Dissertation

submitted to the

Combined Faculties of the Natural Sciences and Mathematics

of the Ruperto-Carola-University of Heidelberg, Germany

for the degree of

Doctor of Natural Sciences

Put forward by

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born in: Catania, Italy

Oral examination: 18 April 2017

Imprints of Quantum Gravity on Large Field Inflation and Reheating

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In this thesis we investigate the feasibility and phenomenology of transplanckian field displacements during Inflation as well as the production of very light fields during Reheating.

We begin by focusing on realisations of axion inflation in the complex structure moduli sector of Type IIB String Theory (ST) flux compactifications. Firstly, we analyse the problem of backreaction of complex structure moduli on the inflationary trajectory in a concrete model of axion monodromy inflation. Secondly, we propose a realisation of natural inflation where the inflaton arises as a combination of two axions. In both cases we find sufficiently flat inflationary potentials over a limited, but transplanckian field range. However, our realisation of axion monodromy inflation requires a potentially large, though realisable, number of tunings to ensure that the inflationary shift symmetry is only weakly broken.

The consequences of the Weak Gravity Conjecture (WGC) for axion monodromy inflation are then explored. We find that the conjecture provides a bound on the inflationary field range, but does not forbid transplanckian displacements. Moreover, we provide a strategy to generalise the WGC to general p-form gauge theories in ST.

Finally, we focus on the physics of the early post-inflationary phase. We show that axion monodromy inflation can lead to a phase decomposition, followed by the radiation of potentially detectable gravitational waves. We also propose a strategy to evade the overproduction of Dark Radiation in the Large Volume Scenario of moduli stabilisation, by means of flavour branes wrapping the bulk cycle of the compactification manifold.

Spuren von Quantengravitation in *Large-Field* Inflation und *Reheating*

In dieser Doktorarbeit untersuchen wir sowohl die Realisierbarkeit und Phänomenologie von transplanckschen Feldauslenkungen wärend der Inflation als auch die Produktion sehr leichter Felder in der *Reheating*-Phase.

Zunächst konzentrieren wir uns auf Realisierungen von Axion-Inflation im Moduli-Sektor der komplexen Struktur von Fluss-Kompaktifizierungen in Typ-IIB-Stringtheorie (ST) auf Calabi-Yau-Mannigfaltigkeiten. Als Erstes analysieren wir in einem konkreten Modell für Axion-Monodromie das Problem der Rückkopplung von Moduli der komplexen Struktur auf die inflationäre Trajektorie. Danach schlagen wir eine Realisierung von natürlicher Inflation vor, in dem das Inflaton aus einer Kombination von zwei Axionen hervorgeht. In beiden Fällen finden wir ausreichend flache inflationäre Potenziale über einen begrenzten aber transplanckschen Feldbereich. Allerdings setzt unsere Realisierung von Axion-Monodromie eine möglicherweise große, aber realisierbare, Anzahl an Feinabstimmungen voraus, welche sicherstellen, dass die inflationä re *Shift*-Symmetrie lediglich schwach gebrochen wird.

Anschließend untersuchen wir die Konsequenzen der *Weak Gravity Conjecture* (WGC) für Axion-Monodromie-Inflation. Wir schlussfolgern, dass transplancksche Feldbereiche beschränkt aber nicht verboten sind. Außerdem beschreiben wir eine Strategie zur Verallgemeinerung der WGC in ST.

Zum Schluss betrachten wir die Physik der post-inflationären Phase. Wir zeigen, dass Axion-Monodromie-Inflation zu einer Phasentrennung führen kann, auf die die Ausstrahlung von potenziell nachweisbaren Gravitationswellen folgt. Zudem schlagen wir eine Strategie zur Umgehung der generischen Überproduktion von dunkler Strahlung im *Large Volume*-Szenario vor, in der *flavour*-Branes, die auf den *Bulk*-Zykel der kompaktifizierten Mannigfaltigkeit gewickelt sind, eine zentrale Rolle spielen.

Navegar é Preciso

Fernando Pessoa

Acknowledgements

I would like to thank Arthur Hebecker for more then one reason: first of all, for being a mentor rather than just a supervisor. Secondly, for continuously sharing his ideas and for patiently teaching me so much physics. Finally, for his constant support and advise concerning my choices during my PhD.

I am especially grateful to my collaborator Lukas Witkowski: not only for the physics and methodology which he has taught me, but for very generously advising me on talks and applications as well. I am also indebted to Patrick Mangat, for sharing many ideas on physics and beyond and for his immense patience in having me as a collaborator and moreover an officemate.

Important parts of this thesis would have not been realised without the collaboration of Jörg Jäckel and Alexander Westphal: I would like to thank both of them for the time they devoted to our common projects as well as for their advise. I am grateful to Tilman Plehn, who is also a reader of this thesis, for his scientific advise and support since I joined the ITP in Heidelberg. I would also like to thank Marco Farina, Craig Lawrie, Viraf Mehta and Lukas Witkowski for their important feedback on some parts of this thesis and Philipp Henkenjohann for his help with the German translation of the title and abstract of this thesis.

I would like to thank the following physicists for useful discussions on the content of this thesis and on related topics: Stephen Angus, Thomas Bachlechner, Robert Brandenberger, Michele Cicoli, Joseph Conlon, Frederik Denef, Gia Dvali, Christophe Grojean, Kate Eckerle, David Hindmarsh, Sascha Leonhardt, M.C. David Marsh, Renata Kallosh, Sebastian Kraus, Viraf Mehta, Jose Francisco Morales, Dominik Neuenfeld, Eran Palti, Fernando Quevedo, Diederik Roest, Marco Scalisi, Michael Schmidt, Michael Spannowsky, Stefan Theisen, Timo Weigand, Clemens Wieck, Timm Wrase.

I am very grateful to Matthew Kleban, for giving me the opportunity to collaborate with him at New York University (NYU), for the time he spent discussing physics with me and for his support. I am indebted with Duccio Pappadopulo at NYU, Marco Farina at Rutgers University and Andrea Tesi at the Enrico Fermi Institute, for giving me the opportunity to collaborate with them on work which is not discussed in this thesis. I would like to thank Duccio and Marco also for adding a relevant dose of fun to my days at NYU and for their scientific (and moral) support. A special thanks goes to Oliver Janssen at NYU for collaborating with me: I have learnt a lot of physics from and together with him and I have highly appreciated his optimism and motivation.

I am grateful to Riccardo Rattazzi for introducing me to the world of particle theory and for providing important advise during these years.

I have been lucky to enjoy a great scientific atmosphere at the Institute for Theoretical

Physics in Heidelberg thanks to the other members of the String Theory group: Oskar Till, Craig Lawrie, Ling Lin, Florent Baume, Daniel Kläwer, Fengjun Xu, Christoph Mayrhofer, Jacob Moritz, Christian Reichelt, Stefan Sjörs, Philipp Henkenjohann, Pablo Soler, Martin Bies and our fantastic secretaries Melanie, Sonja and Cornelia. The very same has been true at NYU and in particular I would like to thank: Gabriele, Anna-Maria, Guzman and Valentino.

During the completion of this PhD, I was financially supported by: the DAAD (2014-2015), which I would like to thank also for creating a wonderful network of scholars; the DFG Graduiertenkolleg GRK 1940 "Physics Beyond the Standard Model" and the Heidelberg Graduate School of Fundamental Physics (HGSFP). I am especially grateful to the last two programs for generously supporting my research stay at New York University.

Beyond the scientific community, there are many other people that I would like to thank. First of all, my family: my parents, my sister and my grandparents. I thank them for supporting me in every possible way through the last twenty-seven years and in particular throughout my scientific education.

Furthermore, I am grateful to those people who have kept me on the right track in these last years. Among them, I am particularly indebted with Nicole for her continuous encouragement and support in the last months, and with Andrea, Anna, Asier, Clara, Daniela, Daniele, Daniel, Elia, Federica, Matteo, Rossella, Toni. Danke! Gracias! Grazie!

Contents

Introduction

The inflationary Universe

Very much like sailors of ancient times, cosmologists today direct their gaze to the sky in search of a guiding light to the understanding of the physics of fundamental interactions. Their North Star is the Cosmic Microwave Background (CMB), accidentally discovered more than fifty years ago and more recently mapped by the WMAP [1] and Planck [2] satellites. The CMB is the first of all possible snapshots of our Universe: taken approximately when the latter had cooled sufficiently to make it favourable for electrons to be bound in hydrogen atoms, so that photons could stream freely instead of scattering continuously off electrons. Planck and WMAP measured the temperature of CMB photons across the sky, finding that it is homogeneous up to fluctuations of one part in one hundred-thousand!

A lot about the history of the Universe was of course known before the era of CMB precision measurements. In particular, only three fundamental observations are needed to reconstruct most of its evolution: it is flat, currently expanding and contains (dark) matter and radiation. The theory of General Relativity then implies that the *Hubble radius*, the size of the observable Universe, is increasing with time. Measurements of the redshifts and distances of galaxies by means of Type Ia supernovae revealed an unexpected addition to these basic ingredients: the Universe today is accelerating [3, 4]. This observation provides evidence for a tiny and positive constant energy density, referred to as the *cosmological constant* or *dark energy*, whose origin is arguably the greatest conundrum of contemporary physics. Overall this picture is known as the ΛCDM model and it is confirmed by the Planck satellite to a very high accuracy: the Universe is composed by roughly 69% dark energy, 26% dark matter and 5% ordinary baryons [2], while the contribution of radiation today is negligible.

Despite its outstanding success, such a standard cosmological model leaves essential questions unanswered: Why is the Universe so flat? Why is the temperature of the CMB so homogeneous? Why does it exhibit such tiny fluctuations? Very special initial values of curvature and temperature seem to be required to produce the *best of all possible worlds* we live in.

This rather unsatisfactory situation can be partially ameliorated by postulating an initial phase of accelerating expansion, during which the Hubble sphere shrinks. Similarly to its old self, the young Universe would then be filled with a source of constant energy density. This is, in short, the answer given by the theory of *Inflation* [5, 6]: the Universe is flat and homogeneous because its ripples and anisotropies were stretched and flattened early on. Crucially, extremely tiny quantum fluctuations in the early Universe were also stretched during inflation, in such a way that they induced measurable inhomogeneities in the temperature of the CMB.

So far observations have confirmed that the language of the primordial Universe is that of quantum field theory. A constant energy density can thus be provided by a light scalar field with an almost constant potential and negligible kinetic energy. The expansion of the Universe helps in achieving the latter condition, as it damps the motion of the inflaton field along its potential. So-called *slow-roll* inflation [7, 8] occurs if the dynamics of the inflaton is dominated by friction. In this picture the scalar field is almost perfectly homogeneous and anisotropies arise from space-dependent (quantum) field fluctuations [9–12]. In particular, the power spectrum of such scalar fluctuations (i.e. the Fourier-expanded correlations between the fluctuations at two different locations) is directly related to the observed CMB temperature power spectrum. This elegant scenario predicts an almost scale-invariant (meaning that its amplitude does not depend on the particular Fourier mode) and gaussian (implying that the n-point correlations, with n odd, are vanishing) spectrum of fluctuations.

The most spectacular prediction of the inflationary Universe is however the existence of primordial gravitational waves, also referred to as tensor modes. These are perturbations in the geometry of spacetime which propagate similarly to electromagnetic waves. Despite being fundamentally the same as the gravitational waves emitted by binary systems of neutron stars and/or black holes, recently detected by the interferometer aLIGO [13], observable inflationary tensor modes would have much smaller amplitude and frequencies. Thus they are not detectable by means of interferometry, at least at the time of writing. Rather, they are indirectly accessible through measurements of the polarisation of CMB photons: tensor modes are indeed the only primordial source of B-mode (divergence-free) polarisation (see e.g. [14] and refs. therein), while scalar fluctuations induce purely E-modes (curl-free).

The Universe of the ΛCDM model is then finally produced during *reheating*: after inflation the energy density residing in the scalar field is distributed to matter and radiation by the decay of the inflaton field. Furthermore, scalar field fluctuations translate into density perturbations, which are responsible for the large-scale structures of the observed Universe.

Planck provided further convincing support to the inflationary picture by measuring the deviation of the scalar power spectrum from scale invariance, expressed by its so-called *tilt* $n_s = 0.9677 \pm 0.0060$ (68% CL) [15] (with $n_s = 1$ corresponding to perfect scale invariance, see the caption of figure 1) and by strongly constraining non-gaussianities. However a detection of primordial tensor modes still eludes us, with Planck and BICEP2-Keck Array (BKP) setting only an upper bound on the ratio of tensor-to-scalar fluctuations, $r \leq 0.08$ (95% CL) [15]. Nevertheless in the near future several ground- and space-based experiments will be able to probe smaller values of the tensor-to-scalar ratio, realistically down to $r \gtrsim 2 \cdot 10^{-3}$ [16].

With regard to inflationary model building, two classes of concrete realisations exist: firstly, non-generic inflaton potentials with very flat regions can be employed; alternatively, generic power-like potentials can be considered. In the former case, the inflaton only needs to traverse small field distances in order for the Universe to undergo sufficient exponential expansion. In the latter case instead, the inflaton has to be displaced across large field intervals, since power-like potentials are generically not flat enough. This option is known as *Large Field Inflation* (LFI) and is particularly attractive for two reasons: first of all, it can be realised by means of simpler, arguably more natural, inflationary models, with possibly only one parameter governing the potential. Secondly and perhaps most importantly, it leads to observable values of r (as shown in figure 1). In fact, a detection of B-modes in the near future would make a strong case for *transplanckian* field displacements [17], meaning that the inflaton traverses distances in field space which are larger than the (reduced) Planck scale $M_P \simeq 10^{18}$ GeV. Not surprisingly, the Large Field Inflation scenario would add an intriguing twist to the inflationary picture.

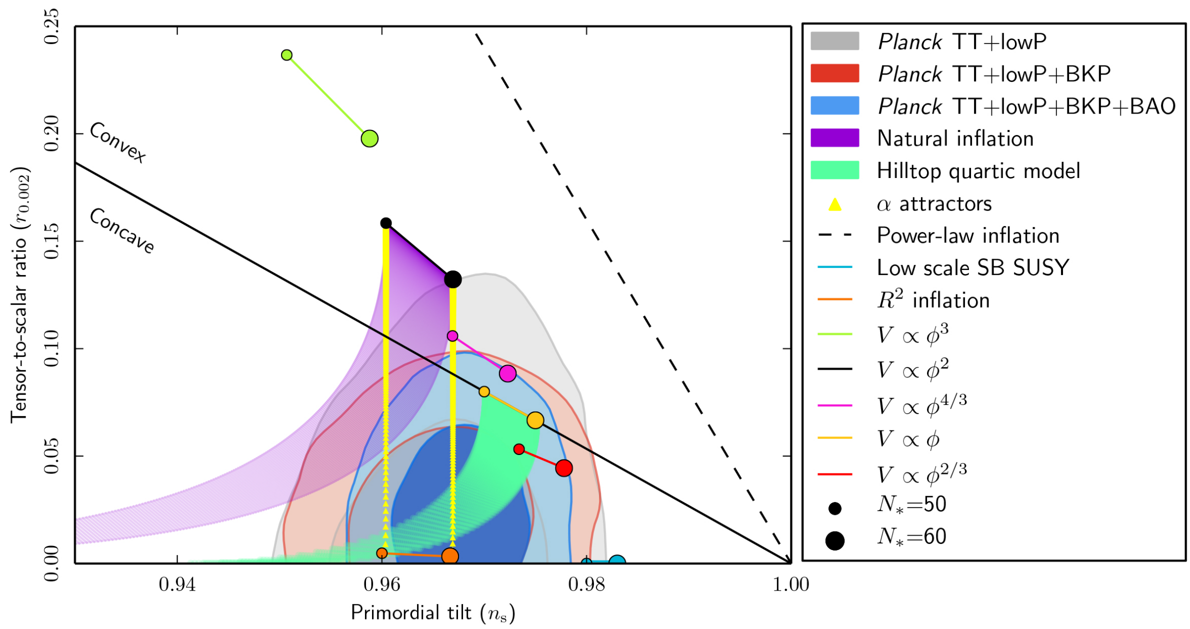

Figure 1: Planck/BKP 2015 constraints on slow-roll inflationary models with potential $V(\phi)$. On the horizontal axis: the tilt n_s of the power spectrum of scalar fluctuations, defined by $k^3\mathcal{P}_s(k) = \mathcal{A}_s \left(\frac{k}{k_\star}\right)^{n_s-1}$. Here $k_\star = 0.05$ Mpc^{-1} is a reference *pivot scale*. On the vertical axis: the ratio of the amplitudes of the tensor and scalar power spectra, $r \equiv \frac{\mathcal{P}_s(k_\star)}{\mathcal{P}_t(k_\star)}$. In this case, $k_\star = 0.002$ Mpc^{-1}. The predictions of several models are plotted as coloured segments, whose points correspond to different values of the number of *efolds* $N_\star$, which parametrises the duration of inflation. The black straight line separates concave from convex potentials. 68% and 95% CL regions are shown. This plot is taken from [15], which should be consulted for more details on the various inflationary potentials.

Inflation and the Planck Scale

The Planck scale is to high energy physics what the Pillars of Hercules were to ancient Mediterranean sailors: the ultimate boundary of the known world. The current understanding of Nature is via quantum field theories (QFT), with fundamental interactions being mediated by gauge fields. The modern point of view is that a given QFT should be interpreted as an effective field theory (EFT): that is, a description valid only up to a certain energy cutoff scale. Above the latter, new ultraviolet (UV) degrees of freedom become dynamical and thus a new theory is needed. The Standard Model (SM) of Particle Physics, which describes the strong and electroweak forces, fits into this scheme. The Large Hadron Collider (LHC) is currently seeking evidence for physics Beyond the Standard Model (BSM) at the TeV scale but so far it appears that the SM remains a valid description of Nature at these energies. Nonetheless, its cutoff scale is necessarily smaller than the Planck scale, where degrees of freedom associated to Quantum Gravity (QG) are expected to become relevant.[1] It is, however, not clear what these degrees of

[1] In fact, there already exists evidence (such as neutrino masses, dark matter, possibly instability of the Higgs vacuum) that the cutoff of the SM is much lower than the Planck scale. However, such

freedom are nor which theory could possibly describe them. While it is true that General Relativity can be used to compute amplitudes of gravitational processes in perturbation theory, at energies close to M_P its predictions for such amplitudes violate basic principles of quantum mechanics; specifically unitarity. Thus, it is one of the main open problems of particle physics to understand how gravity behaves at such high energies and how it is reconciled with quantum mechanics.

In this regard, the desire to describe all interactions in a unified framework motivates an alternative perspective on effective field theories, and in particular on the SM. Namely, above a certain energy cutoff $M_s < M_P$ the fundamental constituents of Nature may be one-dimensional strings, rather than point-like particles. This is the proposal of *String Theory* (ST), which unifies gravity with the strong and electroweak interactions in a framework which is consistent with special relativity and quantum mechanics. Unfortunately, this comes at a price: namely, (super)String Theory is consistent only in ten dimensional spacetime. As astonishing as it sounds, if the six extra-dimensions are extremely small and "curled up" (that is, compact) there is no way to experience them in everyday life and very likely even at high energy particle colliders, such as the LHC.[2] In fact, so far there is no experimental nor observational evidence for String Theory. This should not be seen as a verdict on the proposal itself, as, in principle, its consequences may be visible only at very high energies, possibly close to the Planck scale. Rather, the current situation adds motivation to the interest in primordial cosmology, where the highest energies currently accessible are involved. This leads to the foundational question of *string cosmology*: what are the predictions of String Theory on phenomena taking place in the very early Universe?

This thesis presents some answers to this question, in the context of physics which is directly and indirectly observable in the CMB. In particular, the focus will be on understanding the effects of stringy (and more generally gravitational) degrees of freedom on the effective field theory of inflation and on reheating.

The inflationary mechanism may be particularly concerned with Quantum Gravity. Elementary scalar fields like the inflaton and the Higgs boson are known to pose conceptual challenges to the effective field theory ideology. Their masses and interactions are indeed very finely sensitive to UV physics. In particle physics, this feature translates into the well known *hierarchy problem*: why is the Higgs boson mass so small compared to the Planck scale? Concerning inflation, its UV sensitivity is expressed by the *eta problem*: why is the inflaton field light compared to the Hubble scale? Models of Large Field Inflation are even more problematic: how can the inflaton potential remain sufficiently well-behaved over transplanckian field ranges? Answers to the latter question might shed light on the rather intriguing observational status of LFI: while several of its realisations are compatible with current data, the simplest model, quadratic inflation [19], is very strongly constrained.

Reheating has instead the potential to probe and possibly falsify what is considered a general characteristic of the stringy Universe: namely the existence of a multitude of light pseudoscalar fields beyond the particle content of the SM [20]. The CMB power spectrum can indeed be sensitive to the presence of very light species beyond the three

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phenomena might turn out to be described by rather minimal extensions of the SM.

[2] See however e.g. [18] for possible signatures of ST at the LHC.

families of neutrinos predicted by the SM. Are then current observations already casting doubts on our understanding of effective theories descending from ST?

Troubles and Opportunities from Large Field Inflation

Since most of the content of this thesis is concerned with conceptual and phenomenological aspects of LFI, let us provide a slightly more technical introduction to its realisations and open questions in the context of ST and effective field theory. This will also give us the opportunity to review recent progress.

Let us thus go back to the friction-dominated inflationary dynamics. The latter occurs if the inflaton mass is small compared to the rate of expansion of the Universe during inflation, the *Hubble rate* H [21]. In order to understand the problems related to this requirement in EFT, it is useful to consider a general effective Lagrangian, featuring operators which can be built using powers of the scalar field and of is derivatives:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\frac{m^2}{\Lambda^2}\Lambda^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \sum_n c_n\frac{\mathcal{O}_n[\phi]}{\Lambda^{p_n-4}}, \tag{1}$$

where terms with odd powers of ϕ have not been written explicitly for simplicity. Here $\mathcal{O}_n[\phi]$ are operators of dimension p_n, c_i are dimensionless Wilson coefficients and Λ is the energy cutoff of the EFT. In the case of inflation, the lowest possible value of this cutoff corresponds to the Hubble scale H. Thus, following the discussion above, $H \lesssim \Lambda \lesssim M_P$. Notice that we normalised the inflaton mass by the cutoff, so that the mass term is multiplied by the dimensionless coefficient $a = m^2/\Lambda^2$. According to (1), contributions to physical amplitudes induced by the operators $\mathcal{O}_n[\phi]$ are suppressed at energies below the cutoff.

The theoretical problems of inflation are now apparent. Effective field theories are based on the *naturalness* criterion [22], which dictates that all the dimensionless parameters appearing in the Lagrangian should be $\mathcal{O}(1)$. A given coefficient is allowed to be small only if the Lagrangian enjoys a larger symmetry, in the limit in which that coefficient vanishes. The first and most important dimensionless coefficient in (1) is the mass squared $a = m^2/\Lambda^2$. In the absence of symmetry, the natural expectation is $m^2 \sim \Lambda^2 \gtrsim H^2$. Thus slow-roll inflation cannot occur.

The problem is only more acute for Large Field models, where the inflaton field necessarily takes values larger than the Planck scale. Since $\Lambda < M_P$, higher-dimensional operators in (1) are not suppressed and the effective description of (1) breaks down, since in principle an infinite number of higher-dimensional operators becomes relevant for the low-energy inflaton dynamics.[3]

However, small dimensionless coefficients in (1) are *technically natural* in the sense that a symmetry under continuous shifts of the inflaton $\phi \to \phi + c$, with c constant, is recovered in the limit $(m/\Lambda) \to 0, \lambda \to 0$ (assuming for the moment that we can neglect higher-dimensional operators). This motivates the interest in *axions* as inflaton

[3]Notice however that the energy density during inflation is constant and well below the Planck scale: in particular $V_{\text{inf}} \sim H^2 M_P^2$. Thus it is not *a priori* clear that Quantum Gravity should have anything to do with LFI.

candidates. The latter are fields which take values on a circle. More precisely, they are (pseudo)scalar fields that enjoy a continuous shift-symmetry at all orders in perturbation theory. However, non-perturbative gauge theory effects (that is, instantons) can generate a periodic potential for the axion and thus only a discrete shift symmetry is left. Axions have been originally investigated in relation to the strong CP problem of QCD [23–26] and as Cold Dark Matter candidates [27–29]. Moreover, a plethora of models of *axion inflation* have been proposed. Among them, the simplest is Natural Inflation [30], where the inflaton has an instanton-induced periodic potential

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] \tag{2}$$

and the axion decay constant f measures the periodicity of the field ϕ. Such a model is compatible with the latest Planck data if $f \gtrsim 7\ M_P$ [15], so that the inflaton rolls down one of the cosine wells along a transplanckian field range. Generalisations of this simple model identify the inflaton with a linear combination of multiple axions with subplanckian periodicities (see [31, 32] for the original proposals).

The dimensionless parameters c_i in (1) are also naturally suppressed in the presence of a shift symmetry. It thus seems to be rather easy to present a model of LFI which is protected against UV corrections. However, the interpretation of higher order operators is that they originate from integrating out the UV degrees of freedom which live above the cutoff Λ. From this perspective the problems of LFI reappear: does the UV theory respect the (discrete or continuous) inflationary shift symmetry? If not, what are the degrees of freedom that are responsible for its violation?

Very interestingly, important hints exist that the answer to the latter question involves Quantum Gravity. More specifically, it is expected that UV degrees of freedom associated to QG interfere with continuous global symmetries [33]. Arguments in favour of this statement are often based on the observation that evaporating black holes can potentially violate charge conservation. Further evidence is provided by perturbative String Theory, in which a theorem holds that forbids continuous global internal symmetries [34] (see also [35] for a recent review). Nevertheless, it is not clear whether black hole arguments extend to (discrete) shift symmetries, which do not have a conserved charge, nor how badly would the symmetry be violated. Furthermore, the role of non-perturbative gravitational effects is also not fully understood (see [36, 37] for recent progress).

Thus the fundamental problem of (Large Field) Inflation can be rephrased as a single question: to what extent does Quantum Gravity respect/violate inflationary shift symmetries?

There exists at least two complementary strategies to look for an answer. First of all, by *direct* inspection of the higher-order operators. Since the latter are induced by UV degrees of freedom, this option requires knowledge of the UV completion of the inflationary model. Secondly, by *indirect* knowledge of the properties of such higher-order operators. This can be obtained by effective field theory arguments about what the fundamental properties of QG should be.

String Inflation

The first strategy naturally leads to String Theory, more specifically to effective theories descending from *string compactifications*. Quantisation of the string leads to a massless spectrum and a tower of massive string modes, whose characteristic energy scale is the *string scale* M_s. At energies below M_s, one can obtain an effective action for the massless modes only. Very importantly, the resulting effective theory is an $\mathcal{N} = 2$ supergravity (SUGRA) in 10D. Effective 4D $\mathcal{N} = 1$ supergravity theories can then be obtained by dimensional reduction on certain six-dimensional compact manifolds, known as *Calabi-Yau (CY) orientifolds* (see [38]).[4] This is essentially an extension of the old proposal of Kaluza-Klein compactification of five-dimensional theories on a circle (see for instance [39] and refs. therein). A further crucial ingredient of 10D String Theory is the existence of D-branes [40], which are higher dimensional dynamical surfaces on which open strings can end. The task of *string inflation* is to embed inflationary models either in string-derived 4D supergravities or directly in 10D setups with D-branes.

An essential feature of string compactifications is the existence of a multitude of scalar fields which are massless in perturbation theory and couple to Standard Model fields only through Planck-suppressed operators. They are referred to as *moduli* and can be related to geometric features of the compactification. A useful and simple example for this relation is provided by the familiar case of five-dimensional spacetime, with the fifth dimension being a circle.[5] The extra-dimension can be "integrated out" by Fourier expanding every field in terms of the coordinate on the circle. In particular, fluctuations of the 5D metric along the extra-dimension generate a massless scalar field in 4D, which is referred to as the *radion* (see e.g. [43]). As its name suggests, the vacuum expectation value (VEV) of this field characterises the size of the fifth dimension. The radion is thus an example of a modulus, since its VEV is not automatically fixed by the compactification.

Six dimensional Calabi-Yau manifolds are more complicated than the simple circle discussed above. In particular, their *moduli spaces* are much richer. Once again, a simpler example might be useful. A rectangular torus can be cut symmetrically by means of a horizontal (vertical) plane. The circle arising at the intersection between the torus and the plane is called a 1-cycle of the torus. Therefore, there are two such cycles in this case. Correspondingly, the shape of a rectangular torus is characterised by the ratio between the radii of these two circles, which is parametrised by its so-called *complex structure* modulus. For a general torus, i.e. not necessarily rectangular, the complex structure modulus is actually complex and its phase corresponds to the angle between the two fundamental cycles. The overall volume of a torus is instead described by a so-called Kähler modulus. More in general, cycles can be defined as submanifolds without boundaries. On Calabi-Yau manifolds there are $2-, 3-$ and 4-dimensional cycles and two types of moduli which are related to fluctuations of the 10D metric along the compact dimensions (see chapter 1 for a technical introduction to moduli of CYs). In close analogy with the torus, complex structure moduli parametrise perturbations of the shape of the manifold. Kähler moduli are instead related to fluctuations of the overall volume of the

[4] A basic review of these concepts is provided in chapter 1.

[5] Slightly more complicated setups are employed for instance in the popular Randall-Sundrum proposal to solve the electroweak hierarchy problem [41, 42].

CY, as well as of two- and four-cycles.

Massless scalar fields are generically incompatible with observations, as they would for instance lead to measurable deviations from General Relativity (see e.g. the discussion in [44]). Furthermore the size of the extra-dimensions fixes the values of the couplings of the 4D theory. Therefore it is imperative to generate a potential for the moduli: this is referred to as *moduli stabilisation* and has been the main technical task of string phenomenology since its birth (see chapter 1 for a basic introduction). Even when their VEVs and masses are fixed, moduli can still pose a serious threat to late cosmology. Since their interactions are Planck-suppressed, moduli generically decay very late to SM fields. This may have several catastrophic consequences, among them: photodissociation of ^4He and D, in such a way that the abundances of these elements would deviate from observations; washout of the baryon asymmetry due to the late release of entropy. These issues are usually referred to as the Cosmological Moduli Problem (CMP) [45–47] and can be evaded if moduli are heavier than ~ 30 TeV.

Moduli stabilisation can be partially achieved by means of higher dimensional gauge fluxes, which can be thought of as the generalisation of the familiar electromagnetic field strength to an antisymmetric tensor with more than two indices. Such fluxes can be used to stabilise complex structure moduli [48]. Kähler moduli can be fixed by means of non-perturbative effects only, as in the KKLT scenario [49], or via an interplay of non-perturbative effects and stringy corrections, as in the Large Volume Scenario (LVS) [50, 51]. Both setups can accommodate low-energy supersymmetry without necessarily facing the CMP.

This illustrates the interplay between supersymmetry as a solution to the hierarchy problem and moduli stabilisation. Crucially, the latter is also very closely tied to what might be considered the most revolutionary implication of String Theory. Compactifications of 10D ST on Calabi Yau manifolds with different choices of higher dimensional fluxes leads to a vast *landscape* of possible 4D vacua [52,53], each of which is characterised by different physics (that is, different 4D parameters). Since the estimated number of such vacua is $\gtrsim 10^{500}$ [52], a rather large number of Universes might have a cosmological constant with its observed value. At the time of writing, the paradigm of the string landscape is arguably the most promising solution to the cosmological constant problem. It is also very closely tied to (eternal) inflation, since the latter is responsible for populating the different vacua (see [54] and refs. therein).

With regard to axion inflation, setups of string compactifications offer a plenitude of fields which enjoy (discrete) shift-symmetries [20, 55]. In the simplest case, axions may descend from dimensional reduction of the aforementioned higher-dimensional gauge potentials. Their low-energy shift-symmetry is thus the relic of a gauge symmetry in the UV theory. After including non-perturbative effects, a discrete shift symmetry is left, which has the potential to evade the arguments against global symmetries in QG. Nevertheless, the existence of a single elementary axion with transplanckian decay constant is severely constrained at the level of explicit string constructions [55,56], where field spaces are limited by the compactness of the extra-dimensions. However, there are at least two ways to obtain larger field ranges. Firstly, by considering two [31] or more [32] axions with subplanckian periodicities, an appropriate combination of them can exhibit a transplanckian decay constant (see subsection 1.2.4).

Secondly, the compact field space of a single axion can be unfolded by weakly breaking the associated shift symmetry. This is the basic idea of *axion monodromy* inflation [57,58], whose inflationary potential is of the type

$$V(\phi) \sim \mu^{4-p}\phi^p + \Lambda^4 \cos\left(\frac{\phi}{f}\right), \tag{3}$$

where the first term breaks the axionic shift symmetry respected by the second term and f is meant to be subplanckian. Thus the original circular field space is effectively transformed into a helix and a transplanckian trajectory can be achieved. Models based on the potential (3) are in principle compatible with current data, for certain values of p and f. For instance, the original model [57] uses $p = 1$ and predicts $r \approx 0.07$. Other powers may also be considered, as we will do in chapters 4 and 5.

Finally, other moduli may also exhibit shift symmetries. For instance, in this thesis we shall be particularly concerned with axions that arise from complex structure moduli. In the simple case of a torus, the discrete shift symmetry of its complex structure is part of the well-known symmetry group of global diffeomorphisms of the torus. In the more relevant case of Calabi-Yau manifolds, complex structure moduli can enjoy shift symmetries in certain geometric limits [59–61] (see also [62] for a recent discussion). However, the understanding of these invariances is not as straightforward as in the toroidal case, thus a specific analysis is particularly important.

