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Testing Spacetime Orientability

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Abstract
Historically, a great deal of attention has been addressed to the question of what it would take to test experimentally the metrical structure of spacetime. Arguably, however, consideration of this question has been at the expense of comparable investigations into what it would take to test other structural features of spacetime. In this article, we critique and expand substantially upon an article by Hadley (Hadley in Class Quantum Gravity, 19:4565–4571, 2002), which constitutes one of the best-known paper-length studies of what it would take to test the *orientability* of spacetime. In so doing, we seek to clarify a number of matters which remain unclear in the wake of Hadley's article, thereby allowing the literature on this topic to progress. More positively, we also present, compare, and evaluate a number of other potential approaches to testing the orientability of spacetime which have arisen in the recent physics literature.

Keywords General relativity · Spacetime orientability · Experimental test · Epistemology ofspacetime

1 Introduction

A central question in the foundations of spacetime theories is this: to which spatiotemporal structures can we gain operational and empirical access—and how exactly is said access secured? Historically, a great deal of attention has focused on the question of our empirical access to the (Lorentzian) metrical structure of our (assumed-to-be) general relativistic world. Synge, for example, endorsed a 'chronometric' approach to addressing this question [51, 52]; in explicit contrast, Ehlers, Pirani and Schild [16] proposed that the trajectories of light rays afford empirical

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access to conformal structure, and the trajectories of freely falling massive particles afford access to projective structure; together (alongside some supposedly innocuous auxiliary assumptions), these fix (up to a constant factor) the metrical structure of spacetime.

These issues are, by now, relatively (albeit not completely—see [1, 36]) well-understood. However, one might find the focus on metrical structure in the foregoing to be unduly blinkered: for surely one can likewise raise questions regarding our empirical access to other aspects of the spatiotemporal structure of the world. For example, one might wonder whether it is possible to access empirically (and thereby test) the *orientability* of spacetime. It is upon this question which we focus in this article.

Perhaps surprisingly, the issue of testing the orientability of spacetime has not received a great deal of attention in the literature. A potted history divides into three phases. First, the question of testing spacetime orientability was raised in a number of influential works on global general relativity from the late 1960s and into the 1970s—in particular [19–22, 25] (we refer to these works more specifically below). After this, little (to our knowledge) was published on these issues until 2002, with the advent of an article by Hadley [26]. In our view, Hadley is to be commended for revitalising such questions; however, critical engagement with his article is sorely required, because many of the details of Hadley's remarks are, on reflection, severely problematic. The third phase of the literature consists in a number recent and novel proposals for testing the orientability of spacetime—see *inter alia* [4, 33, 34] (we discuss such works more systematically in the body of this article).

With the foregoing in mind, we envisage the purpose of the present article to be twofold. First: drawing on the literature from the first of the above stages, we seek to overcome a number of confusions and unclarities which, in our view, arise out of Hadley's work in the second of the above stages, and which without careful attention have the potential to stymie the literature. Second, we seek to both survey and evaluate the various more recent proposals for testing the orientability of spacetime which have been adumbrated in the third stage of the history on this topic. Through achieving both of these goals, we seek to be in a position to make fairly definitive statements regarding the prospects for testing spacetime orientability, at least given the current state of play in physics.

The structure of the article is this. In Sect. 2, we consider different possible definitions of the orientability of spacetime, drawing in particular important distinctions between 'manifold orientability', 'time orientability', and 'space orientability' (throughout this article, we use 'temporal orientability' and 'spatial orientability' interchangeably with 'time orientability' and 'space orientability', respectively; when we say simply 'orientability' we refer collectively to all three notions). Clarifying that these all constitute distinct notions of orientability is important, for Hadley slides in his article between the notions in a way which is liable to confuse. In Sect. 3, we consider the question of whether the orientability of spacetime is to be considered a global or a local property—again, we find Hadley's claims here to be problematic. In Sect. 4, we turn to the main event: how can one test the orientability of spacetime? Here, we build upon the experimental setups proposed by Hadley in order to further the discussion. In Sect. 5, we consider whether results from quantum field theory (QFT) can provide (indirect) evidence for the orientability of spacetime—here, we share Hadley's scepticism, albeit for different reasons than those which he adduces. In Sect. 6, we assess whether Hadley's concluding remarks on testing the orientability of spacetime are acceptable. In Sect. 7, we turn to the prospects for the more recent proposals for testing spacetime orientability which have arisen in the physics literature; generally, we find these to be more promising than the proposals made by Hadley. We close in Sect. 8 by addressing directly the question of whether it is indeed possible to test spacetime orientability.

2 Definitions of Orientability

We begin with three well-known but inequivalent definitions of orientability. The first applies to any differentiable manifold:

Definition 1 (*Manifold orientability*) A differentiable manifold M is *manifold orientable* if and only if it admits a smooth non-vanishing top-ranked form.

For example, 4D Lorentzian spacetime (M, g_{ab}) is orientable if and only if it admits a smooth non-vanishing 4-form.[1] Often in the literature, manifold orientability is called simply 'orientability'; however, in this article we will use the longer nomenclature in order to avoid confusion with the two further notions of orientability introduced below. It is also important to stress that manifold orientability is a topological property, which does not depend upon any additional specific structures defined on the manifold. Of course, one might be able to redefine orientability in terms of these additional structures, but ultimately any such definition would have to be equivalent to the original topological definition of manifold orientability.[2,3]

The next two definitions of orientability concern not merely topological matters, but further geometrical fields defined on a Lorentzian spacetime in particular:

[1] Sometimes, such top-ranked forms are referred to as 'volume forms'; however, it is important to stress that such forms need have little to do with physical volumes, as read out by material fields. Certain such forms are more natural than others in this respect: for example, when working with (pseudo-)Riemannian manifolds, it is quite common to use the volume form with the local coordinate expression $\sqrt{|g|}dx^1 \wedge \cdots \wedge dx^n$, which is the form 'adapted' to volumes given by the (pseudo-)Riemannian metric field. Even in this case, however, the connection to volumes read out by material fields is not guaranteed: this is part of the moral of the 'dynamical approach' to spacetime of Brown and Pooley [7–9], which we discuss further below. For the purpose of this article and the above definition, it does not matter which top-ranked form one chooses—what matters is that the manifold in question *admits* of such a form.

[2] We thank an anonymous referee for pressing us to be clear on this point.

[3] Volume forms are sometimes presented as being necessary for integration on manifolds—see e.g. [53, Appendix B]. Given that non-orientable manifolds by definition do not admit of top-ranked forms ('volume forms'—see footnote 1), one might conclude that integration is not possible on non-orientable manifolds. This, however, is not the case, for so-called 'twisted forms' *are* definable on such manifolds, and one can use these objects to define integrals on such manifolds. For further background on twisted forms, see [11]. We are grateful to an anonymous referee for inviting us to discuss twisted forms.

Definition 2 (*Time orientability*) A Lorentzian spacetime (M, g_{ab}) is *time orientable* if and only if it admits a continuous non-vanishing timelike vector field on M.

Definition 3 (*Space orientability*) A Lorentzian spacetime (M, g_{ab}) of dimension n is *space orientable* if and only if it admits a continuous non-vanishing field of orthonormal $(n-1)$-ads of spacelike vectors on M.

There are three points to note here. First: for all three of these definitions, there exist in the literature equivalent definitions—see [5] for presentations of such definitions and proofs of their equivalence. (We will, indeed, make use of one such alternative but equivalent set of definitions below.) Second: there is some ambiguity in the literature between the above three definitions written using the locution 'it admits', and the definitions written in terms of 'there exists' (for the latter, see e.g. [38, p. 131]). On one reading of the latter locution (which we take to be the intended reading), the two are synonymous. On another reading, the second states that a Lorentzian manifold is time orientable (say) if and only if there actually exists a certain timelike vector field on that manifold (in other words, if and only if there actually exists a certain orientation on the manifold, where the three salient notions of orientation are defined below).[4] This second reading cannot be what is intended in these discussions, for it would adjudicate (for example) that vacuum Minkowski spacetime (M, η_{ab}) is not temporally orientable (for which, on this reading, one would need there to exist an additional vector field ξ^a on M such that $\eta_{ab}\xi^a\xi^b < 0$). Third: one could, in principle, consider versions of the latter two of the above three definitions which are 'topological', in the sense that they say (for example) that a differentiable manifold is time orientable just in case it *admits* a time orientable Lorentzian metric (where, recall, time orientability in turn is to be cashed out in terms of whether the Lorentzian manifold *admits* a continuous non-vanishing timelike vector field). This approach is ultimately not relevant for the points which we wish to make in this article, so we set it aside in what follows.

