(Tuesday Afternoon: Theoretical Session, J. Schwinger presiding.)

Klein presented the first paper in the session on the results which have been obtained by the past year from studying the renormalizable field theories in the low energy limit, that is the theorems on gamma ray scattering, photo meson production, and meson nucleon scattering. These investigations have served to clarify the meaning of the coupling constant in the meson theory, and those experiments which can be expected to determine this coupling constant. The techniques which have been used to date can also be extended to the problem of nuclear forces and other phenomena. Klein first presented a tabular summary of the theorems which have been proved, and then presented a typical demonstration of one of the theorems. These theorems all concern the scattering matrix T(k), where k represents an external boson momentum; T^2 is proportional to the cross section. All the theorems are of the general structure that

$$T_{\ell}(k) = B_{\ell}(k, e, g, \mu, m)\left[1 + 0\left(\frac{k}{m}\right)\right]$$

where T_{ℓ} is the scattering matrix, B_{ℓ} is the Born approximation result, and the mass which occurs is a renormalized mass characteristic of the system. The theorems state that as the momentum of the external boson k goes to zero and the mass of the external boson goes to zero that the scattering matrix is given by this Born approximation result. Thus to carry out a measurement of the coupling constant, one looks for a process which is first order in the electromagnetic or meson field, that is an experiment which is quadratic in the coupling constant. The theorems which have been proved and their authors are listed in Table 1.

Process	Partial Wave	Matrix Element Measured	Author
$\gamma-\gamma$	S (Thomson limit)	e	Thirring
	P	e, μ	Gell-Mann and Goldberg
			Low
$\gamma-\pi$	S } nucleon current	eg	Kroll and Rudermann
	P } nucleon current	$g\mu_p, g\mu_n$	Low; Schwinger and Kle
	meson current	eg	Low; Schwinger and Kle
$\pi-\pi$	P wave	g	Schwinger and Klein

The gamma gamma process is gamma ray scattering or meson scattering from a particle of spin 1/2; Low remarked that the theorems can easily be extended to particles with other spin.

Klein then presented a paraphrase of a unified demonstration of all these theorems given by Schwinger, using as an example the gamma ray theorem. One has for the scattering matrix in this case the expression

$$\delta(p'+k'-p-k)\,(p'k'|T_{\nu\mu}|pk) =$$

$$-\frac{e^2}{(2\pi)^4}\int d^4\xi\, d^4\xi'\, d^4x\, d^4x'\, e^{-ik'\xi'} e^{-ip'x'}\,\bar{u}(p')$$

$$\cdot\,(x'|G^{-1}\left[\frac{\partial^2 G}{\delta eA_\nu(\xi')\,\delta eA_\mu(\xi)}\right]_{A=0} G^{-1}|x)\,u(p)\,e^{ik\xi}e^{ipx}$$

in this expression the δ-function is four dimensional, p and k refer to the proton momentum and photon momentum respectively, G is the Green's function in the absence of the external field. Hence G_{-1} operating on the free particle spinnor u(p) gives zero unless there is some part of the second variational derivative which has a second order pole on the free particle energy shell. If one carries out the variational derivative, one obtains an expression such as that given by Gell-Mann or Goldberger; then it becomes impossible to give a simple proof of the theorem. That is, the proof depends on the simple structure of G in the low energy limit. To continue with the derivation of the Thomson theorem, we consider the free particle Green's function in the absence of the external field which is

$$G = \frac{1}{\gamma p + m} + \delta G(\gamma p)$$

The statement that there is a stable state of motion of the system with mass m implies that the additional contribution δG is analytic in the neighborhood of $\gamma p + m = 0$. In order to obtain the gamma ray scattering to zero order in the frequency, we need only know the dependence of G on a uniform electromagnetic potential. This can be obtained simply by allowing p to be replaced by p−e A. In this case the variational derivative can be replaced by an ordinary derivative and since G is still an analytic function of p−e A, it can give no contribution at $\gamma p = -m$ except from the first term.

In extending the theorem to first order in the frequency, it is more convenient to work with the second order Green's function $2m/(p^2 + m^2)$. Then the theorem states that to first order in the frequency it is sufficient to take

$$G_1 = \frac{2m}{(P - eA)^2 + m^2 - \mu\sigma F/2} \qquad \text{where } \sigma F = \sigma_{\mu\nu} F^{\mu\nu}$$

One can easily see that there would be a contribution to the scattering if there were an additional term of the form $G_1 \, A\sigma F \quad G_1$, that such a term would arise from an expansion to first order of an additional magnetic moment term. Hence if we have defined the magnetic moment term appearing in the denominator correctly, it is impossible for such a term to appear. Of course one also needs that statement that to first order in the frequency, G can only depend on σF and γp; this is not immediately obvious but can be proved by invoking relativistic invariance, gage invariance, and charged invariance. In our expansion of G about G_1, we can fall back on the statement that the next term must be an analytic function of G_1 alone because of our definition of μ. Hence the first term which can occur is $\left[A\sigma F, G_1 \right]_+$ which is second order in the frequency. There cannot be a term in AF. It is possible to have a term in $(\sigma F)^2$, but again in order not to give a contribution to the magnetic moment we can show that the leading term must be $G^2 (\sigma F)^2$, and when one takes a variational derivative this term will not contribute in the order to which we are working.

The essential content of all this is that simply by virtue of our definition of charge, mass, and magnetic moment, we have so determined the structure of G in the neighborhood of the singularity that there can be no further contributions to it. The proof of the p wave theorem for meson scattering is entirely analogous. If one looks explicitly at the structure of the Green's function, one finds that if it is second order in the meson wave function it cannot give p wave scattering, so that the analog to the gamma ray s wave theorem is the meson p wave theorem. One can further show that the Kroll-Rudermann theorem on photomeson production is the consequence of the p wave meson theorem and gauge invariance.

