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IMPACT OF THE RECOIL SCHEME ON THE ACCURACY OF ANGULAR-ORDERED PARTON SHOWERS

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Abstract

In these proceedings we present three possible interpretations of the ordering variable implemented in the `Herwig7` angular-ordered parton shower. Each interpretation determines a different recoil-scheme prescription and we show how it can impact the logarithmic accuracy of the algorithm. We also present comparisons with LEP data.

1 Introduction

General Purpose Monte Carlo (GPMC) generators are fundamental tools for collider phenomenology, as they are able to simulate fully realistic collider events, describing both inclusive and exclusive distributions with high accuracy. GPMC involve several components. The event generation starts with the computation of the scattering process at some hard scale, Q, at a fixed order in perturbation theory (usually at least NLO QCD). The event is then fed to a Parton Shower (PS) algorithm, which handles the emissions of soft and collinear partons. The PS thus evolves the system from the hard scale, Q, down to a soft scale, Λ. At this point we enter the non-perturbative regime: QCD interactions are so strong that the coloured partons are forced to form colour singlets, *i.e.* they hadronize. To properly simulate hadron colliders, we also need to provide a model of the *underlying event*, *i.e.* secondary interactions between initial-state partons that do not participate in the hard interaction.

In these proceedings we will focus on the PS component, which provides a bridge between the perturbative and non-perturbative regimes of QCD and allows the multiplicity of particles in the event to increase, which is a key requirement for performing realistic simulations of collider data. To achieve this task, the PS exploits the factorisation properties of QCD in the soft and collinear limits. When a soft-collinear gluon is emitted from a parton i, the cross section is enhanced and behaves like

$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{\pi} 2C_i \frac{d\epsilon}{\epsilon} \frac{dp_T}{p_T}, \tag{1}$$

where $d\sigma_n$ is the differential cross-section for the production of n particles, C_i is the colour factor associated with the emission from parton i (C_A if i is a gluon, C_F if it is a quark), ϵ is the energy fraction carried by the gluon and p_T is its transverse momentum with respect to the emitter. From eq. (1) we clearly see that we can have two sources of logarithmic divergence: one associated with $\epsilon \to 0$ and one with $p_T \to 0$. Thus, when we generate m emissions we can have at most $2m$ logarithms: these are the leading logarithms (LL). It is a common belief that all of the available PS are able to resum such logarithms since the splitting kernels that are employed to mimic the emission of a gluon always approach eq. (1) in the soft-collinear limit. Many efforts have been made towards reaching next-to-leading log (NLL) accuracy, *i.e.* $2m-1$ logarithms for m powers of α_s. For example, the use of quasi-collinear splitting functions [1] gives the first subleading collinear logarithms. If one also adopts the two-loop expression for the running of α_s and the CMW scheme [2], then all the LL and NLL are included, except for those arising from soft wide-angle gluon emissions.

Due to the increasing precision of experimental measurements, the determination of the formal accuracy of a PS is becoming a serious issue which must be addressed. A recent work [3] introduced an approach to evaluate the logarithmic accuracy based on the ability of the PS to reproduce the singularity structure of multi-parton matrix elements, and the logarithmic resummation results. The authors focus on the process of double gluon emission in $e^+e^- \to q\bar{q}$ events, where the quark, q, is massless and the two gluons are well separated in rapidity so that the emission probability reduces to

$$dP_2 = \frac{1}{2}\prod_{i=1}^{2}\left(\frac{\alpha_s}{\pi}2C_F\frac{dp_{T,i}}{p_{T,i}}\frac{d\epsilon_i}{\epsilon_i}\right) = \frac{1}{2}\prod_{i=1}^{2}\left(\frac{\alpha_s}{\pi}2C_F\frac{dp_{T,i}}{p_{T,i}}dy_i\right), \tag{2}$$