Distinguishing between these different kinds of orientability is essential not only in order to ensure conceptual clarity, but also because the three notions stand in non-trivial relations to one another. Before discussing such relations further, we recall one definition and one result, to which we will appeal more than once in the remainder of this article:[5]

Definition 4 (*Parallelisable manifold*) A differentiable manifold M is *parallelisable* if and only if it admits a set of smooth vector fields $\{V_1, \ldots, V_n\}$ such that, at every point $p \in M$, the tangent vectors $\{V_1(p), \ldots, V_n(p)\}$ form a basis of the tangent space T_pM.

[4] Here we are tracking a distinction between (a) a mathematician's notion of existence (in terms of definability), and (b) a more 'physically relevant' notion of existence in terms of specific structures which are posited in one's mathematics (e.g., a specific orientation) and which are taken to have representational significance.

[5] Whether a manifolds admits of n independent vector fields is not always obvious—for example, it is pointed out at [22, p. 224] that $S^2 \times \mathbb{R}^2$ is such a manifold.

Claim 1 *Every parallelisable manifold is manifold orientable.*

Proof Let M be a parallelisable n-dimensional differentiable manifold. This means that there exist smooth, non-vanishing vector fields $\{V_1, ..., V_n\}$ on M which for every point $p \in M$ form a basis of the tangent space T_pM at p. Then, at every point $p \in M$, there exist differential forms $\{\omega_1, ..., \omega_n\}$ such that they give the basis of the dual space T_p^*M. The wedge product of these 1-forms $\tau = \omega_1 \wedge ... \wedge \omega_n$ is a top-ranked form. Moreover, since $\{\omega_1, ..., \omega_n\}$ form a basis for T_p^*M, $\tau(V_1, ..., V_n) = 1$, so τ is non-vanishing. Since the manifold M thereby admits a non-vanishing top-ranked form τ, it is manifold orientable. □

Then, we have the following result, relating our three given notions of orientability:[6]

Claim 2 *If a Lorentzian spacetime (M, g_{ab}) is time orientable and space orientable, then it is manifold orientable.*

Proof Consider a Lorentzian spacetime (M, g_{ab}) of dimension n which is both time orientable and space orientable. This means that there can be defined on the manifold (i) a field of timelike vectors, and (ii) a field of $(n-1)$-ads of orthonormal spacelike vectors. Together, these give rise to a non-vanishing continuous field of tetrads. Since the Lorentzian spacetime is n-dimensional, and the tetrads are built up from n vector fields, tangent vectors belonging to this field of tetrads form a basis of a tangent space at each point of this manifold. Thus, the manifold is parallelisable, and so is orientable. □

This result allows one to note cases in which time orientability comes apart from space orientability—for there are cases in which a Lorentzian manifold is (say) time orientable but not manifold orientable, and so cannot (by the above result) be space orientable. For explicit presentations of such spacetimes (not further relevant for our purposes in this article), see [40, 45].

(In the remainder of this article, for simplicity we specialise to the case of 4-dimensional Lorentzian manifolds—so, in the case of space orientability in particular, we speak now of 'triads', rather than '$(n-1)$-ads'.) Each of the above three definitions of orientability above brings with it an associated notion of an orientation:[7]

[6] For further discussions on the relations between these notions of orientability, see [22, pp. 227–228].

[7] The following are canonical definitions of manifold/time/space orientations which one finds in the literature (see e.g. [40]), but it's worth registering that there is something lacking in them. Take a manifold/time/space orientation of one's choice, according to these definitions. Then apply (say) some constant scale transformation—is it really correct to say that these are *distinct* orientations? So it would be better to speak in terms of *equivalence classes* of orientations according to the below definitions, where

Definition 5 (*Manifold orientation*) A *manifold orientation* $\mathcal{T}_m$ of a differentiable manifold M is the choice of a smooth non-vanishing top-ranked form on M.

Definition 6 (*Time orientation*) A *time orientation* $\mathcal{T}_t$ of a Lorentzian spacetime (M, g_{ab}) is the choice of a continuous non-vanishing timelike vector field on M. A tuple $(M, g_{ab}, \mathcal{T}_t)$ is a *time oriented spacetime*.

Definition 7 (*Space orientation*) A *space orientation* $\mathcal{T}_s$ of a Lorentzian spacetime (M, g_{ab}) is the choice of a continuous non-vanishing field of othonormal spacelike triads on M. A tuple $(M, g_{ab}, \mathcal{T}_s)$ is a *space oriented spacetime*.

Having presented these definitions, let us return to Hadley. One potential source of confusion which a reader might encounter on reading Hadley's article is that although he begins by asking the following question: "how can time-orientability be tested?" [26, p. 4565], he subsequently draws analogies with Möbius bands, which are usually considered relevant to *manifold* (non-)orientability. (It is possible to define a time-oriented metric on a Möbius band [41, p. 5].) Moreover, none of these notions of orientability are defined precisely in Hadley's article. What is even more puzzling is that in the title of his paper, Hadley advertises his project as relevant to some *general* notion of the orientability of spacetime; however, the conclusion of his article consists in four statements concerning only *time* orientability. It is, therefore, in the interests of conceptual clarity that we have offered the above definitions, to which we will refer back in the remaining sections of this article.

Although the definitions of both time and space orientability presented above are those most commonly found in the literature, Hadley does not engage explicitly with them in his paper. Instead (and as we discuss in much greater detail in the following section), his narration regarding testing orientability is based on the idea of moving some salient objects *qua* probes, such as clocks or hands (depending upon whether one is considering time or space orientability, respectively) along particular closed spacetime trajectories. Since we engage with this idea further below, at this point we need to demonstrate how it is related to the default definitions of both time and space orientability given above. It is to this task which we now turn.

We claim that time and space orientability can be defined also in the following ways. (Here, we follow the lead of Geroch and Horowitz [22, pp. 225–226], although we seek to define the following notions of time and space orientability somewhat more rigorously than in their article.) The equivalence of these two definitions to the original two definitions is proved explicitly below. Before presenting these alternative definitions, we present a general notion of the transport of a rank (r, s) tensor T (here, indices omitted for clarity) along a curve $\gamma \subset M$: (We assume that the

Footnote 7 (continued)
elements of these classes are related by irrelevant transformations, such as scale (more generally, any transformation for which the determinant of the associated transformation matrix is positive will be irrelevant). (One author to speak in terms of equivalence classes is Malament: see [38, p. 132].) We thank an anonymous referee for helpful discussions here.

derivative operator at play in what follows is the Levi-Civita operator ∇ such that $\nabla_a g_{bc} = 0$. Hence, transport is defined with respect to the Levi-Civita connection.)

Definition 8 (*Transport along a curve*) Consider some rank (r, s) tensor T, a curve γ, and let X^a be a vector field tangent to γ. Then, T is transported along γ if and only if $\nabla_X T = \Xi$ for some rank (r, s) tensor Ξ.

Note that the choice of Ξ dictates exactly how the tensor T is transported around the curve. Clearly, without further restrictions on Ξ, this notion of transport is very general—indeed, it is intended to be so. Note also that this definition reduces to the definition of parallel transport in the special case $\Xi = 0$. For our purposes, following [22, p. 225], we insist that the transport of the relevant objects be continuous. With all this in hand, we can now give the following alternative definitions of time and space orientability (cf. [22, pp. 225–226]):

Definition 9 (*Time orientability, loops*) A Lorentzian spacetime (M, g_{ab}) is *time orientable* if and only for any closed curve γ through any point $p \in M$, there is some timelike vector V^a at p such that there is some way of continuously transporting V^a around γ so that the original vector falls into the same lobe of the light cone as the transported vector.