Effective Field Theory Constraints

The effects of Quantum Gravity can also be inferred indirectly, via EFT expectations concerning its behaviour. This strategy has been intensively employed in the last two years. A first argument which might be used to constrain LFI is known as the Weak Gravity Conjecture (WGC) [63]. Heuristically, the WGC states that gravity is the weakest force. More specifically, the (electric) WGC requires that the spectrum of a U(1) gauge theory coupled to gravity should contain a particle of mass m and charge q whose electric interaction is stronger than its gravitational one, namely such that

$$\frac{m}{qeM_P} \lesssim 1, \tag{4}$$

where e is the gauge coupling. Furthermore, the (magnetic) conjecture states that the cutoff of the EFT is parametrically smaller than M_P at weak coupling, contrary to the effective field theory expectation. In particular,

$$\Lambda \lesssim eM_P. \tag{5}$$

Notice that both statements of the conjecture forbid the limit $e \to 0$, in which the gauge symmetry would be essentially indistinguishable from a global symmetry. The motivations for the electric WGC are indeed very closely related to the "no-go" arguments against global symmetries in QG, while the magnetic statement descends from the expectation that the minimally charged monopole in the effective theory should not be a black hole (see section 1.2 for more details). Applications to inflationary models actually employ a generalised version of the WGC, which concerns gauge theories with potentials that have a generic number of indices.[6] In particular, a "gauge" boson with no indices is a (pseudo)scalar and its gauge invariance is just a shift symmetry: in other words it is an axion. The role of the particle is played by the *instanton* which generates a periodic potential for the axion (exactly as in QCD). The particle "mass" corresponds to the instanton action S: the dilute gas approximation, usually necessary to perform computations, requires $S \gtrsim 1$ (see e.g. [64]). The "charge" is the inverse of the axion decay constant f. The dimensionless "mass-to-charge" ratio for instantons and axions is thus given by $(S \cdot f)/M_P$. The electric WGC requires that the latter should be smaller than unity, which implies that the axion decay constant is subplanckian, thereby ruling out Natural Inflation and related scenarios [63]. Recent development focused both on the implications for multi-field models as well as on the refinement of the statement of the conjecture (see [36, 65–84] for a non-exhaustive list of recent work).

A second argument against LFI is based on the existence of non-perturbative effects (analogous to instantons in QCD) associated to gravity [36, 37]. The claim is that such gravitational instantons induce terms in the inflationary potential, which may spoil the required flatness if the field traverses transplanckian distances. Yet another argument against LFI exists, based on well-known bounds on the entropy of black holes [85, 86] extended to other systems (such as de Sitter spacetime), where it is known as the *covariant entropy bound* (CEB) [87]. In this case the claim is that large field models may violate the CEB [88] (see however [73, 89]).

So much for the recent conceptual progress on Large Field Inflation. A third route, which is in a sense orthogonal to the previous two, is purely phenomenological. Namely, models realising LFI may exhibit peculiar signatures which affect the CMB power spectrum and/or reheating. Detection of such signatures may potentially help in validating LFI, especially if associated with a measurement of primordial gravitational waves (see [90–98] for a partial list of recent work on the signatures of axion inflation).

The Contribution of this Thesis

This thesis investigates the interplay between String Theory, Large Field Inflation and Reheating, along the three directions outlined above. Accordingly, it is divided in two parts, separated by an *Intermezzo*. Part I is devoted to the realisation of certain inflationary models in the framework of compactifications of 10D string theory to 4D. On the contrary, Part II concerns the physics of the early post-inflationary era. While the content of Part I is mainly theoretical, Part II adopts a phenomenological perspective. The Intermezzo discusses the Weak Gravity Conjecture, both from a conceptual and phenomenological point of view. The reader who is not familiar with string compactifications can find a basic technical introduction to the topic in chapter 1, where we also review the WGC and its application to models of axion inflation. Furthermore, this thesis is supplemented with three appendices: appendix A reviews the language of differential forms and the essential definitions relevant for complex manifolds; appendix B complements the discussion of

[6]In the language of differential forms, these are called *p-form* gauge theories. We review them in appendix A, while the WGC is technically introduced in section 1.2.

scalar field fluctuations after inflation given in chapter 5, while appendix C presents a scenario of small field inflation which is again related to the content of chapter 5. Given the length of this thesis, we find it convenient to provide now a summary of its main contents and results.

Part I: Large Field Inflation in String Compactifications

The aim of Part I is to present new implementations of certain models of axion inflation with large field displacements in setups of Type IIB ST/F-theory compactified on a CY manifold.[7] In particular, in chapter 2 we focus on axion monodromy inflation. Our main aim is to study the limitations of large field displacements due to backreaction of the compactification geometry. This is indeed one of the ways in which stringy UV degrees of freedom can spoil the approximate inflationary shift symmetry. The problem can be understood as follows. In setups based on compactifications there are many fields beyond the inflaton: the inflationary potential thus generically depends on other moduli. The latter are initially all stabilised. As the inflaton is displaced, the other moduli readjust to maintain a configuration of minimal energy. In other words, they are also displaced from their initial values. The danger then is that, when the inflaton moves along a long trajectory, the displacements of other moduli are large. The effect on the inflationary trajectory can be taken into account by integrating them out, i.e. by minimising the potential with respect to the other moduli beside the inflaton. The result of this procedure is a backreacted potential, which can exhibit dangerous regions of non-monotonicity and even local minima.[8]

Inspired by [101], we thus investigate the problem of backreaction in a $\mathcal{N} = 1$ supergravity setup where the inflaton potential is induced by supersymmetric F-terms: this general framework is known as *F-term axion monodromy inflation* [101–103]. This is one of the few settings where explicit computations can be done. In particular, we consider the inflaton to be the axionic component of a complex structure modulus of a CY manifold. In a certain limit of the CY geometry, known as the Large Complex Structure (LCS) limit [104, 105], this field enjoys a shift symmetry at the level of the Kähler potential. The source of symmetry breaking (monodromy) is provided by the superpotential, which features a linear term induced by higher-dimensional gauge fluxes. In order to make sure that the shift symmetry is only weakly broken, we need to impose a tuning condition on these coefficients and their derivatives with respect to the moduli fields. It is crucial that we keep all the other moduli of the compactification dynamical and perform the analysis of backreaction outlined above. Our conclusions are positive: the displacements of other moduli, while being non-negligible, do not spoil the flatness of the inflationary potential across a limited, but transplanckian, field range, which is in principle suitable for LFI. However, the potentially large number of required tunings

[7] A very brief introduction to F-theory [99] is postponed to chapter 1.1.3. For the time being, it suffices to say that F-theory provides a useful and complementary description of Type IIB orientifolds in terms of a twelve-dimensional theory, where two dimensions are non-physical and used for mathematical convenience. Thus compactifications are performed on eight-dimensional Calabi-Yau manifolds. In this case, complex structure moduli describe also the positions of Type IIB D7-branes.

[8] See instead [100] for positive effects of backreaction on the inflationary potential.

represents a severe downside of our construction, which we believe extends also to similar realisations of axion monodromy inflation.

In chapter 3 we present a new model of LFI with two complex structure axions with subplanckian periodicities, inspired by the proposal of Kim, Nilles and Peloso (KNP) [31] (see also [106, 107]), which we call *F-term winding inflation*. The framework is again that of 4D SUGRA, but the aim is to analyse a different source of shift-symmetry breaking: namely deviations from the LCS geometry, which manifest themselves as exponentially suppressed corrections to the Kähler and superpotential. We use a flux-induced F-term scalar potential to enforce a flat direction which effectively "winds" multiple times around the compact field space of two axions and is thus in principle suitable to realise LFI. The aforementioned instanton-like corrections generate a periodic potential, which realises natural inflation [30] with a transplanckian effective decay constant. We point out that our model is the first in realising a known loophole [68, 69] of the WGC in a setup of string compactifications, thereby evading its constraints (see subsection 1.2.4 for a description of the WGC and its loopholes). In particular, while our flat direction develops a periodic potential with a transplanckian periodicity which violates the WGC, there exist also superimposed oscillations with subplanckian periodicity. The latter are strongly suppressed in our model but are crucial in ensuring that the mild form of the WGC is satisfied.

Intermezzo: Axion Monodromy Inflation and the Weak Gravity Conjecture

Chapter 4 features two of the main actors of Part I. We explore the relation between the WGC and axion monodromy inflation. The latter is not constrained by the WGC for axions and instantons: after all, there is really no true axion in this mechanism, as its would-be shift symmetry is explicitly broken. It is thus the main aim of this chapter to show how the generalised WGC can be applied to constrain models of monodromy. We adopt a purely low-energy perspective (see [69, 108] for a previous, different point of view): our axion monodromy setup is defined by a quadratic potential (generated by a shift-symmetry breaking effect) with superimposed periodic "wiggles" as in (3), which can induce the existence of local minima. Such a setup can be effectively described in terms of domain walls separating the latter "vacua". Exactly as particles can be charged under gauge fields, domain walls can be charged under gauge potentials with three antisymmetric indices (3-forms). Thus, we propose to constrain axion monodromy inflation by applying the WGC to 3-form gauge fields and the associated domain walls.[9] In particular, the magnetic WGC translates into an upper bound on the inflationary field range, as desired. Nevertheless, we find that the maximal allowed field displacement can easily accommodate transplanckian inflation.

The second part of chapter 4 aims at providing evidence for the generalised WGC. We do so by presenting a geometric interpretation of the WGC in the framework of string compactifications. In particular, we show that the electric WGC for particles and gauge fields translates into a requirement on the geometry of the compactification manifold

[9] Notice however that these are *not* the monodromy domain walls described by [109, 110] (see also [111]).

(specifically, on the volumes of 3-cycles) and can thus be straightforwardly generalised to generic $(p+1)$-form gauge theories with p-dimensional charged objects.

Part II: Reheating

Part II is devoted to the phenomenological consequences of string-motivated scenarios for reheating. In chapter 5 we continue our exploration of axion monodromy inflation. We describe a very exciting observational feature of this model (see [90–93, 96] for other possible signatures of axion monodromy inflation): namely, the possibility of radiating gravitational waves *after* inflation. Such waves are very different in both frequency and strength from the tensor modes that can be observed by studying the polarisation of the CMB. In particular, they are potentially detectable by future ground- and space-based interferometers.

The phenomenon responsible for the generation of such gravitational waves is a post-inflationary phase decomposition of the Universe. Indeed, as we have already mentioned, axion monodromy potentials can exhibit local minima for post-inflationary field values.[10] Due to the expansion of the Universe, the inflaton gets eventually stuck in one of these "vacua", before reheating occurs. Crucially, due to field fluctuations, the inflaton can end up in different minima in different regions of the same Hubble patch. This happens for reasonable values of parameters. Such a "dynamical phase decomposition" is then followed by a phase transition, after which only the vacuum of lowest energy survives. The latter process occurs through collisions of cosmic bubbles containing the true vacuum: this violent event sources gravitational radiation [114–116] (see also [117–123] for related contexts in which the same phenomenon can occur). In this sense, the situation is quite analogous to the electroweak phase transition in and beyond the SM (see for instance [124]). In particular, also in our setup collisions may take place in a relativistic bath because of the very large release of energy.

Chapter 6 investigates the production of very light fields belonging to dark sectors during reheating. This so-called Dark Radiation (DR) can be indirectly detected in the CMB, whose temperature power spectrum depends on the effective number of neutrino species at CMB time, commonly denoted by N_{eff}. Most recent observations are in very good agreement with the SM prediction, $N_{eff} = 3.04 \pm 0.18$ (*Planck* TT,TE,EE+lowP+BAO, 68% C.L.) [2]. This observation is potentially very dangerous for setups of string compactifications, which are generically characterised by a plethora of light fields belonging to dark sectors. In order to concretely assess the impact of observations on ST, we focus on the aforementioned Large Volume Scenario (LVS) [50, 51]. Crucially, the latter exhibits at least one DR candidate: an essentially massless axion associated to the largest cycle of the compactification manifold. This axion is copiously produced during reheating, which happens through the decay of the so-called bulk modulus of the compactification [125–128]. Thus Dark Radiation represents a serious problem for the LVS.

Motivated by the severe constraints of DR on the LVS, we propose strategies to boost the production of (MS)SM fields during reheating. In particular, we focus on

[10] The same can occur with other inflationary and non-inflationary mechanisms involving axions (see e.g. [112, 113]).

a setup where reheating occurs via the decay of massive gauge bosons belonging to a dark sector and realising global flavour-like symmetries of the visible sector. In order to avoid the observational constraint, non-minimal values of the couplings or of the number of gauge bosons have to be considered. Our work can thus be considered as a demonstration of the constraining power of cosmological observations on string model building, in stark contrast with the common lore that ST scenarios cannot be tested by current and forthcoming measurements.

* * *

This thesis is based on the publications [129–133]. I maintained involvement in all aspects of the preparations of these publications. Here I wish to point out my main contributions, referring to the chapters of this thesis.

- **Chapter 1** as well as appendix A are reviews of well-known results on string compactifications and the Weak Gravity Conjecture.
- **Chapter 2** is based on [130]. I have performed the quantitative analysis of back-reaction of complex structure and Kähler moduli in CY orientifolds in a model of F-term axion monodromy inflation, contained in sections 2.4 and 2.5. Other sections of this chapter are summaries of parts of [130], included here for completeness.
- **Chapter 3** is based on [131]. I am responsible for the description of the inflationary winding trajectory as well as for the computation of the effective inflationary potential, reported in section 3.2.2 and 3.3 respectively. Furthermore, I contributed an interpretation of previous work in the context of the Weak Gravity Conjecture, contained in section 3.4. Other aspects of [131] are summarised here for completeness.
- **Chapter 4** is based on [132]. My collaborators have provided important ideas on the topics discussed in this publication, but all the computations contained in it as well as the writing are the result of my own work.
- **Chapter 5**, appendices B and C are based on [133]. My collaborators have provided important ideas on the topics discussed in this publication, but all the computations contained in it as well as the writing are the result of my own work.
- **Chapter 6** is based on [129]. The analysis of Dark Radiation in the Large Volume Scenario with flavour branes is my main contribution to this publication and is contained in section 6.4. Furthermore, all the plots contained in [129] and updated for this thesis are the result of my own work. Other aspects of [129] are summarised here for completeness.

Chapter 1

Preliminaries

In Part I of this thesis we will discuss inflationary scenarios in the framework of *string compactifications*. In this chapter we thus aim at introducing the essential ingredients of Type IIB String Theory (ST) and especially of its compactifications on Calabi-Yau manifolds. Furthermore, we introduce the Weak Gravity Conjecture (WGC) and discuss its applications to models of axion inflation, to prepare the reader for the material presented in chapters 3 and 4. The content of this chapter is by no means original, as its aim is to collect well-known results that are relevant to set the stage for the work presented in this thesis. We will often use the language of differential forms and p-form gauge theories, which we briefly review in Appendix A. We will also often set $M_P \equiv \sqrt{\frac{1}{8\pi G_N}} \equiv 1$.

1.1 String compactifications

An excellent presentation of the basic concepts of string theory and string compactifications for the study of inflationary models is provided in [134], which we follow closely in some parts of this section. Further references that we follow closely here are [38, 135, 136]. In particular, we adopt the standard notation of [38, 135, 136], which differs from [134].

1.1.1 Type IIB Effective Action

As discussed in the introduction, String Theory postulates that the fundamental constituents of Nature are one-dimensional strings, instead of point-like particles. Quantisation of the string leads to a spectrum of massless modes and a tower of massive excitations. The characteristic energy scale of a given massive mode is the string scale $M_s = 1/l_s$. We also often express the string scale in terms of the *Regge slope* $(\alpha')^2 \equiv 2\pi l_s$.

At energies below M_s we can then consider the effective field theory of the massless string modes: as usual, higher order operators in this theory are induced by the UV degrees of freedom, i.e. by the string massive modes, and are therefore suppressed by powers of M_s. In 10D there are five kinds of supersymmetric string theories. Correspondingly, five different effective field theories in 10D can be obtained by integrating out the massive string modes. Crucially, these effective theories are nothing else than well-known 10D supergravities (SUGRA). The most important ones for inflationary model building are known as Type IIA and Type IIB SUGRA and have $\mathcal{N} = 2$ supercharges. In particular, in this chapter we focus on the Type IIB theory, since it is the one that is relevant to understand the content of this thesis.

The massless bosonic spectrum of Type IIB ST splits into two branches. On one hand, the so-called NS-NS sector contains the 10D metric G_{MN}, the antisymmetric 2-form B_{MN} and the dilaton Φ. The string coupling g_s is defined as: $g_s = e^{\langle\Phi\rangle}$. As its name suggests, g_s controls the strength of string interactions; in particular loop amplitudes are obviously accompanied by higher power of the string coupling. For the time being we will assume $g_s \ll 1$, i.e. we will work at weak coupling. On the other hand, the R-R sector contains the 10D p-form potentials (see section A.4 for a review of p-form gauge theories). In Type IIB only potentials with p even are present, i.e. C_0, C_2, C_4. We will not discuss the fermionic sector, which contains two gravitini and two dilatini. The Type IIB bosonic action is:

$$
\begin{aligned}
S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}\left[e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}|F_1|^2 - \frac{1}{2}|F_3|^2 - \frac{1}{4}|F_5|^2\right] \\
- \frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3, \qquad (1.1)
\end{aligned}
$$

where κ_{10}^2 is the 10D Newton constant, related to α' by

$$
\kappa_{10}^2 = \frac{1}{4\pi}(4\pi\alpha')^4. \qquad (1.2)
$$

The gauge invariant field strengths F_{p+1}, H_3 in (1.1) are defined as:

$$F_1 = dC_0, \quad F_3 = dC_2 - C_0 H_3 \tag{1.3}$$

$$F_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge dC_2, \quad H_3 = dB_2. \tag{1.4}$$

A further self-duality constraint on F_5 has to be imposed at the level of the equations of motion, i.e. $\star_{10} F_5 = F_5$ (see Appendix A.4).[1] The action (1.1) is presented in the so-called *string frame*: the Ricci scalar is thus explicitly coupled to the dilaton. In order to decouple them, one can perform a rescaling of the 10D metric: $\tilde{G}_{MN} = e^{-\Phi/2} G_{MN}$ and obtain the action in the so-called *Einstein frame*.

The Type IIB action (1.1) enjoys an $SL(2, \mathbb{R})$ invariance. In order to make it manifest, it is convenient to define the axio-dilaton S and the 3-form flux G_3

$$S = C_0 + ie^{-\Phi}, \quad G_3 = F_3 - ie^{-\Phi} H_3. \tag{1.5}$$

The type IIB action in the Einstein frame, in terms of the field S and G_3, reads:

$$\begin{aligned} S_{IIB} = & \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[R - \frac{\partial_M S \partial^M \bar{S}}{2(\mathrm{Im} S)^2} - \frac{1}{2} \frac{|G_3|^2}{\mathrm{Im} S} - \frac{1}{4} |F_5|^2 \right] \\ & + \frac{1}{8i\kappa_{10}^2} \int \frac{1}{\mathrm{Im} S} C_4 \wedge G_3 \wedge \bar{G}_3, \end{aligned} \tag{1.6}$$

and is thus manifestly invariant under the $SL(2, \mathbb{R})$ transformations:

$$S \to \frac{aS + b}{cS + d}, \quad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \tag{1.7}$$

with $ad - bc = 1$ and $a, b, c, d \in \mathbb{R}$.

1.1.2 D-brane action

ST is not only a theory of strings, but also of higher-dimensional dynamical surfaces, known as D-branes. In fact, these objects are crucial in describing the dynamics of the massless modes of open strings. In particular, very much like the motion of particles is characterised by a *worldline* (i.e. the one-dimensional curve describing the motion of the particle as a function of its proper time), Dp-branes are characterised by a (p+1)-dimensional *worldvolume* Σ_{p+1}. The subject of D-branes is enormous. Here we will only recall the action of these objects, which is a direct non-linear generalisation of the action of particles coupled to gauge fields in 4D (see Appendix A.4). An excellent introduction to D-branes is provided in [139].

The action of Dp-branes is made by two terms: the Dirac-Born-Infeld (DBI) action describes the coupling of the open string modes to the NS-NS sector, while the Chern-Simons (CS) action describes the coupling to the R-R sector. Let us then consider

[1] As explained in Appendix A.4, the field strength F_7, F_9 are dual to F_3, F_1 respectively and are thus not included in (1.1). However, there exists another formulation of the Type IIB SUGRA, known as *democratic*, where all the p-form field strengths (with p odd) are included [137, 138], together with the corresponding duality constraints.

a Dp-brane spanning its (p+1)-dimensional worldvolume and let us choose coordinates $\xi^\alpha, \alpha = 0, \cdots, p$ on Σ_{p+1}. In terms of 10D coordinates, the worldvolume Σ_{p+1} is described by the embedding functions $X^M \equiv X^M(\xi), M = 0, \ldots, 9$. The brane action in the string frame is then given by

$$S_{Dp-brane} = S_{DBI} + S_{CS}$$

$$S_{DBI} = -T_p \int_\Sigma d^{p+1}\xi e^{-\Phi(X)} \sqrt{-\det\left(g_{\alpha\beta}(X) + 2\pi\alpha' \mathcal{F}_{\alpha\beta}(X)\right)} \tag{1.8}$$

$$S_{CS} = \mu_p \int_\Sigma \mathrm{Tr}\left(e^{2\pi\alpha'\mathcal{F}}\right) \wedge \sum_n C_n. \tag{1.9}$$

Here $g_{\alpha\beta}$ is the induced metric on the worldvolume Σ_{p+1}, which can be explicitly computed by pulling-back the 10D metric G_{MN} on Σ_{p+1}, i.e. $g_{\alpha\beta} = G_{MN}\partial_\alpha X^M \partial_\beta X^N$. Furthermore, $2\pi\alpha'\mathcal{F}_{\alpha\beta} = B_2 + 2\pi\alpha' F_2$, where F_2 is the field strength of the worldvolume gauge theory (thus, it is not one of the bulk fluxes). All the forms appearing in (1.8) are evaluated on the brane worldvolume, meaning that they are obtained by pulling back the corresponding 10D forms on Σ_{p+1}. Notice that the sum over n in the CS action runs over even values of n in the Type IIB theory and such that the total integrand is a $(p+1)$-form. The Dp-brane tension and charge coincide in the string frame, i.e. $T_p = \mu_p$, and their value is related to the string scale by

$$T_p = \mu_p = 2\pi M_s^{p+1}. \tag{1.10}$$

The DBI action provides a coupling between the D-brane and the 10D metric tensor. Crucially, such interactions, as well as the one described by the CS action, are plagued by divergences, which are referred to as *tadpoles*. They are related to the radiation of quanta from the vacuum. In order for the theory to be consistent, these divergences have to be absent. This can be achieved by engineering *tadpole cancellations*, i.e. by including objects which induce the same tadpole amplitude, but with opposite sign. In this specific case, the gravitational tadpole associated to the positive tension of a Dp-brane can be cancelled by an *orientifold plane* (O-plane), i.e. a non dynamical hypersurface with negative tension. The DBI action of an Op-plane is similar to (1.8) (without the coupling to $\mathcal{F}$) with its tension being $T_p^O = -2^{p-4}T_p$. We will comment more on these objects when describing compactification on CY orientifolds, see below.

1.1.3 Compactification to 4D

The supergravities discussed in the previous subsections are quantum field theories defined on a 10D spacetime M_{10}. In order to extract 4D physics we need to dimensionally reduce the actions (1.1), (1.8) and (1.9). This can be done by considering the following *compactification* ansatz:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times M_6, \tag{1.11}$$

where $\mathcal{M}_4$ is the ordinary four-dimensional spacetime and M_6 is a compact six-dimensional Riemannian manifold. Given this product structure, the 10D metric G_{MN} splits into a 4D metric on $\mathcal{M}_4$ and a 6D metric on M_6. A natural ansatz for the metric would then be

$$G_{MN}dX^M dX^N = \eta_{\mu\nu}dx^\mu dx^\nu + \tilde{g}_{mn}dy^m dy^n, \tag{1.12}$$

where $y^m, m = 1, \ldots, 6$ are coordinates on M_6. This is a solution of the 10D Einstein equations in the absence of sources if and only if $\tilde{g}_{mn}$ is Ricci-flat, meaning that $\tilde{R}_{mn} = 0$. This motivates the interest in Calabi-Yau (CY) manifolds, which are precisely compact Kähler manifolds admitting a Ricci-flat metric (see section A.3 for a review of complex manifolds). However, we are generically interested in solutions of the 10D Einstein equations in the presence of branes and orientifold planes, which act as sources. We will thus consider a more general ansatz with warping:

$$G_{MN}dX^M dX^N = e^{2A(y)}g_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n, \tag{1.13}$$

where $g_{\mu\nu}$ is the metric of a maximally symmetric 4D spacetime (e.g. de Sitter spacetime for the study of inflationary models) and $A(y)$ is a warping factor. Crucially, Einstein equations in the presence of sources do *not* require $\tilde{g}_{mn}$ to be Ricci-flat. Thus, it appears that the motivation for CY manifolds is lost. Nevertheless, compactifications where M_6 is a CY 3-folds are the most studied and the best-understood. This is due to the fact that CY manifolds preserve a certain amount of the original 10D $\mathcal{N} = 2$ supersymmetry (due to the fact that their holonomy group is $SU(m)$, where m is the complex dimension of the compact manifold). In turn, this is important both at the theoretical level, as the resulting 4D effective field theory enjoys the important and simplifying structure of a supergravity theory, and at the phenomenological level, as $\mathcal{N} = 1$ supersymmetry may help in solving the electroweak hierarchy problem. In this thesis, we shall be concerned only with compactification on CY manifolds, whose basic concepts are reviewed in Appendix A.3.

Dimensional Reduction

The strategy to obtain an effective 4D theory now proceeds through dimensional reduction of the actions (1.1), (1.8) and (1.9). For definiteness, let us illustrate this procedure by focusing on the effective action for 4D scalar fields.

Consider a generic p-form A_p defined on $\mathcal{M}_{10} = \mathcal{M}_4 \times M_6$. We make the ansatz

$$A_p = a^i(x)\omega_{p,i}(y), \tag{1.14}$$

where $a^i(x)$ are 4D scalar fields and $\{\omega_{p,i}(y), i = 0, \ldots, b_p\}$ is an harmonic basis of the cohomology group $H^p(M_6)$. As explained in Appendix A.2, Hodge's theorem relates the basis $\{\omega_{p,i}(y)\}$ to a basis $\{\Sigma_p^i\}$ of the homology group $H_p(M_6)$, such that

$$\int_{\Sigma_p^i} \omega_{p,j} = \delta^i_j. \tag{1.15}$$

The next step is to integrate (1.1) and (1.8) on M_6. Some integrals can be performed explicitly, by means of (1.14) and (1.15), others are left implicit at this stage. For instance, the kinetic term for $C_2 = c^i \wedge \omega_{2,i}, i = 0, \ldots, b_2$ becomes (we set the warping factor $A(y) = 0$ for convenience):

$$S \supset -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(\frac{1}{2}|dC_2|^2\right) = -\frac{1}{2}\frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g}k^{ij}(\partial^\mu c_i\partial_\mu c_j), \tag{1.16}$$

where $k^{ij} = \int_{M_6} \omega^i \wedge \star_6 \omega^j$ is a metric on H^p. Notice that dimensional reduction of the Einstein-Hilbert term defines the 4D reduced Planck mass in terms of α' and g_s, as follows:

$$M_P^2 = \frac{V_{M_6}}{\kappa_{10}^2 g_s^2} = \frac{2V_{M_6}}{(2\pi)^7 \alpha'^4 g_s^2}, \tag{1.17}$$

where V_{M_6} is the volume of M_6.

The same strategy that led to (1.16) can be followed to obtain vector and more general n-form fields a_n in 4D. In other words, one starts from the ansatz $A_p = a_n^i \wedge \omega_{i,p-n}(y)$ and integrates the corresponding terms in the action on M_6. The reader who is interested in a concrete computation for this latter case can find one in section 4.3.

Moduli of CY compactifications

String compactifications are characterised by *moduli*, i.e. massless scalar fields which arise from dimensional reduction of: p-forms in the the 10D theory, positions of D-branes and deformations of the hermitian metric. In this subsection we focus on the latter, following closely the review given in [38] (see [60] for the original presentation). So-called *Kähler* and *complex structure* moduli are precisely associated to fluctuations δg of the CY metric which preserve the Ricci-flatness of g, i.e. such that $R(g + \delta g) = 0$. The latter equation translates into a differential equation for δg, whose solutions are associated to $(1,1)$- and $(2,1)$-forms on the CY M_6.

As their name suggests, Kähler moduli arise from those fluctuations of the CY metric that also deform the $(1,1)$ Kähler form $J = ig_{i\bar{j}} dy^i \wedge dy^j$. There are therefore $h^{1,1}$ real scalar fields v^α defined by

$$-iJ_{i\bar{j}} = -iv^\alpha(x)(\omega_\alpha(y))_{i\bar{j}}, \tag{1.18}$$

where $\omega_\alpha, \alpha = 1, \dots, h^{1,1}$ is a basis of $H^{1,1}$ (and of $H^{2,2}$). Similarly, by expanding B_2 in the basis ω_α of 2-forms, we obtain $h^{1,1}$ further scalar fields b^α. The Kähler moduli are the complex scalar fields T^α defined by

$$T^\alpha = v^\alpha + ib^\alpha. \tag{1.19}$$

As we will discuss after introducing orientifolds, the real part of a Kähler modulus is associated to the volume of the corresponding 4(2)-cycle.

Complex structure moduli are instead associated to fluctuations of the metric which also deform the complex structure $\mathcal{J}$ of M_6. In particular, they are $h^{2,1}$ complex scalar fields $z^I, I = 1, \dots, h^{2,1}$ defined by

$$\delta g_{\bar{i}\bar{j}} = ic\, z^I(x)(\chi_I)_{kl\bar{i}} \bar{\Omega}^{kl}_{\bar{j}}, \tag{1.20}$$

where c is a prefactor which depends on the normalisation of the holomorphic 3-form Ω of M_6. Furthermore $\chi_I, I = 1, \dots, h^{2,1}$ is a basis of the cohomology group $H^{2,1}(M_6)$. Complex structure moduli are the crucial ingredient of the inflationary models discussed in chapters 2, 3.

The moduli space $\mathcal{M}$ of geometric deformations of a CY manifold can be locally written as $\mathcal{M} = \mathcal{M}_{cs} \times \mathcal{M}_k$, where $\mathcal{M}_{cs}$ is the complex structure moduli space and $\mathcal{M}_k$ is the Kähler moduli space. We now wish to introduce metrics on these spaces. Crucially, these spaces are both Kähler manifolds. A general result is that the hermitian metric of a Kähler manifold can be locally derived from a scalar function, which is referred to as the *Kähler potential*. In the next subsection we will see that this function indeed corresponds to the Kähler potential of a $\mathcal{N} = 1$ supergravity theory.

Let us then start with complex structure moduli. It can be shown that the metric and Kähler potential on $\mathcal{M}_{cs}$ are given by [60]

$$G_{I\bar{J}} = \partial_{z^I}\partial_{\bar{z}^{\bar{J}}} K_{cs}, \qquad K_{cs} = -\ln\left[i\int_{M_6} \Omega \wedge \bar{\Omega}\right], \tag{1.21}$$

where Ω is the holomorphic 3-form on M_6. On H^3 there exists a real symplectic basis (α_I, β^L) of harmonic 3-forms, meaning that

$$\int_{M_6} \alpha_I \wedge \beta^L = \delta_I^L, \tag{1.22}$$

while all the other wedge products vanish. Correspondingly, there exists a dual basis of 3-cycles (A^I, B_L), such that

$$\int_{A^L} \alpha_I = -\int_{B_I} \beta^L = \delta_I^L. \tag{1.23}$$

We can thus decompose Ω as follows:

$$\Omega(z) = Z^I \alpha_I - \mathcal{F}_L \beta^L, \tag{1.24}$$

where $Z^I, \mathcal{F}_L$ are holomorphic functions, knowns as *periods* of M_6. Therefore the Kähler potential can be written as

$$K_{cs} = -\ln\left[i(\bar{Z}^I \mathcal{F}_I - Z^I \bar{\mathcal{F}}_I)\right]. \tag{1.25}$$

In chapters 2 and 3 we will also use the so-called *period vector* whose components Π^i are defined by

$$\Pi^i \equiv \int_{\Sigma_3^i} \Omega = \int_{M_6} \sigma^i \wedge \Omega, \tag{1.26}$$

where $\Sigma_3^i = (A^I, B_L), i = 0, \ldots, h^{3,0}$ is a basis of 3-cycles of M_6 and $\sigma^i = (\alpha_I, \beta^L)$ is the dual basis of 3-forms on M_6. In terms of the periods defined in (1.24), we have $\Pi = (\mathcal{F}_L, Z^I)$.

Let us now turn to Kähler moduli and define the intersection numbers

$$\kappa_{\alpha\beta\gamma} = \int_{M_6} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma, \qquad \kappa = \kappa_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma. \tag{1.27}$$

It can then be shown that the metric and Kähler potential on $\mathcal{M}_k$ are given by

$$G_{\alpha\beta} = \partial_{s^\alpha}\partial_{s^\beta} K_k \tag{1.28}$$

$$K_k = -\ln\left[\frac{i}{6}\kappa_{\alpha\beta\gamma}(s - \bar{s}^\alpha)(s - \bar{s}^\beta)(s - \bar{s}^\gamma)\right] = -\ln\frac{4}{3}\kappa. \tag{1.29}$$

Finally, let us briefly discuss an important duality between Kähler and complex structure moduli. *Mirror symmetry* (see [61]) and refs. therein) states that for every CY M with

Hodge numbers $h^{1,1}$ and $h^{2,1}$ there exist a *dual* CY $\tilde{M}$ with Hodge numbers $\tilde{h}^{1,1} = h^{2,1}$ and $\tilde{h}^{2,1} = h^{1,1}$. It is conjectured that this exchange of Hodge numbers extends to the entire geometric moduli spaces of M and $\tilde{M}$. In particular, mirror symmetry conjectures that the Kähler moduli of M correspond to the complex structure moduli of $\tilde{M}$ and that the complex structure moduli of M correspond to the Kähler moduli of $\tilde{M}$. Thus, the Kähler potential for complex structure moduli of M_6 can be obtained in terms of the Kähler moduli Kähler potential on the mirror dual $\tilde{M}_6$. In terms of effective theories, mirror symmetry conjectures a duality between Type IIB ST compactified on M and Type IIA ST compactified on $\tilde{M}$.