Low then presented a summary of work that he and Chew have been doing. They had first intended to report on the Kroll-Rudermann theorem for p wave scattering, which has already been mentioned by Klein above. They believe this can be used to determine the coupling constant from the p wave meson nucleon scattering. The theorem states simply that

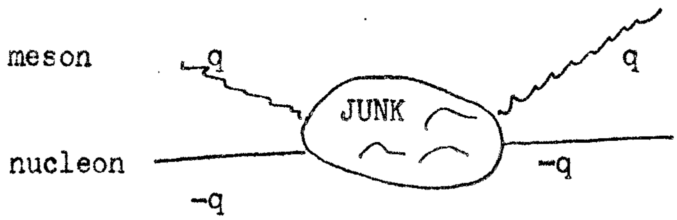
$$e^{i\delta} \sin\delta = \frac{4}{3} f^2 \frac{\eta^3}{\omega} + \text{~~corr~~ CORRECTION}$$

They can prove that the correction term is finite as w goes to zero, and that the coefficient f^2 is the Kroll-Rudermann renormalized coupling constant. The theorem can be stated for all four p states and, including the correction term is of the form

$$\begin{array}{l} \text{State} \\ 11 \\ 13=31 \\ 33 \end{array} \quad \begin{array}{c} \\ \begin{pmatrix} -\frac{8}{3} \\ -\frac{2}{3} \\ \frac{4}{3} \end{pmatrix} \end{array} \qquad \frac{q^3 \cot\delta}{\omega^*} = \frac{1}{f^2}\left[1 - \frac{\omega^*}{\omega_0}\begin{pmatrix} -(1-\frac{x}{2}) \\ -x \\ 1 \end{pmatrix} + \cdots\right]$$

where $\omega^* = (q^2 + \mu^2)^{1/2} + q^2/2M$

In the 33 case it can be proved in the static theory that the coefficient of the correction term is negative. This can also be proved in the relativistic theory, if one neglects pairs and external nucleon recoil. By external nucleon recoil is meant the following. Consider the following diagram.



No matter what occurs in the center of the diagram or how high the momenta, if one can simply drop the term $-q$, which corresponds to the momentum of the nucleon both at the beginning and the end of the diagram, then the sign of the correction term must be negative. Hence if one can somehow find pair damping in the pseudo-scalar meson theory, one can hope to obtain agreement between the theory and experiment. The coefficients of the correction term in the four states as given above are a consequence of the crossing theorem of Gell-Mann and Goldberger as applied to the p wave. By explicit calculation using the cutoff theory, the correction term coefficient x is found to be fairly small. It is presumably small in the relativistic theory also at least at low energy. Hence if the 33 state is enhanced, the 11 state is depressed, and if it is not justifiable to use Born approximation on the 33 state, it is also not justifiable to use it on the 11 state. The above expression is therefore the form which the Kroll-Rudermann theorem and the Gell-Mann Goldberger crossing theorem take for meson nucleon scattering.

So far at least the Kroll-Rudermann theorem for the photo effect does not appear to be as useful, since so far no way has been found to extrapolate it to zero. However it is possible using the cutoff theory to calculate the photo-production explicitly and obtain an expression which, outside of dependence on masses, coupling constant, and magnetic moments, involves only the sine of the measured pion nucleon scattering phase shift. This expression is in remarkable agreement with experiment in the low energy region.

Low then turned to a great calculational advance which has been made in that an easily soluble equation has been found for the pion nucleon scattering. The equations do not produce divergences on interactions so long as pairs are neglected, so that the 11 equation is just as easy to solve as the 33 equation. Unfortunately the equations are non-linear so that at present it is not known whether they possess solutions in general; however in all the special cases that have been tried so far, exact solutions can be found.

For simplicity this equation will be derived using the static theory. In what follows the wave functions which are used are exact wave functions of the entire Hamiltonian. For example an exact incoming wave eigenstate with n mesons and a physical nucleon present will be symbolized by $\Psi_n^{(-)}$ and the matrix element of the interaction we are interested in will be

$$(\Psi_n^{(-)}, V_k \Psi_0)$$

Here $$V_k = \frac{f_0}{\mu} \frac{\vec{\sigma}\cdot\vec{k}}{\sqrt{2\omega_k}} \tau_k v(k)$$

where f_0 is the unrenormalized coupling constant and $v(k)$ is the momentum cutoff function of the theory. The claim is that the S matrix for the scattering is given by

$$(n|S|k) = (n|1|k) - 2\pi i\,\delta(E)\,(\Psi_n^{(-)}, V_k \Psi_0)$$

In order to derive the equation, Low had permission from Wick to use a trick of his which goes as follows. We write

$$\Psi_{\vec{q}}^{(-)} = a_{\vec{q}}^* \Psi_0 + \chi$$

where $a_{\vec{q}}^*$ is the creation operator for the scattering state. Substituting this expression into the Schroedinger equation one obtains directly that

$$\chi = -\frac{1}{H-\omega_q+i\epsilon} V_q \Psi_0$$

Note that although recoil can be included with no trouble, it is impossible to use this trick at present if there are pairs present, since no one knows how to write down correctly the asymptotic eigenstate in this case. Now having obtained an explicit solution, we substitute this into the matrix element giving

$$(\Psi_k^{(-)}, V_k \Psi_0) = (\Psi_0, V_q \frac{1}{H-\omega_q-i\epsilon} V_k \Psi_0) + (\Psi_0, a_q V_k \Psi_0)$$

Since V_k is a function only of σ, τ, k, it commutes with a_q. However, since Ψ_0 is a physical nucleon eigenstate, $a_q \Psi_0$ does not vanish because there are virtual mesons present in the meson cloud surrounding the nucleon. In fact one can show quite easily that

$$a_q \Psi_0 = \frac{1}{\omega_q + H} V_q \Psi_0$$

This expression holds provided that the total Hamiltonian is normalized to be zero operating on a free nucleon state. Thus the scattering matrix becomes

$$T_k(q) = (\Psi_q^{(-)}, V_k \Psi_0) = (\Psi_0, V_q \frac{1}{H-\omega_q-i\epsilon} V_k \Psi_0) + (\Psi_0, V_k \frac{1}{H+\omega_q} V_q \Psi_0),$$