Let us turn now to the alternative definition of spatial orientability. In three spatial dimensions, there exist exactly two classes of triads of orthonormal spacelike vectors such that they cannot be superimposed by any rigid motion [5, p. 23]; call these classes 'left-handed' and 'right-handed'; the property of falling into one of these classes we will call 'handedness'. Given this, we can then give the following alternative definition of spatial orientability:

Definition 10 (*Space orientability, loops*) A Lorentzian spacetime (M, g_{ab}) is *space orientable* if and only if for any curve γ through any point $p \in M$, there is some orthonormal triad of spacelike vectors at p such that there is some way of continuously transporting that triad of vectors around γ so that the triad does not change its handedness.

We claim that these are admissible definitions of both time and space orientability, in the sense that they are equivalent to those introduced earlier (for the special case of $n = 4$, although the equivalence for n can be proved *mutatis mutandis*):

Claim 3 *A Lorentzian manifold is time orientable in the original sense* (*Definition* 2) *if and only if it is time orientable in the loops sense* (*Definition* 9).

Proof Let (M, g_{ab}) be a Lorentzian spacetime.

($\Rightarrow$) Take points $p, q \in M$. Since the Lorentzian manifold admits a continuous non-vanishing timelike vector field, for a curve from p to q, there exists a timelike vector $V^a \in T_p M$ such that continuously transporting V^a along this curve from p to

q does not change the lobe of the light cone into which it falls. Taking now some other curve between p and q, the same point applies. From both curves one can then obtain a closed curve, such that the continuous transport of some timelike vector along this closed curve does not change the lobe of the light cone into which the timelike vector falls.

(⇐) Suppose that for any closed curve γ through any point $p \in M$, there is some timelike vector V^a at p such that there is some way of continuously transporting V^a around γ so that the original vector falls into the same lobe of the light cone as the transported vector. Then, for every such γ, there is some continuous non-vanishing timelike vector field on γ. But the union of all loops γ encompasses every point on M, so M admits of a continuous non-vanishing timelike vector field, as can be seen by taking the union of those vector fields defined on each γ (or those vector fields multiplied by a minus sign, as appropriate).

□

Claim 4 *A Lorentzian manifold is space orientable in the original sense* (*Definition* 3) *if and only if it is space orientable in the loops sense* (*Definition* 10).

Proof Let (M, g_{ab}) be a Lorentzian spacetime.

(⇒) Take points $p, q \in M$. Since the Lorentzian manifold admits a continuous field of orthonormal spacelike vectors, for a curve from p to q, there exists a triad of spacelike vectors $Y_i^a \in T_pM$ $(i = 1, 2, 3)$ such that continuously transporting these Y_i^a along this curve from p to q does not change their handedness. Taking now some other curve between p and q, the same point applies. From both curves one can then obtain a closed curve, such that the continuous transport this triad of spacelike vectors along this closed curve does not change its handedness.

(⇐) Suppose that for any curve γ through any point $p \in M$, there is some orthonormal triad of spacelike vectors at p such that there is some way of continuously transporting that triad of vectors around γ so that the triad does not change its handedness. Then, for every such γ, there is some continuous field of triads of orthonormal spacelike vectors on γ. But the union of all loops γ encompasses every point on M, so M admits of a continuous non-vanishing field of triads of orthonormal spacelike vectors, as can be seen by taking the union of those triads defined on each γ (or those triads with one leg multiplied by a minus sign, as appropriate, in order to render them all of the same handedness). □

To summarise the results of this section, we have: (a) distinguished between manifold, time, and space orientability, (b) clarified the interrelations between these notions, (c) introduced associated notions of orientations, (d) introduced alternative definitions of time and space orientability in terms of the transport of vectors around closed loops. With all of this machinery in hand, we turn now to the question of whether the orientability of a manifold (in each of the above three senses) should be considered a 'local' or a 'global' property of that manifold.

3 Local and Global Properties

One significant preliminary question with which Hadley engages is this: is the orientability of spacetime a local or a global property? In order to answer this question, we follow Manchak [39, p. 11]. First, recall the following definition of local isometry:

Definition 11 (*Local isometry*) Lorentzian spacetimes $\left(M, g_{ab}\right)$ and $\left(M', g'_{ab}\right)$ are *locally isometric* if and only if for each point $p \in M$ there is an open set $O \subset M$ containing p and an open set $O' \subset M'$ such that $\left(O, g_{ab}\right)$ and $\left(O', g'_{ab}\right)$ are isometric, and vice versa with the roles of $\left(M, g_{ab}\right)$ and $\left(M', g'_{ab}\right)$ exchanged.

(This definition has the disadvantage that it is specific to Lorentzian metric theories. It could, however, be adapted straightforwardly to e.g. Newton-Cartan theory—on which see [38, ch. 4].)

With this definition in hand, one can then, again following Manchak, define local and global spacetime properties as follows:

Definition 12 (*Local and global properties*) A spacetime property is *local* if and only if, given any pair of locally isometric spacetimes, one spacetime has the property if and only if the other one does as well. A spacetime property is *global* just in case it is not local.

Using these definitions, let us consider whether the three notions of orientability considered in the previous section qualify as local or as global. (As an aside, note that establishing exactly when locally isometric spacetimes are—or can be—globally inequivalent is a complicated issue, part of what is known as the 'Equivalence Problem' in differential geometry. For further background on this issue, see [2, 30, 31, 37].[8])

One can demonstrate easily that all three notions of orientability are global properties of spacetime [39, p. 62]. Consider a 2-dimensional Minkowski spacetime (M, η_{ab}) in coordinates (t, x). It is obviously time orientable. Consider now a spacetime obtained from this one by removing all points such that $|x| > 1$ and identifying the edge $(t, 1)$ with $(-t, -1)$. Although both spacetimes are locally isometric, the second fails to be time orientable (what we have construed, in essence, is a Möbius band in the temporal direction). Therefore, time orientability is a global spacetime property. A similar example can be constructed for space orientability. Indeed, manifold orientability is also a global property of spacetime. To see this, consider again the same example: although both spacetimes are isometric, the second also fails to be manifold orientable. Therefore, manifold orientability is a global spacetime property.

[8] We are grateful to an anonymous reviewer for helping us to navigate the literature on this topic.

We have thus demonstrated that all senses of orientability discussed above are global properties of spacetime (in the sense used in [39]). Indeed, Hadley himself seems to admit this, when he writes: "orientability is a global rather than a local property" [26, p. 4566]. Given this, however, it is all the more surprising that Hadley uses the term 'orientability' not only with reference to spacetime in its entirety, but also with respect to spacetime *regions*. What is more, as we discuss in the following section, the possibility of speaking about orientability confined to some spacetime region is indeed the core idea underlying his proposed experiments for testing these properties. For this reason, we should dwell on what can be understood by a spacetime region and the localisability of non-orientability to such a region.

The notion of a 'spacetime region' should be understood, roughly speaking, as some subset of points that are in the vicinity of each other, e.g. in the sense of London being in the region of England. In particular, in the way in which Hadley is using this phrase in his article, any such set of points can be considered to be a spacetime region, and surveyed for its orientability. However, since all three varieties of orientability are properties of a *manifold* (a Lorentizan manifold in the case of time and space orientability), the only way in which we can understand the notion of localising non-orientability to such regions is to construe them as sub-*manifolds*, and to consider the orientability (in each of the above three senses) of these sub-manifolds on its own terms. In other words, technically speaking, one would not be considering localising the non-orientability *of* M to some region of M (as already discussed, since orientaibility of M is a global property, this makes little sense), but rather considering whether these regions—again, understood as manifolds unto themselves—are orientable.