where y_i is the rapidity of the gluon. The analysis is restricted to dipole showers, specifically the `Pythia` [4] one, which is the default option of the `Pythia8` [5] generator, and the `Dire` [6] one, available in both `Pythia8` and `Sherpa` [7]. The authors identified regions of phase space where the second gluon emission probability is generated with the wrong colour factor, namely $C_A/2$ instead of C_F.[1] This happens when the second gluon, g_2, is closest in angle to the first gluon, g_1, in the rest frame of the qg_1 (or $\bar{q}g_1$) dipole but closest to q (or $\bar{q}$) in the original $q\bar{q}$ frame. Another consequence is that the first gluon must absorb the transverse-momentum recoil

$$\vec{p}_{T,1} \to \vec{p}_{T,1} - \vec{p}_{T,2}. \tag{3}$$

This also breaks the factorisation of the two emissions, as $p_{T,1}$ can vary quite significantly after the generation of another branching.

Although it is clear that the coherent formalism [8] implemented in the `Herwig7` [9] angular-ordered parton shower prevents the aforementioned subleading colour issue, the impact of the recoil scheme on the accuracy of the algorithm must be investigated. In these proceedings we summarise the findings of Ref. [10], restricting ourselves to the case of a massless final-state parton shower in $e^+e^- \to q\bar{q}$ events.

2 Interpretation of the Ordering Variable

In this section we present the main features of the `Herwig7` angular-ordered (final-state) parton shower, focusing on several possible interpretations of the ordering variable.

[1]This is a subleading colour issue, as $C_F \to C_A/2$ in the large-number-of-colours limit.

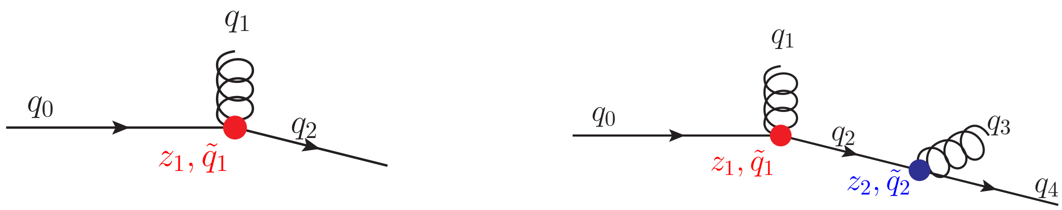

Figure 1: Single (left pane) and double (right pane) gluon emission from a quark line.

2.1 One Emission

We want to generate an emission collinear to the quark, as shown in the left pane of Fig. 1. We denote with p the quark momentum and with n a light-like vector parallel to the momentum of the anti-quark, which is colour connected to the quark in the original two-body configuration. The ordering variable can be equivalently expressed in terms of the transverse momentum ($p_{T,1}$), the virtuality of the emitting quark or the dot-product of the momenta of the emitted partons,

$$\tilde{q}^2 = \frac{p_{T,1}^2}{z_1^2(1-z_1)^2} = \frac{q_0^2}{z_1(1-z_1)} = \frac{2q_1 \cdot q_2}{z_1(1-z_1)}, \tag{4}$$

where z_1 is the light-cone momentum fraction carried by the emitted quark. If we define $\epsilon_1 = 1 - z_1$ we see that in the soft limit, *i.e.* $\epsilon_1 \to 0$

$$|p_{T,1}| \approx \epsilon_1 \tilde{q}_1, \qquad y_1 \approx -\log\frac{\tilde{q}_1}{Q}, \tag{5}$$

where Q is the centre-of-mass energy, and the `Herwig7` emission probability approaches the correct limit

$$dP_{\rm Hw7} = \frac{\alpha_s}{2\pi}\frac{d\tilde{q}^2}{\tilde{q}} C_F \frac{1+z_1^2}{1-z_1} dz_1 \to 2C_F \frac{\alpha_s}{\pi}\frac{d|p_{T,1}|}{|p_{T,1}|} dy_1. \tag{6}$$

2.2 Double Emission

We now want to generate the second gluon emission. If the second gluon is parallel to the anti-quark, the `Herwig7` algorithm identifies the $\bar{q}$ as the emitter, the auxiliary vector n is then chosen to be parallel to the original quark momentum and the generation of the emission is completely independent to the cascade originating from the quark.

