Published by IOP Publishing for Sissa Medialab

Received: *December 9, 2022*
Accepted: *December 14, 2022*
Published: *March 13, 2023*

ICFA Beam Dynamics Newsletter#84 —
Dynamics of high power and high energy cyclotrons

Cyclotrons: Why/how are their dynamics different?

R. Baartman

TRIUMF
4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3, Canada

E-mail: `krab@triumf.ca`

Abstract: For this issue, papers on the topic of cyclotron beam physics have been solicited and chosen to highlight the main areas both of historic interest and of active research. I take the opportunity to outline the differences and similarities between cyclotron dynamics as compared to other accelerator types. As well, I try to introduce the major areas of interest, referring to papers in this issue, as appropriate.

Keywords: Beam dynamics; Beam Optics

2023 JINST 18 T03005

https://doi.org/10.1088/1748-0221/18/03/T03005

Contents

1 Introduction

For isotope production, accelerator driven systems, transmutation of waste, and other applications that require a continuous high power beam, cyclotrons are a competitive choice. Being fixed field and fixed frequency, they have in common with linear accelerators the feature that particles can be accelerated continuously without pulsing. As a result, the normalized beam brightness from the ion source can in principle be maintained through the acceleration process. Megawatt beams of order one milliamp have been demonstrated. By contrast, a rapid cycling synchrotron of say 60 Hz needs to circulate Amps of current to attain the same time averaged beam power. Thus cyclotrons are a natural choice for high power applications. But they have an advantage over linacs that particles continually return to the same RF gap, as many as a thousand times. This optimizes the transfer of energy from RF to the beam, and also makes cyclotrons more compact, but at the cost of a magnet.

2 Orbits and isochronism

Cyclotrons are fixed field accelerators; synchrotrons are fixed orbit. To accommodate the full energy range, cyclotrons require fields extended in the radial direction while synchrotrons require only sufficient radial extent to accommodate the beam emittance and a few percent energy spread. Typically in compact cyclotrons where the beam is injected into the centre, the particle energy changes by a factor of a thousand (momentum by a factor 30) between injection and extraction.

The fixed orbit nature of the synchrotron or storage ring enables a huge simplification of the optics: the reference orbit can consist entirely of straight paths and circular arcs, as the guiding magnets have fields that are piecewise constant on that orbit. Transfer matrices of individual elements are known analytically, so all the linear optics; tunes and Courant-Snyder parameters, can be calculated simply and quickly. Not so with cyclotrons as their optics designs are dominated by the need for isochronism. This requires the orbit length for given particle energy to be precisely proportional to its speed (β), and this in turn requires the magnetic field averaged along the closed orbit be precisely proportional to energy, i.e. γ. We thus have

$$\langle B \rangle = \frac{B_c}{\sqrt{1 - (R/R_\infty)^2}}, \tag{2.1}$$

where B_c is a constant. This is singular at $R = R_\infty$; the radius corresponding to the speed of light.

There is no oscillatory longitudinal motion; the particles remain at fixed phase with respect to the RF. A phase slip larger than 90° would cause particles to decelerate back down to the injection energy so that sets a limit to the accumulated isochronism error. The required level of precision of RF frequency and magnetic field is thus given by the number of RF periods from injection to extraction (turns times harmonic number), and this is typically one part in 10^4 to 10^5.

A fixed field accelerator (FFA) that has some of the features of synchrotrons is the scaling FFAG. These have orbits of fixed shape, that are scaled with energy according to the scaling power law ($B \propto R^k$) first given in Symon et al. [37]. The advantage is that tunes can be constant and no betatron resonances are crossed. However, they cannot be made isochronous as the orbit length does not scale correctly with particle speed. Thus a scaling FFA is essentially an FM cyclotron, and having modulated frequency, it must be pulsed rather than continuous, losing orders of magnitude in beam power.

The EMMA FFA [30], consisting entirely of quadrupoles, is a linear field counter-example. Not following the scaling needed for fixed optics, nor the field scaling needed for isochronism, it deviates to lowest order parabolically in having a too-low field at injection, particle bunches falling behind the RF in phase, then at mid energy having a too-high field, allowing bunches to overtake the RF, and finally at the top energy, falling behind again. This acts as a viable cyclotron, but only if the energy gain per turn is sufficiently high to prevent the accumulated phase slip being larger than 90°. For example, applied to a proton accelerator for 600 MeV, it would require more than 200 MV of RF per turn.