This being said, the following result does, perhaps, afford a more precise means of speaking of the localisability of the non-orientaibility of a manifold:

Claim 5 *Let M be an n-dimensional manifold, and let $S \subset M$ be an n-dimensional submanifold of M. If S is not manifold orientable, then M is not manifold orientable.*

Proof Suppose for contradiction that M is manifold orientable but that S is not manifold orientable. Since M is manifold orientable, it admits a non-vanishing n-form ω. As a result, $\omega|_S$ is a non-vanishing n-form on S. Thus, S is manifold orientable. Contradiction, so M is not manifold orientable. □

The idea would be this (throughout this paragraph, by 'orientability' we mean specifically manifold orientability, for concreteness and simplicity). If a given non-orientable manifold M contains a non-orientable proper sub-manifold $S \subset M$, then (by stipulation) the non-orientability of M can be localised (at least) to S. In our view, this is a legitimate and precise way of speaking; it is, indeed, our best attempt to make sense of statements by Hadley such as "Mathematically, orientability is a global property of spacetime. ... However, a non-orientable region could be microscopic in size" [26, p. 4566]. However, it is not clear that these observations are of any help when it comes to testing the orientability of M by way of probing sub-regions of M, as Hadley discusses in his proposed experimental setups in the following section of his article, and as we evaluate below. One reason for this is that some non-orientable manifolds (in the sense of manifold orientability) only contain non-orientable submanifolds which are in no interesting sense more 'local'—for example, the only non-orientable proper sub-manifolds of the Möbius band (setting aside removing finite sets of points) are 'thinner' Möbius bands; in this case, the non-orientability of the original manifold is not localisable in a way which would allow for the performance of only local experiments to adjudicate on this property. Another reason is that, even if a non-orientable manifold M contains a non-orientable proper sub-manifold S, it cannot be precluded *ab initio* that testing the (non-)orientability of M requires carrying out experimental tests outside of S. Thus, again, more needs to be said to bridge the gap between speaking of sub-manifolds in this way, and questions of testing the orientability of spacetime—what is the relevance of the former to the latter supposed to be?

4 Testing Orientability

Before addressing the question of how to test temporal orientability (which Hadley declares to be his main project in the relevant section of his article), Hadley first considers the same question with respect to testing spatial orientability. We follow suit, by first considering spatial orientability in Sect. 4.1, before turning to temporal orientability in Sect. 4.2. We also present some reflections on testing manifold orientability in Sect. 4.3.

4.1 Testing Spatial Orientability

Hadley first considers how one might test experimentally the spatial orientability of a Lorentzian manifold. (NB: as already flagged, Hadley does not define either of these notions—but we will take it that he intends the canonical definitions given above.) The setup is this. Consider some Lorentzian spacetime (M, g_{ab}), and some persisting spatial region $\mathcal{R}$ which "is not [space] orientable" [26, fig. 1]. (NB: there is a typo in the caption of this figure in Hadley's article.) By this, Hadley means that a handed object such as (naturally) a hand (which, following Hadley, we will idealise as an orthonormal triad of spacelike vectors) sent into this region will return with reversed handedness. There are several points to make about whether such an experiment really does afford the possibility of testing spatial orientability.

The first is this. Hadley speaks of $\mathcal{R}$ as being the region in which spatial non-orientability is 'localised' (see [26, Fig. 1]); however, we have already seen in the previous section that there are reasons to question the coherence of localising a global property of spacetime. If one considers both definitions of time (and space) orientability, it is evident that they do not refer to regions. In particular, in the case of Definitions 2 and 3, one considers vector fields which are defined on the entire manifold, and in the case of Definitions 9 and 10 orientability is defined with reference to closed curves, i.e. loops. Therefore, in the charitable reading of Hadley's experiments, one can talk about the orientability with respect to *loops*, but there is no straightforward sense in which one can speak of the orientability of *regions* (except, as already discussed, when those regions are considered as sub-manifolds). In some region one can have loops that are both orientation-preserving and orientation-reversing, e.g. on the Möbius band. Moreover, even if one finds a loop which reverses (say) time orientation, and takes a region which is given by this loop, it does not mean that the non-time-orientability is confined to this region. Therefore finding a loop which partially passes through region $\mathcal{R}$ and does not preserve orientation does not constitute evidence for the non-orientability of $\mathcal{R}$, which is a central premise of Hadley's experiments.

Setting this aside, the second (more philosophical/foundational) point to be made is the following. One might wonder how the connection between the behaviour of one particular physical system (e.g. a hand, idealised—as mentioned above—as a triad of orthonormal spacelike vectors) and a property of a Lorentzian spacetime is to be substantiated. In particular, there are two questions:

1. How is one to know that *other* physical systems behave in a similar manner?
2. Even granting this, what is the connection between the behaviour of such objects and properties of spacetime?

On the first question: one has to make a universal extrapolation to the effect that all other handed objects behave in a similar manner having traversed $\mathcal{R}$. Depending upon the number of probes sent into this region, and the results returned by these probes (do the change handedness on return, or do they not?), one will secure a greater or lesser degree of confidence that handedness change on traversing $\mathcal{R}$ is indeed a universal property of material fields. Without dismissing it is unproblematic, let us simply grant the first universal extrapolation in what follows. Turning to the second question: even if all probes do behave as described above on traversing $\mathcal{R}$, what is the connection between these probes and the structure of spacetime? If one thinks of the structure of spacetime as completely autonomous of the behavior of material fields, then one might argue: very little. However, it is not obvious that this view—dubbed in [48] an 'unqualified geometrical approach'—is compelling, for it would seem to render miraculous the coincidence of (say) spacetime symmetries and symmetries of the laws governing material fields (cf. [49]).

Two more compelling views here are the following: first, what was dubbed in [48] a 'qualified geometrical view', according to which material bodies behave as they do in virtue of coupling in their dynamical equations to pieces of autonomous spatiotemporal structure; second, a 'dynamical view', most famously associated with the writings of Brown and Pooley [7–9] (see [10, 29] for recent reviews; the position is also anticipated in the physics literature in articles such as [18][9]), according to which spatiotemporal structure *just is* a codification of the behaviour of material bodies. For the purposes of this article we need not get further into the weeds of

[9] We are grateful to an anonymous referee for pointing us to this latter reference.

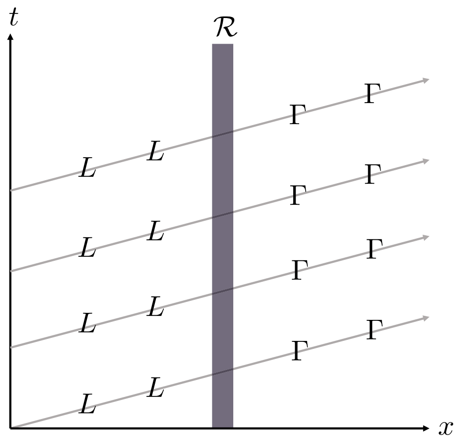

Fig. 1 A 'God's eye view' of a situation in which handed objects (here, Ls, standing in for left hands) are emitted from source, traverse some region $\mathcal{R}$, and leave that region as objects of the opposite handedness (here, Γs, standing in for right hands). In this case, one might say that the behaviour of these objects is a result of their coupling to a spatiotemporal structure which is not spatially orientable (*per* the qualified geometrical view), or that such spatiotemporal structure is a codification of the behaviour of these material objects (*per* the dynamical approach)

philosophical discussions of the nature of spacetime; suffice it to say that, on either of these approaches, one may well have a means of answering the latter of the two questions posed above.

To illustrate this point, set aside for one moment the question of *testing* the orientability of spacetime, and assume a 'God's eye view', from which one sees that all handed objects traversing region $\mathcal{R}$ reverse their handedness (See Fig. 1). In this scenario, one can either account for this handedness switch of these material bodies, as per the above 'qualified geometrical approach', by appeal to their coupling to a background spatiotemporal structure; or, alternatively and as per the above 'dynamical approach', simply state that a non-space orientable spacetime *codifies* this behaviour of material bodies. In either case, one can infer properties of spacetime from such behaviour.