Calabi-Yau orientifolds

Compactifications of type IIB $\mathcal{N} = 2$ 10D SUGRA on CY manifolds lead to 4D $\mathcal{N} = 2$ SUGRAs. Such theories do not have fermions in chiral representations of the gauge groups, which is a crucial feature of the SM. Thus, they are not phenomenologically appealing. However, $\mathcal{N} = 1$ SUGRA IN 4D can be achieved if M_6 is a CY *orientifold*, i.e. by modding out the spectrum of the compactified theory by a certain symmetry transformation. Namely, for type IIB, we consider the following two options for such transformations:

$$\mathcal{T}^1 = (-1)^{F_L}\Omega_p\sigma, \qquad \mathcal{T}^2 = \Omega_p\sigma, \tag{1.30}$$

Here F_L is the space-time number of left-moving fermions and Ω_p is the parity transformation on the string world-sheet theory. While the latter concepts have not been reviewed in this chapter (see e.g. [136] for details), for our purposes only the action of these symmetries on the type IIB bosonic spectrum is important. In particular, fields are either even or odd under $(-1)^{F_L}$ and Ω_p, as follows

$$\Omega_p \qquad \phi, g, C_2 \text{ are even} \qquad\qquad C_0, B_2, C_4 \text{ are odd} \tag{1.31}$$

$$(-1)^{F_L} \qquad \phi, g, B_2 \text{ are even} \qquad\qquad C_0, C_2, C_4 \text{ are odd} \tag{1.32}$$

Furthermore, σ is an isometric and holomorphic involution (meaning $\sigma^2 = \mathbb{1}$) acting solely on M_6. The Kähler form J is left invariant by σ. The two possibilities in (1.30) arise from the sign of the holomorphic 3-form under σ, i.e. $P(\sigma)\Omega = -\Omega$ or $P(\sigma)\Omega = +\Omega$ respectively, where P denotes the pull-back of σ.

The transformations (1.30) are crucially related to the O(rientifold)-planes allowed in the 10D theory. Those objects are the fix-point sets of σ. In particular, the transformation $\mathcal{T}^1$ correspond to compactifications with $O3$ and $O7$ planes, while $\mathcal{T}^2$ to compactifications with $O5$ and $O9$ planes. Due to the necessity of tadpole cancellation, only $D3/D7$ ($D5/D9$) branes are allowed in a compactification with $O3/O7$ ($O5/O9$) planes. Both compactifications lead separately to 4D $\mathcal{N} = 1$ SUGRA, but the simultaneous presence of e.g. $O3$ ($D3$) and $O5$ ($D5$) planes generically breaks supersymmetry completely.

Another important aspects of orientifolds for the content of this thesis is the action of the involution on the cohomology groups $H^{(p,q)}$. The latter split under the action of $P(\sigma)$ (since the latter is holomorphic, it respects Hodge decomposition), i.e. $H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}$, where $\pm$ denote forms that are even or odd under σ respectively. One can then define the orientifold Hodge numbers as $h_\pm^{(p,q)}$ as the dimensions of $H_\pm^{(p,q)}$ respectively.

Crucially, only those fields that are left invariant by $\mathcal{T}^1$ or $\mathcal{T}^2$ survive in the low-energy spectrum. Here, we report only the scalar spectrum of Type IIB compactifications with $O3/O7$ planes (see e.g. [38] and refs. therein for details):

- **Axio-dilaton**: C_0 and Φ combine in a complex 4D scalar

$$S = C_0 + ie^{-\Phi}. \tag{1.33}$$

- **p-form axions**:

  → $2h_-^{1,1}$ real scalars b^α, c^α associated to the decomposition of $B_2 = b^\alpha(x)\omega_\alpha(y), C_2 = c^\alpha(x)\omega_\alpha(y), \alpha = 1, \dots, h_-^{1,1}$. They can be combined in complex fields

$$G^\alpha \equiv c^\alpha - Sb^\alpha. \tag{1.34}$$

  → $h_+^{(1,1)}$ real scalars ϑ^i associated to the decomposition of

$$C_4 = \vartheta^i(x)\tilde{\omega}_i(y), \quad i = 1, \dots, h_+^{2,2}. \tag{1.35}$$

- **Kähler moduli**: only the moduli associated to $H_+^{1,1}$ survive in the 4D theory. Therefore, there are $h_+^{1,1}$ real Kähler moduli t^i in 4D. Their correct complexification (such that they can be interpreted as Kähler coordinates, see e.g. [38]) is not straightforward and involves the fields ϑ^i and G^α as follows:

$$T_i \equiv \frac{1}{2}\kappa_{ijk}t^j t^k + i\vartheta_i + \frac{1}{4}e^\Phi \kappa_{i\alpha\beta}G^\alpha(G - \bar{G})^\beta. \tag{1.36}$$

- **Complex structure moduli**: only the moduli associated to $H_-^{2,1}$ survive. Therefore, there are $h_-^{2,1}$ complex structure moduli in 4D $z^I, I = 1, \dots, h_-^{2,1}$.

In total, there are therefore $h_-^{2,1} + h_+^{1,1} + h_-^{1,1} + 1$ moduli in the 4D spectrum.

Before moving to the 4D effective action, we would like to make two important comments. Firstly, notice that we have called *axions* the scalar fields descending from the p-form potentials B_2, C_2 and C_4. In fact, also C_0 should be considered an axion. This naming is justified: fields descending from dimensional reduction of p-forms on q-cycles enjoy the remnants of the higher-dimensional gauge invariance. In this specific case, $q = p$ and the leftover invariance is simply a continuous shift symmetry, e.g. $b^\alpha \to b^\alpha + c$. It turns out that this symmetry remains valid at all orders in α' and g_s [140], but it is broken by non-perturbative effects to a familiar axionic discrete shift symmetry, e.g. $b^\alpha \to b^\alpha + 2\pi f$, where f is the axion decay constant. The latter can be determined in terms of α' and g_s by dimensional reduction. However, in this thesis we shall not be concerned with axions descending from p-forms, therefore we do not discuss them any longer (see e.g. [134] for a detailed discussion).

Secondly, let us discuss the geometric interpretation of Kähler moduli. Thanks to de Rham's theorem (see Appendix A.2) we can associate to every p-form on M_6 a dual p-cycle. On a six-dimensional manifold 2- and 4-cycles are dual, in the sense that $H_2(M_6) \simeq H_4(M_6)$. Thus a p-form can be associated to a p-cycle, or equivalently to a $(d-p)$-cycle.

In particular, the real part of the Kähler modulus T_i has the interpretation of describing the volume of the 4-cycle Σ_i which is dual to ω_i of (1.35). We will thus often refer to the real part of T_i as a 4-cycle *volume modulus* and denote it by τ_i. The latter is then related to the 2-cycle volumes t_i by means of

$$\tau_i = \frac{1}{2}\kappa_{ijk}t^j t^k, \tag{1.37}$$

where κ_{ijk} are the triple intersection numbers defined in (1.27).

4D Effective Theory

We are now ready to describe the effective theories resulting from compactifications of 10D Type IIB ST on CY orientifolds with $O3/O7$ planes. In the absence of further sources of supersymmetry breaking, these theories are 4D $\mathcal{N} = 1$ supergravities. Their scalar spectrum was given in the previous subsection. Additionally, there is a metric field $g_{\mu\nu}$ and gauge potentials A^a_μ. The full effective action of type IIB CY orientifolds was derived in [141] (see also [38] for a detailed review).

The Lagrangian of a 4D SUGRA theory is expressed in terms of three functions of the moduli fields, which we collectively denote by φ^A: the Kähler potential $K(\varphi^A, \bar{\varphi}^{\bar{A}})$, the holomorphic superpotential $W(\varphi^A)$ and the holomorphic gauge kinetic functions f_{ab}. Using subscripts to denote derivatives with respect to fields, i.e. $K_A = \partial_{\varphi^A} K$ and ignoring D-terms, we have:

$$\mathcal{L} \supset -K_{A\bar{B}} D^\mu \varphi^A D_\mu \bar{\varphi}^{\bar{A}} + V_F + \frac{1}{2}\text{Re} f_{ab} F^a_{\mu\nu} F^{\mu\nu,b} + \frac{1}{2}\text{Im} f_{ab} F^a_{\mu\nu} \tilde{F}^{\mu\nu,b}, \tag{1.38}$$

where D_μ here denotes the gauge covariant derivative, $F^a_{\mu\nu}$ is the field strength of A^a_μ, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\gamma\delta}F^{\gamma\delta}$ and V_F is the F-term scalar potential, given by

$$V_F = e^K (K^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} - 3|W|^2), \tag{1.39}$$

where $D_A W = \partial_A W + K_A W$ is the Kähler covariant derivative. As anticipated in the previous subsections, the Kähler potential appearing in (1.38) is related to the "geometric" Kähler potentials of (1.21), (1.28). In particular, we have

$$K = K_{cs}(z, \bar{z}) + K^Q(S, T, G) \tag{1.40}$$

$$K_{cs} = -\ln\left[-i\int_Y \Omega \wedge \bar{\Omega}\right] \tag{1.41}$$

$$K^Q = -\ln\left[-i(S - \bar{S})\right] - 2\ln\left[\text{Vol}(S, T, G)\right], \tag{1.42}$$

where $\text{Vol} = \frac{1}{6}e^{-\frac{3}{2}\Phi}\kappa_{\alpha\beta\gamma}v^\alpha v^\beta v^\gamma$ is the volume of M_6. Unfortunately the latter is in general only implicitly defined as a function of S, T and G. For the simple case of only one real Kähler modulus v, we have $K^Q = -3\ln(T + \bar{T})$. Notice that, in terms of the period vector defined in (1.26) the complex structure moduli Kähler potential can be written as (here we follow [142])

$$K_{cs} = -\ln\left[-i\Pi^\dagger \cdot \Sigma \cdot \Pi\right], \tag{1.43}$$

where $\Pi^\dagger$ is the hermitian conjugate of Π and

$$\Sigma = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}. \tag{1.44}$$

We shall make use of (1.43) in chap. 2.

A comment is in order before moving to the next section. The aforementioned *mirror symmetry* relates complex structure moduli descending from a Type IIB compactification on M_6 to Kähler moduli descending from a Type IIA compactification on the mirror dual CY $\tilde{M}_6$. Thus, while we do not discuss the Type IIA theory and its compactification in this chapter, it is important to report the associated Kähler moduli Kähler potential:

$$K^{IIA} = -\ln\left[\frac{i}{6}\tilde{\kappa}_{\alpha\beta\gamma}(\tilde{t}-\bar{\tilde{t}})^\alpha(\tilde{t}-\bar{\tilde{t}})^\beta(\tilde{t}-\bar{\tilde{t}})^\gamma\right], \tag{1.45}$$

where $\tilde{t}^\alpha, \alpha = 1, \ldots, \tilde{h}_-^{1,1}$ are the Kähler moduli of $\tilde{M}_6$.

1.1.4 Flux compactifications

In the previous section we described how moduli arise from a compactification of Type IIB ST on Calabi-Yau 3-folds. In particular, we derived the 4D effective theory, which is a $\mathcal{N} = 1$ SUGRA. From (1.38), it is clear that the couplings of such effective theory are moduli-dependent functions. Therefore, in order to obtain physical predictions, we need to stabilise them, i.e. to give them a VEV. For inflationary purposes, it is moreover important that moduli acquire and mantain a positive mass-squared during inflation, otherwise dangerous instabilities arise in the theory. The aim of this section is thus to review the strategy and the main results on *moduli stabilisation* in type IIB orientifolds.

Before moving to the details, let us remind the reader that fluxes $F_{p+1} = dC_p$, satisfying the Bianchi identity $dF = 0$, obey certain *quantisation conditions*. Namely, the flux through a $(p+1)$-dimensional surface S_{p+1} without boundary is quantised. Our convention is that

$$\frac{1}{(2\pi)^p\alpha'^p}\int_{S^{p+1}} F_{p+1} \in \mathbb{Z}. \tag{1.46}$$

Stabilisation of complex structure moduli

Let us begin by describing how the axio-dilaton and complex structure moduli can be stabilised at the classical supergravity level, by solving 10D Einstein equations, following [48]. We will be concerned with compactifications of Type IIB ST on CY 3-folds with O3/O7 planes, with background 3-form fluxes F_3 and H_3. Namely, we add fluxes which are not induced by local sources, such as D-branes, but rather live in the "bulk" of the compactification. We anticipate that their effect for the 4D effective SUGRA is to induce the so-called Gukov-Vafa-Witten (GVW) superpotential:

$$W(S, z^I) = \int_{M_6} \Omega \wedge G_3, \tag{1.47}$$

where $G_3 = F_3 - SH_3$. Crucially, since G_3 and Ω depend on S and the complex structure moduli, the GVW superpotential generates a scalar potential for these fields, which can thus be stabilised.

At leading order in α' and g_s, the action of the 10D theory is the one given in subsection 1.1.1. We now consider warped compactifications with metric ansatz

$$G_{MN}dX^M dX^N = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n. \tag{1.48}$$

Poincaré invariance in 4D requires G_3 to have components solely on M_6. The self-dual flux F_5 is instead taken to be

$$F_5 = (1 + \star_{10})d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{1.49}$$

The next step is to look for solutions of the 10D Einstein equations and the 10D Bianchi identity for F_5. Here we report only the results of such analysis.

First of all, the integrated (over M_6) Bianchi identity for F_5 (i.e. Gauss' law) leads to the *tadpole-cancellation condition*:

$$\frac{1}{2\kappa_{10}^2 T_3}\int_{M^6} H_3 \wedge F_3 + Q_3^{loc} = 0, \tag{1.50}$$

where T_3 and Q_3 are respectively the tension and the total charge associated to D3-branes which act as local sources.

Secondly, the 10D Einstein equations require the presence of objects with negative tension if G_3 is not vanishing. This is yet another piece of evidence for the presence of O-planes, in particular O3 and O7 planes. Furthermore, by combining Einstein equations with the Bianchi identity for F_5 and using (1.49), one finds the following conditions on $G_3, \alpha(y)$ and $A(y)$, assuming that only O3/O7 planes are present:

$$\star_6\, G_3 = iG_3, \qquad e^{4A(y)} = \alpha. \tag{1.51}$$

Therefore, G_3 is *imaginary self dual* (ISD). In what follows we shall therefore focus only on so-called *ISD solutions* with O3/O7 planes (and D3/D7 branes).

Finally, the stabilising potential for the complex structure moduli and the axio-dilaton is just the F-term scalar potential, computed according to (1.39) and by means of (1.47) and (1.40). In particular, the SUSY VEVs of these moduli are determined by imposing the F-term conditions $D_A W = 0$, where A runs over all moduli T_i, G_α, S, z^I. In chapters 2, 3 we will compute the F-term scalar potential for certain inflationary models using the GVW superpotential (1.47). Notice in particular that, after imposing flux quantisation on F_3 and H_3 according to (1.46), the superpotential reads

$$W = \int_{M_6}(F_3 - SH_3)\wedge\Omega = (N_F - SN_H)^i\Pi_i, \tag{1.52}$$

where $N_F, N_H \in \mathbb{Z}$ are the so-called *flux numbers* and Π_i is the period vector, defined via the decomposition of Ω in terms of the symplectic basis of $H^3(M_6)$, see (1.26).

Let us understand why Kähler moduli are *not* stabilised by the F-term potential (1.39). The Kähler moduli Kähler potential (1.40) is of *no-scale* type, meaning that

$$K^{A\bar{B}}\partial_A K \partial_{\bar{B}} K = 3, \tag{1.53}$$

where A, B run over T^i, G^α. In this case, since the superpotential (1.47) does not depend on the Kähler moduli, the F-term scalar potential (1.39) reduces to

$$V_F = e^K K^{I\bar{J}} D_I W D_{\bar{J}}\bar{W}, \tag{1.54}$$

where I, J run over the complex structure moduli z^I. Thus there is simply no potential for Kähler moduli at leading order in α' and g_s.

Generalisation to F-theory

So far we have described compactifications of Type IIB String Theory on CY orientifolds with D3/D7 branes and O3/O7 planes. We have done so under the implicit assumption that the backreaction of the latter objects on the geometry of the background can be neglected. In particular, this implies that the axio-dilaton S does not vary strongly across the manifold. However, it turns out that this assumption is not fully justified, in that e.g. $D7$ brane and $O7$ planes do induce a non-negligible backreaction on the background geometry. In particular, close to a $D7$ brane the string coupling g_s diverges (see e.g. [143] and refs therein).

Thus, a study of compactifications with varying axio-dilaton profile is required. This can be very elegantly addressed in F-theory [99]. In this framework, the axio-dilaton of Type IIB is interpreted as the complex structure modulus of a fictitious (i.e. non-physical) torus. Therefore, F-theory is a twelve-dimensional theory, without D-branes. In particular, brane position moduli of Type IIB belong to the complex structure moduli sector of F-theory on Calabi-Yau *fourfolds* (i.e. eight-dimensional manifolds). The weak coupling limit of F-theory reproduces Type IIB ST, and can be obtained by compactifying on *elliptic fibrations*.

A presentation of F-theory goes beyond the aim of this preliminary chapter. Here we limit ourselves to provide formulae which are relevant for moduli stabilisation, since they will be used in chapter 2. The reader who is interested in understanding the subject can consult the excellent introductions given in [144] and [143].

F-theory exhibits a unique 4-form "bulk" flux G_4. Complex structure moduli can be stabilised by means of the generalised Gukov-Vafa-Witten superpotential

$$W_4 = \int_X G_4 \wedge \Omega_4, \tag{1.55}$$

where Ω_4 is the holomorphic 4-form on the fourfold X. The period vector Π^α of the fourfold can be defined in analogy with (1.26). Thus, after quantisation of G_4, the superpotential can be written as

$$W_4 = N_\alpha \Pi^\alpha. \tag{1.56}$$

Similarly, the complex structure moduli Kähler potential can be obtained by direct generalisation of (1.43)

$$\begin{aligned} \mathcal{K}_{cs} &= -\ln\left[-i\int_X \Omega_4 \wedge \overline{\Omega}_4\right] \\ &= -\ln\Big(\Pi_\alpha(z,u) Q^{\alpha\bar\beta}\overline{\Pi_\beta}(\bar z,\bar u)\Big)\,, \end{aligned} \tag{1.57}$$

where $Q^{\alpha\bar\beta}$ is an intersection matrix which generalises the matrix Σ defined in (1.44). We shall make use of these formulae in chapter 2.

Stabilisation of Kähler moduli

Let us now continue on moduli stabilisation in Type IIB CY orientifolds. The no-scale structure of the Kähler potential can be broken at higher order in α' and/or g_s or by non-perturbative contributions to the Kähler potential. Here we only give a lightning review of those terms which are important for the content of this thesis. More details can be found in any review on moduli stabilisation, e.g. [134, 145]. The most well-known and studied options are:

- **Perturbative corrections** to K: in particular those induced by α' curvature corrections in 10D. Taking them into account, the Kähler moduli Kähler potential becomes [146]
$$K = -2\ln\left[\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right], \tag{1.58}$$
where $\xi = -\frac{\chi(M_6)\zeta(3)}{2(2\pi)^3}$ and $\zeta(3) \approx 1.202$.

- **Non-perturbative corrections** to K: in particular those related to

  1 **Gaugino condensation**, induced by e.g. stacks of N_c D7-branes wrapping a four cycle (see [147]). The gauge theory living on the branes can induce a SUSY-QCD-like non-perturbative contribution to the superpotential, of the form: $W_{np_1} = \mathcal{A}(z^I, S)e^{-\frac{2\pi T}{N_c}}$, where T is the Kähler modulus corresponding to the volume of the four cycle wrapped by the branes.

  2 **Euclidean D3-branes** (E3-branes), which are generalisations of 4D instantons (see [148]). By wrapping a given four cycle, they induce a superpotential of the form: $W = \mathcal{A}(z^I, S)e^{-2\pi T}$, where T is the Kähler modulus corresponding to the volume of the four cycle wrapped by the E3-brane.

  Both terms have the same dependence on T.

Stabilisation of Kähler moduli, among which the modulus related to the overall volume of the CY, is a notoriously difficult task. There currently exists two main proposals to achieve it: the so-called KKLT [49] setup uses a standard flux superpotential with the addition of non-perturbative corrections, as described above. Since the latter are small, it requires a tuning of the leading order superpotential, such that the two contributions are comparable and a SUSY AdS minimum exists. The Large Volume Scenario (LVS) [50, 51] (see also [142]) uses instead the interplay between non-perturbative corrections to the superpotential and α'-corrections to the Kähler potential to achieve a non-SUSY AdS minimum at large volume of the compactification manifold. In this thesis we shall only be concerned with the latter setup, of which we now review the basic formulae.

The Large Volume Scenario (LVS)

Here we follow the review of the LVS given in [149]. The strategy to stabilise Kähler moduli á la LVS is as follows. First of all, bulk fluxes fix the complex structure moduli and the dilaton, via the F-term scalar potential (1.39). For the time being, we assume that this moduli are stabilised at a high scale, such that they can be integrated out, yielding a constant flux superpotential W_0.[2] Secondly, non-perturbative corrections to

[2]However, in chapter 2 we will see that during inflation this assumption may not be completely valid, i.e. complex structure moduli may be displaced from their value at the minimum.

the superpotential are included. As explained above, they can descend from E3 branes wrapping internal four-cycles of M_6. Therefore, the superpotential in the LVS is given by

$$W = W_0 + \sum_i A_p e^{-a_i T_i}, \tag{1.59}$$

where $T_i, i = 1, \dots, h_+^{1,1}$ are the Kähler moduli describing the volume of the associated four-cycles. Furthermore, the Kähler potential in the LVS contains α'-corrections of the type mentioned above (1.58), i.e. it is given by

$$K = K_{cs} - \ln\left[-i(S-\bar{S})\right] - 2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right). \tag{1.60}$$

As explained above, both non-perturbative and α'-corrections induce a potential for Kähler moduli. In particular, it can be shown that (1.60) induces a term $\sim 1/\mathcal{V}^3$, while (1.59) induces a correction $\sim e^{-a_i T_i}/\mathcal{V}^2$. A minimum of the F-term potential can be found if those terms are comparable, which can happen if the volume of one of the four-cycles is much smaller than the overall volume of the CY. We assume that this is the case, and denote by T_s the corresponding Kähler modulus, and by $\tau_s \equiv (1/2)(T_s + \bar{T}_s) \ll \tau_{i\neq s}$ its real part, describing the volume of the small cycle. We also assume that the only non-vanishing intersection number involving T_s is its own triple-intersection κ_{sss}. In this case, the volume $\mathcal{V}$ takes the form $\mathcal{V} = \tilde{\mathcal{V}} - c\tau_s^{3/2}$, where $\tilde{\mathcal{V}}$ does not depend on τ_s and c is a prefactor which depends on κ_{sss}. In particular, the compactification manifold is usually taken to be a so-called *Swiss-Cheese Calabi-Yau* with a volume given by

$$\mathcal{V} = \eta\left(\tau_b - \sum_i \gamma_i \tau_{s,i}^{3/2}\right), \tag{1.61}$$

where τ_s are small-cycle volume moduli and τ_b is the so-called "bulk"-volume modulus, such that at the minimum $\mathcal{V} \approx \eta\tau_b^{3/2}$. In chapter 6 we will also consider compactification manifolds of slightly different type.

The F-term scalar potential derived from (1.59) and (1.60) is then given by

$$\begin{aligned} V_F(\mathcal{V}, \tau_s) &= V_{np,1} + V_{np,2} + V_{\alpha'} \\ &= V_{0,F}\left(\frac{\alpha\sqrt{\tau_s}e^{-2a_s\tau_s}}{c\mathcal{V}} - \frac{\beta|W_0|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi\gamma|W_0|^2}{g_s^{3/2}\mathcal{V}^3}\right), \end{aligned} \tag{1.62}$$

where α, β and γ are positive constants and $V_{0,F}$ is an overall g_s-dependent prefactor. The next step is to minimise the potential (1.62) with respect to $\mathcal{V}$ and τ_s. For $a_s\tau_s \gg 1$, one obtains the following values for $\mathcal{V}$ and τ_s at the minimum of V_F:

$$\tau_s = \frac{1}{g_s}\left(\frac{\xi}{2c}\right)^{3/2}, \qquad \mathcal{V} = \frac{\beta|W_0|c}{2\alpha g_s^{1/2}}\left(\frac{\xi}{2c}\right)^{1/3} e^{\frac{a_s}{g_s}\left(\frac{\xi}{2c}\right)^{2/3}}. \tag{1.63}$$

Therefore we see that the overall volume is fixed at an exponentially large value. Finally, the value of the potential at the minimum is

$$V_F = -\frac{3M_p^4\sqrt{g_s}e^{K_{cs}}}{128\pi^2}\frac{c|W_0|^2}{\mathcal{V}^3}\left(\frac{\xi}{2c}\right)^{1/3}. \tag{1.64}$$

We shall make use of the formulae provided in this subsection in chapters 2, 3 and 6.

Finally, let us provide the scaling of the relevant mass scales with respect to the volume of the compactification manifold. We focus on the case of a Swiss-Cheese CY with volume given by (1.61). We then have

$$\begin{aligned} m_{\tau_b} &\sim \frac{g_s^2|W_0|}{\mathcal{V}^{3/2}} M_P, & m_{\tau_{s,i}} &\sim \frac{g_s|W_0|}{\mathcal{V}} M_P \\ m_{3/2} &\sim \frac{g_s^2|W_0|^2}{\mathcal{V}}, & m_{KK} &\sim \frac{g_s}{\mathcal{V}^{2/3}} M_P \\ m_{z^I} &\sim \frac{g_s^2}{\mathcal{V}} M_P, & m_{a_b} &\sim \exp(-\mathcal{V}^{2/3}) M_P \approx 0, \end{aligned} \tag{1.65}$$

where $m^{3/2}$ is the gravitino mass, m_{a_b} is the mass of the imaginary part of the bulk Kähler modulus and m_{KK} is the mass of the first massive Kaluza-Klein mode, which should be interpreted as the cutoff of the effective 4D effective theory. It is thus apparent that the lightest modulus in the LVS is the bulk modulus τ_b. Correspondingly, the imaginary partner a_b is essentially massless. Therefore the existence of an almost massless (pseudo)-scalar field is a generic prediction of the LVS. Chapter 6 is devoted precisely to the phenomenological implications of such an ultra-light field.

Supersymmetric models of particle physics, such as the MSSM, can be embedded in the LVS. However, the mass spectrum depends on the specific way in which the visible sector is realised. Therefore, we provide more details direcly in chapter 6, where we investigate explicit realisations of the visible sector. Let us just notice that the most studied embeddings of the MSSM in the LVS are characterised by soft susy-breaking masses

$$m_{soft} \sim \frac{M_P}{\mathcal{V}^2}. \tag{1.66}$$

Therefore TeV scale SUSY is achievable for $\mathcal{V} \approx 3 \cdot 10^7$ in units of the string length. Furthermore, the gravitino mass remains high enough to avoid the Cosmological Moduli Problem (CMP) [45–47].

Let us conclude this section on string compactifications by mentioning the last crucial step of moduli stabilisation: the so-called *uplifting* to de-Sitter (dS) spacetime. Indeed, as mentioned above, both the Large Volume and the KKLT scenarios lead to a negative energy density at the minimum, meaning that moduli are stabilised in a AdS background. In order to produce the observed Universe, in particular its inflationary phase, we obviously need to add a positive contribution to the energy density, such that the final minimum corresponds to 4D dS spacetime. This is a highly non-trivial task and, while several proposals exist in the literature, a lot of work remains to be done to solidly establish a go-to mechanism to achieve dS vacua in string theory. We discuss some possibilities in chapter 2 (see e.g. [134] and refs. therein for further details).

1.2 The Weak Gravity Conjecture

In the previous section we set the ground for the study of inflationary models in setups of string compactifications. We saw that ST explicitly provides the UV degrees of freedom which may affect the flatness of the inflationary trajectory, i.e. the moduli and the other fields characterising a given compactification. In such setups it is thus possible to directly address the compatibility between LFI and Quantum Gravity/String Theory. We will do so in chapters 2 and 3.

This section is instead devoted to certain effective field theory expectations about QG. These may be used to constrain LFI as well. In particular, here we introduce the Weak Gravity Conjecture (WGC) [63] for particles and gauge fields, and describe its generalisation to p-form gauge theories. We will explore its implications for axion monodromy inflation in chapter 4.

1.2.1 The single-particle case

Consider a 4D $U(1)$ gauge theory coupled to gravity, with gauge coupling e. The WGC imposes the following constraints on its spectrum and cutoff:

- **Electric WGC**: the spectrum of the theory should exhibit a charged particle with charge q and mass m, such that

$$\frac{m}{qeM_P} \lesssim 1. \tag{1.67}$$

  In other words, there should exist a particle for which *gravity is the weakest force*. Notice that a charged Black Hole (BH) in GR satisfies the Reissner-Nordstrom bound $M \geq QM_P$ and in particular a so-called *extremal* BH saturates this bound as well as (1.67).

- **Magnetic WGC**: the effective field theory is valid up to a cutoff scale Λ which satisfies the bound

$$\Lambda \lesssim eM_P. \tag{1.68}$$

  At weak coupling, i.e. for $e < 1$, Λ is thus *parametrically* smaller than the naively expected cutoff M_P.

A $U(1)$ theory coupled to gravity which does not satisfy (1.67) and (1.68) is claimed to be inconsistent. This is a highly non-trivial statement. Indeed, from the point of view of effective field theory, Lagrangians (and thus models) can be built by simply considering operators that are consistent with Poincaré invariance and gauge symmetry. The situation is believed to be quite different in a UV complete theory of Quantum Gravity (and in particular in String Theory). In particular, out of all the effective field theories which can be written down, only some of them are believed to belong to the so-called *Landscape* of theories which admit a UV completion which is consistent with QG. In the framework of ST, this means that not every effective theory can arise from a string compactification. The effective theories that do not enjoy a consistent UV completion including QG are said to belong to the *Swampland* [63, 150, 151] (see also [152]).

Several motivations have been invoked for this conjecture, though none of them might be considered completely convincing (see [79] for a brief critical review of the arguments in favour of the WGC). As mentioned in the introduction, QG is expected to violate explicitly global symmetries [33]. In the limit $e \to 0$, a $U(1)$ gauge symmetry cannot be distinguished from a global symmetry, therefore it should be constrained by the same arguments against global symmetries in QG. Indeed the original argument in favour of the WGC is that, in a $U(1)$ theory coupled to gravity which does not satisfy the WGC, a large number of stable planckian extremal black holes can be produced. This is believed to be problematic [153–155] (see however [79]). The electric WGC thus states that extremal black holes should be allowed to completely decay into light charged particles. Another argument is based on ST: namely, it is argued that there exists no example of effective theories derived from ST which violate the conjecture. Despite its current status, in this thesis we shall assume that the WGC applies and find its implication for models of axion monodromy.

There exist several different versions of the electric WGC, some of which specify which particle in the spectrum should satisfy the bound (1.67) [63, 69, 73, 75, 78, 156]. In this thesis we shall be concerned only with the following two versions of the electric WGC

- **Strong form of the WGC**: the lightest charged particle in the spectrum of the effective field theory should satisfy the bound (1.67).
- **Mild form of the WGC**: no condition is imposed on the particle which should satisfy the bound (1.67).

We will later describe how certain inflationary models violate the strong form but are compatible with the mild form of the WGG.

The magnetic WGC is instead motivated by the rather natural expectation that the minimally charge magnetic monopole in the theory should not be a black hole. The existence of a magnetic monopole is in turn motivated by the requirement that $U(1)$ gauge fields should be compact in QG [33]: thus magnetically charged BH solutions exist. Since the entropy of such configurations is large, it is natural to assume that there exists fundamental magnetically charged objects such that the charge of the BH arise from the charge of these elementary objects [63,67,79]. Demanding that the radius of the magnetic monopole ($\sim 1/\Lambda$) is larger than its Schwarzschild radius leads to the bound (1.68). In chapter 4 we will provide further motivation for the magnetic WGC, based on String Theory.

1.2.2 The Convex Hull Condition

The electric WGC can be extended to gauge theories with multiple $U(1)$ factors [65]. In this case, we associate a charge-to-mass-ratio vector $\mathbf{z}_i = \mathbf{q}_i M_P/m_i$ to every particle i in the theory. The components of the charge vector $\mathbf{q}_i$ are the charges of the i-th particle with respect to the various $U(1)$ factors. An extremal black hole has $|Z_{BH}| = 1$. Such an object can decay kinematically if the following *Convex Hull condition* is satisfied:

Convex Hull Condition (CHC): *the unit ball is contained in the convex hull spanned by the vectors* $\pm\mathbf{z}_i$.

The CHC condition is best understood pictorially, at least for theories with only two $U(1)$ factors. Let us then consider two such theories with the same gauge group $G = U(1)_a \times U(1)_b$ but with different spectra. Therefore the charge-to-mass-ratio vector of

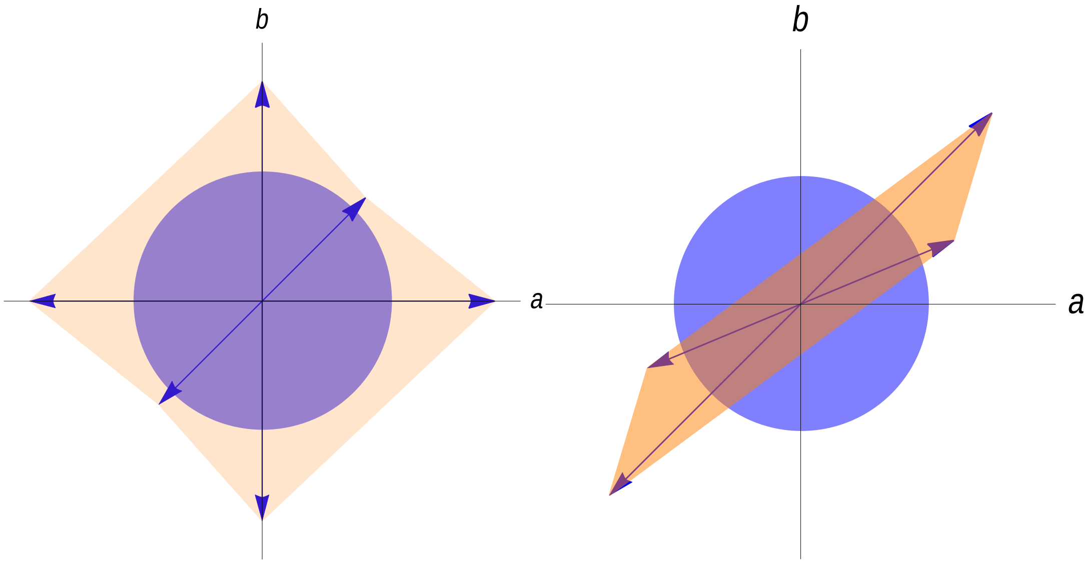


Figure 1.1: The convex hull in two theories with different spectra but same gauge group $G = U(1)_a \times U(1)_b$: (a) there are three particles in the spectrum. The convex hull spanned by their charge-to-mass-ratio vectors contains the unit ball, thus the CHC is satisfied. (b) there are two particles in the spectrum. The convex hull does not contain the unit ball, thus the CHC is not fulfilled.

each particle in the spectrum of theory has two components. In figure 1.1 we show the convex hull for the two theories. The first theory, whose convex hull is shown in figure 1.1 (a), fulfils the CHC, while the second theory, whose convex hull is shown in figure 1.1 (b), does not. Therefore, the second theory is claimed to be incompatible with Quantum Gravity. Notice that the CHC is not just a trivial generalisation of the WGC: in particular, it is not sufficient that all the charge-to-mass-ratio vectors lie outside the unit ball to ensure that the condition is satisfied, as the example in figure 1.1 (b) shows.