One might think that the cyclotron's need for a nonlinear relationship between average field and radius results in highly nonlinear optics, but such is not the case. In fact the nonlinearity in cyclotrons is smaller than that which arises in a synchrotron from the correction of chromaticity. The main difference is that restricting the orbits in a cyclotron to being piecewise constant radius arcs as they are in synchrotrons, is an unhelpful constraint in achieving isochronsim. Moreover, magnetic fields in a sectored compact cyclotron typically vary from nearly independent of azimuth at injection to having separated hill and valley fields near extraction. The orbits thus are found and characterized by numerical integration rather than matrix multiplication.

The feature of cyclotron orbit dynamics is that it is based on a magnetic field map rather than a set of elements and their parameters: length, radius of orbit, and field index or quadrupole strength. While synchrotron optics evolved to the point where they could be contructed entirely of elements whose transfer matrices are known analytically, this simplification technique was not available to cyclotron designs. The reason is that the average field and its gradient are given by the isochronism condition above rather than by transverse focusing.

The off-energy closed orbit (the periodic dispersion) in a synchrotron can be found from the linear optics again by a matrix technique. By contrast, in an N sector cyclotron, for any given energy, the closed orbit is found by Newton's root-finding iterative procedure [14, 16]. A particle is integrated along with its first order differentials, through an angle of $2\pi/N$. This results in both the orbit coordinates and the periodic transfer matrix. The matrix is used to adjust the initial coordinates to that which would have closed the orbit, and the process repeated. After 2 or 3 iterations, this yields the closed orbit to sufficient precision, and its tunes and the Courant-Snyder parameters can then be found from the transfer matrix. Crucially, the integration also yields the orbit period. The designer adjusts ('shims') the magnetic field to achieve a constant period over the complete momentum range, which as stated can be as large as a factor of 30. This in general results in tunes that vary with energy; in particular, the radial tune will be nearly equal to γ since the radial focusing depends mainly on the field gradient and this in turn results to lowest order from the isochronism scaling condition [27]. As Planche shows, to flatten the radial tune requires unconventional magnet designs, but is in principle still possible [31].

An interesting and subtle difference between the two types is that the cyclotron orbits are usually most conveniently analyzed using the azimuth (θ) as independent variable whereas the synchrotron uses the Frenet-Serret coordinate system with orbit length as independent variable. In the cyclotron case, though, if preferred, the transverse optics can be re-analyzed by integrating the equations of motion along the reference orbit, using the same basic equations of motion as used for synchrotrons [6]:[1]

$$x'' + \frac{1-n}{\rho}x = 0, \text{ and } z'' + \frac{n}{\rho}z = 0, \tag{2.2}$$

where primes are derivatives with respect to the path length s, x and z are horizontal and vertical, and ρ is the local orbit curvature. the local field index

$$n = -\frac{\rho}{B}\left.\frac{\partial B}{\partial x}\right|_{x=0}. \tag{2.3}$$

The only difference is that in the synchrotron case both functions $n(s)$ and $\rho(s)$ are known *a priori*, whereas in the cyclotron case, they are not.

3 History of strong focusing and isochronism

As originally invented, cyclotrons were only approximately isochronous and could not reach proton energies higher than $\sim 12\,\text{MeV}$. Not having any azimuthal field variation, they could only focus in both transverse directions if the magnetic field was monotonically decreasing with radius. But relativity requires the opposite. In particular, the isochronism scaling law, $\langle B\rangle \propto \gamma$, results in a vertical tune $\nu_z^2 = -\beta^2\gamma^2$; strongly defocusing. In 1938, L.H. Thomas [38] presented a way of

---

[1] Note that these equations were known before the invention of the quadrupole. Nevertheless, they apply in that case too even though $\rho = \infty$, since n/ρ is independent of ρ.

increasing vertical focusing, by varying the field azimuthally. But it was not until the discovery of the 'strong focusing' effect that the energy limit was overcome.