The situation, however, is complicated when one does not assume such a 'God's eye view' perspective. When one releases a probe of a certain handedness and later witnesses it return with the opposite handedness, there are numerous questions to be addressed. First: how is one to infer, on the basis of that probe alone, that the handedness change of the object was due to its traversing the (spatially distant) region $\mathcal{R}$? More importantly, even setting this aside, it is not clear how the behaviour of a single probe warrants an inference to whether the spacetime manifold in its entirety admits a continuous non-vanishing triad of spacelike vectors. Indeed, suppose that at every instant one emitted probes towards $\mathcal{R}$, which were subsequently returned with opposite handedness (see Fig. 2). In that case, insofar as the entire region to the left of $\mathcal{R}$ is populated with probes of the same handedness, one might think that one can infer that spacetime—or at least the region to the left of $\mathcal{R}$ (construed as a

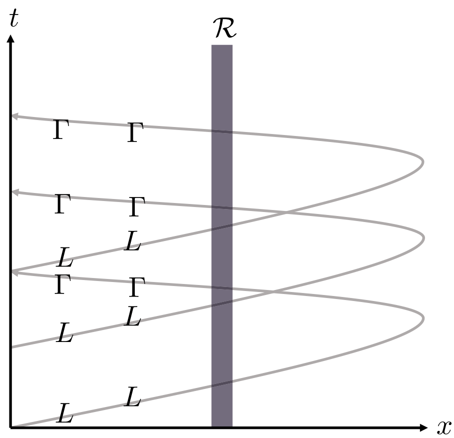

Fig. 2 A situation in which an observer at $x = 0$ emits handed objects (here, Ls, standing in for left hands), which traverse region $\mathcal{R}$ and which subsequently return to the observer as objects with opposite handedness (here, as Γs, standing in for right hands). In this case, the region between $x = 0$ and $\mathcal{R}$ is known to be populated with Ls, in which case an inference on the basis of the observer's neighbourhood to the spatial orientability of the manifold appears justifiable

sub-manifold unto itself)—*is* orientable. Indeed, note that the haecceitistic identity of these probes is *irrelevant* for drawing this conclusion.

4.2 Testing Temporal Orientability

Turn now to Hadley's discussions regarding testing the temporal orientability of spacetime. Roughly, the setup is the same as above: one supposes that there is some region $\mathcal{R}$ in which the temporal non-orientability can be 'localised' (just as above, it is questionable that this makes sense, given that temporal orientability is again a global property of spacetime—but we will likewise set this concern aside in the following). Then, one idealises a clock as a timelike vector (the length of which corresponds to the rate of ticking of the clock: an idea going back at least to Weyl—see [3] and references therein). Setting aside the above philosophical concerns regarding, *inter alia*, the universality of the recorded effects in the experiments countenanced by Hadley—concerns which apply *mutatis mutandis* in the case of testing temporal (rather than spatial) orientability—Hadley raises additional considerations in the case of testing temporal orientability, upon which we will now focus.

Hadley first considers an approach to testing temporal orientability according to which a future-directed timelike vector (idealising a clock ticking with positive intervals) is sent to region $\mathcal{R}$; if this probe returns but is now past-directed (idealising a clock ticking with negative intervals—i.e., a clock ticking backwards), then one might think that one has (modulo the concerns raised in the previous section) tested the temporal orientability of spacetime (and found the spacetime to not be temporally orientable).

Hadley, however, maintains that such an experiment does not successfully test the temporal orientability of spacetime, writing: "This is not a demonstration of non-time-orientability, because in this experiment, the clock increases in value and then decreases. At some point in the path it attains a maximum reading and at that point it does not define a time direction" [26, p. 4568]. It is not entirely clear what Hadley means by this, but plausibly the idea is that, if this probe is emitted as a future-directed timelike vector, and returns as a past-directed timelike vector, then (assuming continuity) at some point it must have been of zero length, thereby 'not defining a temporal direction'. We have three responses to this line of thought. First: it is not clear why this failure precludes one from concluding that the spacetime under consideration does not admit a future-directed timelike vector field—which is the issue of relevance to temporal orientability. Second: surely an analogous argument would apply in the case of spatial orientability: if a triad of vectors is emitted with one handedness, and returns with another, then (again, assuming continuity), for some leg of the triad there must have been some point at which it vanished, thereby failing to define a spatial orientation at that point. Thus, it is not clear why Hadley considers the case of testing temporal orientability to be different from the case of testing spatial orientability in this regard. Third: Hadley has said nothing to justify this (implicit) assumption of continuity: could the temporal orientation of the probe not switch discontinuously as it traverses its path through $\mathcal{R}$ and back to the origin?

Instead of the above, Hadley maintains that a true test of temporal orientability would take the following form. Once again, send a probe (i.e. a clock, idealised as a timelike vector) into region $\mathcal{R}$; in this case, the clock would return by literally moving backwards in time; in other words, the situation could be construed as a clock-anticlock annihilation event (see [26, Fig. 3]). Hadley claims that this cannot, however, be understood as affording direct evidence of the temporal non-orientability of spacetime, for this reinterpretation in terms of a clock-anticlock annihilation event is invariably available; moreover, such tests thereby require access to anticlocks—i.e., clocks comprised of antiparticles—which are not (it is fair to say!) easy to come by in the actual world [26, p. 4569].

Hadley's reasoning here strikes us as a red herring, for it is not at all clear why one requires the second of his two setups in order to infer the existence (or lack thereof) of a continuous timelike vector field on the manifold. For this purpose (notwithstanding the various general concerns we have raised in the foregoing), in our view, Hadley's first proposed experiment is perfectly sufficient.

We will close by making two side remarks on Hadley's discussions of temporal orientability. First: Hadley speaks of, for example, cases in which "the observer sees the time values increasing on the clock" [26, p. 4568], and moreover claims that "the existence or otherwise of a time reversing region is dependent upon the observer" [26, p. 4570]—but it is not obvious that speaking of observer-dependent effects does anything but muddy the waters from the point of view of assessing whether certain experimental setups provide evidence for temporal orientability, given that this is, of course, a frame- and observer-independent notion. Second (and, of course, less importantly): the second of the two setups considered by Hadley—in which objects traverse region $\mathcal{R}$ and 'thereafter' proceed to move backwards in time—is (for what it is worth!) a central plot point of the recent Hollywood movie *Tenet* [43]: while, of course, stimulating in itself, to repeat: the connections between such setups and temporal orientability in the technical sense given above remain unclear on the basis of Hadley's article.

4.3 Testing Manifold Orientability

Hadley's proposed experiments for testing temporal or spatial orientability rest on the idea of transporting vectors around closed loops. There is no reason why such proposals cannot be extended to tetrads; thus, there is no reason why such experiments could not also (at least in principle) be used to test the *manifold* orientability of spacetime. (Relatedly, one can also test this property via Claim 2, having tested antecedently both temporal and spatial orientability.) Of course, the concerns raised above regarding (a) the significance and meaningfulness of relativising orientability to regions such as $\mathcal{R}$, and (b) the general connections between material bodies and the structure of spacetime, will still apply in this case. Moreover: insofar as one of the legs of the tetrads under consideration in this case will correspond (at least on a Lorentzian manifold) to a temporal direction, Hadley might argue that ascertaining whether there is orientability in this direction requires recourse to his second type of experiment discussed in the case of temporal orientability. For the reasons already articulated, we are sceptical that any such recourse is required.

5 Considerations from QFT

All of the foregoing regards the possibility of what one might call a *direct* test of the orientability of spacetime: particular experimental setups which yield affirmative/negative answers on that matter. In Sect. 4 of his article, Hadley turns to what one might call *indirect* tests of spacetime orientability: particular theoretical constructions, motivated and developed on the basis of (aggregates of) other empirical data, which nevertheless yield an affirmative/negative answer to the question of whether spacetime is or is not orientable.

In particular, Hadley focuses on our modelling of the behaviour of spinorial matter in curved spacetimes (in these passages, Hadley is drawing upon earlier work by Geroch [19–21]). Standardly, it is taken to be the case that, in order to describe the behaviour of (say) spin-$\frac{1}{2}$ particles in curved spacetimes, one must generalise the flat-spacetime Dirac equation to those settings using tetrads e^a_μ (and their inverses), writing (in the massless case)

$$i\gamma^a e^\mu_a \nabla_\mu \psi = 0. \tag{1}$$

Note that tetrads can be defined only on parallelisable manifolds. But now recall from above that every parallelisable manifold is orientable. Thus, as Hadley points out (albeit without explicit reference to parallelisability), one might cite the foregoing as indirect evidence that spacetime is manifold orientable: the existence of fermions implies parallelisability, which in turn implies manifold orientability. However, there are several points to make here.

First: Hadley seems to have switched here from discussions of temporal orientability to discussions of manifold orientability. Since we have already seen that these notions are independent, it is not clear that the above observations bear any relevance to the matter of testing the temporal orientability of spacetime (except via connections such as that articulated in Claim 2), as he claims.

