3\varphi\sin\varphi\cos\psi\sin\psi\,d\varphi\wedge\,d\psi\wedge\,D\phi_1\wedge\,D\phi_2\wedge\,D\phi_3\,.
\tag{3}
$$

We consider the flux quantization conditions for the four-form flux. The integral of the four-form flux over any four-cycle in the internal space is an integer,

$$
\frac{1}{(2\pi l_p)^6}\int *G_{(4)}\,\in\,\mathbb{Z}\,,
\tag{4}
$$

where l_p is the Planck length.

[1]We have corrected some typographical errors in $G_{(4)}$ in the main body of paper, [1].

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First, we consider the $*G_{(4)\xi\varphi\psi\phi_1\phi_2\phi_3\phi_4}$ component and we obtain

$$\begin{aligned}\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\varphi\psi\phi_1\phi_2\phi_3\phi_4} &= \frac{1}{(2\pi l_p)^6}\int \frac{16}{g^6\Delta^2}\left(y+2y^{-1/2}\Delta\right)\cos^5\xi\sin\xi\, d\xi\wedge \mathrm{vol}_{S^5}\wedge d\phi_4 \\ &= \frac{1}{4\pi^2 l_p^6 g^6} \equiv N\,, \end{aligned} \tag{5}$$

where $\mathrm{vol}_{S^5}=\pi^3$ and $N\in\mathbb{N}$ is the number of M2-branes wrapping the two-dimensional manifold, Σ. This integration contour corresponds to the interval, $\mathsf{P}_1\mathsf{P}_2$, in figure 2.

Second, we consider the following three components and obtain

$$\begin{aligned}&\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_2\phi_3\phi_4yz}\\ &=\frac{1}{(2\pi l_p)^6}\int\left(-\frac{48\mathcal{C}}{g^5y^3}\cos^3\xi\sin\xi\cos^2\varphi\sin(2\psi)\right)\cos^2\varphi\, d\xi\wedge d\psi\wedge d\phi_2\wedge d\phi_3\wedge d\phi_4\wedge dy\wedge dz\\ &=-N\frac{6g\mathcal{C}}{y_1^2}\cos^2\varphi\cos^2\varphi\,, \end{aligned}\tag{6}$$

$$\begin{aligned}&\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_1\phi_2\phi_4yz}\\ &=\frac{1}{(2\pi l_p)^6}\int\left(-\frac{48\mathcal{C}}{g^5y^3}\cos^3\xi\sin\xi\cos^2\varphi\sin(2\psi)\right)\sin^2\varphi\, d\xi\wedge d\psi\wedge d\phi_1\wedge d\phi_2\wedge d\phi_4\wedge dy\wedge dz\\ &=-N\frac{6g\mathcal{C}}{y_1^2}\cos^2\varphi\sin^2\varphi\,, \end{aligned}\tag{7}$$

$$\begin{aligned}&\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_1\phi_3\phi_4yz}\\ &=\frac{1}{(2\pi l_p)^6}\int\left(-\frac{48\mathcal{C}}{g^5y^3}\cos^3\xi\sin\xi\cos^2\varphi\sin(2\psi)\right)\left(-\sin^2\varphi\right) d\xi\wedge d\psi\wedge d\phi_1\wedge d\phi_3\wedge d\phi_4\wedge dy\wedge dz\\ &=N\frac{6g\mathcal{C}}{y_1^2}\cos^2\varphi\sin^2\varphi\,. \end{aligned}\tag{8}$$

In order to have identical results from these components, we fix $\psi=\frac{\pi}{4}$. Plugging l_p from (5), we obtain

$$\begin{aligned}-\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_2\phi_3\phi_4yz} = -\frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_1\phi_2\phi_4yz} &= \frac{1}{(2\pi l_p)^6}\int *G_{(4)\xi\psi\phi_1\phi_3\phi_4yz}\\ &= N\frac{3g\mathcal{C}}{2y_1^2}\equiv K\,,\end{aligned}\tag{9}$$

where $K\in\mathbb{N}$ is another integer.

From (3.41) in the main body of the paper, [1], and y_1 from (9), we find

$$b=\frac{K^2\left(3g\left(N-2K\right)\right)}{6\sqrt{6}g\left(g\,\mathcal{C}NK\right)^{3/2}}\,, \tag{10}$$

and also find

$$y_1^2=\frac{3g\,\mathcal{C}N}{2K}\,. \tag{11}$$

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Then, by plugging $y_1(b)$, (11), in (3.42) in the main body of the paper, [1], with the expression of $\mathcal{E}(b)$ in (3.41) in the main body of the paper, [1], we also find another expression for b,

$$b = -\frac{\mathcal{C}l\left(3g^2\mathcal{C}^2N^2 - 8g\,\mathcal{C}KN + 4K^2\right)}{\sqrt{6}K^2}\left(\frac{K}{g\,\mathcal{C}N}\right)^{5/2}. \tag{12}$$

Finally, identifying (10) and (12) we can solve for $\mathcal{C}$ and then for b in terms of the quantum numbers, N and K,

$$\mathcal{C} = \frac{12Kl + N}{6gNl}\,, \qquad b = \frac{\sqrt{Kl}\,(8Kl + N)}{2g\,(12Kl + N)^{3/2}}\,. \tag{13}$$

Employing the results, we obtain the Bekenstein-Hawking entropy to be

$$S_{\text{BH}} = \frac{16\sqrt{2}b\mathcal{C}}{\pi^2 l_p^9 g^7}\int_{y_{\min}}^{y_{\max}} \frac{y}{(1-y^2)^2}dy = \frac{8\sqrt{2}b\,\mathcal{C}}{\pi^2 l_p^9 g^7}\frac{1}{y_1^2 - 1} = \frac{64\sqrt{2}\pi}{3}\sqrt{\frac{K^3Nl}{12Kl + N}}\,. \tag{14}$$

If we set $K \sim N$, the Bekenstein-Hawking entropy scales as $S_{\text{BH}} \sim N^{3/2}$ as the ABJM theory. This result should match (3.63) in [2].

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