1.2.3 Generalisation to (p+1)-form gauge theories

The electric WGC was extended to gauge theories with p-form gauge potentials (reviewed in Appendix A.4) already in the original proposal [63].[3] The objects charged under a $(p+1)$-form gauge field are p-dimensional *membranes* (or simply branes), exactly as point-like particles can be charged under 1-form gauge fields (see section A.4. The tension T_p of a brane is analogous to the mass of a particle. Notice that the charge e_p of a p-brane in d dimensions is generically dimensionful. In particular, its energy dimension is $[e_p] = [E]^{(p+2)-d/2}$. The WGC then states that

Electric WGC: *the spectrum of a $(p+1)$-form gauge theory with gauge coupling e_p coupled to gravity should exhibit an electrically charged p-brane with charge q_{el} and tension*

[3]See however [75] for subtleties concerning the cases $p = d-1, d-2$. Furthermore, in section 4.3 we will present a strategy to justify the generalisation of the electric WGC for effective theories descending from dimensional reduction of 10D ST.

$T_{p,el}$ *such that*

$$\frac{T_{p,el}}{e_p q_{el} M_P} \lesssim 1 \tag{1.69}$$

Similarly, also the existence of a dual magnetically charged $d-(p+4)$ brane is required, with tension $T_{d-(p+4)}$ and magnetic coupling $e_{mag} \sim 1/e_{p,el}$ satisfying the same bound as in (1.69). The extension of the magnetic WGC (1.68) to p-branes (and in particular to 2-branes in 4D) will be discussed in chapter 4 and will play a key role in constraining models based on axion monodromy.

The generalised WGC is in particular assumed to be valid for the case $p=-1$. [4] A 0-form gauge field is a (pseudo)scalar field ϕ which is invariant under continuous shifts. In other words, it is an *axion*. The corresponding "−1-dimensional" object is the instanton coupled to the axion via the term $\frac{\phi}{f}\int F\tilde{F}$, where F is the field strength of some QCD-like gauge theory. The charge of the instanton is simply $1/f$, i.e. the inverse of the axion decay constant, while its "mass" is its action S. Therefore, the WGC for axions and instantons reads

$$S\frac{f}{M_P} \lesssim 1. \tag{1.70}$$

Instantons induce a potential for the axion which is generically computable only in the dilute gas approximation [64], i.e. for $S>1$. Therefore the electric WGC (1.70) requires $f \lesssim M_P$.

The generalisation to a theory with multiple axions $\mathbf{a}=(a_1,\ \dots,\ a_n)$ and instantons is straightforward and results in a Convex Hull Condition which is very similar to the one that we already discussed. In particular, the charge-to-mass-ratio-vectors are replaced by the vectors $\mathbf{z}_i = \frac{1}{S_i \mathbf{f}_i}$, where the S_i and $\mathbf{f}_i$ appear in the axion potential as follows

$$V \sim \sum_i \mathcal{A}_i e^{-S_i} \cos\left(\frac{1}{2\pi}\mathbf{q_i}\cdot\mathbf{a}\right), \tag{1.71}$$

where $\mathbf{q}_i = (1/f_1,\ \dots,\ 1/f_n)_i$. The CHC then requires the unit ball to be contained in the convex hull spanned by the vectors $\mathbf{z}_i$.

1.2.4 Constraints on axion inflation

The electric WGC (1.70) turns out to be a fatal constraint on models of LFI based on a single axion, such as Natural Inflation [30]. In the latter, the inflaton is an axion ϕ with the usual instanton-induced potential

$$V(\phi)\sum \Lambda^4\left[1\pm\cos\left(\frac{\phi}{f}\right)\right]. \tag{1.72}$$

Such a model is compatible with current Planck data (and realises LFI) only if $f>M_P$ [15]. The electric WGC (1.70) thus casts serious doubts on its feasibility.

[4] The extension of the WGC to axions and instantons descending from dimensional reduction of gauge potentials and D-branes in ST can be justified by means of string dualities [69] or by constraining the geometry of the compactification manifold, as we will do in section 4.3.

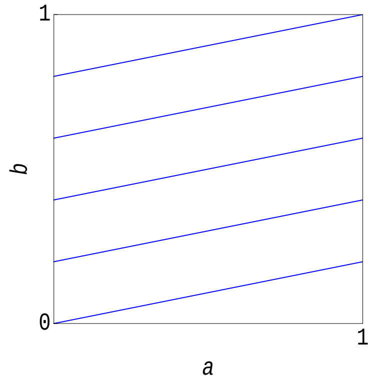


Figure 1.2: Winding inflationary trajectory in the field space of two axions, as can be obtained from (1.73).

Inflationary models employing a single axion descending from dimensional reduction of a higher dimensional gauge potential, such as *Extranatural Inflation* [157] are also severely constrained by the WGC, more specifically by the magnetic statement (as pointed out in [67]).

Concerning multi-field models, it has been argued that the CHC constrains, and may even rule out, models of *decay constant alignment*, in the spirit of the proposal by Kim, Nilles and Peloso (KNP) [31] (see also [106,158,159]). The latter is based on the following idea. Consider a 4D theory with two axions a, b and potential (w.l.o.g.):

$$V_{KNP} = V_1 \left(1 - \cos\frac{b}{f_1}\right) + V_2 \left(1 - \cos\frac{a - Nb}{f_2}\right).$$

In particular, let us assume that $V_1 \sim e^{-S_1} \gg V_2$ and $f_1 \sim f_2 < M_P$. At energies below V_1, the second term in (1.73) fixes the combination $(a - Nb)/f_2 \approx 0$. Integrating out b we obtain an effective potential for a:

$$V_{eff} \approx V_1 \left[1 - \cos\left(\frac{a}{f_{eff}}\right)\right],$$

with $f_{eff} = Nf_1/f_2$. Thus, if $N \gg 1$, the effective axion decay constant can be made transplanckian. Pictorially, this corresponds to realising inflation along a *winding* trajectory in the field space of the axions a and b, as shown in figure 1.2. Let us now rephrase the model of (1.73) in the appropriate language to apply the CHC. To this aim, let us define vectors $\mathbf{z}_1 = \mathbf{q}_1 M_P/S_1$ and $\mathbf{z}_2 = \mathbf{q}_2 M_P/S_2$, with $\mathbf{q}_1 = (0, 1/f_1)$, $\mathbf{q}_2 = (1/f_2, -N/f_2)$. Let us take $S_1 \lesssim S_2$. The CHC for two axions can be written as

$$(|\mathbf{z}_1|^2 - 1)(|\mathbf{z}_2|^2 - 1) \geq (1 + |\mathbf{z}_1 \cdot \mathbf{z}_2|)^2. \tag{1.73}$$

It can be shown that, for $N \gg 1$, the latter condition is violated. The problem is best understood pictorially: for large N the vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ are almost aligned, while their moduli are not large enough to enclose the unit ball. This is shown in figure 1.3. Let us now discuss a strategy to achieve decay constant alignement, without violating the

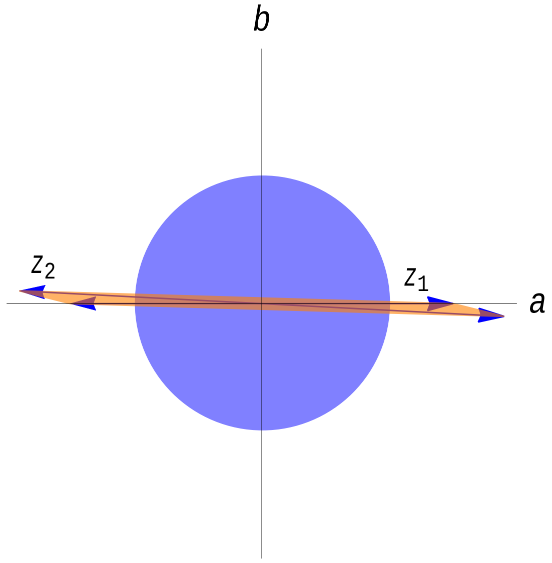


Figure 1.3: Convex Hull spanned by the aligned vectors $\mathbf{z}_1$ and $\mathbf{z}_2$. The unit ball is not contained in the convex hull, therefore the CHC is violated.

WGC. This is referred to as the *spectator loophole* and was pointed out in [68,69]. Let us add to the potential (1.73) another instanton-induced term with $S_3 \gg S_1, S_2, f_3 < M_P$

$$V_{Loophole} \sim e^{-S_1}\left(1 - \cos\frac{b}{f_1}\right) + e^{-S_2}\left(1 - \cos\frac{a - Nb}{f_2}\right) + e^{-S_3}\left(1 - \cos\frac{a}{f_3}\right). \quad (1.74)$$

After integrating b out, the potential for a becomes:

$$V_{Loophole} \sim e^{-S_1}\left(1 - \cos\frac{a}{f_{eff}}\right) + e^{-S_3}\left(1 - \cos\frac{a}{f_3}\right), \quad (1.75)$$

with $f_{eff} \sim N f_1/f_2$. As before, by taking $N \gg 1$, f_{eff} can be made transplanckian. The first term in (1.75) can then be used to realise LFI. The second term is of crucial importance: it is induced by an instanton which has $f_3 < M_P$ and $S_3 > 1$. Thus, this objects fulfils the electric WGC. Furthermore, since $S_3 \gg S_1$, the second term is suppressed compared to the first one and does not spoil inflation. In other words, it generates oscillations with a small period and a small amplitude on the flat inflationary trajectory, which arises from the large-period oscillations induced by the first term in (1.75). In the language of the CHC, the situation is shown in figure 1.4, where we introduced the vector $\mathbf{z}_3 = \mathbf{q}_3 M_P/S_3$. Since the unit ball is contained in the convex hull spanned by the three vectors, the CHC is fulfilled.

It thus appears that aligned models can be easily made compatible with the WGC. Indeed, instantons with larger actions ("heavier") are generically present in models of axion inflation, since the instanton-induced potential is of the form

$$V(\phi) \sim \sum_n \mathcal{A}_n e^{-nS} \cos\left(n\phi/f\right).$$

However, we would like to remark that the model in (1.75) fulfils only the *mild* form of the WGC. Indeed, while it is true that the bound (1.70) is satisfied by one instanton in

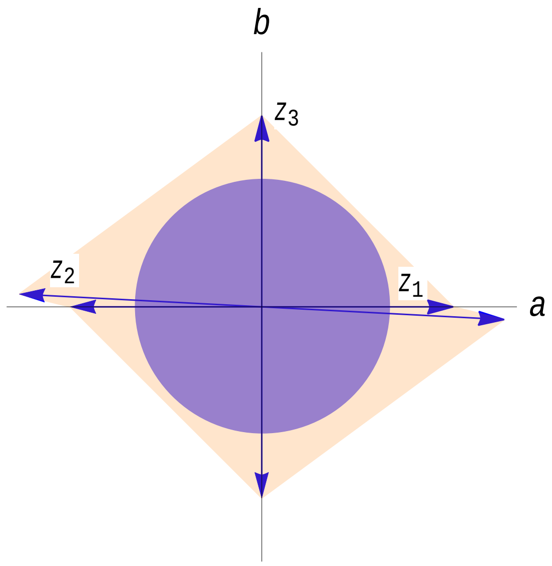


Figure 1.4: Convex Hull spanned by the vectors $\mathbf{z}_1, \mathbf{z}_2$ and $\mathbf{z}_3$. The unit ball is contained in the convex hull, therefore the CHC is satisfied.

the theory, the latter is not the object with the smallest action (the "lightest"). Thus, the spectator loophole does not evade the strong form of the WGC. The discussion that we presented is particularly relevant for chapter 3, where we will propose a model of decay constant alignement in a setup of string compactifications, which realises the spectator loophole.

At this point, it is important to notice that there exists yet another loophole in the no-go arguments based on the WGC [67, 68]. This loophole exploits the fact that the WGC for electrically charged objects can be satisfied for $f > 1$ if S is taken to be sufficiently small, i.e. $S \ll 1$. However, for $S < 1$ we can in general not calculate the instanton induced potential and it hence remains unclear whether inflation can be realized. In concrete models, the potential can be calculable in spite of $S < 1$ and inflation might still work. Such a possibility was suggested in [67] (see also [68]), based on the model of [160].[5]

Before moving to the next chapter of this thesis, a final comment on the WGC is in order. At the time of writing, it is not clear which version, if any, of the conjecture should be used to constrain models of LFI at the level of effective field theory, nor which version holds in String Theory (see [36, 65–79, 84, 131] for the various proposals). In this thesis, we will apply the WGC mostly in its mild form. It remains a very interesting and important task to assess whether there exists a convincing a constraining formulation of the WGC.

[5]Consistency with the magnetic form of the WGC constrains models exploiting the loophole of [67,68] even further. As was pointed out in [67] a transplanckian field range can only be achieved in a field space of two or more axions, e.g. by employing a mechanism like alignment [160].

Part I

Large Field Inflation in String Compactifications

Chapter 2

Moduli backreaction in axion monodromy inflation

Compactifications of 10D String Theory are characterised by the presence of moduli fields. Some of them enjoy (approximate) shift symmetries and can thus be used as inflaton candidates. However, the flatness of the inflationary potential might be spoiled by the presence of many other moduli fields, which can affect the dynamics of the inflaton field. In this chapter we investigate the *backreaction* of moduli on a transplanckian trajectory, in a string-derived SUGRA setting known as *F-term axion monodromy* [101–103]. In particular, we focus on complex structure moduli of a Type IIB/F-theory Calabi-Yau orientifold. In this chapter, we set $M_P \equiv 1$, unless otherwise stated.

This chapter is based on the publication [130].

2.1 Introduction

In the previous chapter we introduced the essentials of string compactifications. In particular, we showed that dimensional reduction of the Type IIB action leads to the presence of *moduli* fields in 4D. These massless scalar fields can correspond to geometric deformations of the six-dimensional compactification manifold, to generalised gauge potentials in 10D or to the positions of D-branes. Here we will focus on the effects of geometric moduli on the feasibility of Large Field Inflation.

Inspired by [101], we propose a realisation of LFI in the *complex structure moduli* sector of a Type IIB/F-theory CY orientifold, meaning that the inflaton field is identified with the imaginary (so-called *axionic*) part of a complex structure modulus. In certain limits of the geometry of the extra-dimensions, complex structure can indeed exhibit a shift symmetry at the level of the Kähler potential. One such limit is known as *Large Complex Structure* (LCS) limit [104, 105] and is realised when the real partner (*saxion*) of the complex structure axion is stabilised at large values, i.e. $\text{Re}(z_0) \gg 1$. This limit does not need to be imposed on every complex structure modulus of the compactification. Rather, one can consider a so-called *partial LCS* regime in which only some moduli are stabilised in the LCS limit.

Throughout this chapter, we will work in a specific class of supergravity implementations of Axion Monodromy Inflation, known as *F-term axion monodromy* [101–103]. The basic underlying idea of all these constructions is as follows. One considers settings where at least one modulus enjoys a shift-symmetric Kähler potential. This shift symmetry as well as the related periodicity of the axionic part of the modulus is then weakly broken by the superpotential, e.g. due to an appropriate flux choice. This gives rise to an enlarged axion field range with a slowly rising potential, suitable for large-field inflation.

As usual in string inflation, moduli stabilisation is a critical issue. This problem was analysed in some detail in [101] concerning Kähler moduli while, concerning complex structure moduli, high-scale flux-stabilisation was assumed in a somewhat simple-minded way. The crucial aspect that will be investigated in this chapter is that of *backreaction* of complex structure moduli during inflation. Indeed, as the inflaton rolls along its potential, its motion can be affected by the dynamics of other moduli fields.

The content of this chapter is organised as follows: in section 2.2 we provide a technical introduction to the problem of backreaction in a generalisation of the model of [101]. In section 2.3 we briefly review how to realise the model in a F-theory setup. The main results of this chapter are obtained in sections 2.4.2-2.5, where we analytically quantify backreaction and compute its effects on the inflationary potential. In section 2.6 we provide a numerical analysis in support of our analytical findings. In section 2.7 we report an estimate of the number of string vacua which can accommodate our model. Finally, we summarise our findings in section 2.8 and discuss possible future directions of research.

2.2 The problem of backreaction

The aim of this section is to briefly introduce the problems of backreaction and tuning in models of F-term axion monodromy inflation. In particular, we will discuss setups where the inflaton is the imaginary part of a complex structure modulus u in the LCS limit, in the spirit of [101] (see also [161–163]). As we have already mentioned, we do not require the other complex structure moduli $z \equiv z^i$ to be in the LCS regime. The moduli z and u can be complex structure moduli (or D7-brane position moduli) of type IIB orientifolds, or complex structure moduli of F-theory fourfolds. In the threefold case we consider the axio-dilaton as a component of z. As we will see in more detail later, the "partial large complex structure regime" which we adopt in this chapter is sufficient to ensure that the complex structure modulus u enjoys a shift symmetry at the level of the Kähler potential, which takes the form

$$\mathcal{K}_{cs} = \mathcal{K}_{cs}(z, \bar{z}, u + \bar{u}). \tag{2.1}$$

The source of symmetry breaking is provided instead by the superpotential, which we take to be linear in u:

$$W = w(z) + a(z)u \tag{2.2}$$

In section 2.3 we will explain why such an assumption can be made without loss of generality. The linear term in (2.2) lifts the otherwise flat direction $y \equiv \mathrm{Im}(u)$. In order for inflation to be viable and not to interfere with moduli stabilisation, the shift symmetry must be only weakly broken. This condition translates into the requirement that the term $a(z)u$ be small along the inflationary trajectory. At the SUSY locus $z = z_\star$, this can be achieved by a tuning of the coefficient $a(z)$. Specifically, the function $a(z)$ is a sum of many terms, each depending on several other moduli. Their SUSY vacuum expectation values depend on a high-dimensional integral flux vector. The coefficient $a(z)$ can therefore be small as a consequence of a fine cancellation between these various terms, as it is customary in the landscape paradigm.

Inflation and moduli stabilisation are obtained via the F-term supergravity scalar potential

$$V = e^{\mathcal{K}}(\mathcal{K}^{I\bar{J}} D_I W \overline{D_J W} + \mathcal{K}^{T_\gamma \bar{T}_\delta} D_{T_\gamma} W \overline{D_{T_\delta} W} - 3|W|^2) \ , \tag{2.3}$$

where $\mathcal{K} = -2\ln\mathcal{V} + \mathcal{K}_{cs}(z, \bar{z}, u + \bar{u})$ and the capital indices run over all moduli z and u. Notice the crucial presence of the Kähler moduli T_γ. In studying the potential (2.3) we will assume that the latter moduli are stabilised according to the Large Volume Scenario [50]. In this framework the Kähler potential for the Kähler moduli enjoys a well-known no-scale structure, meaning that the last two terms in (2.3) cancel at leading order (see the discussion in subsection 1.1.3). Therefore from now on we will consider only the scalar potential

$$V = e^{\mathcal{K}}(\mathcal{K}^{I\bar{J}} D_I W \overline{D_J W}) \ . \tag{2.4}$$

Most importantly, we consider all fields as dynamical, i.e. we do not integrate out all z at this stage. To simplify the argument, we continue our analysis for only two fields, labelled by z and u. The two F-terms entering (2.4) are then given by

$$D_u W = D_u w + a + \mathcal{K}_u a u \ , \tag{2.5}$$

$$D_z W = D_z w + (\partial_z a + \mathcal{K}_z a) u \ . \tag{2.6}$$

The values of u and z at the minimum of the F-term potential are obtained by solving the equations

$$D_u W = 0, \quad D_z W = 0. \tag{2.7}$$

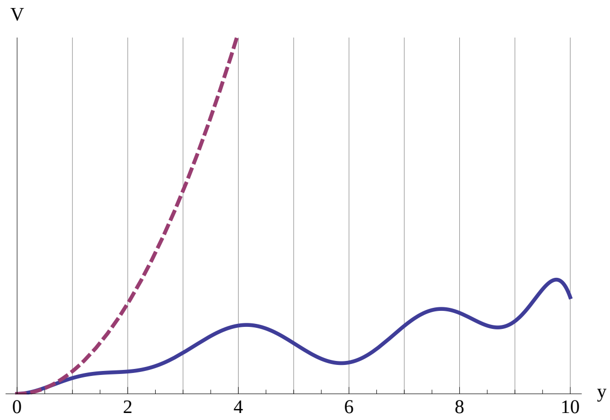


Figure 2.1: 'Naive' inflaton potential (dashed red line) and a possible effective inflaton potential after backreaction is taken into account (solid blue line). Note that the effective inflaton potential is not automatically sufficiently flat over transplanckian regions to realise large field inflation. This picture is taken from [130] and was prepared by my collaborators.

The latter can be interpreted as conditions on the derivatives of the Kähler potential at the minimum:

$$D_uW = 0 \Rightarrow \mathcal{K}_u|_{min} = -\frac{a}{w+au}\Big|_{min} \tag{2.8}$$

$$D_zW = 0 \Rightarrow \mathcal{K}_z|_{min} = -\frac{\partial_z w + \partial_z a \cdot u}{w+au}\Big|_{min}. \tag{2.9}$$

The inflaton potential will get contributions from both F-terms and takes the form:

$$V = e^{\mathcal{K}}\Big[\mathcal{K}^{u\bar{u}}|\mathcal{K}_u a|^2 + K^{z\bar{z}}|\partial_z a + \mathcal{K}_z a|^2 + \mathcal{K}^{u\bar{z}}(\mathcal{K}_u a)(\overline{\partial_z a + \mathcal{K}_z a}) + h.c.)\Big]_{min}\Delta y^2 + \dots, \tag{2.10}$$

where we expanded around the SUSY minimum $\{u = u_\star, z = z_\star\}$ and $\Delta y \equiv y - y_\star$. The ellipses stand for terms due to backreaction of z , which will be studied in detail in section 2.4.1.

The result (2.10) reveals one of the main insights of this chapter. In order to ensure that the shift symmetry of y is only weakly broken, one needs to tune small not only the coefficient $a(z)$, but its derivatives $\partial_z a$ as well. Indeed the second term in the square brackets in (2.10) contains terms which are proportional to $|\partial_z a|$ only. Notice that these terms are obviously not automatically small when $a(z)$ is small. Therefore an additional amount of tuning is required. In particular, for every modulus z^j we will require $|\partial_{z^j} a|$ to be small. This implies that for n moduli we have to require $(n+1)$ tunings.

It is easy to see that one cannot get away with fewer tunings. The argument is as follows. Find the basis in which the Kähler metric is diagonal. In this basis the inflationary potential is a sum of positive terms (in essence, the mixed terms $\sim \mathcal{K}^{z\bar{z}}, \mathcal{K}^{z\bar{u}}$ in (2.10) disappear). Each of these terms is a combination of a and $\partial_{z^i} a, i = 1, \dots, n$. Therefore, $(n+1)$ different terms have to be tuned small to ensure the flatness of the inflationary direction. These combinations of a and $\partial_{z^i} a$ involve elements of the original inverse Kähler metric as coefficients. It is conceivable that these terms could take small values in some region of the moduli space. This corresponds to special geometries of moduli space where, at particular points, certain elements of the metric blow up. Since we do not know whether such situations can occur, in particular given that one complex structure modulus (the inflaton) must be stabilised in the large complex structure limit, we choose not to consider this option in the following. Thus, for the case of n moduli z^i, we fine tune the quantities $|a|$, $|\partial_{z^1} a|$, $|\partial_{z^2} a|$, $\ldots |\partial_{z^n} a|$.

The rest of this chapter is devoted to two important problems related to the potential (2.10) and to the associated tunings:

1 **Backreaction**: the term $a(z)u \subset W$, while lifting the otherwise flat inflationary direction, also induces a cross-coupling between the moduli z and u. This feature of the F-term potential can have dramatic consequences on the inflationary potential. This can be understood as follows: The moduli z^i and u are originally fixed at values $u_\star$, $z^i_\star$ at the SUSY locus. Now consider a variation Δy of the imaginary part of u. Since we keep the moduli z^i dynamical, a variation in the value of y induces a deviation δz^i in the values of the moduli z^i. In turn, the displacements δz^i lead to additional terms in (2.10), which are generically not quadratic in Δy. If these contributions are large, the inflationary direction can be spoiled. In the worst-case scenario, the effective inflationary potential may not even be monotonic, especially over a large (transplanckian) field range. This is illustrated in figure 2.1.

   The main aim of this chapter is therefore to assess whether the tuning of a and $\partial_z a$ is sufficient to control the backreaction of the complex structure moduli z on the inflationary potential. This can be done analytically, as we will describe in detail in subsection 2.4.2. As further evidence for our results, we will also provide a numerical example in section 2.6.

2 **Feasibility of tuning**: A secondary aim of this chapter is to understand whether the tuning of a and $\partial_z a$ is realisable in the string landscape. In section 2.7 we will therefore briefly review the strategy to compute how many string vacua are compatible with such a tuning. While we will discuss the important conclusions of this analysis, we will not provide detailed computations, which can be found in [130] and in [164].

Before moving on to the detailed analysis of backreaction, let us briefly review the origins of the linear symmetry-breaking term in the superpotential (2.2) both from a Type IIB and from an F-theory perspective.

2.3 Structure of the model from F-theory

In the previous section we introduced the basic ingredients of a model of axion monodromy in the complex structure moduli sector of type IIB orientifolds or F-theory fourfolds. In particular, we considered a modulus u which enjoys a shift-symmetry at the level of the Kähler potential (2.1) and appears linearly in the superpotential (2.2). In this section we will take a step backward and ask whether this structure can be obtained in a type

IIB/F-theory framework. This will serve as a motivation for the model (2.2). While we will discuss the main results of this investigation, we will not provide a detailed analysis. The latter can be found in [130] and in [164].

Let us then start by considering the implementation of our model in a type IIB Calabi-Yau (CY) orientifold X. As in the previous section, we denote by $z^I, I = 0, \dots, n$ the $n+1$ complex structure moduli of X. In particular, we take $z^0 \equiv iu$, with u in the LCS regime. In type IIB threefolds with quantised three-form fluxes F_3 and H_3 and holomorphic three-form Ω_3 , the superpotential is obtained via the Gukov-Vafa-Witten formula [48]

$$W = (N_F - SN_H)^\alpha \Pi_\alpha, \tag{2.11}$$

where N_F and N_H are the flux numbers of F_3 and H_3 respectively. Here $S = i/g_s + C_0$ is the axio-dilaton and Π is the so-called period vector of X, given by [105, 165]

$$\Pi_\alpha = \begin{pmatrix} 1 \\ z^I \\ \frac{1}{2}\kappa_{IJK} z^J z^K + f_{IJ} z^J + f_I + \sum_p A_{Ip} e^{-\sum_J b_{pJ} z^J} \\ -\frac{1}{3!}\kappa_{IJK} z^I z^J z^K + f_I z^I + g + \sum_p B_p e^{-\sum_J \tilde{b}_{pJ} z^J} \end{pmatrix} . \tag{2.12}$$

The geometric data of the CY X enter the superpotential via the triple intersection numbers κ_{IJK} of the 4-cycles of the mirror dual CY threefold $\tilde{X}$. Due to the LCS regime in u, the non-perturbative terms $e^{-2\pi u}$ are suppressed in (2.12). Under reasonable assumptions, the terms f_{IJ}, f_I and g can be neglected. It is then apparent that W is generically a polynomial of degree three in u. In particular, the superpotential exhibits the following structure:

$$W = w(S,z) + a(S,z)u + \frac{1}{2}b(S,z)u^2 + \frac{1}{3!}c(S)u^3. \tag{2.13}$$

Notice that we do not require the other moduli $z^i, i = 1, \dots, n$ and S to be in the LCS regime.

Since the shift symmetry of u is broken by the coefficients a, b and c, we must require that they are all small. However, on any type IIB CY orientifold at weak coupling (i.e. $g_s \ll 1$) with at least one modulus u in the LCS regime the following two conditions cannot be simultaneously satisfied:

1. *The coefficients a, b, c and their derivatives with respect to z and S are tuned sufficiently small to allow for inflation.*
2. $\mathrm{Re}(u)$ *can be stabilised using the classical supergravity F-term scalar potential.*

In particular, it is not possible to set b and c to zero and obtain a linear superpotential, as in (2.2).[1]

[1] Loopholes to this statement may exist, see [130] for details.

The consequences of this no-go result can be evaded by considering F-theory fourfolds with quantised four-form flux G_4. In this case, the following straightforward generalisation of (2.11) holds:

$$W = N^\alpha \Pi_\alpha^{fourfold} \, , \tag{2.14}$$

where $\alpha = 1, \ldots, b_4(X)$ and N is the flux number of G_4. We assume again $z^0 \equiv u$ to be in the LCS regime. Crucially, the period vector is now quartic in the complex structure moduli $z^I, I = 0, \ldots, h^{3,1} - 1$. Therefore, W will generically be a polynomial of order four in u. It can now be shown that, by an appropriate choice of intersection numbers, a linear superpotential in u can be consistently obtained.

Furthermore, the Kähler potential for the complex structure moduli of an F-theory fourfold is given by

$$\mathcal{K}_{cs} = -\ln\left(\Pi_\alpha(z,u) Q^{\alpha\bar{\beta}} \overline{\Pi_\beta}(\bar{z},\bar{u})\right) , \tag{2.15}$$

where $Q_{\alpha\bar{\beta}}$ is the intersection matrix. If u is in the LCS regime, the non-perturbative terms in the period vector are suppressed and consequently $\mathcal{K}_{cs}$ is invariant under $u \to u + ia$, i.e. $\mathcal{K}_{cs} = \mathcal{K}_{cs}(z, \bar{z}, u + \bar{u})$.

The important conclusion of this section is that the model with Kähler potential (2.1) and superpotential (2.2) can be consistently obtained in the framework of F-theory fourfolds. Reassured by this statement, we can now proceed to the analysis of backreaction in this model. From now on, in order to simplify the notation, we abbreviate $f_I \equiv \partial_{z^I} f, I = 0, \ldots, n$ and $f_i \equiv \partial_{z^i} f, i = 1, \ldots, n$ for any function f.

2.4 Backreaction and the effective inflaton potential

In this section we will study the backreaction on the complex structure moduli z^i, $\bar{z}^i$ as well as on $x \equiv \mathrm{Re}(u)$, if we displace $y \equiv \mathrm{Im}(u)$ by some finite distance Δy from the minimum. In particular, we will derive the effective inflaton potential once backreaction is taken into account.

2.4.1 Analytical formulation

The starting point are a Kähler potential and a superpotential of the form

$$W = w(z) + a(z)u, \quad \mathcal{K} \equiv \mathcal{K}(z, \bar{z}, u + \bar{u}) \, , \tag{2.16}$$

from which we can determine the F-term scalar potential

$$V = e^{\mathcal{K}} (\mathcal{K}^{I\bar{J}} D_I W \overline{D_J W}) \, . \tag{2.17}$$

Most importantly, we do not assume that any of the z^i are integrated out. On the contrary, we take all z^i as well as u to be dynamical. To quantify backreaction the strategy is as follows. We expand the potential in δz^i, $\delta \bar{z}^i$ and δx to quadratic order about the minimum. As long as the displacements δz^i, $\delta \bar{z}^i$ and δx remain small during inflation this expansion is a good approximation to the full potential and higher order terms can be ignored. For every value of Δy the potential is then a quadratic form in the displacements of the remaining fields. As such, it admits a global minimum at each value of Δy for some $\delta z^i(\Delta y)$, $\delta\bar{z}^i(\Delta y)$ and $\delta x(\Delta y)$, which we calculate explicitly. In the following we will show that the displacements $\delta z^i(\Delta y)$, $\delta\bar{z}^i(\Delta y)$ and $\delta x(\Delta y)$ are indeed small for a wide range in Δy such that our analysis is self-consistent. By substituting these solutions into the expression for the potential we can then derive the effective inflaton potential.

We now perform the steps outlined above explicitly. To begin, we wish to expand the scalar potential (2.17) to quadratic order in x, z^j and $\bar{z}^j$ about their values at the minimum. For this, it will be sufficient to expand the covariant derivatives $D_I W$ to first order. Indeed the inverse Kähler metric and the exponential prefactor do not contribute at quadratic order, as shown by varying the F-term potential twice:

$$\begin{aligned}\delta^2 V_F = \delta^2\Big(e^{\mathcal{K}}\mathcal{K}^{I\bar{J}}\Big)\Big[D_I W\overline{D_J W}\Big]_{min} + \delta\Big(e^{\mathcal{K}}\mathcal{K}^{I\bar{J}}\Big)\delta\Big(D_I W\Big)\Big[\overline{D_J W}\Big]_{min}\\ + \delta\Big(e^{\mathcal{K}}\mathcal{K}^{I\bar{J}}\Big)\delta\Big(\overline{D_J W}\Big)\Big[D_I W\Big]_{min} + \Big[e^{\mathcal{K}}\mathcal{K}^{I\bar{J}}\Big]_{min}\delta^2\Big(D_I W\overline{D_J W}\Big),\end{aligned} \tag{2.18}$$

and imposing the minimum condition $D_I W = 0$.