Second: In 1965, Ogievetsky and Polubarinov developed an alternative approach to modelling fermions in curved spacetimes which does not require tetrads (we call this alternative approach the 'OP spinor formalism'). Instead—and here we follow Pitts' discussion of the Ogievetsky-Polubarinov approach [46] rather than the original work [44], the latter of which uses a square root of the metric rather than the object $\hat{r}^{\mu\nu}$ discussed below—the 'orthodox' curved spacetime Dirac operator $\gamma^a e^\mu_a \nabla_\mu$ is replaced with the operator $\gamma_\mu \hat{r}^{\mu\nu} \nabla_\nu$, where $\hat{r}^{\mu\nu}$ is the 'symmetric square root of the conformal metric density' $\hat{g}^{\mu\nu}$. For an explicit presentation of this (non-linear) geometric object, see [46]—but the salient point here for our purposes is easy to state: if one works in the OP spinor formalism, then one does not require tetrads, and hence (on the assumption of the universality of physical laws) parallelisable manifolds, and hence orientable manifolds, in order to model fermions in curved spacetimes. Thus, this alternative formalism demonstrates that the existence of fermions does *not* provide uncontroversial indirect evidence for the (manifold) orientability of spacetime. (Indeed, one might find the OP formalism attractive, in the sense that it does not involve making—perhaps surprising—assumptions about global topology in order to do local physics, even assuming the universality of physical laws.) Of course, this still leaves open the possibility that spinors in the OP formalism might not be definable on all manifolds (including, perhaps, non-orientable manifolds) for other reasons—in this sense, the possibility of spinors on all manifolds remains an open question—however, it is to show that the argument running from the spinor formalism to parallelisability to orientability does not go through completely uncontroversially.[10]

Third: Hadley is also critical of the above argument from the existence of fermions to the (manifold) orientability of spacetime; however, the specific arguments which he adduces are different from those which we have provided above. Instead, Hadley writes that "the argument relies on a realist interpretation of the wavefunction and the false assumption that a wavefunction is defined at each spacetime point. In fact a wavefunction is a function defined on a 3*N*-dimensional configuration space where *N* is the number of particles" [26, p. 4570]. We have three points to make on this argument from Hadley. (I) To the extent that Hadley claims to be dealing with *classical* theories, it is not clear why or how these observations regarding quantum mechanical wavefunctions is relevant. (II) One should surely take it that realism about the objects of physics is a presupposition of all of these debates—in this sense, it is, again, not clear why the observation that this argument presupposes realism is specifically problematic. (III) Even setting aside the above two points, Hadley is not correct that the above argument presupposes "the false assumption that a wavefunction is defined at each spacetime point". The reason for this is that there exist, in the foundations of quantum mechanics, many different ways of understanding the physical status of the wavefunction. According to 'wavefunction realism', the wavefunction is indeed an object on a (very high) 3*N*-dimensional space, construed of as physically real (see e.g. [42] for a recent book-length presentation of

[10] We are grateful to an anonymous referee for discussion on the content of this paragraph.

this view). However, an alternative approach—'spacetime state realism'—maintains that the wavefunction is a (density matrix valued) field on spacetime. (For a defence of this view, see [54]; for a review and analysis of this debate in its entirety, see [55, ch. 8].) Hadley claims that the indirect argument to orientability from the existence of fermions presupposes problematically the second of these views (although, to be explicit: it is not clear why this is indeed so), yet does not engage with interpretative options in the foundations of quantum mechanics which render this view (at least *prima facie*) viable.

6 Hadley's Concluding Remarks

In the closing section of his article, Hadley arrives at the following conclusions [26, p. 4571]:

> The following statements can all be supported by the arguments above.
>
> (i) Spacetime is not time-orientable. Particle-antiparticle annihilation events are evidence of this.
> (ii) A failure of time-orientability and particle-antiparticle annihilation are indistinguishable. They are alternative descriptions of the same phenomena.
> (iii) Time-orientability is untestable.
> (iv) Non-time-orientability cannot be an objective property of spacetime because the outcome of our test would depend upon the observer.

Based on our above discussions, let us evaluate each of these in turn. First: claim (i) is problematic, for it seems to contradict Hadley's previous claims made in the body of his article, that since (he argues) experiments testing temporal non-orientability can always be reconstrued as clock-anticlock (*a fortiori* particle-antiparticle) annihilation events, we *cannot* infer the temporal non-orientability of spacetime from such experiments. Further, claims (i) and (ii) are in tension with claim (iii): contrary to this point, it is not the case, for Hadley, that temporal orientability is *untestable*, but rather that such tests always have alternative *interpretations*. Finally, claim (iv) seems straightforwardly to contradict the technical definition of temporal orientability, and should therefore, in our minds, be set aside.

7 Other Approaches to Testing Orientability

Our central goal in this article up to this point has been to clear up the conceptual confusions implicated in Hadley's discussion of the possibility of testing the orientability of spacetime, in order to allow the literature on these issues to move forward. As of yet, however, we have mentioned little about other possible means of testing the orientability of spacetime. In this section, we present and assess three candidate approaches to doing so—some drawn from very recent literature in physics.

7.1 Parity Violation

One of the better-known arguments to the effect that one may be able to test the orientability of spacetime via local physical experiments appeals to the well-known fact that the weak interactions violate parity symmetry (see e.g. [19, 25, 57]). Assuming—to use now philosophers' parlance—that every dynamical symmetry should have an associated spacetime symmetry (cf. [14, ch. 3]), one can conclude that the fact that the dynamics of the weak interactions violate parity symmetry implies that there must be an associated piece of spacetime structure which violates said symmetry: this, of course, is an orientation (for philosophical discussion of this argument, also with reference to the substantivalism/relationism debate, see [27, 28, 47]). Since an orientation presupposes orientability (in the relevant sense), parity violation of the weak interactions would at least appear to imply the orientability of space (and thereby—via results such as Claim 2—to also have implications for temporal and manifold orientability).

Is it so? This question has been taken up in [15, pp. 144–145], and more recently in [5, Sect. 6.3]; here it suffices for us to summarise some of the most important and salient issues:

1. To think that local physical experiments have some bearing upon the orientability of spacetime presupposes some connection between local dynamics and spatiotemporal structure—cf. our above discussion of dynamical versus geometrical approaches to spacetime. (Since the discussions there carry over to the present case, we will not labour such issues further here.)
2. One must assume that the results of one's local physical experiments hold everywhere in the manifold, if one is to make this extrapolation to the spatial orientability of that manifold. Earman [15, p. 145] calls this an application of Dicke's strong equivalence principle (cf. [13, pp. 4–5]); in our view, it is more straightforward to view it as an application of inductive reasoning of the kind already discussed in Sect. 4.
3. As Earman has pointed out [15, pp. 145–147], whether one can directly read off parity violation (and so spatial orientability) from experiments such as that of Wu [56] (*viz*., the classic experimental setups which are typically taken to demonstrate parity non-conservation) in fact implicates one in substantive assumptions regarding other symmetries: in particular, charge inversion ('C') and time inversion ('T') symmetries. This dampens—but does not completely undermine—the force of what can be inferred from such experiments.

In sum: granting certain assumptions regarding the connections between dynamics and spacetime, and granting certain inductive extrapolations from the results of local physical experiments to the entire manifold, one can indeed make inferences regarding the orientiability of spacetime from the results of experiments such as that of Wu (which typically, and most straightforwardly, are taken to demonstrate parity non-conservation); that being said, and as Earman has elaborated, one's drawing of such inferences is not completely devoid of conceptual difficulties.