At first, it was thought that for cyclotrons and FFAGs at relativistic energy, sufficiently strong focusing could only be possible if reverse bends were interleaved between positive ones, thus dramatically and uneconomically increasing the size of the machine. But in 1954, D.W. Kerst [24] invented the 'spirally-ridged' sector magnet. In his words:

> *The high momentum spread allowed in Symon's ring magnet, where the alternating gradient is provided by field reversal in alternate magnet segments, allows acceleration from a low momentum to high momentum within a relatively small radial span, without change in the magnetic field excitation. The disadvantage of this design is that the resulting accelerator has a circumference about five times that of an equivalent accelerator of the same maximum field. The spirally ridged pole magnet described here is an attempt to retain the desirable features of Symon's design (D.C. magnet excitation, high momentum content, large injection aperture) and at the same time to reduce the circumference by eliminating the regions of reverse field. This is accomplished by constructing a magnet pole contour of high positive n (high field at the outer radius) and superimposing on it spiral ridges to provide the alternate gradient focussing...*

Further, from Symon et al. [37]:

> *...A particle going around the machine experiences a gradient first of one sign then of the opposite sign as it crosses the periodic field ridges and troughs at a small angle* [i.e. large spiral angle]*, so there is AG focusing of the betatron oscillations.*

"Alternating gradient" focusing and "strong focusing" are now used interchangably and the invention of the latter is usually attributed to Ernest Courant [10]. The older "alternating gradient" appellation is explained by the fact that all accelerators up to and including the CERN PS and Brookhaven AGS used only dipoles with transverse field gradient. Quadrupoles were invented by T. Kitagaki in 1952 [25] and did not "catch on" immediately. But the older name still applies: whether the focusing comes from dipoles with alternating field indexes, spiralled dipole edges giving alternating signs to edge focusing, or quadrupoles of alternating polarity; in all three cases, there is a transverse gradient that alternates in sign, and this is the main ingredient of "strong focusing".

The first cyclotron to use the Thomas azimuthal field variation to achieve vertical focusing in spite of the field rising with radius as needed for accurate isochronism, was the Delft 12 MeV cyclotron in 1958 [19]. It resulted in a factor of 10 reduction in RF voltage needed to achieve top energy. Many other existing cyclotrons were retrofitted, with sectored poles, some radial and some spiralled. By 1964 there were 17 cyclotrons with spiral edged poles of which 8 were operating and 9 under construction [35]. The highest energy at that time was 60 MeV, but designs had been started for two cyclotrons at above 500 MeV; these eventually came to fruition in the mid-70s as the TRIUMF and PSI meson factory cyclotrons. The TRIUMF case in particular relies completely on strong focusing to achieve a real vertical tune at top energy. It has a spiral angle that reaches an extreme 72°. This is related to the fact that the H^- ions place an upper limit of 0.56 Tesla on the magnetic field, and therefore limit the azimuthal field variation (the 'flutter') to a very low value. More details found in paper by Koay in this issue [29].

2023 JINST 18 T03005

There is in fact no limit to the energy achievable with cyclotrons. Though there have been designs up to 15 GeV [8], none were built. However, the required energy gain per turn grows with energy, faster than linearly. This is related to the ‘$B \propto \gamma$ while $R \propto \beta$’ scaling law. To keep orbits separate, there has to be sufficent radius gain per turn. We have $dE/dR \propto d\gamma/d\beta = \beta\gamma^3$. Let us assume the magnetic field has some realistic upper limit; then momentum $\beta\gamma \propto BR$ means machine radius $\sim \beta\gamma$ and the needed energy gain per radius is proportional to γ^2. Thus beyond some energy, the machines are too large in circumference, and the total needed energy gain per turn is such that only one or a few turns are needed for full energy. At that point, the cyclotron has no economic advantage over a linear accelerator. The current highest γ is the PSI cyclotron ($\gamma = 1.6$), and it has only 182 turns [17]. This suggests $\gamma \sim 10$ as a rough upper limit for cyclotron economic viability. More recent designs exist up to $\gamma = 3$, and one is included in this issue [39], but none have been built beyond $\gamma = 1.6$: the PSI cyclotron and it has already been running for over 50 years [17, 36].

Besides the necessities of extraction (see [29]), for any given design, the energy limit is related to the betatron tune. A difficult feature of cyclotrons is that the radial tune usually varies with radius. With the simplest sector focusing, the radial tune is approximately equal to γ. Planche has found a way that in principal can overcome this feature, but no magnets have been designed to check its feasibility [31]. As Kleeven shows [27], the number of sectors of a cyclotron must be chosen to avoid the half-integer intrinsic stopband at top energy. Thus, roughly $N > 2\gamma$, and usually it must be significantly larger due to the stopband width. As the contribution to ν_z^2 due to isochronism is $-\beta^2\gamma^2$, the amount of field variation (‘flutter’) needed to achieve net vertical focusing grows sharply with energy, and this broadens the half integer intrinsic stopband $2\nu_x = N$, bringing the energy limit downward. For lowered flutter, the vertical tune can be recovered by raising the spiral angle; to lowest order, the relation is

$$\nu_z^2 \approx -\beta^2\gamma^2 + F(1 + 2\tan^2\xi) \tag{3.1}$$

where F is the flutter defined as the mean-squared magnetic field on the orbit, and ξ is the spiral angle. For the extreme TRIUMF case of $\xi = 72°$, the spiral angle is seen to amplify the effect of flutter by a factor 20, demonstrating again the efficacy of strong focusing. Thus for any chosen top energy, there is a minimum number of sectors, and an optimal combination of flutter and spiral angle. See [27] for details.