The covariant derivatives are given by:

$$\begin{aligned}D_u W &= a + \mathcal{K}_u(w + ax + iay),\\ D_{z^i} W &= w_i + a_i(x + iy) + \mathcal{K}_i(w + ax + iay).\end{aligned} \tag{2.19}$$

Recall that a subscript i corresponds to a derivative w.r.t. z^i: $f_i \equiv \partial_{z^i} f$. The values $u_\star$, $z_\star$ of the complex structure moduli at the minimum are found by imposing the conditions:

$$D_u W = 0, \quad D_{z^i} W = 0. \tag{2.20}$$

The latter can be solved in terms of the derivatives of the Kähler potential at the minimum:

$$\begin{aligned}D_u W = 0 \Rightarrow \mathcal{K}_u|_\star &= -\frac{a}{w + au}\bigg|_\star\\ D_{z^i} W = 0 \Rightarrow \mathcal{K}_i|_\star &= -\frac{w_i + a_i u}{w + au}\bigg|_\star \quad .\end{aligned} \tag{2.21}$$

We now write $z^j = z^j_\star + \delta z^j$ and $u = u_\star + \delta x + i\Delta y$ and expand (2.19) to linear order in δx, δz^j and $\delta\bar{z}^j$. Note that we will not perform an expansion in Δy. On the contrary, our result will be exact in Δy. This is absolutely crucial as Δy will take transplanckian values during inflation and is not a small quantity. In the following it will also be useful to absorb the term $au_\star$ into a quantity w_*:

$$w_* \equiv W(z, u_\star) = w(z) + a(z)u_\star\ . \tag{2.22}$$

Expanding (2.19) to linear order in δx, δz^j and $\delta\bar{z}^j$ we find:

$$\begin{aligned}
D_u W =& \Big[a_j + \mathcal{K}_{uj} w_* + \mathcal{K}_u w_{*j} + i(\mathcal{K}_{uj} a + \mathcal{K}_u a_j)\Delta y\Big]_\star \delta z^j \\
&+ \Big[\mathcal{K}_{u\bar{j}} w_* + ia\mathcal{K}_{u\bar{j}}\Delta y\Big]_\star \delta\bar{z}^j \\
&+ \Big[\mathcal{K}_u a + \mathcal{K}_{ux} w_* + i\mathcal{K}_{ux} a\Delta y\Big]_\star \delta x + i\Big[\mathcal{K}_u a\Big]_\star \Delta y + O(\delta^2), \qquad (2.23)\\
D_{z^i} W =& \Big[w_{*ij} + \mathcal{K}_{ij} w_* + \mathcal{K}_i w_{*j} + i(a_{ij} + \mathcal{K}_{ij} a + \mathcal{K}_i a_j)\Delta y\Big]_\star \delta z^j \\
&+ \Big[\mathcal{K}_{i\bar{j}} w_* + i\mathcal{K}_{i\bar{j}} a\Delta y\Big]_\star \delta\bar{z}^j \\
&+ \Big[a_i + \mathcal{K}_{ix} w_* + \mathcal{K}_i a + i\mathcal{K}_{ix} a\Delta y\Big]_\star \delta x + i\Big[a_i + \mathcal{K}_i a\Big]_\star \Delta y + O(\delta^2). \qquad (2.24)
\end{aligned}$$

Here we used the subscript $\star$ to make it explicit that the quantities in square brackets are evaluated at the minimum, but we will suppress it in what follows.

If the displacements $\delta z, \delta\bar{z}, \delta x$ are small, the leading term in the potential is quadratic in Δy. This term is therefore the *naive* inflationary potential and reads:

$$V_{naive} \sim \Big[\mathcal{K}^{u\bar{u}}|\mathcal{K}_u a|^2 + \mathcal{K}^{i\bar{j}}(a_i + \mathcal{K}_i a)\overline{(a_j + \mathcal{K}_j a)} + (\mathcal{K}^{u\bar{j}}(\mathcal{K}_u a)(\overline{a_j + \mathcal{K}_j a}) + h.c.)\Big] (\Delta y)^2 \qquad (2.25)$$

In order for Δy to be a suitable direction for inflation, we require that the naive potential is almost flat. From (2.25), this requirement is satisfied if $|\mathcal{K}_u a|$ and $|a_j + \mathcal{K}_j a|$ are small. This can be achieved by tuning all the parameters $|a|, |a_j| \ll 1$.[2] In order to obtain compact expressions, we introduce the following quantities:

$$\begin{aligned}
\eta_u &= i\mathcal{K}_u a \\
\eta_j &= i(a_j + \mathcal{K}_j a). \qquad (2.26)
\end{aligned}$$

At this point, it is important to notice that (2.21) imposes $\mathcal{K}_u \sim a$. The latter implies that $\mathcal{K}_u$ is as small as a at the minimum, while $\mathcal{K}_i$ and the elements of the Kähler metric are not parametrically small. Introducing the small parameter

$$\epsilon \equiv |a| \ , \qquad (2.27)$$

it follows that $\eta_u \sim \epsilon^2$ while the second term in η_j is only proportional to ϵ. In this and the following subsection we assume that a_i is tuned in such a way that $\eta_i \sim \epsilon^2$ as well. Under these assumptions η_u and η_i are parametrically of the same size. This turns out to be useful for our explicit computations. We discuss the generic case of hierarchical η's in section 2.4.3.

We can now simplify our expressions (2.23). We will later show that the displacements $\delta x, \delta z^j, \delta\bar{z}^j$ are small to the extent that η_u, η_j are small. In particular, when $\eta_u \sim \eta_j \sim \epsilon^2$ we will find that $\delta x \sim \delta z^j \sim \delta\bar{z}^j \sim \epsilon^2$. It follows that e.g. $a_j \delta z^j \sim \epsilon^3$ while $\mathcal{K}_{uj} w_* \delta z^j \sim \epsilon^2$. To simplify further, we can then neglect those terms in (2.23) that are smaller than $O(\epsilon^2)$.

[2] In the case of k complex structure moduli entering a, these are $k+1$ tunings. As we have discussed in section (2.2), one cannot get away with fewer tunings.

Let us be more precise about the latter statement. In (2.23) and (2.24) there are terms of the form $O(\epsilon^3)\Delta y$. Those terms are negligible compared to those of $O(\epsilon)$ as long as $\Delta y \ll \epsilon^{-1}$. We shall therefore restrict the field displacement to $0 < \Delta y \ll \epsilon^{-1}$. This is also motivated by the following argument. In order not to interfere with Kähler moduli stabilisation we need to impose $au \sim \epsilon u \ll w$ in (2.16). This constraint then implies the same restriction on the field range. We thus arrive at:

$$D_u W \simeq \Big[\mathcal{K}_{uj} w_*\Big]\delta z^j + \Big[\mathcal{K}_{u\bar{j}} w_*\Big]\delta \bar{z}^j + \Big[\mathcal{K}_{ux} w_*\Big]\delta x + \eta_u \Delta y, \tag{2.28}$$

$$\begin{aligned} D_{z^i} W \simeq & \Big[w_{*ij} + \mathcal{K}_{ij} w_* + \mathcal{K}_i w_{*j} + i(a_{ij})\Delta y\Big]\delta z^j + \Big[\mathcal{K}_{i\bar{j}} w_*\Big]\delta \bar{z}^j \\ & + \Big[\mathcal{K}_{ix} w_*\Big]\delta x + \eta_i \Delta y \quad . \end{aligned} \tag{2.29}$$

Note that at leading order $D_u W \sim D_{z^i} W \sim \epsilon^2$ and $V \sim \epsilon^4$. We can now understand why it was sufficient to expand the covariant derivatives to first order in δx, δz^j and $\delta \bar{z}^j$. It is easy to check that higher order terms would be subleading both in the covariant derivatives as well as in V. For what follows it will be useful to write the expressions (2.28) and (2.29) more compactly using the notation:

$$D_I W = (A_{Ij} + B_{Ij}\Delta y)\delta z^j + C_{Ij}\delta \bar{z}^j + G_I \delta x + \eta_I \Delta y. \tag{2.30}$$

Here the index I runs over u and all z^i, where $I = 0$ is identified with u and $I = i$ with $i = 1, \dots, n$ corresponds to z^i. A summation over the index j is implied. While being simple, the notation (2.30) obscures some of the structure evident in (2.28) and (2.29). In particular, note that

$$B_{0i} = \partial_u \partial_{z^i} a = 0 \qquad \text{for } i = 1, \dots, n \ , \tag{2.31}$$

$$B_{ij} = B_{ji} = \partial_{z^i} \partial_{z^j} a \qquad \text{for } i, j = 1, \dots, n \ , \tag{2.32}$$

$$G_i = 2A_{0i} \qquad \text{for } i = 1, \dots, n \ . \tag{2.33}$$

In the following, it will be convenient to work with real fields only. Writing $z^i = v^i + iw^i$ and $\bar{z}^i = v^i - iw^i$ we can rewrite (2.30) in terms of the displacements δv^j and δw^j:

$$D_I W = (A_{Ij} + C_{Ij} + B_{Ij}\Delta y)\delta v^j + i(A_{Ij} - C_{Ij} + B_{Ij}\Delta y)\delta w^j + G_I \delta x + \eta_I \Delta y \ . \tag{2.34}$$

We are now in a position to write down the F-term potential at quadratic order in the displacements, starting from its definition,

$$V_F = e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} D_I W \overline{D_J W}, \tag{2.35}$$

and insert our expressions (2.34). The resulting potential can be written as a quadratic form:

$$V_F = \frac{1}{2}\mathbf{\Delta}^T \mathcal{D}(\Delta y)\mathbf{\Delta} + [\mathbf{b}(\Delta y, \eta_I)]^T \mathbf{\Delta} + \mu^2 (\Delta y)^2 \ , \tag{2.36}$$

whose individual terms we will now explain. For one, $\mathbf{\Delta}$ is a vector with $(2n+1)$ entries containing the displacements $\mathbf{\Delta} = (\delta x, \delta v^i, \delta w^i)^T$. Also, $\mu^2 = e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \eta_I \bar{\eta}_{\bar{J}}$ is the squared mass of the naive inflaton potential. Furthermore, $\mathcal{D}$ is the real symmetric

$(2n+1) \times (2n+1)$ matrix of the second derivatives of the scalar potential with respect to the displacements $\delta x, \delta v^i, \delta w^i$. Explicitly, it is given by

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{xx} & \mathcal{D}_{xv^j} & \mathcal{D}_{xw^j} \\ \mathcal{D}_{v^i x} & \mathcal{D}_{v^i v^j} & \mathcal{D}_{v^i w^j} \\ \mathcal{D}_{w^i x} & \mathcal{D}_{w^i v^j} & \mathcal{D}_{w^i w^j} \end{pmatrix} , \tag{2.37}$$

with:

$$\begin{aligned}
\mathcal{D}_{xx} &= 2\, e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} G_I \overline{G_J} \ , \qquad (2.38) \\
\mathcal{D}_{xv^i} = \mathcal{D}_{v^i x} &= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \left[G_I \overline{(A_{Ji} + C_{Ji} + B_{Ji}\Delta y)} + (A_{Ii} + C_{Ii} + B_{Ii}\Delta y)\overline{G_J} \right] \ , \\
\mathcal{D}_{xw^i} = \mathcal{D}_{w^i x} &= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \left[-i G_I \overline{(A_{Ji} - C_{Ji} + B_{Ji}\Delta y)} + i(A_{Ii} - C_{Ii} + B_{Ii}\Delta y)\overline{G_J} \right] \ , \\
\mathcal{D}_{v^i v^j} = \mathcal{D}_{v^j v^i} &= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \Big[(A_{Ii} + C_{Ii} + B_{Ii}\Delta y)\overline{(A_{Jj} + C_{Jj} + B_{Jj}\Delta y)} + \\
&\qquad + (A_{Ij} + C_{Ij} + B_{Ij}\Delta y)\overline{(A_{Ji} + C_{Ji} + B_{Ji}\Delta y)} \Big] \ , \\
\mathcal{D}_{v^i w^j} = \mathcal{D}_{w^j v^i} &= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \Big[-i(A_{Ii} + C_{Ii} + B_{Ii}\Delta y)\overline{(A_{Jj} - C_{Jj} + B_{Jj}\Delta y)} + \\
&\qquad + i(A_{Ij} - C_{Ij} + B_{Ij}\Delta y)\overline{(A_{Ji} + C_{Ji} + B_{Ji}\Delta y)} \Big] \ , \\
\mathcal{D}_{w^i w^j} = \mathcal{D}_{w^j w^i} &= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \Big[(A_{Ii} - C_{Ii} + B_{Ii}\Delta y)\overline{(A_{Jj} - C_{Jj} + B_{Jj}\Delta y)} + \\
&\qquad + (A_{Ij} - C_{Ij} + B_{Ij}\Delta y)\overline{(A_{Ji} - C_{Ji} + B_{Ji}\Delta y)} \Big] \ .
\end{aligned}$$

The elements of the vector $\mathbf{b} = (b_x, b_{v^i}, b_{w^i})^T$ are given by the first derivatives of the F-term potential (evaluated at the minimum, i.e. at $\mathbf{\Delta} = 0$). Explicitly, we have

$$\begin{aligned}
b_x &= \left[\partial_{(\delta x)} V\right]_\star = e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \left[G_I \overline{\eta_J} + \eta_I \overline{G_J} \right] \Delta y \ , \qquad (2.39) \\
b_{v^i} &= \left[\partial_{(\delta v^i)} V\right]_\star = e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \left[(A_{Ii} + C_{Ii} + B_{Ii}\Delta y)\overline{\eta_J} + \eta_I \overline{(A_{Ji} + C_{Ji} + B_{Ji}\Delta y)} \right] \Delta y \ , \\
b_{w^i} &= \left[\partial_{(\delta w^i)} V\right]_\star = e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} \left[i(A_{Ii} - C_{Ii} + B_{Ii}\Delta y)\overline{\eta_J} - i\eta_I \overline{(A_{Ji} - C_{Ji} + B_{Ji}\Delta y)} \right] \Delta y \ .
\end{aligned}$$

We can now determine the displacements δx, δv^i and δw^i as functions of Δy by minimising the potential (2.36). The unique minimum at each value of Δy is found by solving

$$\mathcal{D}\mathbf{\Delta}_{min} = -\mathbf{b} \quad . \tag{2.40}$$

We find

$$\Rightarrow \quad \mathbf{\Delta}_{min} = -\mathcal{D}^{-1}\mathbf{b} = -\frac{\mathrm{adj}[\mathcal{D}]}{\det[\mathcal{D}]}\, \mathbf{b}, \tag{2.41}$$

where adj$[\mathcal{D}]$ is the adjugate matrix of $\mathcal{D}$. By substituting the solution $\mathbf{\Delta}_{min}$ back into (2.36) we arrive at the effective potential

$$V_{eff}(\Delta y) = -\frac{1}{2}\mathbf{b}^T(\Delta y)\mathcal{D}^{-1}(\Delta y)\mathbf{b}(\Delta y) + \mu^2 \Delta y^2. \tag{2.42}$$

This is the main result of this section. We have derived an expression for the effective potential with backreaction taken into account, i.e. V_{eff} is the potential along the flattest trajectory away from the SUSY minimum. Note that it still remains to be checked whether this potential is suitable for inflation. Further, recall that the above is only valid as long as backreaction of complex structure moduli is weak, such that terms cubic in δx etc. can be ignored. In the following section we will show that this can be achieved by tuning all η_I small.

However, before analysing (2.42) further we can already make the following observation: even if backreaction is under control (i.e. the displacements δx etc. are small) the effect of backreaction onto the inflaton potential is not negligible. Without backreaction the potential would be just given by $\mu^2(\Delta y)^2 = e^{\mathcal{K}}\mathcal{K}^{I\bar{J}}\eta_I\overline{\eta_J}(\Delta y)^2$, which is quadratic in the small quantities η_I. Note that all entries of the vector $\mathbf{b}$ (2.39) are linear in the small quantities η_I, while $\mathcal{D}$ does not depend on η_I at all. As a result, the first term in (2.42) containing the effects of backreaction is quadratic in η_I. As there are no other small parameters in our setup we find that the first term in (2.42) is not parametrically suppressed w.r.t. the naive inflaton potential. On the contrary, both terms in (2.42) are equally important and the effective potential can differ significantly from the naive inflaton potential.

In the next section, we will analyse the effective potential in more detail. In particular, we will find:

- For small and intermediate Δy the effective potential does in general not behave like a simple monomial in Δy. While the naive inflaton potential is quadratic by construction, backreaction will change this behaviour for intermediate Δy.

- However, for large enough Δy the effective potential can again be approximated by a parabola $V_{eff} = \mu_{eff}^2(\Delta y)^2$. We are thus left with a sizable interval in field space where the effective potential is essentially quadratic. Thus it is in principle suitable for realising quadratic large field inflation.

2.4.2 Quantifying backreaction

In this section we wish to determine $\boldsymbol{\Delta}_{min}(\Delta y)$ and check that backreaction can indeed be controlled. By substituting $\boldsymbol{\Delta}_{min}(\Delta y)$ into (2.36) we will also be able to study the effective potential as a function of Δy.

To perform the next steps analytically and in full generality is not practical. The inverse matrix $\mathcal{D}^{-1}$ and thus $\boldsymbol{\Delta}_{min}$ will typically be complicated expressions in the parameters A_{Ii}, B_{Ii}, C_{Ii}, G_I and η_I, which will obscure the points we wish to make in this section.

To circumvent these complications, one can study backreaction and the effective potential numerically, and we will do so in section 2.6. Here we adopt a different approach. In particular, we wish to show that by tuning η_I small backreaction of complex structure moduli can be controlled. For this analysis the exact numerical values of the parameters A_{Ii}, B_{Ii}, C_{Ii} and G_I as well as $\mathcal{K}^{I\bar{J}}$ are not important; all we need to know is that they are not tuned small. Thus, to simplify the following calculations, we assume

$$|A_{Ii}| \sim |B_{Ii}| \sim |C_{Ii}| \sim |G_I| \sim \mathcal{K}^{I\bar{J}} \sim \mathcal{O}(1) \ , \tag{2.43}$$

$$|\eta_I| \sim \epsilon^2 \ll 1 \ . \tag{2.44}$$

Then the matrix $\mathcal{D}$ and the vector $\mathbf{b}$ are given by:

$$\mathcal{D} = e^{\mathcal{K}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1)+\mathcal{O}(1)\Delta y & \dots & \mathcal{O}(1)+\mathcal{O}(1)\Delta y \\ \mathcal{O}(1)+\mathcal{O}(1)\Delta y & (\mathcal{O}(1)+\mathcal{O}(1)\Delta y)^2 & \dots & (\mathcal{O}(1)+\mathcal{O}(1)\Delta y)^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{O}(1)+\mathcal{O}(1)\Delta y & (\mathcal{O}(1)+\mathcal{O}(1)\Delta y)^2 & \dots & (\mathcal{O}(1)+\mathcal{O}(1)\Delta y)^2 \end{pmatrix}, \qquad (2.45)$$

$$\mathbf{b} = e^{\mathcal{K}} \begin{pmatrix} \mathcal{O}(1) \\ \mathcal{O}(1)+\mathcal{O}(1)\Delta y \\ \vdots \\ \mathcal{O}(1)+\mathcal{O}(1)\Delta y \end{pmatrix} \epsilon^2 \, \Delta y. \qquad (2.46)$$

It is now straightforward to determine the dependence of $\mathcal{D}^{-1}$ on Δy. Recall that for a geometry with $n+1$ complex structure moduli $\mathcal{D}$ is a $(2n+1)\times(2n+1)$ matrix. Then one obtains:

$$\mathcal{D}^{-1} = \frac{e^{-\mathcal{K}}}{\mathrm{pol}^{4n}(\Delta y)} \begin{pmatrix} \mathrm{pol}^{4n}(\Delta y) & \mathrm{pol}^{4n-1}(\Delta y) & \dots & \mathrm{pol}^{4n-1}(\Delta y) \\ \mathrm{pol}^{4n-1}(\Delta y) & \mathrm{pol}^{4n-2}(\Delta y) & \dots & \mathrm{pol}^{4n-2}(\Delta y) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{pol}^{4n-1}(\Delta y) & \mathrm{pol}^{4n-2}(\Delta y) & \dots & \mathrm{pol}^{4n-2}(\Delta y) \end{pmatrix}, \qquad (2.47)$$

where $\mathrm{pol}^d(\Delta y)$ symbolises a polynomial of degree d in Δy. More precisely, $\mathrm{pol}^d(\Delta y) = \sum_{m=0}^{d} p_m (\Delta y)^m$ with coefficients p_m which depend on A_{Ii}, B_{Ii}, C_{Ii}, G_I and $\mathcal{K}^{I\bar{J}}$.

To arrive at (2.47) we had to rely on several assumptions. For one, to be able to invert $\mathcal{D}$ it has to be non-degenerate. In addition, if $\mathcal{D}$ has a non-trivial substructure, it is certainly possible that there are cancellations when calculating the determinant and adjugate of $\mathcal{D}$. Then the polynomials appearing in $\mathcal{D}^{-1}$ would be of a lower degree than naively expected. We checked numerically that cancellations typically do not occur and hence it is justified to write $\mathcal{D}^{-1}$ as in (2.47).

We are now in a position to determine the displacements δx, δv^i and δw^i as functions of Δy:

$$\mathbf{\Delta}_{min} = \begin{pmatrix} \delta x \\ \delta v^i \\ \delta w^i \end{pmatrix}_{min} = \begin{pmatrix} \mathrm{pol}^{4n}(\Delta y) \\ \mathrm{pol}^{4n-1}(\Delta y) \\ \mathrm{pol}^{4n-1}(\Delta y) \end{pmatrix} \frac{\epsilon^2 \, \Delta y}{\mathrm{pol}^{4n}(\Delta y)}, \qquad (2.48)$$

where in the above δv^i and δw^i represent all moduli of this type.

We can make the following observations. For one, the displacements δx, δv^i and δw^i are proportional to the small parameter ϵ^2. Thus they are in principle small to the extent that ϵ^2 is small. We used this fact in the previous section to neglect terms of the form $\epsilon\delta x$ etc. in $D_I W$. However, given the expression (2.48) we can say much more about the dependence of δx, δv^i and δw^i on Δy. In particular, we can identify three regimes where the displacements behave differently:

1. $\Delta y \ll 1$: In this regime the polynomials in (2.48) will be dominated by their constant terms. It is then easy to see that $\delta x \sim \delta v^i \sim \delta w^i \sim \epsilon^2 \Delta y$. The displacements increase linearly with Δy, but they remain small in this regime. Backreaction is under control.

2. $\Delta y \sim O(1)$: no term in particular is expected to dominate in the polynomials of (2.48). The displacements then behave as generic functions of Δy, possibly with regions of positive and negative slope. While the displacements are still suppressed by ϵ^2, they can get enhanced in this regime if the term in the denominator of (2.48) (i.e. the determinant of $\mathcal{D}$) becomes small. In this case backreaction is not completely under control and higher order terms in δx etc. cannot always be ignored.

3. $\Delta y \gg 1$: here the polynomials are dominated by the monomial with the highest degree: $\text{pol}^d(\Delta y) \sim (\Delta y)^d$. We then find the following: $\delta v^i, \delta w^i$ approach a constant, while δx increases linearly with Δy. In particular, $\delta v^i \sim \delta w^i \sim \mathcal{O}(1)\epsilon^2$ while $\delta x \sim O(1)\epsilon^2\Delta y$. The most dangerous modulus in this regime is then δx, as it increases linearly with Δy. We can ignore higher order corrections in δx to the potential as long as $\delta x \ll 1$, which requires $\Delta y \ll 1/\epsilon^2$. This condition is automatically satisfied as we are working under the assumption $0 < \Delta y \ll 1/\epsilon$. Therefore in this regime higher order corrections in δx are negligible.

In quadratic inflation one is interested in the regime of large displacements along the inflationary direction. As we have just shown, in this particular regime backreaction is completely under control up to maximal distances $\sim O(1/\epsilon)$. The parameter ϵ cannot be set to any arbitrary value, as this will affect both the phenomenology of inflation as well as the severity of tuning in the landscape. We will discuss this briefly in section 2.7. Let us here anticipate that it is feasible to have $(\Delta y)_{max} \sim O(10^2)$ in units of the Planck mass. The important point is that there exist a regime of large field displacements where our assumptions about backreaction are justified. Therefore in this regime the approximation of the potential to quadratic order in δx, δz^i and $\delta \bar{z}^i$ is valid.

We now turn to the effective potential, which we already encountered in (2.42):

$$V_{eff} = -\frac{1}{2}\mathbf{b}^T\mathcal{D}^{-1}\mathbf{b} + \mu^2\Delta y^2 \ .$$

In the previous section we already observed that both terms scale as ϵ^4 and thus backreaction is not negligible. Here we will study its dependence on Δy.

Many observations from our analysis of the Δy-dependence of $\mathbf{\Delta}_{min}$ also apply here. We will be particularly interested in the regime $1 \ll \Delta y \ll 1/\epsilon$. As we just argued, our expansion of the potential to second order is a good approximation of the F-term scalar potential (2.35) in this regime. In this region of field space, the inverse matrix $\mathcal{D}^{-1}$ and the vector $\mathbf{b}$ are easy to write down:

$$\mathcal{D}^{-1} \simeq e^{-\mathcal{K}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1)\Delta y^{-1} & \dots & \mathcal{O}(1)\Delta y^{-1} \\ \mathcal{O}(1)\Delta y^{-1} & \mathcal{O}(1)\Delta y^{-2} & \dots & \mathcal{O}(1)\Delta y^{-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{O}(1)\Delta y^{-1} & \mathcal{O}(1)\Delta y^{-2} & \dots & \mathcal{O}(1)\Delta y^{-2} \end{pmatrix}, \tag{2.49}$$

$$b \simeq \ e^{\mathcal{K}} \begin{pmatrix} \mathcal{O}(1) \ \epsilon^2 \ \Delta y \\ \mathcal{O}(1) \ \epsilon^2 \ \Delta y^2 \\ \vdots \\ \mathcal{O}(1) \ \epsilon^2 \ \Delta y^2 \end{pmatrix}. \tag{2.50}$$

In the regime of large Δy the effective potential is then given by inserting the two above expressions (2.49) and (2.50) into (2.42):

$$V_{eff} \simeq \Big(- O(1)e^{\mathcal{K}}\epsilon^4 + \mu^2\Big)\Delta y^2 \equiv \mu_{eff}^2 \Delta y^2, \tag{2.51}$$

where $\mu^2 = e^{\mathcal{K}}K^{I\bar{J}}\eta_I\overline{\eta_J} \sim e^{\mathcal{K}}\epsilon^4$. Some comments are in order. First, we find that for large Δy the effective potential is a sum of two terms quadratic in Δy. The first one is due to backreaction on $\delta x, \delta z^i, \delta\bar{z}^i$ as one moves along Δy. The second term is the naive Δy potential. The computation that we performed shows that those two contributions are of the same order of magnitude. Therefore we observe that, even though backreaction is under control in the regime under consideration, its effect on the potential is certainly not negligible.

Secondly and most importantly, in the regime $1 \ll \Delta y \ll 1/\epsilon$ the potential is well approximated by a positive quadratic function. It is therefore in principle suitable for realising quadratic inflation. Notice however that the effective mass μ_{eff} is numerically smaller than the naive mass μ.

Our result can be compared to previous studies of backreaction in axion monodromy inflation. In [100, 161] it was found that backreaction of the inflaton potential on heavier moduli can flatten the inflaton potential at large field values. To be specific, for models of inflation with φ^p-potentials this can manifest itself in the reduction of the power p at large field values. In our case we do not observe a reduction in the power p: our inflaton potential is quadratic for both small and large inflaton field values and flattening reduces the inflaton mass instead. This particular manifestation of flattening is a direct consequence of the mathematical structure of the supergravity scalar potential once we implement all the tuning conditions. Most importantly, the flattening we observe has the same physical origin as the effect described by [100, 161]: it arises from integrating out heavier moduli.

By canonically normalising the inflaton we can then also determine the physical inflaton mass. Note that the inflaton direction is mainly given by y: at large Δy the moduli z^i are essentially fixed and $\delta x \sim \epsilon\Delta y$ only varies weakly with y. Thus, to leading order we can identify the inflaton with Δy. The effective Lagrangian for Δy reads:

$$\mathcal{L}_{eff} = \mathcal{K}_{uu}(\partial\Delta y)^2 - V_{eff}(\Delta y) = \mathcal{K}_{uu}(\partial\Delta y)^2 - \mu_{eff}^2(\Delta y)^2. \tag{2.52}$$

Therefore, at leading order the inflaton is simply obtained via the rescaling $\varphi = \sqrt{2\mathcal{K}_{uu}}\Delta y$ and the inflaton mass is given by $m_\theta^2 = \mu_{eff}^2/\mathcal{K}_{uu}$. The constraint $\Delta y \ll 1/\epsilon$ can now be translated into a constraint on the maximal initial displacement of φ. The field range of the inflaton is limited to $\varphi \ll \sqrt{2\mathcal{K}_{uu}}/\epsilon$.

This section can thus be summarised as follows: by tuning small $n+1$ parameters $a, \partial_{z^1}a, \ldots, \partial_{z^n}a$, we can ensure that there exists a large range in field space in which backreaction is under control and the inflationary potential is in principle suitable for quadratic inflation.

2.4.3 Backreaction for less severe tuning

In the previous section we imposed the tuning $|a_i + \mathcal{K}_i a| \sim \epsilon^2$ (recall that $\epsilon \equiv |a|$) to ensure that the symmetry-breaking terms in the effective lagrangian for Δy are small enough to allow for inflation. However in principle the possibility of a less severe tuning exists. Namely, one can require only $|a_i| \sim |a| = \epsilon$, such that now $|a_i + \mathcal{K}_i a| \sim \epsilon$. In order to establish whether such a tuning is sufficient to allow for inflation, an analysis of backreaction has to be performed. The strategy is exactly the same as in the previous sections.

In this section, in order not to disturb the flow of the presentation, we will only highlight the differences with respect to the analysis already performed in the previous sections. The detailed computations can be found in [130]. Here we focus on the regime $\Delta y \gg 1/\epsilon$. One can check *a posteriori* that backreaction of complex structure moduli is under control only in this regime, thereby justifying our assumption. The analysis then reveals the following features:

1 Assuming $\delta z \sim \delta x \sim \epsilon$, we find that the F-term scalar potential (2.17) vanishes at order ϵ^2 (and, automatically, also at order ϵ^3). Therefore, we need to go beyond leading order and consider $\delta z = \delta z_1 + \delta z_2$ with $\delta z_1 \sim \epsilon$ and $\delta z_2 \sim \epsilon^2$.

2 In this case, the potential does not vanish at order ϵ^4. By minimising it, one finds that for $\Delta y \gg 1/\epsilon$ backreaction is under control and in particular

$$\delta z_1 \sim \epsilon \ , \quad \delta z_2 \sim \epsilon^2 \ , \quad \delta x \sim \epsilon \ , \tag{2.53}$$

thereby justifying *a posteriori* our assumptions.

3 In the regime $\Delta y \gg 1/\epsilon$, the leading contribution to the effective inflationary potential including backreactions is quadratic in Δy. In particular

$$V_{eff} = \mu_{eff}^2 (\Delta y)^2 \sim e^{\mathcal{K}} |\epsilon|^4 (\Delta y)^2 \ . \tag{2.54}$$

Therefore, in the regime $\Delta y \gg 1/\epsilon$, inflation can in principle be achieved with the less severe tuning $|a_i + \mathcal{K}_i a| \sim \epsilon$. However, in this regime the superpotential $W = w + au$ is dominated by the linear term, in contrast with the case $|a_i + \mathcal{K}_i a| \sim \epsilon^2$. Therefore W changes significantly with Δy. In the Large Volume Scenario, the overall volume of the compactification scales as $\mathcal{V} \propto |W|$. Thus, it is clear that backreaction of Kähler moduli cannot be neglected in this case. In particular, the inflationary direction will arise as an admixture of Δy and of the volume modulus. The danger is thus that for large field displacements Kähler moduli may be destabilised. We comment on this important issue in the next section.

2.5 Kähler moduli and backreaction

In this section we briefly comment on the consequences of large displacements of Δy for Kähler moduli stabilisation. The discussion is based on moduli stabilisation according to the LVS [50], which was reviewed in chapter 1. In this framework, complex structures moduli are integrated out and give rise to a constant tree level superpotential W. The scalar potential for the Kähler moduli arises through the interplay of α'-corrections in the Kähler potential and non-perturbative corrections in the superpotential. This effective potential for Kähler moduli admits a non-supersymmetric AdS minimum at exponentially large volume:

$$\mathcal{V} \propto |W| e^{2\pi\tau_s}, \tag{2.55}$$

where τ_s is the real part of the Kähler modulus of the small cycle. After minimisation, the LVS scalar potential behaves as $V_{LVS} \sim -|W|^2/\mathcal{V}^3$.

In our setup the tree level superpotential is linear in one of the complex structure moduli, i.e. $W = w + au$. As long as $au \ll w$, the superpotential is approximately constant and the modulus u does not play any role in the stabilisation of the volume. However, large Δy displacements can make the linear term dominant with respect to w. In this case W, hence the volume according to (2.55), runs with Δy. Thus the complex structure modulus u can potentially interfere with the Kähler moduli, through the volume of the Calabi-Yau manifold.[3] Moreover in this case, as we will show, the dominant contribution to the potential for Δy comes from the LVS potential. Then our study of the complex structure F-term potential is not sufficient to establish whether the Δy direction is suitable for realising quadratic inflation.

In what follows we do not wish to perform a complete analysis of the issue that we have just presented. Rather, we would like to describe more specifically how this problem affects our work and suggest that inflation might nevertheless work.

Let us then separately discuss the two setups that were presented in subsections 2.4.1 and 2.4.3 respectively. The first case, where $|a| \sim \epsilon$, $|a_i + \mathcal{K}_i a| \sim \epsilon^2$, is not affected by the discussion above. Indeed, it was assumed that the inflaton displacement is restricted to the region $\Delta y \ll 1/\epsilon$. In this regime we have $a\Delta y \ll w \sim O(1)$ and the superpotential is always dominated by the constant term.

