Supersymmetric Quantum Spin Model and Quantum Hall Effect

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In this work, we introduce a supersymmetric extension of quantum antiferromagnetic states, *i.e.* supersymmetric valence bond solid (SVBS) states [1]. The SVBS states interpolate valence bond solid (VBS) states and (nearest neighbor) resonating valence bond (RVB) states. In particular, in 1D chain, the SVBS state interpolates AKLT state and Majumdar-Ghosh dimer state. Based on the $UOSp(1|2)$ graded symmetry, the truncated Hamiltonian whose ground state is the SVBS can also be derived explicitly.

In [2], the supersymmetric quantum Hall effect was developed, and Laughlin-Haldane wavefunction was derived as $\Psi = \prod_{i<j}(u_i v_j - v_i u_j - r\eta_i\eta_j)^m$, where u and v are Grassmann even and η Grassmann odd. (In [2], $r = 1$.) The supersymmetric Laughlin-Haldane wavefunction is invariant under the $UOSp(1|2)$ transformations generated by $l_a = \frac{1}{2}\begin{pmatrix}\sigma^a & 0\\ 0 & 0\end{pmatrix}$, $l_\alpha = \frac{1}{2}\begin{pmatrix}0 & \frac{1}{x}\tau_\alpha \\ -x(\epsilon\tau)^t_\alpha & 0\end{pmatrix}$, with $r = x^2$, $\tau_1 = (1,0)^t$, and $\tau_2 = (0,1)^t$. There are remarkable analogies between VBS states and the Laughlin-Haldane wavefunction [3]. Based on the analogies, we construct SVBS states as

$$|\mathrm{SVBS}\rangle = \prod_{<ij>}(a_i^\dagger b_j^\dagger - b_i^\dagger a_j^\dagger - r f_i^\dagger f_j^\dagger)^M|0\rangle,$$

where a and b denote the Schwinger boson operators while f the fermion (hole) operator. With use of the spin-hole coherent state, 1D SVBS state ($M = 1$) is represented and expanded as

$$\begin{aligned}\Psi_{\mathrm{SVBS}} &= \prod_i (u_i v_{i+1} - v_i u_{i+1} - r\eta_i\eta_{i+1})\\ &= \Phi - r\frac{\eta_i\eta_j}{u_i v_{i+1} - v_i u_{i+1}}\Phi + \frac{r^2}{2}\left(\frac{\eta_i\eta_j}{u_i v_{i+1} - v_i u_{i+1}}\right)^2\Phi + \cdots + (-r)^{L/2}\prod_{i=1}\eta_i\cdot(\Phi_{\mathrm{MG}:odd} - \Phi_{\mathrm{MG}:even}),\end{aligned}$$

where $\Phi = \prod_i(u_i v_{i+1} - v_i u_{i+1})$ is the AKLT state and $\Phi_{\mathrm{MG}:even,odd} = \prod_{i:even,odd}(u_i v_{i+1} - v_i u_{i+1})$ are two degenerate Majumdar-Ghosh dimer states. Thus, two different valence bond states are realized in the two extremal limits of the SVBS state, *i.e.* $r \to 0,\ \infty$. With finite r, the SVBS state shows the superconducting property: the expectation value of the two fermion operators does not vanish representing the superconducting order. With more fermion coordinates, the SVBS states interpolate various kinds of RVB states, in general.

[References] [1] Arovas, Hasebe, Qi, Zhang, PRB 79 (2009) 224404. [2] Hasebe, PRL 94 (2005) 206802. [3] Arovas, Auerbach, Haldane, PRL 60 (1988) 531.