4 Magnetic field map and resonance crossings

The cyclotron’s vertical component of the magnetic field is mapped on the geometric median plane, but this is not sufficient in particular for large cyclotrons. One needs also to know the up-down asymmetry as this will result in radial magnetic fields that steer the circulating beam off the median plane. Moreover, passage through coupling resonances with such skew fields can lead to vertical emittance growth and beam loss. The resonance $\nu_x - \nu_z = 1$ is one such example, driven by the first harmonic of the up-down asymmetry. (See [34].) The ‘Walkinshaw’ resonance $\nu_x - 2\nu_z = 0$ is usually an even more dangerous one as it is intrinsic and driven by the average radial second derivative. As the average isochronous field necessarily scales as γ while radius scales as β, this derivative cannot be corrected except at the sacrifice of isochronism.

In a technique developed by Gordon [15], the magnetic field is split into a symmetric part $B(R,\theta)$ and an asymmetric part $C(R,\theta)$. It can be measured to lowest order by measuring the difference in $B_z(R,\theta)$ between the median plane and slightly off it. These two parts can be expanded by Maxwell equations to give all the three field components in the median plane. The technique can be found in the thesis of Mort's final graduate student, Dong-O Jeon [21], and has been applied here by Lige Zhang [39] to analyze some self-consistency issues in the TRIUMF magnetic field data.

Mort Gordon was a remarkable man. He developed orbit dynamics theory and magnetic field expansion methods that resulted in orbit codes many of which are still in daily use today. He was also completely blind. To write papers, he would dictate to his wife, though she knew no physics. Some but not nearly all of his work was published in the now defunct journal *Particle Accelerators*. Dong-O Jeon writes as a personal reminiscence:

> *As a PhD student of Morton Gordon, I had the honor of witnessing and learning from a truly amazing person and physicist. I became his student when he was in his 60's. He said he had lost his vision in one eye around the age of 13 and had become completely blind around the age of 30. I can only imagine how stressful it was as he became gradually blind. Nevertheless, he was a most optimistic and cheerful person. He was always meek and kind, never losing his wit and humour.*
>
> *Though he was blind, he had no difficulties in teaching electro-magnetism classes or doing research. It seemed to me as if the whole of Jackson's textbook was in his head. He appointed one student to hang a few strings on a blackboard before the class, to prevent him from writing equations over top of each other. Between those strings, he wrote down all of the complex equations out of his memory, which was an amazing scene. As he wrote down the equations, he also dictated them precisely, using the words 'bra' and 'ket' to denote opening and closing parentheses. Though I was a foreigner, I was able to listen and write down the equations on my notebook without even looking at the blackboard.*
>
> *He said his wife read and recorded all of the equations and papers for him on a recorder, despite not understanding physics. How did he solve complex equations? Many times I saw him solving equations by simply dictating to or listening to a recorder playing back and forth.*
>
> *Morton made significant contributions to orbit codes and magnetic field descriptions, especially in cyclotrons. He was a giant in the field of cyclotron dynamics; he overcame his physical handicap and was an excellent researcher, mentor, and advisor.*

A personal recollection follows. As a newly-employed young physicist, I was tasked to try to understand the emittance growth from passing through the $\nu_r = 3/2$ resonance in the TRIUMF cyclotron. Measurement completely disagreed with a theory based upon the rate of growth inside the stopband. Mort Gordon was visiting and we asked his advice. He told us to forget about the exponential growth in the stopband and just use the equations of motion with varying tune and integrate through, since the change in β-function on either side of the stopband is too rapid to be adiabatic. This can be done analytically for linearly changing tune, and results in Fresnel integrals. It completely agreed with measurement [4]. I later discovered that this 'rapid-passage' case was independently known in the synchrotron community and can be found e.g. in Gilbert Guignard's compendium of formulae [18].