The second setup requires more attention. The complex structure moduli scalar potential is under explicit control only for $\Delta y \gg 1/\epsilon$. In this regime $a\Delta y \sim \epsilon\Delta y \gg w$, when $w \sim O(1)$. As we argued above, Kähler moduli stabilisation is certainly an important issue in this case. We focus on the relevance of the LVS potential for the candidate inflationary direction Δy. The starting point is the potential

$$V_{tot}(\Delta y) = V_{eff}(\Delta y) + V_{LVS}(\Delta y) + V_{uplift}(\Delta y), \tag{2.56}$$

where $V_{eff} \sim |\epsilon|^4 (\Delta y)^2/\mathcal{V}^2$ is the effective potential computed in section 2.4.3 and $V_{LVS} \sim |W|^2/\mathcal{V}^3$. We have also included a term to uplift to a dS vacuum. Notice that V_{LVS} and V_{uplift} depend on Δy through W and the volume, according to (2.55). In particular, the effective potential V_{eff} is suppressed with respect to V_{LVS} by $\epsilon^2\mathcal{V}$, because $V_{LVS} \sim |W|^2/\mathcal{V}^3 \sim \epsilon^2(\Delta y)^2/\mathcal{V}^3$ in the regime $\Delta y \gg 1/\epsilon$. In order to remain in the LVS framework, we tune ϵ such that $\epsilon^2\mathcal{V} \ll 1$. [4] It is therefore clear that in this setup the relevant potential for Δy comes from the interplay of the LVS and the uplift

[3]The interplay between Kähler and complex structure moduli in complex structure moduli inflation has been also recently studied in [163]. The authors consider a somewhat different scenario, based on a racetrack scalar potential for the Kähler moduli. They obtain constraints on the running of W from the destabilisation of the volume. Given these conditions, they point out the difficulties associated with large field inflation in a model with one complex structure modulus.

[4]Given a certain size of ϵ, this bounds the volume $\mathcal{V}$. The limited size of $\mathcal{V}$ in large field models of this type has also been discussed in [101]. A more general study of bounds on the volume can be found in [166].

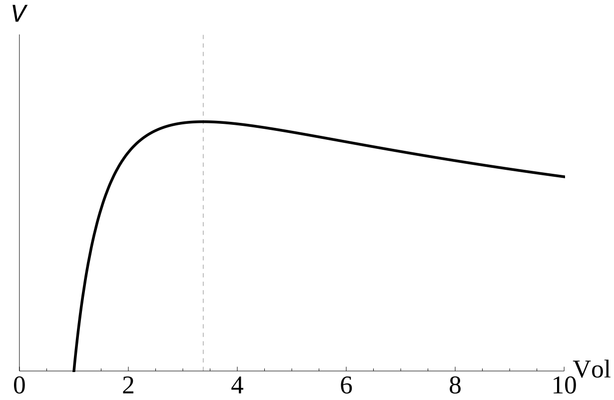


Figure 2.2: Total potential (2.58) as a function of the volume $\mathcal{V} \sim e^A|a|\Delta y$. The dashed line intersects the potential at its maximum. Inflation could take place in the region on the left of the extremum. We normalised the x-axis such that $\langle\mathcal{V}\rangle = 1$.

potentials, i.e. $V_{tot}(\Delta y) \simeq V_{LVS}(\Delta y) + V_{uplift}(\Delta y)$. One can now perform a study of this potential, which necessarily depends on the functional form of the desired uplift. We focus on a scenario where the latter is provided by some hidden matter fields which develop non-vanishing VEVs through minimisation of their F- and D-term potentials [167] (see also [168] for a recent discussion). In this case the total scalar potential (2.56), neglecting V_{eff}, is given by [168]:

$$V_{tot}(\mathcal{V}) \propto \frac{e^{-4\pi\tau_s}}{\mathcal{V}}\left[\mathcal{V}^{1/3}\delta - \sqrt{\ln\left(\frac{\mathcal{V}}{W}\right)}\right], \tag{2.57}$$

where δ is a numerical factor depending on the $U(1)$ charges of the big cycle modulus and the matter fields and (2.55) was used. At the minimum one imposes $\langle V_{tot}\rangle = 0$ to achieve a Minkowski vacuum. Therefore at the minimum $\langle\mathcal{V}\rangle^{1/3}\delta = \ln\Big(\langle\mathcal{V}\rangle/|W|\Big)$. The total potential (2.57) can thus be rewritten as

$$V_{tot}(\mathcal{V}) \propto \frac{e^{-4\pi\tau_s}}{\mathcal{V}}\left[\mathcal{V}^{1/3} - \langle\mathcal{V}\rangle^{1/3}\right]. \tag{2.58}$$

This potential is monotonically rising from 0 to $V_{max} = (3/2)^3\langle\mathcal{V}\rangle$, then decreases and vanishes asymptotically (see figure 2.2).

Since $\mathcal{V} \sim e^{2\pi\tau_s}|a|\Delta y$, the total potential rises monotonically as a function of Δy up to $(\Delta y_{max}/y_\star) \simeq 3.4/|a|$, where $y_\star$ is the value of the y at the minimum. The inflationary range can be now found by canonically normalising Δy, i.e. by defining $\varphi = \sqrt{\mathcal{K}_{uu}}\Delta y$. We conclude that for $\varphi \leq 3.4/(|a|x_\star)$ the potential (2.58) is monotonically rising. Notice that generically this is a sizable range, despite the fact that x is stabilised in the LCS regime, as we have tuned $|a|$ small.

The results of this section can be summarised as follows. We found that in the setup described in subsection 2.4.1 the complex structure moduli do not affect Kähler moduli stabilisation. On the contrary, the setup described in subsection 2.4.3 implies an interplay between the volume modulus and the inflationary direction Δy in the regime of large field displacements. We found that in this case the LVS and uplift potentials give the dominant contribution to the total potential for Δy. By focusing on D-term uplifting from hidden sector matter fields, we showed that the total potential (2.58) is still monotonically rising throughout a sizable range for Δy. As such, it might be suitable for realising large field inflation. Rather than focusing on a more detailed analysis of the potential, which is left for future work, we now provide some numerical examples of the effective inflationary potential including backreaction.

2.6 A numerical example

The aim of this section is to provide numerical support for the findings of subsections 2.4.2 and 2.4.1. We focus on a setup with only four complex structure moduli (i.e. with u and three more moduli z^1, z^2 and z^3). In particular, by assuming random values for the coefficients in the scalar potential expanded to second order in $\delta z^i, \delta \bar{z}^i$ and δx, we determine numerically the displacements $\delta z^i, \delta \bar{z}^i$ and δx by minimising the scalar potential as a function of Δy. Furthermore, we compute the effective potential for Δy including backreaction.

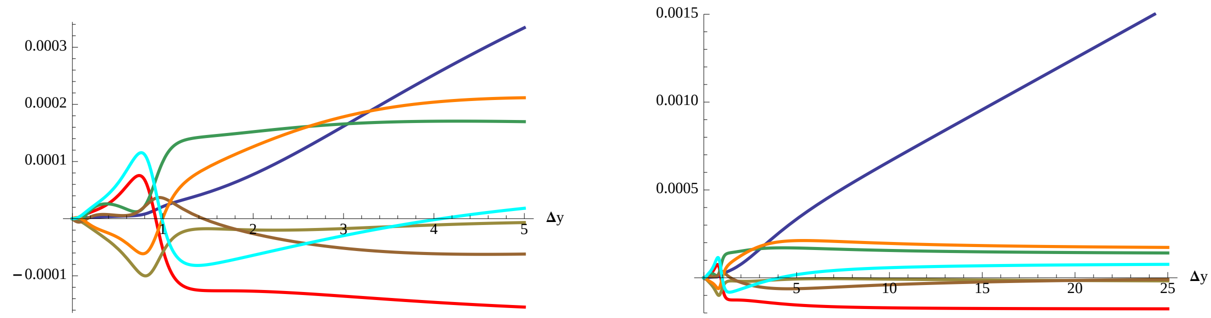
Before presenting our findings, two comments on the validity of our example are in order. First of all, since we generate the coefficients of the scalar potential randomly, we cannot be sure that our results can be reproduced by means of an explicit choice of geometry and flux numbers (for a study along these lines see e.g. [162]). Secondly, our example is particularly simple, in that we consider a small number of complex structure moduli, which in turn implies that the number of available fluxes is also low. Therefore, the possibility of tuning parameters may be severely constrained in such a setup. Despite this warnings, we believe that our numerical analysis, while not completely realistic, is useful in that its qualitative features are not significantly affected by the precise numerical input. Another numerical example with similar findings is presented in [130].

In order to understand the behaviour of the displacements $\delta z^i, \delta \bar{z}^i$ and δx, we assign values $\mathcal{O}(10^{-4})$ to all quantities which need to be tuned small. The remaining quantities are assigned values $\mathcal{O}(1)$.

In figure 2.3 we display the real and imaginary parts of the displacements $\delta z^i = \delta v^i + i\delta w^i$, as well as the behaviour of δx. We find that for large $\Delta y \gtrsim 5$ the displacements δv^1, δw^1, δv^2 and δw^2 approach a small constant value of order $\sim 10^{-4}$. Also, δx asymptotes towards a linear function of Δy with slope and offset of order $\sim 10^{-4}$.

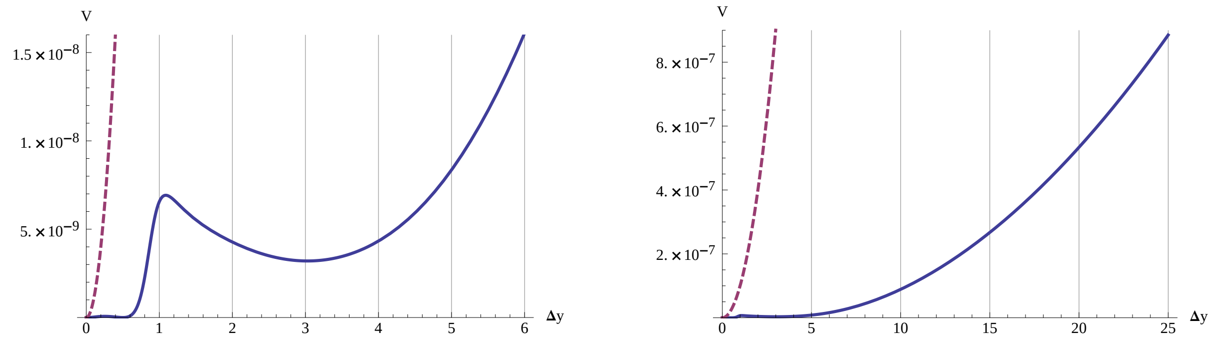
The result for the effective inflaton potential (figure 2.4) exhibits the expected behavior for large Δy: For $\Delta y \gtrsim 5$ the potential approximates a parabola of the form $1.65 \cdot 10^{-9}(\Delta y)^2$. However, we find an interesting behaviour for intermediate Δy: the potential exhibits a local minimum with non-zero V for $\Delta y \approx 3$. By adding terms of the form $\mathcal{O}(1)(\delta x)^3$ etc. to V we can also check explicitly that in the region of interest ($\Delta y < 100$) higher order terms in the expansion of V can be ignored. Interestingly we find that higher order terms do not destroy the local minimum at $\Delta y \simeq 3$.

We conclude that inflation could in principle be realised in this model. For $\Delta y \gtrsim 5$ the potential is essentially quadratic and can support chaotic inflation. The inflaton would roll down the potential until it reached the local minimum at $\Delta y \simeq 3$ where inflation



(a) (b)

Figure 2.3: Plots of the displacements δx (blue), δv^1 (red), δw^1 (ochre), δv^2 (green), δw^2 (brown), δv^3 (orange) and δw^3 (cyan) vs. Δy. This picture is taken from [130] and was prepared by my collaborators.



(a) (b)

Figure 2.4: Plots of the effective inflaton potential (blue, solid) and the 'naive' inflaton potential (red, dashed) vs. Δy. This picture is taken from [130] and was prepared by my collaborators.

would end.

We can now make an interesting observation based on the fact that the local minimum has a positive vacuum energy. Recall that Kähler moduli stabilisation following the Large Volume Scenario leads to an AdS minimum, which needs to be uplifted to give a dS vacuum. If our analysis in this chapter can be successfully combined with Kähler moduli stabilisation à la LVS, the positive vacuum energy of the local minimum could provide the necessary uplift. Our vacuum would then be identified with the minimum we observe at $\Delta y \simeq 3$. We take this finding as a hint that the sector of complex structure moduli as studied in this chapter can in principle give rise to metastable dS vacua.

2.7 Feasibility of tuning in the string landscape

In the previous sections of this chapter, we focused on the backreaction of the complex structure moduli of F-theory fourfolds on the inflationary trajectory. In particular, we understood that the breaking of the shift symmetry of $y \equiv \mathrm{Im}(u)$ can be in principle controlled. More specifically, we started by tuning the coefficient of the quadratic term in the F-term scalar potential. We then established the existence of a large (transplanckian) field range along which this coefficient remains small, thereby allowing for slow-roll inflation.

In this section we will briefly discuss whether the tunings of $|a|$ and its derivatives are feasible from the point of view of the string landscape. In other words, we will provide the number of string vacua which are compatible with the requirements $|a| \ll 1$, $|a_i + \mathcal{K}_i a| \ll 1$. We will not review in detail how to compute the number of susy flux vacua in type IIB orientifolds and F-theory fourfolds. Rather, we will only report the final results of such a computation, whose details for our specific setup can be found in [130] and in [164].

The computation is based on the following formula derived in [169]:

$$\mathcal{N}(L \leq L_\star) = \frac{(2\pi L_\star)^{2m}}{(2m)!\sqrt{\det \eta}} \int_{\mathcal{M}} d^{2m}z \ \det(g) \ \rho(z), \tag{2.59}$$

which counts the number of supersymmetric flux vacua compatible with the tadpole condition $L \leq L_\star \equiv \chi(X)/24$ on a CY fourfold $X = (T^2 \times Y)/\mathbb{Z}_2$. Here ρ is the density of supersymmetric vacua per unit volume of the moduli space $\mathcal{M}$ and η is the so-called intersection form on X (see [169] for details). Furthermore, $m = h^{2,1}_-(Y) + 1$.

In our case, the number of supersymmetric flux vacua $\mathcal{N}$ which satisfy our tuning requirements will depend on: the number $J_t/2 - 1$ of complex structure moduli which a depends on; the number J_f of components of the flux vector N^α that we set to zero in order to achieve a linear superpotential in u in the F-theory limit; the Betti number b_4 of X. The appropriate generalisation of (2.59), described in [130], leads to the following results for $\mathcal{N}$:

1 Less severe tuning described in section 2.4.3, i.e. $|a| \sim \epsilon$, $|a_i + \mathcal{K}_i a| \sim \epsilon$:

$$\mathcal{N}(L \leq L_\star, |a_I| \lesssim \epsilon) \sim \frac{(2\pi L_\star)^{b_4/2-(J_f+J_t)/2}}{(b_4/2 - (J_f + J_t)/2)!} \cdot (\pi\epsilon^2)^{J_t/2}, \tag{2.60}$$

2 More severe tuning described in section 2.4.2, i.e. $|a| \sim \epsilon$, $|a_i + \mathcal{K}_i a| \sim \epsilon^2$:

$$\mathcal{N}(L \leq L_\star, |a| \lesssim \epsilon, |D_i a| \lesssim \epsilon^2) \sim \frac{(2\pi L_\star)^{b_4/2-(J_f+J_t)/2}}{(b_4/2 - (J_f + J_t)/2)!} \cdot \pi^{J_t/2}\epsilon^{2J_t-2}. \tag{2.61}$$

Let us focus on case 2, since it is more constraining in terms of remaining flux vacua. In order to understand the magnitude of (2.61), we now specify numerical values for the parameters ϵ, J_t, J_f and b_4. In subsection 2.4.2 we have seen that backreaction is under control only for $\Delta y < 1/\epsilon$. Quadratic inflation requires a field range $\mathcal{O}(15)$ for the canonically normalised inflaton field $\phi = \sqrt{2\mathcal{K}_{u\bar{u}}}\Delta y$. This implies

$$\epsilon \lesssim \frac{1}{15\sqrt{2}\mathrm{Re}(u)}. \tag{2.62}$$

Let us then assume $\mathrm{Re}(u) \sim \mathcal{O}(1)$, which is enough to suppress the non-perturbative contributions in u to the Kähler potential. Furthermore, let us take $L_\star = 972$, $b_4 = 23320$, $h^{3,1}(X) = 3878$ (see [144]). This choice leads to $\sim 10^{1700}$ F-theory flux vacua, without any further tuning requirement. Let us now take $J_t = 600, J_f \ll b_4$. Then approximately $\mathcal{N} \sim 10^{350}$ out of the original 10^{1700} vacua are left after imposing our tunings. Notice that larger values of J_t exponentially suppress this value of $\mathcal{N}$.

The conclusion of this section is as follows: while the tunings of subsection 2.4.2 greatly reduce the available F-theory landscape, for $J_t \lesssim 600$ the latter is still large enough to allow for a multiverse solution to the cosmological constant problem.

2.8 Summary and conclusions

Let us conclude this chapter by summarising our findings and by providing possible directions for future research.

We aimed at assessing backreaction of complex structure moduli in one specific realisation of F-term axion monodromy inflation [101]. In this particular setup, the inflaton candidate is the imaginary part of a complex structure modulus in the Large Complex Structure regime (i.e. $\mathrm{Re}(u) > 1$). We considered both type IIB CY orientifolds and F-theory CY fourfolds. In the type IIB case, $y \equiv \mathrm{Im}(u)$ corresponds to the axionic part of the Kähler modulus of the mirror-dual type-IIA model. We do not require the other moduli $z \equiv z^i, i = 0 \ldots n$ to be in the LCS regime as well: we therefore adopt a so-called "partial large complex structure limit".

As a consequence of the LCS regime, non-perturbative contribution in u to the complex-structure-moduli Kähler potential are suppressed. Therefore, $\mathrm{Im}(u)$ enjoys the required shift symmetry at the level of the Kähler potential. For inflationary purposes, this flat direction has to be lifted. We did so by introducing a linear symmetry breaking term in the superpotential, i.e. by considering $W = w + au$. While realisations of such a model are highly constrained in a type IIB framework, we showed that they can arise in F-theory fourfolds. Large field slow roll inflation requires the potential for y to be sufficiently flat over transplanckian field ranges, i.e. for $\Delta y \gtrsim 1$. A necessary condition to ensure this feature is clearly $a \sim \epsilon \ll 1$. From a stringy point of view, this can be achieved by a flux-tuning, i.e. by a delicate cancellation between several larger contributions to a. Obviously this necessarily implies that a is a function of other fields of the compactification, in particular $a \equiv a(z)$.

Starting from the model just described we obtained the following results:

1 **The model requires additional tunings**. In other words, the tuning of a is not sufficient to guarantee a weak breaking of the original shift-symmetry. In particular, also the derivatives $\partial_{z^i} a$ have to be tuned small. The reason of this requirement is as follows: In a realistic compactification all the complex structure moduli remain dynamical during inflation, therefore they can be displaced from their values at the supersymmetric minimum. Thus, the variation of a with respect to z becomes relevant. More technically, the mass terms in the F-term scalar potential all have to be tuned small. There are two possibilities to do so:

– **Case 1**: We impose $|a| \sim \epsilon \ll 1$ and $|a_i + \mathcal{K}_i a| \sim \epsilon^2 \ll 1$.
– **Case 2**: We impose $|a| \sim \epsilon \ll 1$ and $|a_i + \mathcal{K}_i a| \sim \epsilon \ll 1$.

We believe that the need of additional tunings is a general feature of models of F-term axion monodromy inflation, although not all of them are easy to analyse due to lack of a complete analytical presentation.

2 **Backreaction of complex structure moduli cannot be neglected**. In other words, the displacements $\delta z, \delta\bar{z}, \delta x$ of other complex structure moduli from their vacuum expectation values lead to significant contributions to the inflationary potential. Crucially, the displacements can themselves be small and their effects computable. While this happens only in a restricted range of field displacement Δy, the latter is transplanckian, thus in principle suitable for large field inflation. More technically, we reached this conclusion by expanding the F-term scalar potential to quadratic order in the displacements and to order ϵ^4. We then found the values of $\delta z, \delta\bar{z}, \delta x$ which minimise the potential for each value of Δy. The result of this procedure is:

– **Case 1**: Backreaction is under control in the regime $1 \ll \Delta y \ll 1/\epsilon$.
– **Case 2**: Backreaction is under control in the regime $1/\epsilon \ll \Delta y \ll 1/\epsilon^2$.

Furthermore, by "integrating out" the displacements $\delta z, \delta\bar{z}, \delta x$ we provided an expression for the effective inflationary potential including backreaction. In both cases, such a potential is quadratic in Δy in the ranges where backreaction is under control and is thus in principle suitable to realise chaotic inflation. A peculiarity of case 2 is that backreaction of Kähler moduli cannot be neglected. While inflation is still in principle possible in this case, the inflationary potential is subdominant with respect to the effects descending from the backreaction od Kähler moduli, thus the phenomenology is more complicated in this case. In support of our analytical analyses, we provided an agreeing numerical example.

3 **The required tuning is in principle realisable in the F-theory landscape**. We briefly reviewed how to estimate the number $\mathcal{N}$ of supersymmetric F-theory vacua which are compatible with the tuning of a and its derivatives. As expected, we found that the latter greatly reduces the available landscape of vacua. In particular, in case 1, $\mathcal{N}$ is suppressed by a factor ϵ^{2J_t-2}, where $J_t/2$ is the number of required tunings. Nevertheless, if such a number is not too large (i.e. $\lesssim 600$) then $\mathcal{N}$ is still large enough to allow for a landscape-type solution of the cosmological constant problem. Therefore our highly tuned setup is feasible in the framework of type IIB/F-theory compactifications.

Despite the positive conclusions of our investigation, it is clear that more work is necessary in order to assess the effects of backreaction of the geometry on inflation. In particular, the $4D$ supergravity language that we used throughout the chapter does not capture certain stringy effects which may significantly affect our findings. Specifically, we have in mind α'-corrections to the Kähler potential arising from 7-branes.[5] In this direction, the authors of [170] showed that on CY fourfolds a certain class of F-theory α'-corrections does not modify the functional form of the Kähler potential (see also [171]). In addition, in [102] and [172] it was argued that a more complete analysis of large-field inflation with D7-branes requires the incorporation of higher-derivative corrections to the 4d supergravity description coming from DBI terms. Unfortunately at the moment it is not completely understood how these α'-corrections should be included in the Kähler potential and we are therefore unable to assess their relevance for our work. We hope that, as in [102, 172], these corrections will lead to an important but not dangerous flattening of the inflationary potential at large field values.

A more thorough understanding of the backreaction of Kähler moduli in models of axion monodromy deserves further study as well.

Finally, we noticed an interesting feature of the backreacted potential which deserves further study. Namely, at small field values, close to the supersymmetric minimum, the effective potential may exhibit local minima, providing an opportunity for dS uplifting. These wells may also lead to extremely interesting deviations from the standard phenomenology of chaotic inflation. In particular, the reheating phase may be particularly rich in these models, similarly to scenarios that will be discussed in chapter 5.

[5]See [130] for further comments on α'-corrections at the $\mathcal{N} = 2$ level on the type IIA dual of our setup.

Chapter 3

Winding Inflation and deviations from the Large Complex Structure regime

Models where the inflaton is identified with a combination of two axions with subplanckian decay constants represent a possibility to achieve trans-planckian inflation [31]. In this chapter, we present an implementation of this strategy in the complex structure moduli sector of Type IIB Calabi-Yau orientifold compactifications. The flatness of the inflaton potential can be spoiled by instanton-like oscillations which correspond to deviations from the shift-symmetric geometry (Large Complex Structure). We focus precisely on these corrections and elucidate the relation of our model with the Weak Gravity Conjecture. We set $M_P \equiv 1$, unless otherwise stated.

This chapter is based on the publication [131].

3.1 Introduction

Inflationary model building in string compactifications can shed light on the compatibility between shift symmetries and Quantum Gravity (QG). On one hand, in string compactifications it is possible to explicitly take into account various sources of symmetry breaking and assess their effect on the inflationary potential. One such effect is backreaction of other moduli of the compactification, which was studied in the previous chapter. The severe tuning of fluxes which we had to require to keep backreaction under control can be interpreted as evidence for the difficulty of realising LFI compatibly with quantum gravity. As mentioned in the introduction of this thesis, such difficulties may also be expressed by certain effective field theory arguments against the robustness of shift symmetries in quantum gravity [63, 88]. Among them, the Weak Gravity Conjecture (WGC) [63], reviewed in section 1.2 is arguably the most promising in terms of constraints on large field axion inflation. Despite important progress (see [36, 65–79, 84, 131] for a partial collection of relevant results), at the time of writing it is not clear which version, if any, of the WGC should be used to constrain models where a transplanckian trajectory arises in the field space of two or more axions [31, 32].

This conundrum represents another opportunity for inflationary model building in string compactifications: that is, to present counterexamples to one or the other version of the WGC. In this sense, model building becomes a tool to select and refine certain low-energy expectations on the behaviour of Quantum Gravity, rather than an explicit description of the UV problems of large field inflation.

According to the discussion above, in this chapter we adopt two attitudes: on one hand, we investigate the consequences of deviations from the LCS regime for inflation. On the other hand, we comment on the relation between those deviations and the WGC. We propose a model where large field inflation occurs along a winding trajectory in the field space of two axions, in the spirit of the KNP proposal [31] and [106, 107] (see also chapter 1.2.4 for a review of the basic idea). In particular, we will consider axions arising from complex structure moduli in the Large Complex Structure (LCS) regime, as in chapter 2. However in this particular case we will not focus on monomial inflaton potentials. Rather, the phenomenology of our model is that of natural inflation (see [173] for a previous realisation of natural inflation in the complex structure moduli sector). Very interestingly, our model implements a loophole of the WGC pointed out in [68, 69] (see chapter 1.2.4 for a description of such loophole) and may thus suggest that only the so-called mild form of the conjecture is respected by string theory.

This chapter is structured as follows: in subsection 3.2.1 we introduce the basic ingredients of our model. In subsection 3.2.2 we discuss in detail the stabilisation of complex structure moduli in our setup. In section 3.3 we compute the effective inflaton potential, including the backreaction of other complex structure moduli. In section 3.4 we explain how our model realises a loophole of the WGC and discuss relation to previous work. Finally, we summarise the results of the chapter in section 3.5.

3.2 The model

In this section we introduce a model which realises a winding trajectory in the field space of two axions. The latter are identified with the axionic components of two complex structure moduli u and v. Here we find it convenient to define the axionic component as the *real* part of the corresponding complex structure modulus, rather than the imaginary part as in chapter 2. After introducing the ingredients of the model, we describe how the inflationary trajectory arises from moduli stabilisation.

3.2.1 Kähler potential and superpotential

As already described in chapter 2, complex structure axions enjoy a shift symmetry when the corresponding saxions are stabilised in the Large Complex Structure (LCS) regime. Here we consider two complex structure moduli u and v in the LCS regime, while the other complex structure moduli need not be in the same geometric limit. Thus the Kähler potential is at leading order invariant under shifts of $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$, while it generically does not exhibit any flat direction with respect to other complex structure moduli. The latter, as well as the axio-dilaton, will be collectively denoted by z. More explicitly:

$$\mathcal{K}_{cs}^{LCS} = -\log\left[\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v})\right] \tag{3.1}$$

The crucial difference with respect to the model of chapter 2 is that we now add to (3.1) corrections the LCS geometry. These come in the form of exponentially suppressed terms in the Kähler potential, in a sense analogous to non-perturbative corrections. In this specific instance, we only retain the leading "instantonic" terms, which can be taken to behave as $e^{-2\pi i u}$ and $e^{-2\pi i v}$ (see [131] for details). We also assume that the F-term conditions stabilise u and v such that

$$e^{-2\pi \mathrm{Im}(u)} \ll e^{-2\pi \mathrm{Im}(v)} \ll 1 \ . \tag{3.2}$$

As we will show later, inflation proceeds along a direction in which the condition (3.2) remains true. Therefore, it is consistent to include the corrections to the LCS regime for v only. The terms of the form $e^{2\pi i u}$ will instead be neglected. As a result, we will consider the following Kähler potential for complex structure moduli:[1]

$$\mathcal{K} = -\log\Big(\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v}) + \Big[\mathcal{B}(z, \bar{z}, v - \bar{v})e^{2\pi i v} + \text{c.c.}\Big]\Big) \ . \tag{3.3}$$

At the level of the Kähler potential (3.3), only one shift-symmetric direction remains, i.e. $\mathrm{Im}(u)$.

The second ingredient of our model is the following flux-induced superpotential:

$$\mathcal{W} = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v} \ , \tag{3.4}$$

where $N \in \mathbb{Z}$ and we consider $N \gg 1$. Notice that, in agreement with our discussion around (3.2), the superpotential (3.4) includes exponentially suppressed terms in v, but

[1] Further details about the structure of the quantum-corrected Kähler potential are given in [131], based on [105, 174].

not in u. Notice that u and v appear only linearly in (3.4). The absence of quadratic and cubic terms can be achieved by setting to zero some three-form flux numbers. The discussion is very similar to the one provided in chapter 2.3, therefore we do not repeat it here (see also [131]). Similarly, $N \gg 1$ can be ensured by choosing certain flux numbers large. From (3.3) and (3.4), we can already anticipate the existence of a flat direction, which is closely aligned with $\mathrm{Re}(u)$.

3.2.2 The inflationary winding trajectory

In this section, we show in detail how the inflationary trajectory arises from the Kähler potential (3.3) and the superpotential (3.4). We start by discussing moduli stabilisation in our setup. First of all, we stabilise Kähler moduli according to the Large Volume Scenario (LVS) [50]. As reviewed in subsection 1.1.4, at leading order the theory for Kähler moduli is of no-scale type. which implies:

$$V = e^{\mathcal{K}} \left(\mathcal{K}^{I\bar{J}} D_I \mathcal{W} \overline{D_J \mathcal{W}} + \mathcal{K}^{T_\rho \bar{T}_\sigma} D_{T_\rho} \mathcal{W} \overline{D_{T_\sigma} \mathcal{W}} - 3|\mathcal{W}|^2 \right) \tag{3.5}$$

$$= e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} D_I \mathcal{W} \overline{D_J \mathcal{W}} \ , \tag{3.6}$$

with $I, J = 0, \ldots, n-1$ running over all complex structure moduli and the axio-dilaton. The potential is thus minimised for $D_I \mathcal{W} = 0$ for all I. Subleading terms due to α' and non-perturbative corrections stabilise the Kähler moduli. We will comment again on the stabilisation of Kähler moduli at the end of section 3.3, after we compute the effective inflationary potential.

We now focus on the complex structure moduli u and v, in particular on their axionic directions $\mathrm{Re}(u)$, $\mathrm{Re}(v)$. Ignoring the exponential terms in (3.3) and (3.4), only the combination $\mathrm{Re}(u) - N\mathrm{Re}(v)$ appears in the scalar potential (3.5), i.e. $V \equiv f\left[\mathrm{Re}(u) - N\mathrm{Re}(v)\right]$. Furthermore, the function f is minimised at some argument x_0, i.e. at $\mathrm{Re}(u) - N\mathrm{Re}(v) = x_0$. The latter equation defines a flat direction or "valley" on the plane parameterised by $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$. This flat direction is independent of the metric on field space, i.e. $\mathcal{K}$. It can be parameterised by any combination of $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$, except the fixed combination $\mathrm{Re}(u) - N\mathrm{Re}(v)$. In particular, we find it convenient to define the fields ψ and ϕ by:

$$\phi \equiv u \ , \qquad \psi \equiv u - Nv \ . \tag{3.7}$$

With respect to these new variables, the flat direction is parameterised by $\mathrm{Re}(\phi)$ with $\mathrm{Re}(\psi)$ being fixed. Note that varying $\mathrm{Re}(\phi)$ at fixed $\mathrm{Re}(\psi)$ is *not* the same as varying $\mathrm{Re}(u)$ at fixed $\mathrm{Re}(v)$, i.e. the valley is not identical to the coordinate axis $\mathrm{Re}(u)$.

Definitions of ϕ which differ from the one in (3.7) are equally valid. One could e.g. choose the new variable such that it describes a direction which is orthogonal to ψ in field space. However, this is a metric dependent statement. Therefore, the definition of such orthogonal variables would involve elements of the Kähler metric and complicate the following computations. We prefer to redefine variables according to (3.7).

After the change of variables, the exponential term reads:

$$e^{-2\pi \mathrm{Im}(v)} = e^{-2\pi \frac{\mathrm{Im}(\phi) - \mathrm{Im}(\psi)}{N}} \equiv \epsilon \ll 1, \tag{3.8}$$

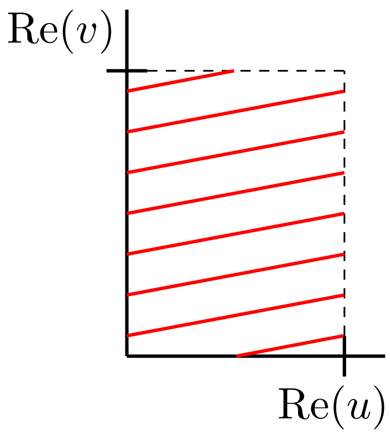


Figure 3.1: Inflaton trajectory in Re(v)-Re(u)-plane. The winding trajectory is a result of stabilising one direction in Re(v)-Re(u)-space by an F-term potential due to bulk fluxes. This picture is taken from [131] and was prepared by my collaborators.

where we defined the small parameter ϵ for notational convenience. To be precise, $\epsilon \equiv e^{-2\pi \text{Im}(v_0)}$, where $\text{Im}(v_0)$ is the value stabilised by the leading order (non-exponential) potential. As we will see, shifts in v or, in the new variables, in ψ, ϕ, will be small enough during inflation. Thus, ϵ is now a parameter.