If both gluons are collinear to the quark, then the requirement that they have a large rapidity separation suppresses the contribution arising from the $g \to gg$ splitting and both gluons will be generated from the quark line with colour factor C_F. The angular-ordering condition $\tilde{q}_2 < z_1 \tilde{q}_1$ dictates that the gluon with the smallest rapidity is emitted first, as shown in the right pane of Fig. 1. The first emitted quark now becomes off-shell gaining a virtuality $q_2^2 = z_2(1-z_2)\tilde{q}_2^2$ and the relations in eq. (4) are no longer valid, as it is impossible to preserve simultaneously $p_{T,1}$, q_0^2 and $q_1 \cdot q_2$. The quantity that we preserve determines the recoil-scheme prescription.

2.2.1 Transverse-Momentum-Preserving Scheme

The original choice [11] was to preserve the transverse momentum so that we can always write

$$p_{T,i} = z_i(1-z_i)\tilde{q}_i. \tag{7}$$

In the soft-collinear limit, the transverse momentum and the rapidity of each gluon always reproduce eq. (5). Thus, two gluons that are well separated in rapidity are effectively emitted independently as required.

To preserve the transverse momentum, the virtuality of the previous emitter must increase

$$q_0^2 = z_1(1-z_1)\tilde{q}_1^2 \to z_1(1-z_1)\tilde{q}_1^2 + \frac{z_2(1-z_2)\tilde{q}_2^2}{z_1}. \tag{8}$$

This tends to produce too much hard radiation in the non-logarithmically-enhanced region of phase space, overpopulating the tail of certain distributions.

2.2.2 *Virtuality-Preserving Scheme*

It was then suggested that the virtuality should be preserved [12]: the transverse momentum of the first emission is then reduced

$$p_{T,1}^2 = (1-z_1)\left[z_1^2(1-z_1)\tilde{q}_1^2 - z_2(1-z_2)\tilde{q}_2^2\right]. \tag{9}$$

This choice does not guarantee the existence of a positive solution. It is easy to see that, even if both emissions are soft, if the first one is much softer than the second one then there will be a negative solution, thus breaking the factorisation of multiple gluon emissions that are well separated in rapidity.

However, it was found that by setting the transverse momentum to 0 whenever a negative solution was encountered, the agreement with the experimental data is much better than in the p_T-preserving scheme.

2.2.3 *Dot-Product-Preserving Scheme*

Motivated by the desire to implement a scheme that is able to produce independent soft gluon emissions but does not overpopulate the non-logarithmically-enhanced regions, the last recoil scheme implemented [10] preserves the dot-product of the emitted partons and features intermediate properties between the p_T- and q^2-preserving schemes. After n emissions, the transverse momentum of the first gluon is modified to

$$p_{T,1}^2 = (1-z_1)^2\left[z_1^2\tilde{q}_1^2 - \sum_{i=2}^{n}(1-z_i)\tilde{q}_i^2\right]. \tag{10}$$

Using the angular-ordering condition $z_i\tilde{q}_i > \tilde{q}_{i+1}$ it can be proven that $p_{T,1}$ cannot became negative. Furthermore, if all the emissions are soft

$$p_{T,1}^2 \to \epsilon_1^2(\tilde{q}_1^2 - \sum_{i=2}^{n}\epsilon_i\tilde{q}_i^2) \approx \epsilon_1^2\tilde{q}_1^2, \tag{11}$$

i.e. subsequent soft emissions do not affect the transverse momentum of the previous ones.