5 Cyclotron types

There are two main types of cyclotrons: ‘compact’ and ‘ring’. In compact cyclotrons the magnetic field extends to the cyclotron’s centre. The injected beam generally comes in on the central axis and is inflected to some small first turn radius, the injection energy typically being only a few tens of keV. The magnet pole is therefore monolithic and the current-carrying field coils encircle the cyclotron. Generally, but not exclusively such a cyclotron is small. The exception is the TRIUMF cyclotron, which has in common with compact cyclotrons that the first turn radius is comparable to the magnet gap, because the resonators lie inside the gap making it anomalously large. Besides TRIUMF, all of the high current low energy isotope production cyclotrons are of this type.

Ring cyclotrons have no magnetic field at $R = 0$, so must inject at a larger radius and typically an energy of at least 1 MeV. But the sectors can be separated, each having its own coil. This is a large advantage since there can be field-free drifts between sectors where resonators can be placed and the large field variation (‘flutter’) is effective in creating comparatively stronger vertical focusing; Thomas focusing using radial sectors, or strong-focusing using spiralled ones.

Ring cyclotrons are essentially lattices with combined function dipoles and so are somewhat more similar to synchrotrons than compact ones are. The radial sector case is actually amenable to analysis since such consist of only dipoles, each with well-defined field index, and drifts. Thus matrix optics can be used after all. Gordon first analyzed this special case [12]. Each sector is a purely radial pie-shaped piece, with constant field on the intended closed orbit: sector and orbit shapes can be found from geometry. The field $B(R)$ varies only with radius in any given sector to make it exactly isochronous. This case is instructive as the tunes and energy limits can be found analytically. Many cyclotrons exist that are based on this design: the PSI Injector 2 [36], the two separated sector GANIL cyclotrons [23], and the four cyclotrons of RIBF [20], and others.

A strong focusing case along similar lines but using spiraled sectors is by Botman et al. [7]. But in the general case, iso-gauss contours of the magnetic field nowhere coincide with orbits, so this `TRANSPORT`-style analysis is an approximation at best.

6 Centre region focusing and the space charge limit in compact cyclotrons

Cyclotrons are capable of accelerating from a very low energy; low enough that the injected beam can be sourced from a small HV power supply. In compact cyclotrons, the first turn radius is typically on the order of 1 cm. This is smaller than the magnet gap and thus is in a region of flat magnetic field, where there is no vertical focusing and the radial tune is near 1. The trick to maintaining vertical focusing is to rely on the accelerating RF fields. As is well-known, the RF fields can focus either longitudinally (speeding up particles that are late and slowing down leading particles) or transversely, but not both; there is a sum rule deriving ultimately from Gauss’ law. In a linac, one sacrifices transverse focusing to gain longitudinal RF focusing and uses externally applied transverse fields to maintain focal strength. In cyclotrons, it is the opposite: RF forces that would defocus longitudinally (early particles recieve more energy gain than late ones) are used to focus transversely. The reason this works is that in an isochronous machine, a particle’s energy change will not result in a phase change. Only particles that are off-crest and on the falling RF energy gain are focused vertically. This continues for a few turns; as the RF focusing diminishes with increasing energy, magnetic focusing takes over as sector focusing grows with radius.

However it is also at lowest energy that space charge has largest effect. The weakest vertical focusing on the first few turns thus sets the space charge intensity limit. This limit is explored elsewhere in this issue; see [32] for more details.

7 Extraction and phase acceptance

The usual cartoon depicts a beam in a cyclotron spiralling outward and then bending away from the magnet into an extraction beamline. This leads one to think that extraction is a simple matter. But relying on conservative force fields means that any extra force used to extract the beam will also perturb the orbits of the last few turns, not the final one alone. When turn separation is much smaller than the magnet gap, there must be an additional field, usually electric, and a septum separating and shielding it from those orbits. In order for the septum to survive, there must be sufficient separation between turns. This sets a lower limit on the energy gain per turn. Additionally, a clever trick is to set the beam into a coherent oscillation to increase the turn separation of the final turn [22].

Past the outer turn of the cyclotron, the magnetic field begins to fall and so becomes non-isochronous. Baumgarten [5] shows, though, that with sufficient energy gain per turn, it is still possible extract beam without a septum. A condition for this is that the beam be extracted before it has slipped in phase to the point it begins decelerating, combined with a sufficiently sudden onset of the field falloff with radius. Under these conditions, beam tends to extract at all azimuths, so it is still necessary to impose a local gradient bump to localize the extraction. These conditions are most easily attained in high energy cyclotrons where energy gain per turn can be designed to be very large. Nevertheless it can be made to work for small cyclotrons, albeit with lower extraction efficiency; it had been invented and designed at IBA [11, 26].