7.2 Quantum Electrodynamic Fluctuations

In this section, we wish to expose to the philosophical community another (very recent!) proposal for a possible local experimental test of spacetime orientability, elaborated in [4, 33, 34].[11] On this approach, one begins by considering quantum electrodynamic fields on manifolds with distinct spatial topologies. When one computes the two-point function $\langle E_i(\mathbf{x}, t)E_i(\mathbf{x}', t')\rangle$ for the electric field E_i, one obtains (see [6])

$$\langle E_i(\mathbf{x}, t)E_i(\mathbf{x}', t')\rangle = \frac{\partial}{\partial x_i}\frac{\partial}{\partial x'_i}D(\mathbf{x}, t;\mathbf{x}', t') - \frac{\partial}{\partial t}\frac{\partial}{\partial t'}D(\mathbf{x}, t;\mathbf{x}', t'). \tag{2}$$

In Minkowski spacetime with the usual simply-connected spatial topology $\mathbb{E}^3$, the Hadamard function $D(\mathbf{x}, t;\mathbf{x}', t')$ takes the form

$$D(\mathbf{x}, t;\mathbf{x}', t') = \frac{1}{4\pi^2\left(\Delta t^2 - |\Delta\mathbf{x}|^2\right)}, \tag{3}$$

where $\Delta t := t - t'$ and $|\Delta\mathbf{x}|^2 := \left(x - x'\right)^2 + \left(y - y'\right)^2 + \left(z - z'\right)^2$. Importantly, however, this function *differs* for manifolds with other, alternative spatial topologies, including non-orientable topologies—see [33, p. 5]. This, in turn, can lead to tangible empirical consequences, in terms of e.g. the mean square velocity dispersion of charged particles [33, p. 6].

We see nothing problematic with the mathematical and physical reasoning deployed by the authors of the above-described works; thus, we agree that the foregoing seems to afford a means of testing the orientability of spacetime via local experiments. Indeed, this approach appears superior to that discussed in Sect. 7.1, insofar as one does not need to make the inductive extrapolation that such results obtain everywhere in order to arrive at the conclusion that spacetime is non-orientable (in contrast to point (2) in Sect. 7.1); moreover, there do not appear to be other straightforward ways of interpreting such results which are consistent with spacetime orientability (in contrast with point (3) in Sect. 7.1). (That being said, the general points about dynamical versus geometrical approaches to spacetime outlined in point (1) of Sect. 7.1 continue to hold in this case.)

There is one further point to make here. As the authors of [33] note, "In the physics at daily and even astrophysical length and time scales, we do not find any sign or hint of nonorientability" [33, p. 12]. This, however, does not necessarily imply (to continue with the kinds of case countenanced in [4, 33, 34]) that the spatial topology of the universe is indeed $\mathbb{E}^3$, for it may be that the scales over which such local manifestations of non-orientability arise are too small (e.g., sub-Planckian) or too large (e.g., cosmological) or otherwise experimentally problematic (e.g., behind black hole horizons) for the effects of non-orientability to be detectable. This

[11] We are grateful to an anonymous referee for drawing our attention to this work.

notwithstanding, effects such as that discussed in this section do appear to afford an *in principle*—if not in practice—means of testing spacetime orientability. (For some further discussion related to this point, see [5, ch. 7].)

7.3 Circles in the Cosmic Microwave Background

We turn now to a third and final possible means of testing spacetime orientability via local experiments. It has long been understood that signatures of certain topological properties of spacetime (including non-orientability) may manifest themselves in the structure of the CMB (see [17, 32, 35, 50]). In particular, one expects that, in non-trivial spatial topologies, there will arise correlated circles of temperature fluctuations in the microwave background. Although no such circles have been observed up to this point, the authors of [23] point out that experimental results up to the present day are still consistent with non-trivial spatial topologies. That is, they pose—and ultimately answer in the affirmative—the following question:

> Assuming that the negative result of the general search ... can be confirmed through a similar analysis made with data from Planck and future CMB experiments, an important remaining question that naturally arises here is whether there still are nearly flat, but not exactly flat, universes with compact topology that would give rise to circles in the sky whose observable parameters λ and θ would fall outside the parameter range covered by this more general search. [23, p. 2]

Here, there are parallels with e.g. the experimental search for SUSY in particle detectors: experimental null results may whittle the region of parameter space in which the target phenomenon is possible, but they do not necessarily falsify the possibility of that phenomenon. Of course, there are interesting questions in this vicinity regarding the point at which one may simply reject the postulation of the phenomenon in question, should one continue to obtain such null results. This, however, is tangential to our purposes in this question: the point is that observations of the CMB do have the *potential* to give evidence of the non-orientability of spacetime; moreover, such experimental approaches would appear to have the same advantages over the methods discussed in Sect. 7.1 (i.e., those appealing to parity violation) as those discussed in Sect. 7.2 (i.e., those making use of certain local quantum electrodynamical effects), in the sense that they appear to warrant the conclusion that spacetime is non-orientable from a single local spacetime region; moreover, they are not straightforwardly amenable to re-interpretation as results in some orientable spacetime.

8 Close

Our first goal in this article has been to improve in various ways upon Hadley's analysis of the question of whether it is possible to test the orientability of spacetime. First: by providing precise definitions of three different notions of orientability—*viz*., manifold, temporal, and spatial. Second: by clarifying the sense in which orientability can be considered a global versus a local property of spacetime. Third: by arguing that while Hadley's experimental setups are *prima facie* sensible proposals for testing the orientability of spacetime, (a) they place undue focus on *regions*, (b) they do not engage sufficiently with various foundational and philosophical questions regarding the connection between the outcomes of such experiments and the nature of spacetime, and (c) they take a particular form in the case of temporal non-orientability which is, in our view, a red herring. Fourth: by clarifying whether results from QFT (and, in particular, the existence of spinors) provide indirect evidence for the orientability of spacetime—on this front, we share Hadley's scepticism, albeit for very different reasons than those which he adduces.

Our second goal in this article has been more positive: to systematise, evaluate, and compare various other proposals for testing the orientability of spacetime. Having now done so, we see that—modulo in particular certain philosophical assumptions regarding the connection between spacetime and the dynamics of material bodies, as well as certain inductive extrapolations that (i) the results which one has observed for one type of material field apply to all other material fields, and (ii) the results of physical experiments which one secures in one spacetime region obtain also in all others—it is indeed in principle (if not in practice: recall e.g. our discussion of relevant scales in Sect. 7.2) possible to test various of the different salient notions of orientability which we have considered in this article.

As already discussed in the introduction to this article, the question as to whether it is possible to test the orientability of spacetime is but one (significantly underexplored) topic in the general field of how we are to gain operational and empirical access to the nature of spacetime: surely an important matter to be addressed, if we are to be confident in our ability to grasp the fundamental nature of the physical world. (In general, this field goes under the name of the 'epistemology of spacetime': see [12, 24] for reviews.) We hope that our constructive dialogue with Hadley's article, as well as our systematisation of the other extant literature on this topic, will help to further discussions of these issues; and, of course, we invite other authors to consider further ways in which such topological and geometrical properties of spacetime (and others besides) can be tested experimentally.

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Declarations

Conflict of interest There are no conflict of interest associated with this article.