In terms of our new variables the leading parts of $\mathcal{K}$ and W are

$$\mathcal{K} = K(z, \bar{z}, \text{Im}(\phi), \text{Im}(\psi)) + \mathcal{O}(\epsilon) \ , \tag{3.9}$$

$$\mathcal{W} = w(z) + f(z)\psi + \mathcal{O}(\epsilon) \ . \tag{3.10}$$

The conditions $D_I \mathcal{W} = 0$ will in general stabilise all of the moduli z, both $\text{Im}(\phi)$ and $\text{Im}(\psi)$ as well as the combination $\text{Re}(\psi)$.[2] In other words, the appearance of ψ in W leads to the breakdown of the two shift symmetries in u and v to one remaining shift symmetry. This shift-symmetric direction is parametrised by $\text{Re}(\phi)$, which does not appear in $\mathcal{K}$ and $\mathcal{W}$. It is our inflaton candidate, which has a flat potential at this stage.

It is easy to see from (3.7) that, at fixed $\text{Re}(\psi)$, the field $\text{Re}(\phi)$ parametrizes a direction that is nearly aligned with $\text{Re}(u)$. Indeed, we have $\delta\text{Re}(u) = N\delta\text{Re}(v)$ such that, at large N, u changes much more strongly than v. Such a flat direction thus corresponds to a winding trajectory shown in figure 3.1 and can be very long.

The idea of achieving a long direction due to winding trajectories in the field space of two axions was proposed in [31] and subsequently many large-field inflation models in string theory have employed this mechanism [106, 107, 175–179]. However, in contrast to previous proposals the winding trajectory in our case arises from an F-term potential due to fluxes. For example, in [107] non-perturbative contributions to the potential were employed to realise a winding trajectory similar to ours (see also the discussion in section 3.4).

[2] Here $D_I \mathcal{W} = 0$ for all I gives rise to $2n + 1$ real equations for $2n + 2$ real moduli. Note that $D_\phi \mathcal{W} = K_\phi \mathcal{W} = 0$ gives rise to only one real equation $K_\phi = 0$.

3.3 Effective inflationary potential

We now include the subleading terms $\sim e^{2\pi i v} = e^{2\pi i \frac{\phi-\psi}{N}} = \epsilon\; e^{2\pi i \frac{\mathrm{Re}(\phi)-\mathrm{Re}(\psi)}{N}}$ and determine the inflaton potential to order ϵ^2. We also include the backreaction on the remaining moduli in our analysis (see chapter 2 for an introduction to the problem of backreaction in string inflation).

We can write $\mathcal{K}$ and $\mathcal{W}$ of (3.3) and (3.4) as

$$\mathcal{K} = K + \delta K + \mathcal{O}(\epsilon^2) \tag{3.11}$$
$$\mathcal{W} = W + \delta W + \mathcal{O}(\epsilon^2)\ , \tag{3.12}$$

where $\delta K \sim \delta W \sim \epsilon$. We thus have

$$D_I\mathcal{W} = (\partial_I W + K_I W) + (\partial_I \delta W + K_I \delta W + \delta K_I W) + \mathcal{O}(\epsilon^2)\ , \tag{3.13}$$

where the index I runs over all superfields z, ψ and ϕ. Given our specific structure for $\mathcal{K}$ and $\mathcal{W}$ from (3.3) and (3.4) we find that all covariant derivatives can always be brought into the form

$$\begin{aligned} D_I\mathcal{W} &= A_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi}) \\ &\quad + \epsilon\left[B_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi})e^{\frac{2\pi i\phi_1}{N}} + C_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi})e^{-\frac{2\pi i\phi_1}{N}}\right] + \mathcal{O}(\epsilon^2)\ , \end{aligned} \tag{3.14}$$

where $\phi_1 \equiv \mathrm{Re}(\phi)$ and A_I, B_I, C_I are complex functions of z, $\bar{z}$, ψ, $\bar{\psi}$ and $\mathrm{Im}(\phi)$ which can be easily calculated for a specific example. Notice that we have reabsorbed the phases $e^{-2\pi i\psi_1/N}$ and $e^{2\pi i\psi_1/N}$ in the complex prefactors B_I and C_I respectively.

The potential is given by

$$V = e^{K+\delta K}(K^{I\bar{J}} + \delta K^{I\bar{J}})D_I\mathcal{W}\overline{D_J\mathcal{W}} + \dots\ . \tag{3.15}$$

However, we will find that during inflation $D_I\mathcal{W} \sim \epsilon$. Thus, to determine V at order ϵ^2 we can ignore δK and $\delta K^{I\bar{J}}$ in the prefactor and only work with

$$V = e^K K^{I\bar{J}} D_I\mathcal{W}\overline{D_J\mathcal{W}} + \mathcal{O}(\epsilon^3)\ . \tag{3.16}$$

Without loss of generality we can use an orthogonal transformation O_I^J to diagonalise the Kähler metric. Its eigenvalues λ_I can then be reabsorbed in a redefinition of the rotated vectors $O_I^J(D_J\mathcal{W})$. The potential becomes

$$V = e^K \sum_{I=1}^{n+1} |v_I|^2\ , \quad \text{with } v_I = \sqrt{\lambda_I}\ \sum_J {O_I}^J(D_J\mathcal{W})\ , \tag{3.17}$$

where λ_I are the eigenvalues of the original Kähler metric and O is an orthogonal matrix which diagonalises the Kähler metric. Most importantly, v_I will still have the same structure as $D_I\mathcal{W}$:

$$v_I = \tilde{A}_I + \epsilon\left[\tilde{B}_I e^{\frac{2\pi i\phi_1}{N}} + \tilde{C}_I e^{-\frac{2\pi i\phi_1}{N}}\right]\ . \tag{3.18}$$

We then split v_I into real and imaginary parts to write the potential as

$$V = e^K \sum_{\alpha=1}^{2n+2} w_\alpha^2 \ , \tag{3.19}$$

where

$$w_\alpha = \begin{cases} \mathrm{Re}(v_\alpha) & \alpha = 1, \ldots, n+1 \\ \mathrm{Im}(v_{\alpha-(n+1)}) & \alpha = n+2, \ldots, 2n+2 \end{cases} \tag{3.20}$$

In particular, the w_α now have the structure

$$w_\alpha = a_\alpha + \epsilon \left[b_\alpha \cos\left(\frac{2\pi\phi_1}{N}\right) + c_\alpha \sin\left(\frac{2\pi\phi_1}{N}\right)\right] \ , \tag{3.21}$$

where a_α, b_α and c_α are functions of the $(2n+1)$ moduli $\mathrm{Re}(z^i), \mathrm{Im}(z^i), \mathrm{Re}(\psi), \mathrm{Im}(\psi), \mathrm{Im}(\phi)$, which we will denote by $\{\xi^i\}$.

We now determine the inflaton potential including backreaction on the moduli $\{\xi^i\}$. At leading order we found that the minimum of the potential was at $D_I W = 0$ for all I. In terms of the real parameters this corresponds to $a_\alpha = 0$ for all α. Including the $\mathcal{O}(\epsilon)$ corrections giving rise to the inflaton potential, the minimum of the potential will now be shifted. The moduli $\{\xi^i\}$ will be displaced from their values at the original SUSY minimum by a small amount $\delta\xi^i$. We will find that this displacement is as small as $\delta\xi^i \sim \epsilon$. To determine the potential up to order ϵ^2 it will thus be sufficient to expand the parameter a_α in $\delta\xi^i$ about the SUSY minimum:

$$a_\alpha = 0 + M_{\alpha i}\delta\xi^i + \mathcal{O}(\epsilon^2) \ . \tag{3.22}$$

We only need to keep the leading terms of b_α and c_α as any further expansion would only produce subleading terms.

Note that a_α is a vector with $(2n+2)$ entries which are linear combinations of $(2n+1)$ variables. This has the following consequences: we can always perform a rotation on $w_\alpha \to w'_\alpha = R w_\alpha$ such that $a'_\alpha = R a_\alpha$ is a vector whose last component is zero: $a'_{2n+2} = 0$. We also change coordinates $\delta\xi^i \to (\delta\xi^i)' \equiv a'_i$. The potential becomes

$$\begin{aligned} V = e^K \sum_{i=1}^{2n+1} \left\{ (\delta\xi^i)' + \epsilon \left[b'_i \cos\left(\frac{2\pi\phi_1}{N}\right) + c'_i \sin\left(\frac{2\pi\phi_1}{N}\right)\right]\right\}^2 \\ + e^K \epsilon^2 \left(b'_{2n+2} \cos\left(\frac{2\pi\phi_1}{N}\right) + c'_{2n+2} \sin\left(\frac{2\pi\phi_1}{N}\right)\right)^2 . \end{aligned} \tag{3.23}$$

The effect of backreaction can be read off from the above expression. The moduli adjust such that $(\delta\xi^i)'$ cancel the first $(2n+1)$ contributions to the potential completely. The effective inflaton potential is then given by the last term alone. Dropping the indices on b'_{2n+2} and c'_{2n+2} we find:

$$V_{\mathrm{inf}} = e^K \epsilon^2 \left(b' \cos\left(\frac{2\pi\phi_1}{N}\right) + c' \sin\left(\frac{2\pi\phi_1}{N}\right)\right)^2 \ , \tag{3.24}$$

which one can rewrite as

$$V_{\rm inf} = e^K \epsilon^2 \lambda^2 \left\{ \sin\left(\frac{2\pi\phi_1}{N} + \theta \right) \right\}^2 \,. \tag{3.25}$$

The effective potential (3.25) realises natural inflation. In order to determine the associated effective axion decay constant, let us canonically normalise the inflaton. In terms of ϕ and ψ, the kinetic terms of u and v read:

$$\mathcal{L} \supset \mathcal{K}_{u\bar{u}}(\partial\phi_1)^2 + \mathcal{K}_{v\bar{v}}(\partial\phi_1)^2/N^2 + \mathcal{K}_{u\bar{v}}(\partial\phi_1)^2/N + c.c. \tag{3.26}$$

Therefore, for large N, we have $\mathcal{K}_{\phi\bar{\phi}} = \mathcal{K}_{u\bar{u}} + O(1/N)$ and the canonically normalised inflaton field is defined by: $\varphi = \sqrt{2\mathcal{K}_{u\bar{u}}}\phi_1 + O(1/N)$. By means of the latter relation, we determine the effective potential for φ:

$$V_{\rm inf}(\varphi) \sim e^K \epsilon^2 \lambda^2 \left[1 - \cos\left(\frac{\varphi}{f} + 2\theta \right) \right] , \tag{3.27}$$

where we have used a trigonometric identity to bring (3.25) to the standard form of natural inflation. The axion decay constant in (3.27) is given by

$$f \sim \frac{N}{4\pi {\rm Im}(u)} . \tag{3.28}$$

The most recent CMB observations [15] require $f > 6.8$ (at 95% CL), which according to (3.28) implies the following requirement on the stabilisation of the saxion $\mathrm{Im}(u)$:

$$\frac{N}{{\rm Im}(u)} > 85 . \tag{3.29}$$

3.3.1 Comments on Kähler moduli stabilisation

As mentioned in subsection 3.2.2, Kähler moduli are stabilised à la LVS in our setup. It remains to be checked that the inflationary potential (3.27) is always subleading compared to the LVS potential (1.62). If this were not the case, then Kähler moduli would probably be destabilised, thereby affecting the inflationary trajectory in a potentially catastrophic way.[3] A detailed analysis of this aspect of our model can be found in [131] and in [164]. Here we just point out that the stabilisation requirements can be met in our setup for reasonable vales of parameters and allowing a mild tuning of the coefficient $f(z)$ (to one part in ten) in (3.4).

3.4 Relation to the WGC

In this section, we wish to show that the model developed in this chapter realises a known loophole of the mild form of the WGC.[4]

[3] See instead [100, 130] and in particular [180] for a discussion of Kähler moduli backreaction on the inflationary trajectory, leading to a flattening of the potential.

[4] For a discussion of the relation of our model to other effective field theory expectations on QG (e.g. arguments based on gravitational instantons), see [131] and [164].

Our focus in this section is the extension of the WGC to axions which descend from dimensional reduction of gauge potentials and associated instantons, discussed already in [63] and in a stringy setup in [69], and reviewed in subsection 1.2.3. Unfortunately, our setup is rather different. Our axion is the real part of a 10D metric fluctuation in the LCS regime, while the instanton arises as a correction to the LCS geometry, and is therefore a purely geometric effect. There is no gauge potential associated to φ, therefore it is not clear to us how to extend the results of [69] to this specific setup (see [131] for a more detailed discussion on this point).

Nevertheless, we will assume that the WGC for axions and instantons applies to our case as well and discuss how our model evades its constraints. Let us therefore begin by recalling the loophole of the WGC pointed out in [68, 69]. Consider an axion ϕ with the following instanton-induced potential:

$$V = \Lambda_1^4 e^{-m}\left[1-\cos\left(\frac{\phi}{f}\right)\right] + \Lambda_2^4 e^{-M}\left[1-\cos\left(\frac{k\phi}{f}\right)\right] , \qquad (3.30)$$

where $k \in \mathbb{Z}$. Here m and M are the actions of two instantons. The potential in (3.30) is a sum of two periodic terms: the first one with periodicity f and instanton action m and the second one with a smaller periodicity f/k and instanton action M. Let us then assume $M \gg m$, which implies that the first term dominates with respect to the second contribution. If we take $f > 1$ the first term realises a realistic model of natural inflation but violates the WGC. The second term does not violate the WGC as long as $f/k < 1$. The first term describes long oscillations, upon which there are short oscillations induced by the second term. These "fast" wiggles are potentially dangerous for slow-roll inflation, in that they can spoil the flatness of the trajectory. However, since $M \gg m$, they are completely suppressed and do not affect the inflationary potential. As argued in [68, 69], the model in (3.30) is compatible with the mild form of the WGC for axions and instantons. The latter requires the existence of an instanton with action S and an axion with decay constant f, such that $f \lesssim 1/S < 1$. Crucially, this instanton does not have to be the lightest one, as the strong form would require. This is precisely the case of the potential (3.30).

Very interestingly, this is also the case for the model considered in this chapter. Indeed our setup features two axions $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$ with two associated instanton induced terms $e^{2\pi i u}$ and $e^{2\pi i v}$. We then assumed that the saxions can be stabilised such that $e^{-2\pi \mathrm{Im}(u)} \ll e^{-2\pi \mathrm{Im}(v)}$. Our inflationary axion is given by $\phi = \mathrm{Re}(u)$ at fixed $\mathrm{Re}(\psi) = \mathrm{Re}(u) - N\mathrm{Re}(v)$. The canonically normalised field is given by $\varphi = \sqrt{2K_{\phi\bar{\phi}}}\phi_1$ and couples to the instantons as follows:

$$A_1 e^{-2\pi v_2} e^{2\pi i \frac{\varphi}{f_{eff}}} \quad \text{and} \quad A_2 e^{-2\pi u_2} e^{2\pi i \frac{N\varphi}{f_{eff}}} , \qquad (3.31)$$

where $f_{eff} = N\sqrt{2K_{\phi\bar{\phi}}}$. When $f_{eff} > 1$ (which can be achieved by taking $N \gg 1$), this realises the loophole described below (3.30). In particular, the first term in (3.31) is responsible for large field inflation but violates the WGC, while the second term satisfies the WGC and does not affect the inflationary potential.

This feature distinguishes our model from previous proposals, such as [107]. The latter model realises axion alignment with two periodic terms and two axions. Therefore, as explained in [68], it violates the WGC. Moreover, the authors employ a superpotential of the form:

$$\mathcal{W}_{inf} = A_1 e^{-\frac{2\pi\psi}{f_1}} e^{-\frac{2\pi\phi}{f_2}} + A_2 e^{-\frac{2\pi\psi}{f_3}}, \tag{3.32}$$

with f_2 superplanckian and f_1, f_3 subplanckian. In their model, the imaginary part of ϕ is the inflaton candidate. It appears only in one term in the superpotential, with a large decay constant. Therefore, there seems to be no other term in ϕ_1 which satisfies the WGC and which is more suppressed than the inflationary one. Note that in principle one can enforce the WGC by adding other non-perturbative terms in ϕ, and making sure that a hierarchy in the decay constants can still be realised.

3.5 Summary and conclusions

Inflationary model building in string theory can help understand which effective field theory argument, if any, can be used to constrain large field inflation.

In this chapter, we presented a new model which realises natural inflation along a winding trajectory in the field space of two axions. The basic ingredients of the model are the following:

- **Axions**: the real parts of two complex structure moduli u and v of a CY threefold in the LCS regime. Other complex structure moduli do not need to be in the LCS regime.

- **"Instantons"**: exponentially suppressed corrections to the LCS geometry. We assumed that the imaginary parts (saxions) of u and v are stabilised such that $e^{-2\pi \mathrm{Im}(u)} \ll e^{-2\pi \mathrm{Im}(v)}$. Accordingly, only the instanton-like terms in v were included in the Kähler potential and in the superpotential.

- **Stabilisation**: occurs via minimisation of a flux-induced F-term scalar potential. Neglecting exponentially-suppressed effects, the saxions $\mathrm{Im}(u)$ and $\mathrm{Im}(v)$, as well as the combination $\mathrm{Re}(u) - N\mathrm{Re}(v)$ are fixed at the minimum.

- **Winding trajectory**: at leading order the scalar potential features a flat direction which is closely aligned to $\mathrm{Re}(u)$. This represents a winding trajectory in the field space of $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$.

- **Inflationary potential**: exponentially suppressed terms in $\mathrm{Im}(v)$ were employed to lift the flat direction. In particular, the induced potential realises natural inflation. By an appropriate choice of fluxes, the inflaton decay constant can be made transplanckian.

Crucially, our model evades the constraints coming from the Weak Gravity Conjecture (WGC) by realising a well-known loophole of the conjecture. More precisely, the model satisfies the mild form of the WGC, but violates the strong form. The crucial assumption that allowed us to evade the WGC is that a certain hierarchy between instanton-like corrections to the LCS geometry can ensured by means of moduli stabilisation. While we do not see any fundamental obstruction, it remains to be checked that this scheme can be implemented in an explicit setup.

Very interestingly, the model presented in this chapter may be considered as evidence that only the mild form of the conjecture is respected by string theory.

Intermezzo

Constraints on Large Field Inflation from Effective Field Theory

Chapter 4

Axion Monodromy and the Weak Gravity Conjecture

Transplanckian field displacements might be constrained by means of effective field theory expectations about the behaviour of Quantum Gravity. Among these “folk theorems”, the Weak Gravity Conjecture (WGC) [63] has recently played a central role. In this chapter, we apply the generalised version of the conjecture to models of axion monodromy inflation. Furthermore, we provide a stringy strategy to extend the electric WGC for particles and gauge fields to any $(p + 1)$-form gauge potential with associated p-dimensional charged objects.

This chapter is based on the publication [132].

4.1 Introduction

So far we have analysed the problem of Large Field Inflation from a stringy model building perspective. In doing so, we adopted the following attitude: by studying realisations of inflationary models in a UV-complete setup, we aim at understanding the effects of certain UV degrees of freedom on a transplanckian trajectory. In particular, in chapter 2 we investigated limitations due to backreaction of the compactification geometry. In chapter 3 we instead noticed that corrections to the geometry of the compactification manifold induce instanton-like terms in the inflationary potential, thereby realising natural inflation. In this latter case the UV degrees of freedom may play a positive role, by providing opportunities to evade the consequences of the Weak Gravity Conjecture (WGC).

The latter has been used in the literature to constrain models of axion inflation from an effective field theory perspective. While it is not clear yet which version, if any, of the WGC should hold, several constraints have been derived for models of N-flation [32] and decay constant alignment [31] (see subsection 1.2.4 for a brief review and [36,65–79,84,131] for a partial list of such constraints).[1] The implications of the WGC for models of axion monodromy inflation are less clear (see however [69]).

This chapter consists roughly speaking of two parts. In the first part we aim at deriving constraints on axion monodromy inflation from the WGC. Our attitude will be quite different from the one adopted in the previous chapters. Here we will use the language of effective field theory, rather than analyse explicit string constructions. Our analysis is also relevant for the recently proposed *relaxion* solution to the hierarchy problem [181] (see also [182–189]), which also requires transplanckian field displacements.

In this regard, developing an idea of [69], the authors of [108] have applied the WGC for domain walls to relaxion monodromy. Their analysis rests on interpreting the monodromy as being due to the gauging of the discrete shift symmetry of an axion by a 3-form potential à la Kaloper-Sorbo (KS) [109, 110] (see also [111]). The WGC for the original 3-form gauge theory says that this system comes with light domain walls which, in turn, threaten the slow-roll field evolution in the resulting monodromy model. In what follows, we will advocate a different point of view on constraints coming from $4D$ membranes in models of axion monodromy (inflation and relaxation).

In the second part of this chapter we aim at making progress in a more conceptual direction. Namely, we will describe some insight concerning extensions of the WGC to generic p-dimensional object in d-dimensions (see [69, 73, 75, 108] for previous work in this direction).

This chapter is structured as follows. Section 4.2 is devoted to phenomenological considerations. In particular, in subsection 4.2.1 we describe the presence of domain walls in $4D$ effective field theory models of axion monodromy and deduce the constraints coming from the electric WGC. In subsection 4.2.2 we assume the magnetic WGC for domain walls and extract the consequences for Axion Monodromy Inflation. In subsection 4.2.3 we motivate the extension of the magnetic WGC to domain walls and in subsection 4.2.4 we comment on the relation to KS membranes. Section 4.3 is devoted to a geometric

[1] Other effective field theory arguments may or may not constrain large field inflation: in particular see [36, 37] for gravitational instantons and [88, 89] for arguments based on entropy.

interpretation of the WGC. Finally, we offer our conclusions in section 4.4.

4.2 Axion monodromy and Domain Walls

In this section we aim at obtaining constraints on models based on axion monodromy (inflation or relaxation). We begin by pointing out the existence of light domain walls in those models. Interestingly, these are different from the membranes inherent to the KS approach to axion monodromy. They belong purely to the effective field theory regime and do not descend from a higher dimensional gauge theory. We apply the WGC to these low energy domain walls and then discuss the relation of our result to the recent analysis of [108].

4.2.1 Light domain walls

We start by adopting a naive $4D$ effective field theory point of view of axion monodromy models. The Lagrangian of such a model is given by:

$$\mathcal{L} = (\partial\phi)^2 - V(\phi/f), \tag{4.1}$$

and the inflationary (or relaxion) potential generically consists of a polynomial part and an oscillatory term, e.g.:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \alpha\cos\left(\frac{\phi}{f}\right) \tag{4.2}$$

In writing a cosine term in (4.2), we are assuming that the axion ϕ couples to instantons.[2] The results that we will derive in the following subsections rely on the presence of this oscillatory term. Let us remark that, in models of cosmological relaxation, such a contribution is crucial. Furthermore, in models of axion monodromy and relaxation one typically has (and for relaxation actually needs) $\alpha \equiv \alpha(\phi)$, see e.g. [93, 190, 191].

The cosine term generates 'wiggles' on top of the quadratic potential. For suitable values of m, α and f, namely for $\alpha/(m^2 f^2) > 1$, the potential is characterised by the presence of local minima, see figure (4.1). In this chapter we focus precisely on this case. Slow-roll inflation starts at large values of ϕ, where the quadratic potential is dominant and there are no local minima. Eventually, the field reaches the region where the wells become relevant and minima appear. We wish to constrain the model with potential (4.2) by focusing on this latter region. 'Wiggles' are related to the existence of domain walls: once the inflaton (relaxion) gets stuck in one of the cosine wells, there is a nonvanishing probability to tunnel to the next well, which is characterised by a smaller value of the potential. This happens by the nucleation of a cosmic bubble created by a Coleman-De Luccia instanton, containing the state of lower energy and its rapid expansion.

In order to understand this point, let us adopt a coarse-grained approach: namely, let us consider a model with $V(\phi)$ as in figure (4.1) at spatial distances which are larger than the inverse "mass" $V''(\phi)^{-1}$. At these distances, ϕ is non-dynamical. The "wiggles"

[2] Even if axions without coupling to gauge theory or stringy instantons exist, the presence of gravitational instantons (see [36] and references therein) appears unavoidable.

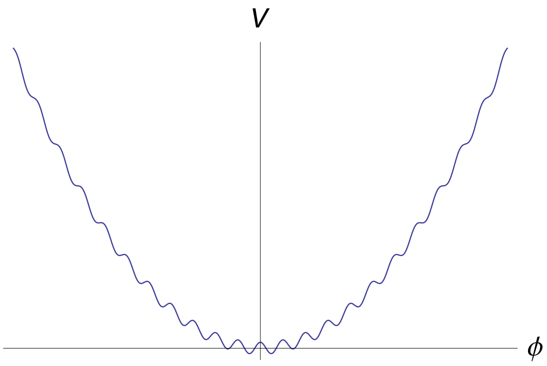


Figure 4.1: Monodromy potential, as in (4.2). Here $\alpha/(m^2 f^2) \simeq 50$.

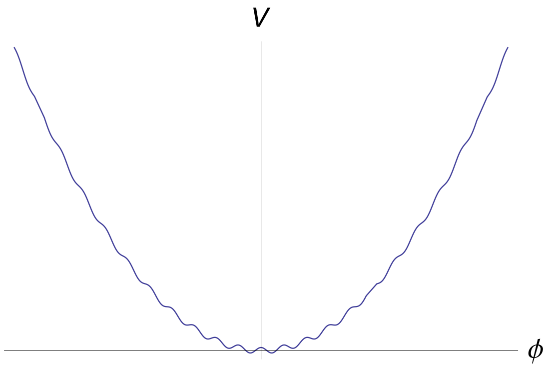


Figure 4.2: Monodromy potential, as in (4.2). Here $\alpha/(m^2 f^2) \simeq 25$.

are therefore invisible, and what remains is a set of points, corresponding to the local minima of the original potential. These points are naturally labelled by an integer index n. Therefore the energy of the corresponding configurations is just:

$$E \simeq (1/2)m^2\phi_{min}^2 \simeq \frac{1}{2}m^2(2\pi f)^2 n^2, \quad n \in \mathbb{Z}. \tag{4.3}$$

Such a discrete set of vacua can be described in terms of a four-form field strength $F_4 = dA_3$.[3] Indeed, due to gauge symmetry, the theory of a free 3-form potential in $4D$ has no dynamics (as e.g. in [52, 192–194]). The points corresponding to the local minima of the original potential for ϕ are separated by domain walls. Therefore, the 3-form lagrangian which provides an effective description of the axion system is:

$$\mathcal{L} = \frac{1}{2e^2}\int F_4^2 + \int_{DW} A_3, \tag{4.4}$$

where we have included a phenomenological coupling of A_3 to domain walls. It is easy to see that F_4 changes by e^2 across a membrane, such that $F_4^2/(2e^2) \equiv (1/2)e^2n^2$. By comparison with (4.3), we find: $e = 2\pi m f$.

The tension of the domain wall, i.e. the surface tension of the bubble containing the state of lower energy, can be estimated as the product of the characteristic thickness $b \sim \Delta\phi/\sqrt{V}$ and height of the domain wall $V \sim \alpha$ (see e.g. [64]). One obtains: $T_{DW} \sim \sqrt{V}\Delta\phi \sim \alpha^{1/2} f$. As we make the domain walls lighter, i.e. as we lower the value of α, the wiggles become less pronounced, see figure (4.2).

In order to ensure that the inflaton (or relaxion) can slowly-roll for a sufficiently large distance, one needs to make sure that the height of the wiggles, i.e. the tension of the domain walls, is small enough.[4]

The crucial point is that lowering the tension of these domain walls goes precisely in the same direction as required by the WGC. Let us recall that, in its original form [63], the

[3] Jumping ahead, we note that our use of a four-form flux is therefore different from the approach of [109, 111], where F_4 is introduced as the field strength of a 3-form which gauges the shift symmetry of the $4D$ dual of ϕ. This will be discussed in subsection 4.2.4.

[4] Nevertheless, the last stages of inflation (or relaxation) may arise as continuous nucleation of cosmic bubbles.

conjecture concerns $4D$ $U(1)$ gauge theories with coupling e and gravity. The electric side of the conjecture requires that a particle of mass m_e exists such that: $eM_P/m_e \gtrsim 1$. The statement can in principle be extended to any $(p+1)$-form gauge theory in d dimensions, with p-dimensional electrically charged objects. The generalisation to domain walls, i.e. $p = 2$ in $4D$, is actually not straightforward and may present subtleties (see [75, 108]). For the moment, we assume that the conjecture is valid for domain walls; we motivate our assumption in detail in section (4.3). Therefore, we have the following constraint on the tension and coupling of the domain wall:

$$\text{WGC:} \quad T \lesssim eM_P. \tag{4.5}$$

Applied to inflationary (relaxion) models, this condition leads to $T \lesssim mfM_P$. The conjecture requires a small tension, which is what is needed to have slow-roll inflation (or relaxation).

Therefore, we are unable to constrain Inflation/Relaxation models by this logic.

4.2.2 Constraints from the magnetic WGC

In the previous subsection we have seen that the electric side of the WGC, as applied to light domain walls, does not constrain models based on axion monodromy. However, there exist two versions of the conjecture.[5] The aim of this subsection is to show that the magnetic side imposes a non-trivial constraint on the field range in models of Axion Monodromy (inflation or relaxation).

We start by providing a statement of the magnetic WGC in the form of a constraint on the cutoff Λ of a gauge theory. To this aim, let us proceed by dimensional analysis. We consider a $(p+1)$-form gauge theory with coupling $e_{p,d}$ in d dimensions with electrically charged Dp-branes and magnetically charged $D(d-(p+4))$ branes. The magnetic WGC simply states that the minimally charged magnetic brane should not be a black brane. The tension of a black brane is $T^{BH}_{d-(p+4)} \sim M_d^{d-2}R^{p+1}$, where R is the Schwarzschild radius of the black brane and M_d is the Planck scale in d dimensions. The tension of a magnetically charged brane can be estimated by integrating the field strength outside the core, as in the familiar case of the magnetic monopole. In d dimensions and for a $p+1$-form, the coupling has dimensions $[E]^{(p+2)-d/2}$. Therefore:

$$T_{d-(p+4)} \sim \frac{\Lambda^{p+1}}{e^2_{p,d}}. \tag{4.6}$$

The magnetic WGC then requires:

$$\text{magnetic WGC:} \quad T \lesssim T^{BH} \Rightarrow \Lambda^{2(p+1)} \lesssim e^2_{p,d}M_d^{d-2}. \tag{4.7}$$

Although this derivation does not go through in $4D$ for $p = 2$ (since we cannot make sense of $D(-2)$ branes), we conjecture "by analytical continuation" in (p, d) that the constraint applies. We therefore obtain:

$$\Lambda \lesssim e^{1/3}M_P^{1/3}. \tag{4.8}$$

[5] There exists yet another version of the conjecture, demanding that the states satisfying the WGC are within the validity range of the effective field theory [73]. In this chapter, we do not consider it, since, in string models, this appears not to hold if one identifies the KK scale with the cutoff. Also, there are further variants of the electric version ("strong","mild","lattice"), which we do not discuss.

This is the constraint we were after. We will provide more support for it later on.

We now apply (4.8) to axion monodromy models. As we have seen in the previous subsection, the coupling e is related to the axion parameters by: $e = 2\pi m f$. Therefore, we get the condition $\Lambda \lesssim (2\pi m f M_P)^{1/3}$. The relevant constraint is now obtained by requiring that the Hubble scale is below the EFT cutoff, i.e. $H = (V/3M_P^2)^{1/2} = 1/\sqrt{6}\cdot(m/M_P)\phi \lesssim \Lambda$. This gives an upper bound on the field range:

$$\frac{\phi}{M_P} \lesssim \left(\frac{M_P}{m}\right)^{2/3}\left(\frac{2\pi f}{M_P}\right)^{1/3}. \tag{4.9}$$

As it stands, the constraint (4.9), although non-trivial, represent only a mild bound on the field range. With $m/M_P \sim 10^{-5}$, and $2\pi f/M_P \simeq 1$ one gets $\phi/M_P \lesssim 10^3$, which safely allows large field inflation. We expect that our dimensional analysis estimate is modified only by $O(1)$ factors (see section 4.3). However for models with small f the constraint (4.9) may become relevant.

It is a generic expectation that, in models of large field inflation, the field range cannot be parametrically large. The discussion of this section confirms this expectation: In the case of axion monodromy, the magnetic side of the WGC limits the field excursions. However, phenomenologically relevant field ranges are allowed.

Let us now very briefly discuss the corresponding constraint for models of cosmological relaxation [181] based on monodromy [108]. In this case, we take our axion ϕ to be the relaxion, and couple it to the Higgs field of the standard model. Therefore, the relaxion potential is:

$$V_\phi = \frac{1}{2}m^2\phi^2 + \alpha_v \cos\left(\frac{\phi}{f}\right) + (-M^2 + g\phi)|h|^2, \tag{4.10}$$

where M is the cutoff scale and $\alpha_v \equiv \alpha(h = v)$. As discussed in [181], the following constraints apply to this class of models:

$$\Delta\phi \gtrsim M^2/g \quad \text{to scan the entire range of values of the higgs mass.} \tag{4.11}$$

$$H \lesssim \alpha_v^{1/4} \quad \text{to form the low energy barriers.} \tag{4.12}$$

$$H > M^2/M_P \quad \text{for the energy density to be inflaton dominated.} \tag{4.13}$$

Furthermore, the slow roll of ϕ ends when the slopes of the perturbative and non-perturbative potential terms are equal, i.e.: $m^2\phi \sim \alpha_v/f$. This should happen at a generic point in the range of ϕ. Hence, from (4.11), $\phi \sim M^2/g$ and thus:

$$\frac{m^2M^2}{g} \sim \frac{\alpha_v}{f}. \tag{4.14}$$

We can now find the consequences of the magnetic WGC for this class of models. We apply (4.8) and require, as in the inflationary case, $H \lesssim \Lambda \lesssim (2\pi m f)^{1/3}M_P^{1/3}$. We express f in terms of α_v by means of (4.14). By also imposing (4.13), we are able to constrain the cutoff M as follows:

$$M \lesssim \left(\frac{2\pi g}{m}\right)^{1/8}\alpha_v^{1/8}M_P^{1/2}. \tag{4.15}$$

A similar constraint was given in [108] (we review this approach in section 4.2.4), where a more detailed discussion can also be found. Even if α_v is as low as $f_\pi^2 m_\pi^2$ this constraint is not fatal.