The sinusoidal nature of a single accelerating RF harmonic and the frozen phase motion of the particles mean that turns will be parabolic with phase, increasing their effective width. For clean extraction this demands more separation the larger the phase spread. If $\hat{n}$ is the number of turns and V_{rf} the voltage per turn, the requirement from this consideration alone is that the furthest off-crest particle be at larger radius than the on-crest particle of the preceeding turn:

$$\hat{n} V_{\mathrm{rf}} \cos\phi > (\hat{n}-1) V_{\mathrm{rf}} \quad \rightarrow \quad |\phi| < \sqrt{\frac{2}{\hat{n}}}. \tag{7.1}$$

This results in a phase acceptance of only a few degrees for the typical case of a few hundred turns.

Adding a third harmonic can flatten the waveform and accept phases in the range $|\phi| < \left(\frac{8}{3\hat{n}}\right)^{1/4}$, gaining a factor of three or so in phase acceptance at the cost of a second high power resonant RF system.

A surprising finding though is that the limit is alleviated to some extent by a space charge effect that results in some self-bunching, explained in the next section. Imao [20, figure 19] mentions in his paper that the transmission efficiency improves with intensity due to this effect.

There is however, one technique that can result in high efficiency low loss extraction in spite of having no turn separation. That is charge exchange, or stripping extraction. In this technique the phase acceptance is not limited by the demands of extraction, and so can be as large as 60° or more; the only limitation being that the furthest off-crest particle make it past the injection RF gap. This is especially effective for low energy cyclotrons to produce protons up to about 70 MeV, since H^- ions

2023 JINST 18 T03005

can be used. At any higher energy, the magnetic field must be reduced to avoid Lorentz stripping, since the extra electron is very weakly bound. This makes the cyclotron large and non-cost-effective. For higher energies, the economic solution may be to use other hydrogen molecular ions. This is covered by Koay [29].

8 Peculiar effects of space charge in cyclotrons

Ordinarily, in an isochronous cyclotron, the particles are locked in longitudinal position with respect to each other. But this is a rotating frame so with the extra central force of space charge, particles begin to circulate within a bunch. It is analogous to how cyclones form in the rotating atmosphere of the earth when there is a central force field due to an area of low atmospheric pressure. In detail, the physics is that the space charge force on a leading particle forces it to higher energy and therefore outboard of the central particle. Similarly, trailing particles will tend inward, outboard particles will fall back in phase and inboard move forward. The motion is along contours of constant space charge potential: it results in a net rotation. The rate of rotation in units of the revolution frequency is simply the Laslett radial tune shift [3]. The effect has come to be known as the 'vortex effect' and was first posited by Mort Gordon [13], however he did not treat the limit of short, well-separated bunches. In that case, the bunches tend accurately toward circular in the median plane. This was first simulated by Adam [1], proved by Kleeven [28], and observed at PSI [2, 36]. If bunches are too long, longer than approximately 3 times their width [9], they would split into separated circular bunchlets [33]. But if sufficiently short and matched with bunch length equal to radial width at injection, space charge helps to maintain the bunches' small phase extent.

9 Efficient acceleration for heavy ions

Cyclotrons can be used to efficiently accelerate any ion. By having resonator systems that are tunable over a wide range, a single cyclotron can accelerate masses up to the heaviest. It is useful to have a scaling relation to determine highest energy for some ion knowing its mass and charge state. The magnetic field and radius of the final orbit determine a particle's mometum:

$$2\pi\langle BR\rangle := \oint B\mathrm{d}s = 2\pi(B\rho) = 2\pi\beta\gamma mc/q. \tag{9.1}$$

Roughly, $\langle BR\rangle$ is the product of the average magnetic field and the outer orbit radius. Commonly, the desired quantity is the kinetic energy per atomic mass unit, E/A. We can write this as

$$\frac{E}{A} = \left[\left(\frac{\langle BR\rangle}{3.1\,\mathrm{Tm}}\right)^2 \frac{931\,\mathrm{MeV}}{2}\right]\left(\frac{Q}{A}\right)^2 := K\left(\frac{Q}{A}\right)^2, \tag{9.2}$$

where Q particle charge in units of e. To be relativistically correct, the factor of 2 in the denominator should be replaced by $\gamma + 1$. The quantity in square brackets is conventionally called the 'K-value' of the cyclotron.