The first term is normal, corresponding to absorption with a normal energy denominator and then emission of the final meson, whereas the second term corresponds to the final meson being emitted before the initial meson is absorbed. In order to evaluate the energy denominators one introduces the projection operators for the scattering eigenstates, that is the eigenstates of the complete Hamiltonian, by writing

$$\sum_n \Psi_n^{(-)} \rangle \langle \Psi_n^{(-)} = 1$$

Note that when we have the terms corresponding to 1 meson scattering states this gives us immediately an integral equation for the matrix element. Note in particular that the k dependence of the matrix element and of the equation is trivial, which leads to a tremendous simplification for the static theory. If in particular we restrict ourselves to the approximation of keeping only those intermediate states in which there is one meson present asymptotically, one obtains an equation for the q state itself involving only the q state. (That is, if one solves this equation once, one has immediately solved for the momentum dependence of the phase shifts over the entire range where the equation is applicable.)

In this one meson approximation, the equation for the phase shift can be obtained immediately and is

$$e^{i\delta_\alpha(q)} \sin\delta_\alpha(q) = \lambda_\alpha \frac{q^3 v(q)}{\omega(q)} + \frac{q^3 v(q)}{\pi} \int \frac{d\omega_p}{p^3} \left[\frac{\sin^2\delta_\alpha(p)}{\omega_p - \omega_q} + A_{\alpha\beta} \frac{\sin^2\delta_\beta(p)}{\omega_p + \omega_q} \right]$$

The term in this equation which couples the phase shifts of different momentum and isotopic spin is the term which enables this equation to satisfy the Gell-Mann-Goldberger crossing theorem. That is, thanks to this term if you interchange q and k and the isotopic spin τ_q and τ_k and change the sign of ω, the equation is reproduced. Note that in the static theory the 13 and 31 scattering states are identical so that there are only 3 phase shifts. The relativistic theory without recoil has the same property since in Born approximation one has this property and the Gell-Mann-Goldberger theorem insures it for the correction term. The matrix coupling the different phase shifts is

$$A_{\alpha\beta} = \begin{array}{c} \\ 11 \\ 13 \\ 33 \end{array} \begin{array}{c} \begin{array}{ccc} 11 & 13 & 33 \end{array} \\ \begin{pmatrix} \frac{1}{9} & -\frac{8}{9} & \frac{16}{9} \\ \frac{2}{9} & -\frac{4}{9} & \frac{7}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \end{pmatrix} \end{array}$$

and the coefficient is given by Born approximation, that is

$$\lambda_\alpha = \begin{pmatrix} +8/3\ f^2 \\ -2/3\ f^2 \\ 4/3\ f^2 \end{pmatrix}$$

Note that the Gell-Mann-Goldberger crossing theorem states that λ_α must be an eigenvector of $A_{\alpha\beta}$ with eigenvalue -1; that is, for even powers of ω the eigenvalue is $+1$ and for odd powers -1. If one now substitutes into this equation the state in which no mesons are present one obtains simply $(\Phi_0, V_q \Phi_0)$;

but according to all the standard methods of renormalization we have $(\Phi_0, f_0\, \sigma\cdot\tau\, \Phi_0) = f(u, \sigma\cdot\tau u)$.

Hence the equation we have written down above contains the renormalized coupling constant. If the coupling term $A_{\alpha\beta}$ is dropped the equation can be solved exactly, with one numerical integral. This numerical integral turns out to be the original one Chew wrote down to give the phase shifts, which was presented at this conference a couple of years ago. That is Chew's original equation in the zero order approximation in the iteration solution of this equation.

The equation which can be solved exactly has the form

$$h_\alpha(q) = h(\omega_q) = \frac{f(\omega_q)}{\pi} \int \frac{d\omega_p}{f(\omega_p)} \frac{|h_\alpha(p)|^2}{\omega_p - \omega_q - i\epsilon}$$

Its solution is simply

$$h(\omega) = f(\omega) \Big/ \left[1 - \frac{1}{\pi} \int d\omega_p \, f(\omega_p) / (\omega_p - \omega) \right]$$

The approach to solving the equations with the coupling term $A_{\alpha\beta}$ present is to make a reasonable guess at the effect of the coupling and put this into the Born approximation for the non-linear equation; then an equivalent equation can be found which can be solved. It is possible to do the integrals, so that relatively complicated guesses can be done quite easily and the consistency of these checked. This in fact is how the one meson approximation results given in the pion session by Chew were obtained.

It is also possible to find a simple equation for the photo meson production problem and to solve it in a very good approximation. The matrix element which is calculated is $\left(\Psi_q^{(-)}, \int \vec{j}(x) \, e^{i\vec{k}\cdot\vec{x}} \, \Psi_0 \right) = H'$

and is given in this approximation by the expression

$$H' = \frac{ef}{\mu} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \left\{ \vec{\sigma}\cdot\vec{e} + \frac{2\vec{\sigma}\cdot(\vec{k}-\vec{q})\,\vec{q}\cdot\vec{e}}{2\omega^2(1 - v_n \cos\theta)} \right\}$$

$$+ \sqrt{2}\, \frac{ef}{\mu} \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \left[2i\vec{q}\cdot\vec{k}\times\vec{e} + \vec{\sigma}\cdot\vec{q}\times(\vec{k}\times\vec{e}) \right] \frac{(\mu_p - \mu_n)}{12} \left(\frac{\mu}{M} \right) \frac{e^{i\delta_{33}} \sin c}{f^2 q^3}$$

The two terms in the curly braces are the Born approximation: the first of these is the Kroll-Rudermann term and the second is the term which comes from the p wave low energy limit theorem. The only important correction to this result is given in the third term and arises from the 33 state. Low believes that this result is, in fact, much more general than the one meson approximation. Two meson corrections were estimated for the scattering in the forward direction in Born approximation, and are about 15%, but so far these corrections have not been estimated for the photoproduction problem. The anomolous magnetic moments of neutron and proton appear in the formula because the calculation was done using the relativistic theory and the entire current between the two nucleon states; the terms which gave rise to this coefficient then could be identified as the static moment of the nucleon to order $(v/c)^2$. Low does not believe this is the same as introducing a Pauli moment into the calculation since that would give transitions to negative energy states, which have not been allowed for here.