The virtuality of the first emitter still increases, however,

$$q_0^2 = z_1(1-z_1)\tilde{q}_1^2 + \sum_{i=2}^{n} z_i(1-z_i)\tilde{q}_i^2, \tag{12}$$

thus leading, again, to a poor description of the tails of certain distributions, although better than that provided by the p_T-preserving scheme.

To prevent the virtuality of the original quark and anti-quark from becoming too large, we can accept the event with a probability given by

$$r = \sqrt{1 - 2\left(\frac{q_q^2 + q_{\bar{q}}^2}{s}\right) + \left(\frac{q_q^2 - q_{\bar{q}}^2}{s}\right)^2}, \tag{13}$$

where $\sqrt{s}$ is the centre-of-mass energy, q_q^2 is the virtuality developed by the quark and $q_{\bar{q}}^2$ by the anti-quark. The factor r comes from the fact that the original underlying two-body phase space is reduced when the particles increase their mass. With the inclusion of this factor, the phase-space factorisation becomes exact. We can easily see that for soft-collinear emissions $r \to 1$, thus this veto does not affect the logarithmically-enhanced contributions but it can introduce, at most, power corrections.

3 LEP results

In this section we present the results of our simulations obtained with the `Herwig7` generator and compare them with data from LEP.

We begin by showing the thrust distribution (Fig. 2), which can be considered as a proxy for all shape distributions. The p_T- and dot-product preserving schemes overpopulate the tail of the distribution, which corresponds to the non-logarithmically-enhanced region of the phase space. Conversely, the q^2-preserving scheme leads to a worse description of the data for $1 - T \leq 1/3$. When we apply the phase space veto (*i.e.* we accept the event with probability given by eq. (13)), the behaviour of the dot-product scheme improves in the tail, leading the best overall agreement with the data.

The jet-resolution-parameter distribution is shown in the left panel of Fig. 3. As in the previous case, the q^2 scheme is the most accurate in the non-logarithmically-enhanced region (that corresponds to small values of $-\log(y_{23})$), while the p_T scheme provides the best description in the opposite limit, but gives the worst overall agreement. The dot-product scheme with the veto is similar to the q^2 scheme, while without the veto it leads to the best overall agreement with data.

From the right panel of Fig. 3 it can bee seen that none of the schemes are able to reproduce the bottom quark fragmentation function for large values of x_B. Issues related to multiple emissions from heavy quarks, as well as gluons splitting into heavy quarks, are currently subjects of further investigation.

4 Summary and Outlook

Motivated by Ref. [3] we have investigated the impact that the choice of recoil scheme has on the accuracy of the `Herwig7` angular-ordered PS. We found that although the p_T-preserving recoil scheme ensures the independence of successive soft-collinear emissions well separated in rapidity, it produces too much radiation in the non-logarithmically-enhanced region of phase space. The q^2-preserving scheme, on the other hand, avoids overpopulating this region of phase space but breaks the independence of successive emissions and therefore loses logarithmic accuracy. We introduced the dot-product-preserving scheme as an attempt to retain the best features of both schemes, but it still somewhat overpopulates non-logarithmically-enhanced region of phase space. To ameliorate this we went on to introduce a phase-space veto that suppresses events with large-virtuality partons. In these proceedings we did not mention the effect of quark masses, although this is considered to some extent in Ref. [10] and is an ongoing area of research we hope to address further in future publications.