Note that the final energy per nucleon is proportional to the square of the charge state. As with other heavy ion accelerators, there is the usual bootstrap issue that the very high charge states needed to make acceleration efficient are most easily obtained or retained if the particles already have a high energy. The most economical way to reach high energy is in stages therefore with stripping occurring between the stages. A good example, and the highest K cyclotron system is that of RIBF at the RIKEN laboratory [20]. They have a series of four cyclotrons with the final one having $K = 2600$ MeV. Thus for example, they accelerate $^{238}\mathrm{U}^{86+}$ to 345 MeV/amu. This is relativistic at 68% lightspeed; the total kinetic energy per particle is 82 GeV. To increase the intensity of the uranium beam, a new acceleration scheme using charge stripper rings (CSRs) has been proposed as an innovative and cost-effective way to increase charge stripping efficiency [20].

10 Conclusion

I would like to thank all the cyclotron physicists who contributed to this issue. In some cases, my request for a paper from an expert on some aspect of cyclotron dynamics precipitated a large amount of new research that was undertaken in addition to their daily responsibilities. This has resulted in a wide range of interesting papers at the forefront of cyclotron design.

Acknowledgments

TRIUMF receives federal funding via a contribution agreement with the National Research Council of Canada.

References

[1] S. Adam, *Calculation of space charge effects in isochronous cyclotrons*, *IEEE Trans. Nucl. Sci.* **32** (1985) 2507.

[2] S. Adam et al., *Space charge effects in cyclotrons-from simulations to insights*, in *Proceedings of the 14th International Conference on Cyclotrons and their Applications*, Cape Town, South Africa, 8–13 October 1995, pp. 446–449.

[3] R. Baartman, *Space charge limit in separated turn cyclotrons*, in *Proceedings of 20th International Conference on Cyclotrons and their Applications*, Vancouver, BC, Canada, 16–20 September 2013, pp. 305–309.

[4] R.A. Baartman, G.H. Mackenzie and M.M. Gordon, *Amplitude growth from the rapid traversal of a half-integer resonance*, in *Proceedings of the Tenth international conference on cyclotrons and their applications*, East Lansing, MI, U.S.A., 30 April–3 May 1984, pp. 40–43.

[5] C. Baumgarten, *Cyclotron beam extraction by acceleration*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[6] N.M. Blachman and E.D. Courant, *The dynamics of a synchrotron with straight sections*, *Rev. Sci. Instrum.* **20** (1949) 596.

[7] J.I.M. Botman, M.K. Craddock and C.J. Kost, *Transfer matrix techniques for ring cyclotron sector shapes*, in *Proceedings of the Tenth international conference on cyclotrons and their applications*, East Lansing, MI, U.S.A., 30 April–3 May 1984, pp. 32–35.

[8] J.I.M. Botman, M.K. Craddock, C.J. Kost and J.R. Richardson, *Magnet sector design for a 15-GeV superconducting cyclotron*, *IEEE Trans. Nucl. Sci.* **30** (1983) 2007.

[9] A. Cerfon, J. Guadagni, O. Bühler, J.P. Freidberg and F.I. Parra, *Vlasov equation approach to space charge effects in isochronous cyclotrons*, in *Proceedings of the 20th International Conference on Cyclotrons and their Applications*, Vancouver, BC, Canada, 16–20 September 2013, pp. 310–314.

[10] E.D. Courant, M.S. Livingston and H.S. Snyder, *The strong-focusing synchroton — a new high energy accelerator*, *Phys. Rev.* **88** (1952) 1190.

[11] G. D'Agostino and W.J.G.M. Kleeven, *Beam dynamics and space charge studies for the InnovaTron cyclotron*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[12] M.M. Gordon, *Orbit properties of the isochronous cyclotron ring with radial sectors*, *Ann. Phys.* **50** (1968) 571.

[13] M.M. Gordon, *The longitudinal space charge effect and energy resolution*, in *Proceedings of the Fifth International Cyclotron Conference*, Oxford, U.K., 17–20 September 1969, pp. 305–317.

[14] M.M. Gordon, *Computation of closed orbits and basic focusing properties for sector-focused cyclotrons and the design of* `CYCLOPS`, *Part. Accel.* **16** (1984) 39.

[15] M.M. Gordon and V. Taivassalo, *The z4 orbit code and the focusing bar fields used calculations for superconducting cyclotrons*, *Nucl. Instrum. Meth. A* **247** (1986) 423.