To summarize what has been proved by Low and Chew: first it has been shown that if the function of the phase shifts given above is extrapolated to zero for all four p states, this should give the same f^2; second if one can neglect pairs in the relativistic theory then the slope of the extrapolated straight line has the correct sign to give the resonant solution for the 33 state; third it has been proved that a third parameter is needed to give the phase shifts in this linear approximation for the other three p states. Finally this calculation makes it much clearer why the cutoff theory is in such good agreement with the experiment, since the two free parameters of the theory can be used to give the correct intercept and slope for the effective range function of the 33 state.

Serber then reported on a calculation of Lee and Friedman which is a different approximation to the same mathematical problem tackled by Chew and Low, that is the cutoff meson theory. They are interested primarily in doing a calculation of multiple meson production in the Bloch-Nordsiek approximation but in order to get parameters for this calculation, they have calculated the Tomonaga approximation to meson scattering with a finite source. The radial dependence of the wave function is factored out, and its Fourier transform is

$$f(|k|) = N\,k\,u(|k|) \Big/ \sqrt{\omega_k}\,(\omega_k+\lambda)$$

Here N is the normalization constant chosen so that $\sum_k |f(|k|)|^2 = 1$; $u(k)$ is a square cutoff function, and the parameter λ is determined by a variational equation. In order to fit the 33 scattering, the cutoff had to be chosen as 6.2μ, the unrenormalized coupling constant was 0.712, the ratio $Z_2/Z_1 = 0.384$, and hence the renormalized coupling constant is 0.105. It is interesting to recall that in the extreme strong coupling limit Z_2/Z_1 is 1/3. In fact, strong coupling is a good approximation to this part of the problem. This is similar to other results, such as for example Watson found in scattering, that the strong and the weak coupling approximation give the same result except for a numerical factor. Also the location of the peaks of the wave functions agrees with the strong coupling result; however, the wave functions are wider and have longer tails. On the other hand, since for the weak or strong coupling limit the parameter λ is zero, this result is rather far from either limit in that sense.

In order to calculate the anomalous magnetic moments, the electromagnetic field is introduced in a gauge invariant way. The particular method chosen is such that only the term $(J_{meson} \cdot A)$ contributes to the calculation of the anomalous moments. The result is $\pm$ 1.48. If this calculation were done the way Pauli and Dancoff did it, you would add .16 to this result. The strong coupling limit would give the anomalous moment of the proton as 2.76; the difference here is primarily due to the fact that the parameter λ is 3.39 instead of zero. The average number of mesons in the field is of the order of two, but because of the tail on the wave functions the probability amplitudes for more mesons in the field are important.

The results for the p phase shifts as the function of energy are given in Table 2.

T_{lab}	70 Mev	120	140	169	187	210	217
δ_{33}	9.69°	33.19°	52.67°	74.75°	87.75°	100.2°	101.2°
$\delta_{13}=\delta_{31}$	-0.35°		-1.02°	-1.36°	-1.60°	-1.91°	
δ_{11}	2.50°		5.09°	5.72°	5.99°	6.20°	

If one makes a plot of α_{33} using these values and the Chew-Low function, this does not give a straight line and does not extrapolate back to the correct value of the coupling constant. Another anomaly is that α_{11} is positive, and therefore to satisfy the low energy limit theorem and also the Gell-Mann and Goldberger crossing theorem, will have to change sign at some lower energy. Hence these results either indicate that effective range theory is a bad approximation to this problem, or that the Tomonaga approach does not yield adequate results.

Feynman then suggested another approximation approach to this problem. If one neglects nucleon recoil and so on as is being done here, then you are calculating the properties of the following operator

$$\exp - \iint (\sigma_i \tau_j)_t \, (\sigma_i \tau_j)_s \, F(t-s) \, dt \, ds$$

where

$$F(t-s) = \int e^{i\omega_q t} Q^2 c(Q^2)/\omega_q \, d^3Q$$

The Tomonaga approximation consists of replacing this function by an exponential. However, if you actually plot the logarithm of this function, you find that it is made up of the sum of two exponentials, one of which has a very rapid variation determined essentially by the cutoff mass, and superimposed on this a very slowly varying weak tail. Therefore it would seem reasonable to use the exponential approximation for the strongly varying part of the function, which you are not interested in, and after you have taken care of this, treat the weak tail as a perturbation; all the difficulty of the problem and all the labor is involved in the first part which is precisely the part you are not actually interested in.

Breit then reported on an investigation of the corrections to be expected in the coulomb scattering of two protons due to relativistic effects. If one were to calculate in the center of mass system and ignore pairs, presumably one should obtain Møller's matrix element. However if one tries to do this it is not so immediate. One has the equation

$$\left[E(p_1 p_2) - E - i\epsilon\right] C_{p_1 p_2} = \int M_{p_1 p_2 p_1' p_2'}\, \delta(\qquad)\, dp_1'\, dp_2'$$

This would be easy to solve were it not for the fact that the Møller matrix element M contains factors like $1/(p_1 - p_{1'})^2$. Using Dirac's method for calculating the wave function in momentum space and transforming to coordinate space one has $\int e^{ikr}\, d\Omega_k / (k-k_0)^2$. Expanding this one obtains

$$4\pi \sum i^k P_L\left(\frac{\vec{k}\cdot\vec{r}}{k_0 r}\right) \frac{F_L(kr)}{kr}\, Q_L\left(\frac{k^2+k_0^2}{2kk_0}\right)$$

