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Preliminary Draft

Transverse Cooling in Precooler

The merits and implications of including transverse cooling in the precooler are briefly discussed.

I. Reference Case:

Characteristic parameters:

i) $\sim 5 \cdot 10^{-7}\ \bar{p}$ accepted in $\varepsilon_T = (\varepsilon_V \varepsilon_H)^{1/2} \simeq 2.5\pi \cdot 10^{-6}\ m$, $\frac{\Delta p}{p} = \pm 0.02$

ii) $\frac{\Delta p}{p}$ reduced by a factor 7 in ΔT, $2s \lesssim \Delta T \lesssim 3s$

iii) beam spot on target $r_p = 0.15\ mm$,

$\bar{p}$ divergence $\quad x'_{\bar{p}} \simeq 16 \cdot 10^{-3}$

II. Gain from larger ε.

with constant r_p, $\varepsilon \propto x'_{\bar{p}}$. Available information indicates that $x'_p \gtrsim 50$ mrad does not bring substantial further gain.

Approximate $\bar{p}$ yield by

$Y \simeq L\varphi \arctan \frac{L\varphi}{2 r_p^2}$ (for $\beta_{0,\bar{p}} \gg \beta_{0,p}$)

We obtain: $3.4 \lesssim \frac{Y(3\varepsilon_T)}{Y(\varepsilon_T)} \lesssim 5.1$

for $\quad 6\ cm \gtrsim L \gtrsim 10\ mm$

We use simply a factor 3 for the possible increase in yield.

This implies two transverse cooling systems (the use of skew quads may allow working with only one) each reducing ϵ_y, ϵ_x by a factor of 3.

To realize this gain of 3 all cooling times must not exceed that of the reference case.

In general:

$$\frac{1+\alpha}{1+\alpha/3} \leq G \leq 3$$

where $\tau_{unstack,\,decel} = \alpha\,\tau_{cool}$ in the reference case.

<u>Example</u>: if any of the cooling times increase to 6 s (instead of 2) and assuming $\tau_{unstack,\,decel} = 2\,s$, we will skip one MR cycle and G is reduced to 1.5.

III. Rough Performance Estimate for Transverse Systems

The usual equation for transverse cooling is :

$$\frac{d\varepsilon}{dt} = -\frac{W}{N}\left[2g - g^2\left[\Gamma + \eta\right]\right]\varepsilon \qquad (1)$$

Γ = mixing term $\cong Max(1, (h_{max}\,\frac{\Delta f}{f})^{-1} \ln f_{max}/f_{min}) = \Gamma(t)$

$\eta = \frac{\text{noise power}}{\text{signal power}} = \eta(t) = \eta(t=0)\,\frac{\varepsilon(0)}{\varepsilon(t)}$

For simultaneous cooling in all 3 phase planes $\Gamma(t)$ is controlled by the progress of longitudinal cooling. A general solution for (1) can immediately be written down but is not very transparent and optimal gains and cooling times are not obtained without numerical integrations for $\Gamma = \Gamma(t)$, $\eta = \eta(t)$, $\Gamma \sim \eta$.

We distinguish 2 cases:

a) cooling at $\sim$ 4 GeV

b) cooling at $\sim$ 200 MeV, applicable to decelerating precooler

We discuss (i) first (⊥ cooling at 4 GeV)

A system with $W \sim 500$ MHz (250 MHz < f < 750 MHz) appeared rather hopeless. A second case with 1 GHz < f < 4 GHz is discussed in the following.

For the mixing parameter we obtain:

$\frac{\Delta p}{p}$ [10^{-2}]	Γ
$2 \cdot 10^{-2}$	1
10^{-2}	1
$5 \cdot 10^{-3}$	2
$1.5 \cdot 10^{-3}$	4

This case is then (presumably) noise limited, rather than mixing limited:

$$\frac{d\varepsilon}{dt} \simeq -\frac{W}{N}\left[2g - g^2\eta\right]\varepsilon \qquad (2)$$

We invoke the capability to continuously adjust the gain factor to its optimum value:

$g_0 = \frac{1}{\eta}$ leading to

$$\frac{d\varepsilon}{dt} \simeq -\frac{W}{N}\frac{\varepsilon}{\eta} = -\frac{W}{N}\frac{\varepsilon^2}{\eta(0)\,\varepsilon(0)} \qquad (3)$$

with the solution

$$\varepsilon(t)/\varepsilon(0) = \frac{t_o}{t+t_o}, \qquad t_o = \frac{N}{W}\eta(0) \qquad 4)$$

Note: with constant gain $\varepsilon(t) \to \varepsilon(\infty) \neq 0$ exponentially, to reach a given $\varepsilon(t)$ then typically takes ~1.3 to 1.7 times longer than in the adjusted gain case.

For a reduction by a factor 3 we require $\Delta t = 2t_o$

$$\Delta t = \frac{2N}{W}\eta(0) \quad \rightarrow \quad \eta(0) = \frac{\Delta t W}{2N}$$

with $W = 3$ GHz, $2N = 2 \cdot 3 N_r = 3 \cdot 10^8$ we obtain

Δt [s]	$\eta(0)$
2	20
4	40
6	60

These values justify neglecting Γ in the original equation. We must establish the feasibility of obtaining these noise/signal ratios, discuss required kicker power and state lattice requirements for these components.