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References

1. Adlam, E., Linnemann, N., Read, J.: Constructive axiomatics in physics part II: The Ehlers-Pirani-Schild axiomatisation in context (2022) (Unpublished manuscript.)
2. Åman, J.E., d'Inverno, R.A., Joly, G.C., MacCallum, M.A.H.: Progress on the equivalence problem. In: Caviness, B.F. (ed.) EUROCAL 85: Proceedings of the European Conference on Computer Algebra, Linz, Austria, vol. 2, pp. 89–98. Springer, Berlin, Heidelberg (1985)
3. Bell, J.L., Korté, H.: Hermann Weyl, In: Zalta, E.N. (ed.), The Stanford Encyclopedia of Philosophy, (2015). https://plato.stanford.edu/entries/weyl/
4. Bessa, C.H.G., Rebouças, M.J.: Electromagnetic vacuum fluctuations and topologically induced motion of a charged particle. Class. Quantum Gravity **37**, 125006 (2020)
5. Bielińska, M.: Testing spacetime orientability, B.Phil. thesis, University of Oxford, (2021). (Available on request from the author.)
6. Birrel, N.D., Davies, P.C.W.: Quantum Fields in Curved Space. Cambridge University Press, Cambridge (1982)
7. Brown, H.R.: Physical Relativity: Space-Time Structure from a Dynamical Perspective. Oxford University Press, Oxford (2005)
8. Brown, H.R., Pooley, O.: The origins of the spacetime metric: Bell's Lorentzian pedagogy and its significance in general relativity. In: Callender, C., Huggett, N. (eds.) Physics Meets Philosophy at the Plank Scale. Cambridge University Press, Cambridge (2001)
9. Brown, H.R., Pooley, O.: Minkowski space-time: a glorious non-entity. In: Dieks, D. (ed.) The Ontology of Spacetime. Elsevier, Amsterdam (2006)
10. Brown, H.R., Read, J.: The dynamical approach to spacetime theories. In: Knox, E., Wilson, A. (eds.) The Routledge Companion to Philosophy of Physics, pp. 70–85. Routledge, London (2021)
11. Burke, W.L.: Applied Differential Geometry. Cambridge University Press, Cambridge (2008)
12. Dewar, N., Linnemann, N., Read, J.: The epistemology of spacetime. Philos. Compass **17**(4), e12821 (2022)
13. Dicke, R.H.: Experimental Relativity. Gordon & Breach, New York (1964)
14. Earman, J.: World Enough and Space-Time: Absolute Versus Relational Theories of Space and Time. MIT Press, Cambridge (1989)
15. Earman, J.: Kant, incongruous counterparts, and the nature of space and space-time, ratio 13, pp. 1–18. In: van Cleve, J., Frederick, R.E. (eds.) The Philosophy of Right and Left: Incongruent Counterparts and the Nature of Space, pp. 131–151. Kluwer Academic Publishers, Dordrecht, Boston, London (1991)
16. Ehlers, J., Pirani, F.A.E., Schild, A.: The geometry of free fall and light propagation. In: O'Reifeartaigh, L. (ed.) General Relativity: Papers in Honour of J. L. Synge, pp. 63–84. Clarendon Press, Oxford (1972)
17. Ellis, G.F.R.: Topology and cosmology. Gen. Relativ. Gravit. **2**(1), 7–21 (1971)
18. Freund, P.G.O., Maheshwari, A., Schonberg, E.: Finite-range gravitation. Astrophys. J. **157**, 857–867 (1969)
19. Geroch, R.: Singularities in the space-time of general relativity, Ph.D. thesis, Department of Physics, Princeton University (1967)
20. Geroch, R.: Spinor structure of space–times in general relativity. I. J. Math. Phys. **9**, 1739–1744 (1968)
21. Geroch, R.: Spinor structure of space–times in general relativity. II. J. Math. Phys. **11**, 343–348 (1970)
22. Geroch, R., Horowitz, G.T.: Global structure of spacetimes. In: Hawking, S.W., Israel, W. (eds.) General Relativity: An Einstein Centenary Survey, pp. 212–293. Cambridge University Press, Cambridge (1979)
23. Gomero, G.I., Mota, B., Rebouças, M.J.: Limits of the circles-in-the-sky searches in the determination of cosmic topology of nearly flat universes. Phys. Rev. D **94**, 043501 (2016)
24. Gray, J., Ferreirós, J.: Epistemology of geometry. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy (2021). https://plato.stanford.edu/entries/epistemology-geometry/

25. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Space-time. Cambridge University Press, Cambridge (1973)
26. Hadley, M.J.: The orientability of spacetime. Class. Quantum Gravity **19**, 4565–4571 (2002)
27. Hoefer, C., Hands, K., Pions, E.: Chirality arguments for substantival space. Int. Stud. Philos. Sci. **14**, 237–256 (2000)
28. Huggett, N.: Reflections on parity nonconservation. Philos. Sci. **67**, 219–241 (2000)
29. Huggett, N. , Hoefer, C., Read, J.: Absolute and relational space and motion: post-newtonian theories. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy (2021). https://plato.stanford.edu/entries/spacetime-theories/
30. Karlhede, A.: A review of the geometrical equivalence of metrics in general relativity. Gen. Relativ. Gravit. **12**(9), 693–707 (1980)
31. Karlhede, A.: The equivalence problem. Gen. Relativ. Gravit. **38**(6), 1109–1114 (2006)
32. Lachièze-Rey, M., Luminet, J.-P.: Cosmic topology. Phys. Rep. **254**, 135–214 (1995)
33. Lemos, N.A., Rebouças, M.J.: Inquiring electromagnetic quantum fluctuations about the orientability of space. Eur. J. Phys. C **81**, 618 (2021)
34. Lemos, N.A., Müller, D., Rebouças, M.J.: Probing spatial orientability of Friedmann-Robertson-Walker spatially flat spacetime. Phys. Rev. D **106**, 023528 (2022)
35. Levin, J.: Topology and the cosmic microwave background. Phys. Rep. **365**, 251–333 (2002)
36. Linnemann, N., Read, J.: Constructive axiomatics in physics part I: walkthrough to the Ehlers-Pirani-Schild Axiomatisation (2021). (Unpublished manuscript.)
37. MacCallum, M.A.H.: Classifying metrics in theory and practice. In: De Sabbata, V., Schmutzer, E. (eds.) Unified Field Theories of More Than 4 Dimensions Including Exact Solutions. Proceedings of the International School of Cosmology and Gravitation, pp. 352–382. World Scientific, Singapore (1983)
38. Malament, D.B.: Topics in the Foundations of General Relativity and Newtonian Gravitation Theory. University of Chicago Press, Chicago (2012)
39. Manchak, J.: Global Spacetime Structure, Cambridge Elements: Philosophy of Physics. Cambridge University Press, Cambridge (2020)
40. Minguzzi, E.: Lorentzian causality theory. Living Rev. Relativ. **22**(3), 1–202 (2019)
41. Minguzzi, E., Sànchez, M.: The causal heirarchy of spacetimes. In: Baum, H., Alekseevsky, D. (eds.) Recent Developments in Pseudo-Riemannian Geometry, pp. 299–358. European Mathematical Society Publishing House, Zurich (2008)
42. Ney, A.: The World in the Wave Function: A Metaphysics for Quantum Physics. Oxford University Press, Oxford (2021)
43. Nolan, C.: Tenet, Warner Bros. Pictures (2020)
44. Ogievetsky, V.I., Polubarinov, I.V.: Spinors in gravitation theory, soviet physics. J. Exp. Theor. Phys. **21**, 1093ff (1965). (**Russian volume 48, pp. 1625ff**)
45. Physics StackExchange: Is time orientability independent of space orientability?, https://physics.stackexchange.com/questions/666643/is- time-orientability-independent-of-space- orientability. Accessed Sept 2021
46. Pitts, J.B.: The nontriviality of trivial general covariance: how electrons restrict 'Time' coordinates, spinors (almost) fit into tensor calculus, and $\frac{7}{16}$ of a tetrad is surplus structure. Stud. Hist. Philos. Mod. Phys. **43**, 1–24 (2012)
47. Pooley, O.: Handedness, parity violation, and the reality of space. In: Brading, K., Castellani, E. (eds.) Symmetries Phys.: Philos. Reflect. Cambridge University Press, Cambridge (2003)
48. Read, J.: Explanation, geometry, and conspiracy in relativity theory. In: Beisbart, C., Sauer, T., Wüthrich, C. (eds.) Thinking About Space and Time: 100 Years of Applying and Interpreting General Relativity, Einstein Studies series, vol. 15. Birkhäuser, Basel (2020)
49. Read, J., Brown, H.R., Lehmkuhl, D.: Two miracles of general relativity. Stud. Hist. Philos. Mod. Phys. **64**, 14–25 (2018)
50. Starkman, G.D.: Topology and cosmology. Class. Quantum Gravity **15**, 2529–2538 (1998)
51. Synge, J.L.: Relativity: The Special Theory. North-Holland, Amsterdam (1956)
52. Synge, J.L.: Relativity: The General Theory. North-Holland, Amsterdam (1964)
53. Wald, R.M.: General Relativity. University of Chicago Press, Chicago (1984)
54. Wallace, D., Timpson, C.G.: Quantum mechanics on spacetime I: spacetime state realism, Br. J. Philos. Sci., pp. 697-727 (2010)
55. Wallace, D.: The Emergent Multiverse: Quantum Theory According to the Everett Interpretation. Oxford University Press, Oxford (2012)

56. Wu, C.S., Ambler, E., Hayward, R.W., Hoppes, D.D., Hudson, R.P.: Experimental test of parity conservation in Beta Decay. Phys. Rev. **105**(4), 1413–1415 (1957)
57. Zal'dovich, Y.B., Novikov, I.D.: The hypothesis of cores retarded during expansion and the hot cosmological model. J. Exp. Theor. Phys. **6**, 236–238 (1967)

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