Hence when $k = k_0$ there are quadratically divergent terms; so if one tries to solve by an iteration procedure, one would never obtain the phase shifts characteristic of the old-fashioned Gordon-Mott solution in the non-relativistic limit. One could presumably introduce an artificial coordinate intermediate variable and solve the problem in much the same way as was done by Mott and Gordon in the non-relativistic case. However it is simpler to use a connection between the energy and the phase shifts. If one has a quantizing sphere, then the number of wave lengths of the wave functions inside this sphere will be very large compared to the radius of the sphere. If one introduces a small shift in the energy, then in order to keep the same number of nodes in the wave functions and hence the same quantum numbers one gets a connection between the phase shifts and the energy. To first order one obtains a characteristic asymptotic form for the coulomb wave functions namely

$$\sin\left(kr - \eta \ln 2kr - L\tfrac{\pi}{2} + \arg\Gamma(L+1+i\eta)\right)$$

Hence all that is needed to solve the problem is an expression for the change in energy. To first order this change in energy is simply given by the matrix elements of $\frac{1-\vec{\alpha}_I\cdot\vec{\alpha}_{II}}{r}\cos\frac{2\pi r_{pp'}}{\lambda_{st}}$. If one uses wave functions of the same energy, λ is infinite so that the cosine factor does not appear. One obtains in this way a change in phase given by

$$\delta K \cong -\frac{E_I^2+P^2}{\hbar P E_I}\int F_L^2\frac{e^2}{r}\,dr\,;$$

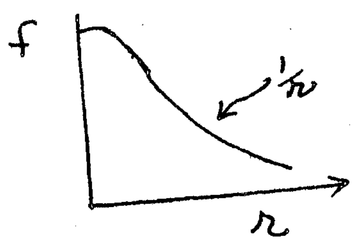
hence one has that

$$\eta_{rel} = \frac{E_I^2+P^2}{2\hbar P E_I}e^2 = \frac{e^2}{\hbar v'},$$

where v' is the velocity of the first proton in the laboratory system, a result already deduced by Garren. This is all there is to it for singlet states.

However for triplet states, one has

$$\int\left(\tilde{\Psi}^*\frac{1-\vec{\alpha}_I\vec{\alpha}_{II}}{r}\psi\right)d\vec{r} \rightarrow \int\tilde{\Psi}^*\Big\{\Big\{\frac{1}{r}+\frac{P^2\frac{1}{r}P^2}{(E_I+M)^4}+\frac{3}{(E_I+M)^2}\left(P^2\frac{1}{r}+\frac{1}{r}P^2\right)$$
$$+\frac{2\hbar}{(E_I+M)^2}\left[\!\left[\frac{3f'}{r}+\frac{1}{(E_I+M)^2}\left[\frac{f'}{r}P^2-\frac{\hbar^2\partial(\frac{f'}{r})}{\partial r}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\right]\right]\!\right](\vec{L}\vec{s})-\frac{2\hbar}{(E_I+M)^2}\left[\left(\frac{f'}{r}-\Delta f\right)s^2+\right.$$
$$\left.\frac{(\vec{r}\vec{s})}{r}\frac{\partial(\frac{f'}{r})}{\partial r}\right]+\frac{2\hbar^2}{(E_I+M)^2}\left[\!\left[\frac{\partial(\frac{f'}{r})}{r\,\partial r}\left\{\vec{L}(\vec{L}\vec{s})\vec{s}+i\hbar\left[(\vec{r}\vec{s})(\vec{p}\vec{s})-(\vec{r}\vec{p})s^2\right]\right\}\right.\right.$$
$$\left.\left.+\frac{f'}{r}\left[p^2s^2-(\vec{p}\vec{s})^2\right]\right]\!\right]\Big\}\Big\}\,\Phi\,d\vec{r}$$



If one disregards (temporarily) $\left[\vec{P},\frac{1}{r}\right]$ and setting $\vec{\xi}=\frac{\vec{P}_I}{E_I+M}$ the combination $1+\xi^4+6\xi^2=\left(\frac{2E_I}{E_I+M}\right)^2\left[1+\frac{P^2}{E_I^2}\right]$ reproducing the factor for singlets giving

$$\eta_{rel} = \frac{E_I^2+P^2}{2\hbar P E_I}e^2 = \frac{e^2}{\hbar v'}$$

by means of

$$\delta K \cong -\frac{\sqrt{M^2+\hbar^2K^2}}{\hbar^2 K}\int\frac{E_I^2+P^2}{E_I^2}\left(\tilde{\Phi}^*\frac{e^2}{r}\Phi\right)d\vec{r}$$

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The essential point to note about this result is that in the triplet scattering there are L.S and tensor-like terms which can couple states of different angular momentum.

Yang raised the question of whether it is true that every conservation law is related to a gauge transformation. This has already been discussed in a published paper in connection with isotopic spin conservation. The essential point is that the conservation law is related to invariance under certain transformations, which implies that there is some indeterminacy in the phase. What has to be asked is whether this indeterminateness of phase should have a local character. This idea can also be applied to the conservation of heavy particles. If you asked what group of transformations generates the conservation of heavy particles the simplest one is the transformation

$$\psi_p \to e^{i\alpha}\psi_p\,, \quad \psi_n \to e^{i\alpha}\psi_n$$

where the phase is the same for both neutrons and protons. If the phase depends on space time it is easy to show and in order to preserve invariance, one needs to introduce a vector field with zero charge and zero mass. In this case there is no complication such as the non-linear terms which arose in the case of isotopic spin. The consequence of this vector field is a heavy particle number associated with any system and a repulsive force between any two objects. If the number of nucleons in the two objects are A_1 and A_2 this force plus the gravitational force will then be