We analyze η for n PU's following Höhl (CERN PS/DL 78-25):

$$\eta = \frac{\xi}{n} \frac{I_N^2}{I_{pu}^2}$$

where ξ represents signal power loss in the combining network, $\xi > 1$.

$$\eta(0) \simeq \frac{8\xi}{n} \frac{10^{NF/10} kT}{S^2 R e^2 N f_0}$$

with $S = 0.5$, $NF = 1.5\,dB$, $R = 50\,\Omega$, $T = 300$, $N = 1.5 \cdot 10^8$, $f_0 = 0.6\,MHz$, $\xi \sim 2$:

$$\eta(0) \simeq \frac{1}{n} \cdot 3.25 \cdot 10^3$$

And therefore:

Δt [s]	$\eta(0)$	n
2	20	~160
4	40	~80
6	60	~54

Remarks:

i) $S = 0.5$ is an ICE value (see Höhl's paper) and may not be applicable to GHz range PU's

ii) the signal combining business should be explored somewhat more carefully

iii) an actual PU design is not available now, a wall current monitor (difference voltage L-R, U-D) might be a reasonable choice. ~~Etc~~

iv) the gap spacing needs investigation; we assume here ~ 10 cm. This may be fairly conservative, (the AA precooling assembly has gaps spaced ~ 2cm)

<u>case b) low energy (200 MeV ⊥ cooling)</u>

In this case the work of the LBL experiment at the ECR should be directly applicable. At present experimental data are not available but we expect that a reduction of transverse emittances by a factor of 3 should be possible in ≤ 0.5 s.

IV Power Requirements

In the noise dominated regime the power depends on $B\rho$, v, $\mathcal{E}(0)$, t_0 and the specific type of kicker chosen. For a kicker with a gap not much wider ~~than~~ (higher) than the beam we estimate:

$$P_t \sim 3 \, \frac{(B\rho v \mathcal{E}_0)^2}{L^2 f_0 R_K} \, \frac{1}{t_0} \qquad \text{for single kicker}$$

$$\sim \frac{3}{n} \, \frac{(B\rho v \mathcal{E}_0)^2}{L^2 f_0 R_K} \, \frac{1}{t_0} \qquad \text{for } n \text{ kickers}$$

For the low energy case a single TW-kicker ($\beta_w = \beta_{\bar{p}}$) is envisaged and power levels of a few times 100 W (200 to 400) are expected for $\Delta t = 0.4$ s, $f_0 = 0.6$ MHz, $\mathcal{E}_0 = 3 \cdot \mathcal{E}_{0,ref}$. $(A_0 = 3\pi \mathcal{E}_0)$

For the high energy case, using a kicker array of ~ 200 simple plates ~ 2.5 cm long in beam direction $P_t \sim 1$ to 2 kW is estimated.

V. Lattice Requirements

a) Common to all systems:

Sufficient path differences ($S_{beam} - S_{signal}$) to overcome amplifier delays.

High energy: assume approximately circular machine

$$S_B - S_S \triangleq c \cdot \Delta\tau_{amp}$$

$$\theta - 2\sin\frac{\theta}{2} \gtrsim \frac{c}{R}\Delta\tau_{amp}$$

typical $\Delta\tau_{amp} \sim 50\,ns$

with $R = 75\,m$ we need an azimuthal separation θ between PU and K of $\gtrsim 0.55\,\pi$

In the interest of minimizing PU-K mixing the kicker should be as close as possible to the pick-up (but of course satisfy the above constraints). In the longitudinal case the correction may be applied one full turn delayed (← investigate) which removes these constraints.

Low energy (200 MeV)

In this case a safe K-PU separation is:

$$\Delta s_{min} = \frac{\beta}{1-\beta} \cdot \tau t_c \cong 20 \ m$$

b) transverse cooling systems

high & low energy:

i) phase advance. $\Delta\psi_{PU,K} = \frac{2k+1}{2}\pi$, $k = 0,1,2,$

ii) dispersion $\eta = 0$ at horizontal pick-up

high energy.

i) PU - straight : $L \gtrsim 20 \ m$, "high β", $\int_{L_{pu}} \frac{ds}{\beta(s)} \lesssim 10°$

try to "fill" PU of ~ 50mm (roughly)

horizontal & vertical PU's may possibly be combined

requiring then only one such straight

ii) K- straights: 2 (vertical & horizontal)

$L \simeq 10 \ m$, probably best to keep smallest

, b satisfying $\int_{L_K} \frac{ds}{\beta(s)} \lesssim 30°$

low energy..

i) PU: 2 (vertical and horizontal)

each ~ 5m long, $\int_{PU} \frac{ds}{\beta(s)} \lesssim 10°$

both pick-up's may be placed in same straight

(provided suitable kicker locations with correct phase advance are provided)

ii) K-straights:

2 (vertical & horizontal)

each ~ 2m long, again ideally small β,

but $\int_{L_K} \frac{ds}{\beta(s)} \lesssim 30°$

c) longitudinal system

The same system may be used at high and low energy [except notch filter]

i) dispersion: $\eta = 0$ at kicker

ii) Length $L_{pu} \lesssim 20\,m$, $L_K \simeq 20\,m$ } as proposed seems reasonable.

iii) β-function: allow reasonable PU, K - ID
if decelerating with increased acceptance:
try to approach $\beta_{max,pu} \approx \beta_{max,K} \rightarrow L_{pu}, L_K$

VI. Tentative Conclusions

i) a total gain of 1.5 to 3 in $\bar{p}$ accumulation may be feasible using stochastic, transverse cooling

ii) the estimates of PU-sensitivity, questions of electronics and kicker design of a ~ 4 GHz high energy cooling system contain substantial uncertainties

iii) the proposed PU-gap spacings (leading to long straight sections) may be conservative (compare AA-ring); this should (if necessary experimentally) be explored

iv) transverse cooling at low energy (i.e. in a decelerating precooler) appears more convincing, since we have at least bench tests of PU's, will have experimental data soon, estimate cooling times < 0.5 s and require much less straight section length.