[16] M.M. Gordon and T.A. Welton, *Computation Methods for AVF Cyclotron Design Studies*, Tech. Rep., ORNL-2765, Oak Ridge National Laboratory, Oak Ridge, U.S.A. (1959).

[17] J. Grillenberger, C. Baumgarten and M. Seidel, *The high intensity proton accelerator facility*, *SciPost Phys. Proc.* (2021) 002.

[18] G. Guignard, *Selection of formulae concerning proton storage rings*, Tech. Rep., CERN-77-10, CERN Yellow Report, CERN, Geneva, Switzerland (1977).

[19] F.A. Heyn and K.K. Tat, *Operation of a radial sector fixed-frequency proton cyclotron*, *Rev. Sci. Instrum.* **29** (1958) 662.

[20] H. Imao, *Upgrade plan with charge stripper rings at RIKEN RI beam factory*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[21] D.-O. Jeon, *Nonlinear Effects in the Vertical Motion of Ions in a Superconducting Cyclotron*, Ph.D. thesis, Michigan State University, U.S.A. (1995).

[22] W. Joho, *Extraction from medium and high energy cyclotrons*, in *Proceedings of Fifth International Cyclotron Conference*, Oxford, U.K., 17–20 September 1969, pp. 159–79.

[23] A. Joubert, *Status report on the ganil facility*, in *Proceedings of Tenth International Conference on Cyclotrons and their Applications*, East Lansing, MI, U.S.A., 30 April–3 May 1984, pp. 3–10.

[24] D.W. Kerst, L.W. Jones, K.R. Symon and K.M. Terwilliger, *A fixed field alternating gradient accelerator with spirally ridged poles*, Tech. Rep., MURA-042, Mid-Western Universities Research Association (1954).

[25] T. Kitagaki, *A focusing method for large accelerators*, *Physical Review* **89** (1953) 1161.

[26] W. Kleeven et al., *The self-extracting cyclotron*, *AIP Conf. Proc.* **600** (2001) 69.

[27] W.J.G.M. Kleeven, *On the energy limit of compact isochronous cyclotrons*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[28] W.J.G.M. Kleeven, *Theory of accelerated orbits and space charge effects in an AVF cyclotron*, Ph.D. thesis, Technische Universiteit Eindhoven (1988).

[29] H.W. Koay, *Lorentz dissociation of hydrogen ions in a cyclotron*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[30] S. Machida, R. Barlow, J.S. Berg, N. Bliss, R.K. Buckley, J.A. Clarke et al., *Acceleration in the linear non-scaling fixed-field alternating-gradient accelerator EMMA*, *Nat. Phys.* **8** (2012) 243.

[31] T. Planche, *Constant-tune cyclotrons*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[32] T. Planche, R. Baartman, H.W. Koay, Y.-N. Rao and L.G. Zhang, *Intensity limit in compact H^- and H_2^+ cyclotrons*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[33] E. Pozdeyev, J.A. Rodriguez, F. Marti and R.C. York, *Longitudinal beam dynamics studies with space charge in small isochronous ring*, *Phys. Rev. ST Accel. Beams* **12** (2009) 054202.

[34] Y.-N. Rao and L.G. Zhang, *Coupling resonance correction and avoidance for the TRIUMF 500 MeV cyclotron*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).

[35] J.R. Richardson, *Sector focusing cyclotrons*, in F.J.M. Farley, ed., *Progress in Nuclear Techniques and Instrumantation*, vol. 1, North-Holland, Amsterdam (1965), pp. 1–101.

[36] T. Stammbach, S. Adam, H. Fitze, W. Joho, M. Märki, M. Olivo et al., *The feasibility of high power cyclotrons*, *Nucl. Instrum. Meth. B* **113** (1996) 1.

[37] K.R. Symon, D.W. Kerst, L.W. Jones, L.J. Laslett and K.M. Terwilliger, *Fixed-field alternating-gradient particle accelerators*, *Phys. Rev.* **103** (1956) 1837.

[38] L.H. Thomas, *The paths of ions in the cyclotron i. orbits in the magnetic field*, *Phys. Rev.* **54** (1938) 580.

[39] L.G. Zhang and Y.-N. Rao, *Redundant field survey data of cyclotron with imperfect median plane*, in *ICFA Beam Dynamics Newsletter#84 — Dynamics of high power and high energy cyclotrons* (2023).