$$g^2 \frac{A_1 A_2}{r^2} - G^2 \frac{M_1 M_2}{r^2}$$

Since M and A are not strictly proportional, the observational consequence of this would be in apparent difference between gravitational and inertial mass for different objects. This was studied experimentally quite extensively by Eotvos up to about 1920 and he found that the ratio between gravitational and inertial mass is constant for all objects with an accuracy of 1 part in 10^{-8}. Since the packing fraction in nuclei varies by a factor of 1000, this means that the repulsive force postulated here can be at most only 10^{-5} of the gravitational force. This accuracy could be sharpened by comparing objects with a very large difference in packing fraction directly, such as hydrogen and oxygen. The negative experimental results seem to indicate either that the idea that every conservation law is associated with a gauge transformation is groundless, or that one needs to look for another type of group to generate the constitution of heavy particles. In reply to a question of Feynman's, Yang noted that the coupling constant associated with this force cannot go to zero since then the phase, which is proportional to the coupling constant, vanishes and one cannot preserve the invariance. In response to a question of Breit's as to whether this violates the principle of equivalence, Oppenheimer noted that there is no difficulty here since the transformation law obeyed by this field is different from that of the gravitational field. He noted however that there might be a much more sensitive test, in that with a vector field velocity dependent effects could be quite large in nuclei, and these might contribute terms which would go the other way from the static term given here. Foldy raised the point that since the sun is largely composed of hydrogen, one might also be able to find a sensitive test in astronomical effects.

Fierz had some remarks to make on the dangers of using certain models to give a clue as to the behavior of quantum electrodynamics. For example Thirring has shown that part of the propagation function of the theory whose Hamiltonian is $H = (\text{grad}\,\phi)^2 + \phi^2 + g\,\phi^3$

cannot be expanded in powers of the so-called coupling constant. However one can see very immediately that such a theory cannot be developed at all in powers of the coupling constant. In such a theory the classical limit is contained, so one can simply look at the classical theory. The energy of such a field is not positive definite, so it is always possible to make the system unstable. One can write down the solutions of the classical theory and show in fact that they can never be developed in powers of the coupling constant. If one used instead a $G\phi^4$ coupling term, then the energy is positive definite but $G = 0$ is an essential singularity of the theory, since if the sign of G is reversed one again gets instability. However one can't conclude anything about quantum electrodynamics from such a model. In the Pauli-Weisskopf theory the energy essentially must be positive definite no matter what you assume about the value of the charge. Because of gauge and energy conservation the fields are always limited. However in Dirac theory there is no classical limit and such arguments cannot be used. So Fierz wanted to warn that it is probably a bad idea to try to draw any conclusions about quantum electrodynamics from such models.

<u>Feynman</u> talked about possible ways you might start to calculate the specific relativistic theory with the interaction given by $g\,\bar{\psi}\,\gamma_5\,\tau\cdot\phi\,\psi$. There is lots of evidence that this theory is wrong and Feynman doesn't believe it, so this is presented in case someone else wants to calculate this theory. In the first place the existence of the strange particles shows that the interaction between π meson and proton is not completely described by this theory, to put it mildly. Secondly the successes of Chew have shown that nature is correctly represented if the interaction is predominately through P waves and the S interaction is extremely small. Yet in this theory almost certainly the S wave will be large. Further Feynman does not like the theory because it is so close to electrodynamics; you just replace γ_μ by γ_5 and introduce a mass into the propagator and nature certainly has a better imagination than that. Finally all honest attempts to solve this theory which don't start with perturbation theory and expect to improve on what you start with (which is evidently false with the coupling constant of the order of 10) but start at the other end, fall flat on their face after a few stages, and one comes to the conclusion as Wentzel has that the consequences of this theory, if they could be worked out, would be completely different from what nature looks like.

The specific approach Feynman reported on was a way to include the contribution due to closed loops before you start the calculation. This is usually ignored in the Tamm-Dancoff calculations which have been done, but can be handled in the following way. The usual action used in the theory is

$$8\pi S = \int \frac{1}{g^2}\left[(\nabla\phi)^2 - \mu^2\phi^2\right] d^4x$$

where the wave function that occurs here is the wave function customarily used multiplied by the coupling constant of the theory. Usually one starts with this action and then calculates the effect of closed loops; this is equivalent to using another action in calculating without the inclusion of closed loops. Hence if this new action is very different from the old one, this takes the carpet out from under anyone who calculates ignoring any closed loops. Consider for example a particle moving from point 1 to point 2 through a fixed (i.e. not quantized) external field. If we denote the action with no field by $T_0(\phi)$, then we have

$$T g^2 = \int T_0(\phi)\, e^{iS(\phi)} \mathcal{D}\phi = \int T_0'(\phi)\, e^{iS(\phi)}\, e^{L[\phi]} \mathcal{D}\phi$$

whereby T_0' we mean the action with no closed loops present and

$$L[\phi] = \int F(\phi, x)\, d^4x$$

Here F is the amplitude to go around any loop completely. If ϕ is small compared to M, then it is a good approximation to expand in power series in ϕ and all the gradiant terms are easily included. On physical grounds one would expect that, if this theory is ever going to work, in some region the gradiant of the wave function should be of the order of μ. For example for the usual static theory the wave function is the order of μ and the ratio of the gradiant of the wave function to the wave function itself is also of the order of μ. The loop contribution is essentially an average of ϕ over a region of the order of $\frac{1}{M}$, so that the first approximation is to assume $\phi =$ constant. If one calculates this one obtains

$$S' = \frac{1}{g_0^2}\left(\phi_\mu^2 - \mu_0^2\phi^2\right) + \frac{8}{\pi}\Big[\left(\Lambda^2 - m^2\ln\frac{\Lambda^2}{m^2}\right)\phi^2 - \frac{1}{2}\phi^4\ln\Big(\frac{\Lambda}{m}$$
$$+\frac{1}{2}\left\{(m^2+\phi^2)^2\ln\left(1+\frac{\phi^2}{m^2}\right) - m^2\phi^2 - \frac{3}{2}\phi^4\right\}\Big]$$

The first correction terms due to the gradient are

$$\frac{8}{\pi}\Big[\Big(\frac{1}{2}\left(\ln\frac{\Lambda^2}{m^2}-1\right) - \frac{1}{2}\ln\left(1+\frac{\phi^2}{m^2}\right)\Big)\phi_\mu^2 - \frac{1}{3}\,\frac{(\phi\cdot\phi_\mu)^2}{m^2+\phi^2}$$

Feynman has also calculated the fourth order corrections, although these were not given.