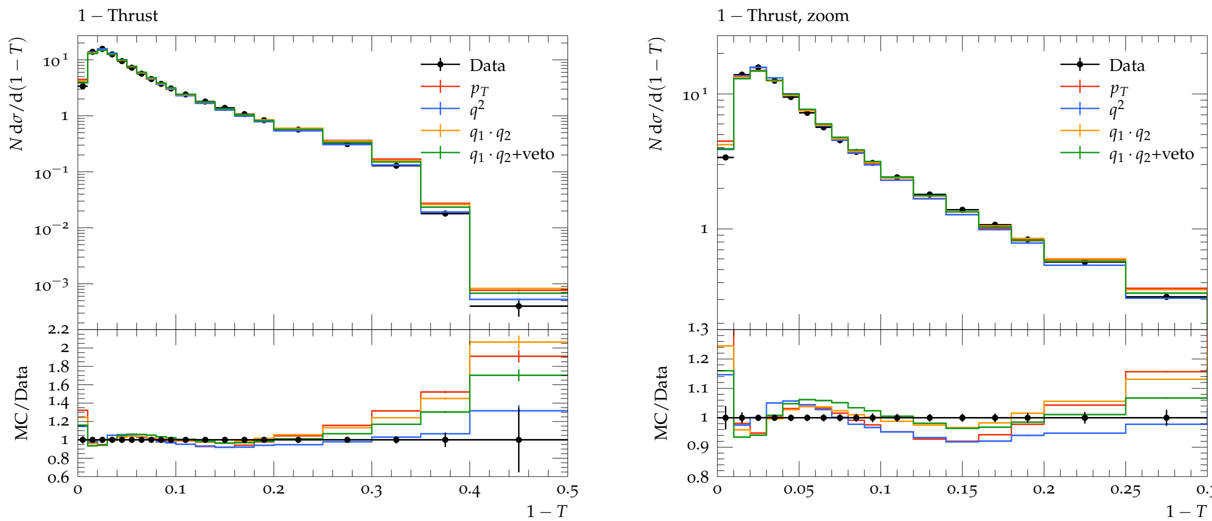

Figure 2: The thrust distribution at the Z-pole compared with data from the DELPHI [13] experiment. The right panel gives an expanded view of the same for small values $1-T$.

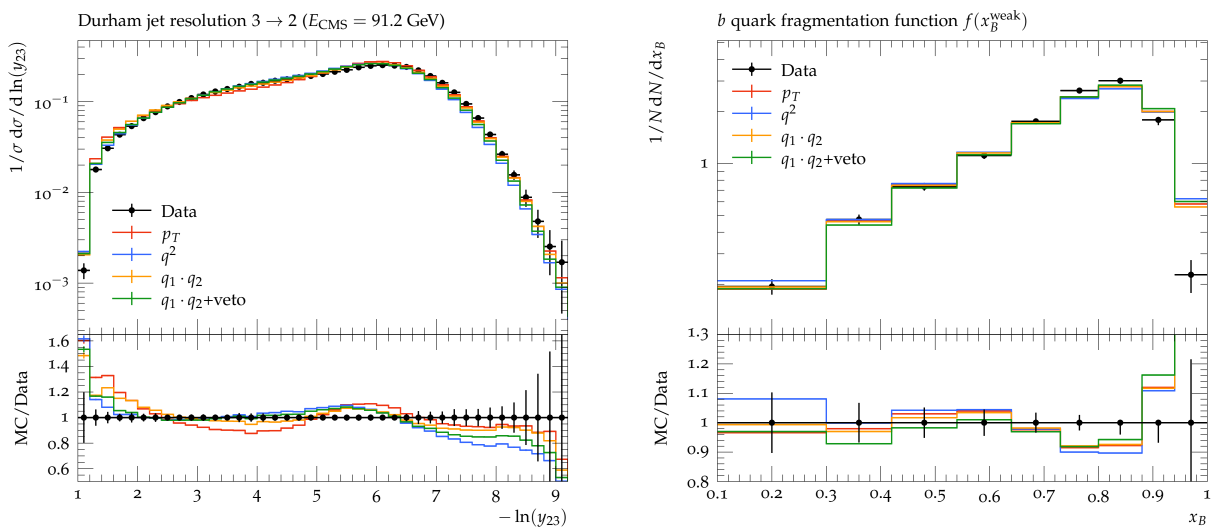

Figure 3: In the left panel the 3-to-2 jet resolution parameter for the Durham algorithm at the Z-pole compared with data from the ALEPH [14] experiment. In the right panel the fragmentation function of weakly-decaying B-hadrons compared with data from DELPHI [15].

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