What are the consequences of having this new action? One had hoped that in some crude sense one would have a new action of the form

$$S' = \frac{1}{g_1^2}\left(\phi_\mu^2 - \mu_1^2\phi^2\right.$$

where g_1 and μ_1 are new constants; but does the new action in fact look like this? Note that the constants which appear here are not yet the experimental constants since further renormalizations have to be carried out, but one can get some idea, presumably, of what is going on if one substitutes the experimental constants in the above expressions. One finds that in fact the results are very different from what one would hope to find and it is sheer wishful thinking to believe that the next renormalization operation would bring about agreement between the modified action and what we are looking for. If one introduces the cutoff, then the new constants are related to the unrenormalized ones by matching the terms as ϕ goes to zero and as the gradient of ϕ goes to zero. These give the two expressions

$$\frac{\mu^2}{g_1^2} = \frac{\mu^2}{g_0^2} - \frac{8}{\pi}\left[\Lambda^2 - m^2\ln\frac{\Lambda^2}{m^2}\right]; \quad \frac{1}{g_1^2} = \frac{1}{g_0^2} + \frac{8}{\pi}\frac{1}{2}\ln\frac{\Lambda^2}{m^2}$$

However if one actually adjusts the new coupling constant in order to fit these equations, then one finds that it is no longer true that to the sixth and higher terms are negligible, that is if you carry out the renormalization you make the original action so small as to allow the correction terms to completely dominate the desired result, and have completely changed the physics of the situation. This situation is illustrated in Table 3 where the renormalized Lagrangian is given for certain order of magnitude for the field and the gradiant of the field in comparison with the various closed loop contributions.

$\sim\phi$ $\sim\nabla\phi$		μ μ^2	μ μM	μM	M^2
Renormalized L	ϕ^2 $(\nabla\phi)^2$	1/16000 1/16000	1/16000 1/400	1/400 1/400	1/400 1/10
Closed Loops	ϕ^2	1/10	1/10	4	4
	$\nabla\phi^2$	1/1600	1/40	1/40	1
	ϕ^4	1/1600	1/1600	1	1
	ϕ^6	1/128,000	1/128,000	1	1
	$(\phi\nabla\phi)^2$	1/64,000	-	1/40	1
	$(\nabla^2\phi)^2$	1/64,000	1/40	1/1600	1
	$(\nabla\phi)^4$	1/64,000	-	-	1

It is clear from this that unless something very peculiar happens with the next renormalization the situation is hopeless. Dyson commented that he had approached the theory of nuclear saturation using very similar ideas and had also run into precisely this kind of situation and these kind of numbers. Feynman summarized by saying that almost certainly if you permit a cutoff and choose it to be, say, of order of 2M and require g^2 to be positive and allow any theoretical mass, then he believes you can disprove the relativistic γ_5 meson theory.

Peierls reported on some general considerations about what the general features of the propagator in the meson nucleon theory must look like, in particular as to where its singularities must lie. One knows of course to begin with that it is the function of γp. In the complex plane it has a pole at M, and also at $M + \mu$ it must have a branch point, where one is at threshold for single meson production, a branch point at $M + 2\mu$, which is threshold for double meson production and so on. If the coupling is strong enough so that there are real isobaric states in the theory, then these will contribute either poles or branch points as well depending on what the selection rules are. What happens if one has virtual isobaric states with a finite lifetime and hence a complex energy? It would seem obvious, off hand, that these should also give poles in the complex plane. However one has against this the following arguments: that in the first place the upper half plane is free of poles as Lehmann has proved, and secondly that one can see from the way the equations look that the whole plane is symmetric about the real axis, so one concludes there can be no poles in the whole complex plane, except along the axis. Thus it seems there are two contradictory conclusions, but these can be reconciled if one remembers that one is dealing with ambiguous functions which involved a square root. While one gets no poles in the functions as they appear in the expressions considered here, if one continues these functions instead on to the other portion of the Reimann surface, it turns out that one indeed finds the poles there corresponding to virtual isobaric states. Some of these poles are fixed and others depend on the resonance levels of the system. It is important to know where these poles are, of course, since they can very drastically limit the region in which expansions are possible.

Peierls then reported on some work of Salam very briefly, and a more detailed writeup is given here.

The investigation which I wish to describe started from the work of Edwards and Peierls, who set out to solve the functional differential equations proposed by Schwinger (Proc. Nat. Acad. Sciences) for the one particle Green's Function. They succeeded in obtaining a solution to the equations if all closed loops are neglected. Dr. Mathews and myself tried to include them and found that the simplest procedure is to start from the definition of the Green's function in terms of Feynman's "sum over paths" formulation, rather than from the differential equation approach. In Feynman's formulation, the Green's function

$\langle 0 | \psi(1)\, \overline{\psi}(2) | 0 \rangle$ is defined as

$$\frac{1}{N} \int \psi(1)\, \overline{\psi}(2)\, e^{iI}\, \delta\psi\, \delta\overline{\psi}\, \delta\phi$$

The integral on the right denotes summation over all fields ψ , $\overline{\psi}$ and ϕ ; I is the total action

$$I = \int (\mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{int})\, d^4x$$

N is a normalizing constant, which equals $\int e^{iI} \delta\psi\, \delta\overline{\psi}\, \delta\phi$. The fields ϕ and ψ are to be treated as classical quantities, except that the fermi fields have a prescribed order and two such fields anticommute. The ψ and $\overline{\psi}$ integrations can readily be performed giving the final result:

$$\frac{1}{N} \int S_F(1,2;\phi)\, e^{iI_b}\, D(\phi)\, \delta\phi$$

Here $S_F(1,2;\phi)$ is the one particle Green's function in an external field ϕ, I_b is the action for a Bose field ϕ and $D(\phi)$ the vacuum to vacuum transition amplitude in our external ϕ field. $D(\phi)$ turns out to be the Fredholm determinant for an integral equation with the kernel $g\, | S_F(1,2)\, \phi(2) |$. (Here we assume $\mathcal{L}_{int} = g\, \overline{\psi}\psi\phi$, for simplicity) The result obtained by Edwards and Peierls, in which closed loops were neglected, is obtained from the present result in replacing $D(\phi)$ by 1.

The n particle Green's function, likewise turns out to be

$$\frac{1}{N}\int \mathrm{Det}\,[S_F(i,j,\phi)]^n e^{iI_0} D(\phi)\,\delta\phi$$

The Fredholm determinant can be written in the form of an exponential $e^{-L(\phi)}$ and this is the form which Feynman used in his talk. This representation is certainly valid if $g^2\int|S_F(1,2)\,\phi(2)|^2 d1\,d2 < 1$.

Peierls reported on some work that Edwards has been doing along these lines. Since in order to use the above approach one needs to know the Green's function in an arbitrary meson field, this approach would at first sight appear hopeless for an actual calculation, but it is in fact not quite so hopeless as it seems. Starting with the simplest case, that is the netural scalar theory with no recoil, one can in fact obtain the linear differential equation of the first order which can be solved exactly, and when the solution is introduced into the functional integration, this can also be carried out exactly. This is the only case where this is possible, however. When one looks at the charged scalar theory with no recoil one cannot write down the solution as a function of ϕ, but it is useful as in ordinary integration to introduce a new variable for ϕ. If one in fact introduces a part of the solution as the variable of integration, the integral becomes an explicit functional integral which still cannot be done exactly; however one can make further algebraic transformations of the remaining variables. The first such transformation gives in fact an exact solution for the case of both weak and strong coupling and hence something that should be reasonable in the intermediate region as well. The qualitative properties of the singularities turn out to be what one would expect from the discussion above. It appears that a similar method will be workable for pseudoscalar theory with recoil, only the algebra gets very much more complex. Klein commented that the functional integration could also be carried out for the pair theory, but that the separation of renormalization effects in this case was less explicit; it did have certain advantages, however, in that the scattering could be obtained explicitly. Peierls noted that in the cases he has studied, if one gets anywhere near an explicit solution one can very easily recognize the renormalization terms.

Oppenheimer added a brief comment about what Peierls had said about the propagator and what Feynman had said about how violent the renormalization program is. This is in connection with theories for which the renormalization can be done explicitly and in which when the cutoff goes to infinity the connection between the old coupling constant and the new is not a happy one. In the case considered by Lee one gets singularities which are not of the kind that Pais and Uhlenbeck worried about and not of the kind that Peierls has worried about. These singularities happen to be tremendous; they move away as the coupling constant goes to zero but with a finite coupling constant they are still around. This is another illustration of the fact that when you go into a theory in a violent way with strong renormalization you have no guarantee that the propagator will retain the analytic character which Peierls described and which is needed.

Feynman remarked that a student of his had calculated the interaction of scalar nucleons with mass zero scalar mesons, and if the self energy of the nucleon is calculated including recoil but with a cutoff, this self energy goes negative for large coupling constants.

Lehmann started his presentation by remarking that in spite of the various statements we have had that something is way wrong with the existing field theories, he feels it is important to find out just where the trouble is in order that this part of the theory can be changed if possible. The problem is complicated since even in the renormalizable theories one has to split off infinite terms. It is therefore difficult to discuss whether solutions to these basic equations exist. He has tried to reformulate the theory so that the theory contains only finite parameters from the start and no renormalization is necessary. In order to do this, it is necessary to avoid use of the field equations in terms of the Hamiltonian function. In theories with no stable bound states, one can express all relevant quantities by means of the vacuum functions, that is the vacuum expectation values of time ordered operators. Usually one sets up equations for these functions using the field equation and they contain renormalization constants. However Lehmann has found it possible to set up these equations from general requirements of Lorenz invariance the causality requirement; that is, that the field operators commute at space like points, and an asymptotic condition on the matrix elements of the field operator in the remote past and the remote future. The advantage of introducing these assumptions explicitly is that one can then relax on some of them if something goes wrong. One expects that if these equations for the vacuum functions have solutions at all, they will have very many solutions since the interaction has not been specified explicitly. He finds a set of time ordered equations which are non-linear; these have some similarity to those found by Low for the meson theory which we have already discussed in this session. (It was made clear in subsequent discussion that although the Low equation as given above is for a specific interaction linear in the meson field, it is readily generalizable to other interactions and can be written down without specifying these in detail.) In order to obtain unique solutions, one can impose some condition on these functions in addition to those general requirements stated above, for example the asymptotic form in momentum space. It is then possible to make a power series expansion of the solution and it can be shown that this power series expansion is identical with the renormalized formulation of field theory. Lehmann feels it should be possible to derive the renormalized expressions from some set of finite equations directly. The real problem, of course, is to find a general proof that one can have solutions which are not restricted to perturbation theory expansions. Lehmann feels this discussion might well be facilitated by having finite equations to work with. In connection with the previous discussion on difficulties with the singularities in the propagators, Lehmann mentioned the case where in addition to the pole corresponding to the real mass of the particle, there is another pole corresponding to a "ghost state" with a mass given by

$$m - \mu\, e^{1/g^2_{\text{Renorm}}}.$$

where the coupling constant appearing in this expression is the renormalized coupling constant. It would be interesting to know if this case corresponds to the situation in quantum electrodynamics. Of course no one has solved this problem in general, but one can treat an approximation in which this can be worked out. In Lees' example where this happens, the exact propagation function is given by the sum of a series of simple bubble diagrams. One can use the same approximation in quantum electrodynamics and one then finds also that for any e^2 such a pole occurs. This could be a possible source of trouble in the theory.

(End of theoretical session.)