Before moving on to the next subsection, we would like to remark on a well-known problem of all the axion models, which also affects our setup. In these models there are always instantons associated to the slowly-rolling axion. If they all contribute to the axion potential, there is no flat direction on which to inflate (relax). It is a non-trivial task to suppress the higher order instantons (our 'wiggles'), and strategies to do so and evade the WGC have been an important focus of recent work (see [131] for a proposal which realises a loophole of the WGC [68, 69]).

Let us now motivate, as promised, our extension of the magnetic WGC to generic p-dimensional objects.

4.2.3 String Theory and the WGC

It has been suggested in [108] that there is no magnetic side of the conjecture for domain walls, a statement which conflicts with our previous discussion. Here we would therefore like to motivate our use of the magnetic WGC. From now on, we work in units where $M_P \equiv 1$.

From the point of view of string theory, there are two possible ways of satisfying the electric WGC. On the one hand, string compactifications may provide light objects whose tension and coupling satisfy the inequality $T \lesssim e$. However, Dp-branes in $10D$ are extremal, i.e. they marginally fulfil the WGC. Under compactifications, the resulting objects are not guaranteed to be extremal, unless SUSY is preserved. Therefore, it is not clear whether objects arising from string compactifications could violate the WGC.

On the other hand, there exists another mechanism by which the conjecture can be satisfied in string compactifications: It is the presence of a maximal scale up to which a 4D effective field theory description is valid. In many cases such a cutoff is set by the KK scale $M_{KK} \sim 1/R$, where R is the typical length scale of the compactification manifold. Above M_{KK}, one has to work with the full 10D theory. In particular, if the tension of the objects descending from string theory is larger than M_{KK}, then these objects simply do not exist in the low energy effective field theory. Therefore, by lowering the KK scale, one can ensure that the WGC is not violated, by simply removing the dangerous objects from the spectrum of the low energy theory. A low cutoff is precisely what is required by the magnetic side of the WGC for a weakly coupled theory.

Explicitly, consider a q-dimensional object descending upon compactification from a p-dimensional brane in 10D. The ratio between its tension and the appropriate power of the KK scale is given by: $\tau_q / M_{KK}^{q+1} \sim M_s^{p+1} R^{p+1} / g_s$. We are assuming that we are in a controlled regime, i.e. either $g_s < 1$ or $R > 1$ or both. Therefore as R increases the corresponding object simply disappears from the 4D theory.

The bottomline of this discussion is that, in many cases, string theory satisfies the WGC by imposing a low cutoff to the 4D effective field theory, not by providing objects which are light enough. In other words, setups of string compactifications satisfy the magnetic side of the WGC and, as a consequence, the electric side as well.

This is the reason why we think that the magnetic constraint is the more fundamental conjecture among the two version of the WGC. Therefore, we assume that the magnetic WGC is valid for any p-form, and in particular for domain walls.

Recently, the electric WGC has been applied to another class of membranes in the context of realisations of axion monodromy models à la Kaloper-Sorbo (KS) (see [109] for the KS proposal, [108] for the recent developments) . In the next subsection we describe the relation of this work to our findings.

4.2.4 Relation to domain walls à la Kaloper-Sorbo

We begin by reviewing the strategy of [69, 108] to constrain nucleation rates in models based on axion monodromy. In this subsection, we follow the notation of [108], which differs from the one used in the previous subsections.

The KS proposal [109] to implement monodromy models in a $4D$ setup is to introduce a 3-form gauge potential A_3 and to couple the corresponding 4-form field strength to the axion:

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}|F_4|^2 + g\phi F_4, \tag{4.16}$$

where $|F_p|^2 \equiv \frac{1}{p!} F_{\mu_1 \dots \mu_p} F^{\mu_1 \dots \mu_p}$. Notice that this setup is different from the dual picture that we have described in section (4.2.1). We used just one scalar field theory with a discretuum of vacua, which corresponds to a gauge theory with the same discretuum of vacua. By contrast, the lagrangian in (4.16) consists of a scalar field theory (first term) and a gauge theory (second term), each with its own set of vacua. The third term couples these two theories. The potential A_3 couples to fundamental $4D$ domain walls via $S \sim q \int_{2-branes} A_3$. The field strength F_4 varies across the membranes and is quantised in units of the membrane charge, i.e. $F_4 = nq\ (\star 1)$. A shift in the value of F_4 is a part of the residual gauge symmetry of the KS lagrangian. Under this symmetry, also the scalar field shifts:

$$\phi \to \phi + 2\pi f, \quad nq \to (n-k)q, \quad n, k \in \mathbb{Z}, \tag{4.17}$$

with the consistency condition $2\pi f g = kq$, and f being the axion periodicity. Due to this residual gauge symmetry, we are left with only one set of vacua, labeled by one integer.

The quadratic potential for ϕ arises from integrating out the field strength F_4:

$$V_{KS} = \frac{1}{2}(nq + g\phi)^2. \tag{4.18}$$

The crucial point is that each value of n corresponds to a different branch of the potential. The gauge symmetry (4.17) provides a way to identify these branches. In this sense, crossing a membrane corresponds to an alternative way to move one step down in the potential. This is different from rolling over or tunneling through a "wiggle" of figure (4.1). The KS membranes can potentially spoil the slow-roll behavior allowed by small "instanton-induced wiggles". As usual, the probability for such tunneling events is described in terms of a nucleation rate for the corresponding bubbles.

Since this probability is exponentially suppressed, one might wonder whether this effect represents a concern for Axion Inflation. The nucleation rate Γ is given by e^{-B}, where $B \sim T/H^3$ in the relaxion regime (see [195]).

In [69], the authors show that a strong suppression of the nucleation rate requires a violation of the WGC. More recently, in [108], the authors follow the same direction to constrain models of relaxion monodromy. In this case, the WGC requires $T \lesssim 2\pi f g$. By requiring $B > 1$, the authors obtain a constraint which is similar to (4.15). In particular, the parametric dependence on α_v is the same. Furthermore, the authors of [108] obtain a stronger constraint by requiring that $B > N$, where N is the number of e-folds. This requirement arises from demanding that there are no domain walls in the part of the universe created during the above N e-folds. Such a constraint cannot be obtained by using our low energy wiggles, because the latter arise only much later, when inflation is in its last stages.

Applied to inflationary models, $T \lesssim eM_P$ and $T \gg H^3$ lead to the same constraint that we have found in (4.9). However, we have obtained it by using a different, arguably simpler, effective field theory point of view, based on the magnetic, rather than the electric WGC. Notice also that the objects that we have described in section (4.2.1) can be naturally lighter than the KS membranes.

We have seen that the Kaloper-Sorbo procedure consists in gauging an axionic theory by a 3-form potential. The original theory of a free 3-form potential has domain walls to which the WGC can be applied. However, gauging corresponds to a discontinuous, qualitative change of the model.[6]

It is hence not clear whether the relevant parameters, i.e. the tension and the coupling, and therefore the consequences of the WGC, remain unchanged. In particular, it may be hard to check the changes of the coupling and tension of the domain walls, since the dualisation procedure described in [111] does not always lead to an explicit determination of the F_4 lagrangian. Therefore, it is desirable to work with constraints which do not appeal to the situation *before* gauging. Crucially, *after* the gauging both the fundamental KS domain walls and the 'wiggle-induced' effective domain walls are present.

We are then left with two possibilities: The first is that the electric WGC has to be separately satisfied by both the KS and by the effective domain walls described in this chapter. In this case the constraint given in [108] and based on the electric WGC for the (heavier) KS domain walls applies. The same constraint arises as a consequence of the magnetic WGC. Everything is consistent and the present chapter provides an alternative derivation of the same constraint.

The other possibility is that the electric WGC needs to be satisfied only by the lightest domain walls. These are the effective domain walls, but due to their lightness no interesting constraint arises. The heavier KS domain walls provide no further constraints. Thus, the magnetic WGC provides the only useful constraint, as described in this chapter.

Our conclusion is that in both cases the field range is constrained according to (4.9). In the first case the latter comes from the electric side applied to KS membranes, as explained in [69, 108], and from the magnetic side applied to "wiggles" membranes. In the second point of view, which we adopt in this chapter, no UV information on the origin of the gauge theory is required and (4.9) follows only from the magnetic WGC.

[6]In particular, the conceivable limiting procedure of taking the gauge coupling to zero and hence going from the gauged to the ungauged case is forbidden by the WGC itself. Also, in string constructions gauging often corresponds to (necessarily discrete) changes in the flux configuration or even in the geometry of the compact space.

4.3 The WGC as a geometric constraint

In this section we want to address the extension of the WGC to domain walls. We will do so in the framework of 10D string theory compactified on a CY manifold.

4.3.1 Previous approaches and our perspective

In [75], the authors provide the following statement of the WGC for any p-form in d dimensions:

$$\left[\frac{\alpha^2}{2}+\frac{p(d-p-2)}{d-2}\right]T_p^2 \leq e_{p,d}^2 q^2 M_d^{d-2}. \tag{4.19}$$

In the absence of a dilaton background, the inequality is degenerate for $p=0$ (axions) and $p=d-2$ (strings). Moreover, for $p=d-1$, i.e. for domain walls, the inequality cannot be satisfied, as already pointed out in [75]. Therefore, one may worry that there is no statement of the electric WGC for domain walls.

An idea to extend the WGC to generic p-dimensional objects, as noticed in [108], is to use string dualities. This follows very closely the strategy of [69], where the conjecture is extended to axions and instantons. In that case, the authors consider type IIB on a CY 3-fold with $D1$ branes and their associated C_2 gauge potential. Wrapping the branes on 2-cycles and compactifying to 4D, one obtains a theory of C_2 axions and $D1$ instantons. This type IIB theory is then T-dualised to type IIA with $D2$ branes and their associated C_3 potential. Since this theory is strongly coupled, one actually uses the M-theory picture, introducing a further compact dimension. Again, by wrapping the branes around 2-cycles and compactifying, one obtains a $5D$ theory with a $U(1)$ gauge field and $M2$ particles. This is the original content of the WGC, which can therefore be applied to this particular 5D setting. Finally, one can translate the constraints obtained on the particles/vector fields side to the axion/instanton side, by using the T-duality relations between IIA and IIB couplings and mass scales.

In [108], the authors propose to implement the very same idea to constrain domain walls. Starting with a 10D theory with $p=d-1$ objects, they propose to T-dualise twice along directions transverse to the branes, so that the dual theory is of the same type but with $p=d-3$ branes. One can then apply (4.19) to the latter setup, then translate the constraints to the domain walls side.

We agree with the authors of [108] that the apparent problems of the WGC for domain walls disappear when considering them in a string theory setup. Notice that the dualisation procedure works for any p-dimensional object in 10 dimensions reduced to a q dimensional object in d dimensions. Indeed the moduli of the theory, i.e. the compactification radius and the string coupling, disappear from the charge-to-tension ratio on both sides of the duality. Were this not the case, we would not be able to extract a sensible constraint on the objects in the $4D$ theory.

This property suggests that the WGC in 10D string theory can be phrased as a constraint on some geometrical data of the particular compactification manifold, independently of the specific p-dimensional object. Once the geometry of the compactification manifold is constrained, one can extract the consequences for any other q-dimensional object in the theory. This is the novel point of this section. Our focus in this section is the electric statement.

Our approach implies that there is no need of T-dualising in order to extend the WGC to objects other than $4D$ particles. In the next subsection, we will verify this statement focusing on the case of domain walls. Let us therefore outline the strategy to extend the conjecture to any p-dimensional object, without using dualities. One starts with a type IIB setting with Dp branes wrapped on p-cycles of the internal manifold X. Upon compactification, this leads to a 4D theory of particles and gauge fields. One then applies the standard WGC to this setting: the result is a constraint on the metric on the space of p-cycles in X. For example, in [69] the authors obtain a constraint on $K_{ab} \sim \int w_a \wedge \star w_b$, where w_a is a basis of $H^2(X, \mathbb{Z})$. Once this constraint is obtained, it is valid for any brane setup on the same CY. One can then consider Dq branes, with $p \neq q$ wrapped on the same p cycles and obtain inequalities for the tension and couplings of the $4D$ theory derived by compactification on X.

4.3.2 Computation

Following our discussion, we now perform an explicit computation to prove our claim. We first focus on obtaining particles in $d = 4$. As a starting configuration we choose type IIB with D3 branes compactified on a CY 3-fold X. Other choices are equally valid. We work in the conventions of [135]. The relevant 10d action reads:

$$S_{10} \supset \frac{1}{2\kappa_{10}^2} \int_{M^{10}} \left[\frac{1}{g_s^2} R_{10} \star \mathbf{1} - \frac{1}{4} F_5 \wedge \star F_5 \right] + \mu_3 \int_{D3} C_4 \tag{4.20}$$

where $\kappa_{10}^2 = (1/2)(2\pi)^7 \alpha'^4$ and $\mu_3 = 2\pi(4\pi^2\alpha')^{-2}$. Now let us perform dimensional reduction, by wrapping the $D3$ on 3-cycles of X. We focus on the gauge kinetic term. We consider a symplectic basis $w_i = (\alpha_a, \beta^b)$ of $H^3(X, \mathbb{Z})$, i.e. s.t.:

$$\int_X \alpha_a \wedge \beta^b = \delta_a^b, \tag{4.21}$$

and the other intersection numbers vanish. By Poincaré duality, one can define the integral charges:

$$q_i^k = \int_{\Sigma_k} w^i = \int_X w^i \wedge w^k, \tag{4.22}$$

where Σ_k is a 3-cycle in X and w^k is its dual form. By (4.21), these charges are either vanishing or unit.

The 4D action is obtained by expanding the five-form flux and the four-form potential in terms of the symplectic basis of $H^3(X, \mathbb{Z})$:

$$F_5 = \sum_{i=1}^{N} F_2^i(x) \wedge w_i(y), \quad C_4 = \sum_{i=1}^{N} A_1^i(x) \wedge w_i(y), \tag{4.23}$$

then integrating over X. Here N is the number of 3 cycles of X. $D3$-branes wrapping a 3-cycle Σ generate particles in the $4D$ theory. For the moment being, let us focus on one such cycle. We will later extend our results to particles descending from different cycles. Let us introduce the metric on the space of 3-forms:

$$K_{ij} \equiv \int_X w_i \wedge \star w_j. \tag{4.24}$$

Before moving to the $4D$ theory, an important remark is in order. This concerns the self-duality of F_5, i.e. $\star F_5 = F_5$. This constraint cannot be implemented at the level of the $10d$ action (4.20). Therefore one actually starts with a more general theory where $F_5 \neq \star F_5$. Nevertheless, the kinetic term in (4.20) is normalised with a prefactor 1/4 instead of 1/2, as will be appropriate after self-duality is imposed. To obtain consistent $10d$ equations of motion, the coupling in (4.20) should actually read:

$$S_{10} \supset \frac{\mu_3}{2}\int_{D3} C_4 + \frac{\mu_3}{2}\int_{\tilde{D3}} \tilde{C}_4, \tag{4.25}$$

where at the moment different branes source the dual potentials (see also [196], footnote n. 6). Self-duality can then be consistently imposed at the level of the $10d$ equations of motion, derived from this action. This goes together with identifying $D3$ and $\tilde{D3}$.

We now consider the $4D$ theory descending from dimensional reduction. In $4D$ there are certain constraints on the field strengths F_2^i and $\tilde{F}_2^i = \star F_2^i$ descending from self-duality of F_5 in $10d$. For the sake of our analysis, we first focus on the set of unconstrained F_2^i, exactly as we we did with F_5 in $10d$. The $4D$ action then reads:

$$S_4 \supset \frac{M_P^2}{2\cdot 4}\int_{M^4} \frac{g_s^2}{V_X} K_{ij} F_2^i \wedge \star F_2^j + q_i^\Sigma \frac{\mu_3}{2}\int_{0-brane} A_1^i. \tag{4.26}$$

Here $M_P^2 = V_X/\kappa_{10}^2 g_s^2$ is the $4D$ Planck mass. The equations of motion arising from (4.26) read

$$d\star F^i K_{ij} = \frac{2V_X}{M_P^2 g_s^2}\mu_3 q_i^\Sigma dj_{\text{0-brane}}. \tag{4.27}$$

From the latter, it is clear that only a certain linear combination of gauge fields is sourced by the particle with charge q_i^Σ. To make this visible in the 4D action, we define the field A_1 and its field strength $F_2 = dA_1$ by

$$A_1^i \equiv A_1 K^{ij} q_j^\Sigma. \tag{4.28}$$

In terms of A_1 and F_2 the 4D action reads

$$S_4 \supset \frac{M_P^2}{2\cdot 4}|\mathbf{q}^\Sigma|^2 \frac{g_s^2}{V_X}\int_{M^4} F_2\wedge\star F_2 + |\mathbf{q}^\Sigma|^2\frac{\mu_3}{2}\int_{0-brane} A_1, \tag{4.29}$$

where $|q^\Sigma|^2 \equiv K^{ij} q_i^\Sigma q_j^\Sigma$. Now we consider the realistic setup where the dual field strength $\tilde{F}_2$ and its associated $\tilde{A}_1$ are also included. Therefore we add to (4.26) the action:

$$\tilde{S}_4 \supset \frac{M_P^2}{2\cdot 4}\int_{M^4} \frac{g_s^2}{V_X} K_{ij} \tilde{F}_2^i \wedge \star \tilde{F}_2^j + \tilde{q}_i^\Sigma \frac{\mu_3}{2}\int_{\widetilde{0-brane}} \tilde{A}_1^i \tag{4.30}$$

where $\tilde{A}_1$ and $\tilde{q}_i^\Sigma$ are analogous to A_1^i and q_Σ^i. The coupling term can be obtained from dimensionally reducing (4.25). Finally we relate F_2^i and $\tilde{F}_2^i$ by dimensionally reducing

$10d$ self-duality of F_5. In particular, $10d$ self-duality implies $F_2^J = \star_4 F_2^K H_K^J$, where the matrix H_K^J is defined by $\star_6 w_K = H_K^J w_J$. In imposing this constraint, we also identify the branes sourcing A_1^i and $\tilde{A}_1^i$. Thus, adding (4.29) and (4.30) corresponds to a doubling of the action (4.29). Therefore, the final theory which we will constrain via the WGC has action

$$S_4 \supset \frac{M_P^2}{2\cdot 2}|\mathbf{q}^\Sigma|^2 \frac{g_s^2}{V_X}\int_{M^4} F_2 \wedge \star F_2 + |\mathbf{q}^\Sigma|^2 \mu_3 \int_{0-brane} A_1. \tag{4.31}$$

In order to extract the $4D$ gauge coupling, we normalise the gauge potential. Finally, we obtain:

$$S_4 \supset \frac{1}{2e^2}\int_{M^4} F_2 \wedge \star F_2 + \int_{0-brane} A_1, \tag{4.32}$$

where we have kept the same notation for the normalised fields and the $4D$ gauge coupling is defined as:

$$e^2 = \frac{2V_X \mu_3^2 |q^\Sigma|^2}{M_P^2 g_s^2}. \tag{4.33}$$

The result of this procedure is therefore a $4D$ theory of a $U(1)$ gauge field with coupling (4.33). The particle descending from the $D3$ brane wrapped on Σ has mass $M^\Sigma = (T_3/g_s)\int_\Sigma \star\mathbf{1} = (T_3/g_s)V^\Sigma$, and $T_3 = \mu_3$.

We are now ready to apply the WGC to the $4D$ theory defined by (4.32) with particles of mass M^Σ:

$$\frac{eM_P}{M^\Sigma} \geq \frac{\sqrt{2}}{2} \Rightarrow \frac{V_X^{1/2}|q^\Sigma|}{V^\Sigma} \geq \frac{1}{2}. \tag{4.34}$$

Before moving to the case of domain walls, let us pause to extract the full meaning of (4.34). The WGC for particles arising from a string compactification translates into a purely geometric constraint on the size and intersections of the cycles of the manifold, in this case 3-cycles. Crucially, all couplings and $4D$ scales have disappeared from the final statement. Despite the presence of volume factors, the charge-to-mass ratio is independent on any rescaling of the $6d$ metric $\tilde{g}_{mn}$. This statement is actually true for any p-cycle: indeed the metric K_{ij} on the dual space of p-forms contains $(3-p)$ powers of the $6d$ metric, so the numerator scales as $\tilde{g}_{mn}^{p/2}$, but so does the denominator.

The conclusion is as follows: the procedure that we have followed works for any p-dimensional object and associated field strength defined on a chosen manifold X and dimensionally reduced to a q dimensional object in $4D$. In particular, (4.34) is a constraint on the 3-cycles of X. As such, it can be applied applied to any other 4D object descending from any p-brane on X wrapped on the same 3-cycles.

We are particularly interested in constraining $4D$ domain walls. In order to apply our previous result, we study the case in which the membranes arise from compactifications of type IIB string theory with $D5$ branes wrapped on 3-cycles. The action is obtained by simply replacing the $D3$ branes with $D5$ branes in (4.20):

$$S_{10} \supset \frac{1}{2\kappa_{10}^2}\int_{M^{10}} \left[-\frac{1}{2}F_7 \wedge \star F_7\right] + \mu_5 \int_{D5} C_6 + S_{DBI}, \tag{4.35}$$

with $\mu_5 = \mu_3/(2\pi\alpha')$. Dimensional reduction to $4D$ goes as in the previous case, therefore we do not repeat the computation. The $4D$ action reads:

$$S_4 \supset \frac{1}{2e_{DW}^2}\int_{M^4} F_4 \wedge \star F_4 + \int_{D2} A_3 \tag{4.36}$$

with:

$$e^2_{DW} = \frac{2V_X\mu_5^2|q^\Sigma|^2}{M_P^2 g_s^2}. \tag{4.37}$$

The tension of the $4D$ domain wall is: $T_{DW} = T_5/g_s V^\Sigma$. The charge-to-tension ratio is:

$$\frac{e_{DW}M_P}{T_{DW}} = \frac{(2V_X)^{1/2}|q^\Sigma|}{V^\Sigma}. \tag{4.38}$$

As expected, (4.38) is the same as (4.34). Therefore the WGC constraint on particles translates into the following inequality for the charge-to-tension ratio of domain walls:

$$\text{WGC:} \quad \frac{e_{DW}M_P}{T_{DW}} \geq \frac{\sqrt{2}}{2}. \tag{4.39}$$

This is the result we were after, namely a WGC for domain walls.

One can give a general inequality for a $(q+1)$-dimensional object in d dimensions descending from a s-brane wrapped on a $(s-q)$ cycle of a CY X, by relating its charge-to-mass ratio to that of particle descending from a p-brane wrapped on the same $(s-q)$ cycle. For consistency $s-q=p$. The WGC then states that the charge-to-tension ratio of the $D(q)$-brane must satisfy the condition:

$$\frac{e_p M_P}{T_p} \geq \sqrt{\frac{d-3}{d-2}}. \tag{4.40}$$

Finally, let us generalise our results to the case of N cycles $\Sigma_k, k = 1, \ldots, N$. Correspondingly, we have a set of charge vectors $\mathbf{q}^{\Sigma_k}$. These vectors belong to $\mathbb{R}^N$ equipped with metric K_{ij} defined as in (4.24). With the same notation as above, consider Dp-branes wrapped on p-cycles of a CY manifold. These lead to particles in d dimensions with mass M_k. The Convex Hull Condition (CHC) for the p-cycles reads:

The convex hull spanned by the vectors $\mathbf{z}^k \equiv \frac{V_X^{1/2}\mathbf{q}^{\Sigma_k}}{V^{\Sigma_k}}$, *must contain the ball of radius* $r = \sqrt{\frac{d-3}{d-2}}$.

Now consider a q-brane in d dimensions obtained by wrapping a $D(s)$-brane on p-cycles of the same CY. The tension and the charge vectors of the $(q+1)$-dimensional objects are respectively T_q^k and $e_q\mathbf{q}^{\Sigma_k}$, where e_q is the prefactor in $(1/2)(1/e_q^2)\int K_{ij}F^i_{q+2}\wedge \star F^j_{q+2} + q_i^\Sigma \int_{\Sigma_k} A^i_{q+1}$ in the effective theory. Assuming the CHC for particles, we obtain the following statement for the q-branes:

The convex hull spanned by the vectors $\mathbf{Z}^k \equiv \frac{e_q \mathbf{q}^{\Sigma_k} M_d}{T_q^k}$ *must contain the ball of radius* $r_q = \sqrt{\frac{d-3}{d-2}}$.

It is important to remark that (4.39) has been obtained without using any string duality: the WGC for particles imposes a constraint on the geometry of CY three cycles. This constraint, applied to objects derived from any p-brane in the 10d setup, translates to a corresponding WGC for these particular objects. This line of reasoning can be applied also to the case of axions and instantons. In that case one starts from a Dp brane wrapped on p cycles, then considers $D(p-1)$ branes wrapped on the same cycles. Obviously this requires a change in the theory, e.g. from type IIB to type IIA/M-theory on the same CY. However, the constraints obtained in the IIB setting are still just geometric constraints on p-cycles of the CY, therefore there is no need of performing a duality between the two theories. It is sufficient to consider a type IIA/M-theory setup with the appropriate branes, and impose on this setup the previously determined geometric constraint. It would be interesting to think about manifolds with backreaction and fluxes. In this case, the transition from IIA and IIB (or other setups) would not be so straightforward.

4.4 Summary and Conclusions

In this chapter we have investigated two different aspects of the Weak Gravity Conjecture. Firstly, we have discussed its consequences for models based on Axion Monodromy (Inflation and Relaxation). Secondly, we have provided a geometric interpretation of the conjecture in the framework of string compactification. We now provide a detailed summary of our results.

In the first part of this chapter, we have adopted an effective field theory point of view. Namely, given a certain scalar potential, we have tried to constrain its use in models of monodromy inflation. In particular, inflaton (relaxion) potentials in models of Axion Monodromy are characterised by the presence of ‘wiggles’ on top of a polynomial potential. The resulting local minima imply the existence of $4D$ domain walls. This is more evident by using an effective description in terms of a four-form flux, whose value changes across these membranes.

We assumed that the WGC can be extended to domain walls. In our setup, its electric version gives an upper bound on the tension of the $4D$ membranes. Crucially, this condition agrees with what is required to realise slow-roll: as the tension decreases, the height of the ‘wiggles’ decreases and slow roll can be seen as a continuous nucleation of cosmic bubbles. Therefore, we conclude that, in this logic, the electric WGC does not constrain models of axion monodromy (Inflation and Relaxation).

For this reason, we focused on the constraints imposed by the magnetic side of the WGC, which we stated as an upper bound on the cutoff of a generic $(p+1)$-form gauge theory (in the spirit of [63]). We then applied the condition to inflationary models, i.e. we required $H \ll \Lambda$. This gives a non-trivial constraint on the field range: $\phi \lesssim m^{-2/3} f^{1/3} M_P^{4/3}$. The latter however allows for large field displacements, but forbids models with a small decay constant.

We then discussed our extension of the magnetic WGC. We argued that string theory lowers the KK scale to fulfil the WGC for objects which descend from compactifications of string theory with Dp-branes, rather than making them light enough. As a consequence, heavy “stringy” objects, which could potentially violate the WGC are confined above the cutoff M_{KK}. Therefore they do not exist from an effective field theory point of view. Of course, low energy light objects are allowed, as is the case for our domain walls. Consequently, the electric side is automatically satisfied. We suggest that the magnetic WGC should be seen as the fundamental constraint among the different versions of the

WGC.

Recently, the electric WGC has been applied to membranes arising from the realisation of Axion Monodromy á la Kaloper-Sorbo (KS), in the context of new realisations of relaxion models [108]. When the tension of these membranes decreases, the probability of tunneling to another branch of the potential increases. Such a transition can spoil slow-roll, as it corresponds to discrete "jumps" in the axion trajectory. The requirement that the tunneling rate is suppressed parametrically leads to the same constraint on the field range that we obtained by studying the domain walls arising from 'wiggles' in the axion potential.

However, KS membranes are different from the low energy domain walls described in this chapter. This may have implications for the various constraints.

There are, in fact, two possibilities. On the one hand, one could impose the WGC separately on the two classes of membranes. In this case, the constraints given in [108] for relaxion models apply and can be extended to inflationary models. The magnetic WGC applied to the low energy domain walls gives the same constraint.

On the other hand, it is possible that only the lightest domain walls have to satisfy the WGC. In this case, the electric WGC applied to the low energy domain walls does not give any constraint. By contrast, the magnetic side gives a bound on the field range, hence playing a central role. As discussed in this chapter, there are reasons related to the KS gauging which make this second possibility relevant.

In the second part of this chapter, we worked in the framework of string compactifications. We started with 10D type IIB with $D3$ branes and compactified to $4D$ by wrapping the branes around 3-cycles of a CY manifold. Therefore, we obtained particles and gauge fields in $4D$. We applied the original WGC to this setup. Very interestingly, the final constraint does not depend on the couplings and moduli of the 10D setup. The electric WGC translates into a purely geometric constraint on the size and intersection of the 3-cycles of the CY. Explicitly:

$$\frac{V_X^{1/2}|q^\Sigma|}{V^\Sigma} \geq A_d,$$

where V_X is the volume of the compactification manifold, V^Σ is the volume of the 3-cycle Σ, $|q^\Sigma|$ is the norm of the harmonic form related to Σ using the metric X, and A_d is a $O(1)$ number given in subsection 4.3.2. The charge-to-tension ratio of any p-dimensional object wrapped on the same 3-cycles is the same as in the left hand side of the inequality above. Therefore, by constraining the geometry of 3-cycles through the $D3$/particles case, we obtain a WGC for any $p-3$-dimensional object in $4D$ arising from compactification of type IIB with Dp-branes wrapped on the same cycles. In particular, by taking $p=5$ we obtain the WGC for $4D$ domain walls. Crucially, we do so without the use of string dualities.

The same procedure applies to any p-dimensional object wrapped on some q-cycle of a CY, to obtain a $p-q$-dimensional object in $4D$. In particular, in the general case the WGC still translates into the inequality above, with Σ being a q-cycle. Therefore, our approach provides a simple strategy to extend the electric WGC to any q-dimensional object, without the use of string dualities.

Let us close our discussion with observing two further consequences implied by the constraint on the tension of the low-energy domain walls from the WGC. [7] We note firstly, that we get a fundamental upper bound on the size of resonant oscillating non-Gaussianity induced by the 'wiggles' in the scalar potential. Following the analyses of [191,197], the magnitude $f_{NL}^{res.}$ of this type of non-Gaussianity with an oscillating shape in k-space is approximately given by $f_{NL}^{res.} \sim bM_P^3/(f\phi)^{3/2}$. Here, $b = \alpha/(m^2 f\phi)$ denotes the 'monotonicity' parameter of the scalar potential with 'wiggles' ($b < 1$ corresponds to $V' > 0$ for $\phi > 0$). We can rewrite this as $b = \alpha f^2/(m^2 f^3 \phi) \sim T_{DW}^2/(m^2 f^3 \phi) < m^2 f^2 M_P^2/(m^2 f^3 \phi) = M_P^2/(f\phi)$ where the inequality arises from the WGC $T_{DW} < eM_P = mfM_P$. Hence, we get a bound $f_{NL}^{res.} \lesssim M_P^5/(f\phi)^{5/2}$, to be evaluated at $\phi = \phi_{60} \sim 10M_P$ for the observable CMB scales. The bound thus finally reads $f_{NL}^{res.} \lesssim 3 \times 10^{-3}\,(M_P/f)^{5/2}$. The typical range for the axion decay constant is $10^{-4}M_P \lesssim f \lesssim 0.1M_P$ (see e.g. [191]). Consequently, for $f \gtrsim 5 \times 10^{-2}M_P$ this fundamental upper bound on $f_{NL}^{res.}$ becomes stronger, $f_{NL}^{res.} \lesssim \mathcal{O}(1)$ for $f \gtrsim 5 \times 10^{-2}M_P$, than the current observational bounds [198].

Secondly, we observe that in a quadratic potential the boundary to slow-roll eternal inflation (defined as the value of $\phi = \phi_\star$ where $\epsilon \sim V$) $\phi_\star \sim M_P^{3/2} m^{-1/2}$ can be higher than our magnetic WGC field range bound $\phi < m^{-2/3} f^{1/3} M_P^{4/3}$ for values of $f \lesssim 10^{-3}M_P$, because COBE normalization of the CMB fluctuations fixes $m \sim 10^{-5}M_P$. Intriguingly, recent analyses such as [2, 93] (see also e.g. [90] for earlier work on the WMAP 9-year data) of the PLANCK data searching for oscillating contributions to the CMB power spectrum and the 3-point-function hint with the highest significance at very-high-frequency oscillating patterns with $f \sim 10^{-4}M_P$. If this were corroborated in the future, then jointly with the magnetic WGC this would rule out slow-roll eternal inflation in quadratic axion monodromy inflation potentials in the past of our part of the universe.

The generalization of both of these observations to more general axion monodromy potentials $V \sim \phi^p$ with 'wiggles' is an interesting problem for the future.

[7] We would like to thank Alexander Westphal for discussions on these points.

Part II

Reheating

Chapter 5

Gravitational Waves from Axion Monodromy

Models of Large Field Inflation might exhibit peculiar phenomenological signatures during and after inflation. These may become very important to select a specific model of LFI, in case of detection of primordial tensor modes. In this chapter we describe how the Universe might be decomposed into different phases *after* axion monodromy inflation. Such a phenomenon would be followed by collisions of cosmic bubbles, associated with the emission of gravitational radiation. Very interestingly, the signal of such an exotic post-inflationary dynamics might be detectable by future space- and ground-based interferometers.

This chapter is based on the publication [133].

5.1 Introduction

The central predictions allowing us to discriminate between inflationary models are the slow roll parameters, most prominently the tilt of the scalar power spectrum $n_s \simeq 0.96$ and the tensor-to-scalar ratio $r \lesssim 0.08$ [2, 199]. However, even with growing precision many models are expected to remain consistent with this limited set of data. Hence it is of great importance to identify additional predictions characterizing specific classes of models.

So far we have extensively focused on one such class: axion monodromy inflation [57, 58]. In chapter 2 we studied certain stringy realisations of this mechanism, while in chapter 4 we focused on its relation with effective field theory arguments about Quantum Gravity. Here we adopt yet another perspective: namely, we focus on its phenomenology. In particular, the aim of this chapter is to describe a new and potentially striking observational signature which is peculiar to axion monodromy inflation. For previous work on the phenomenology of such inflationary models see [90–93, 96, 200] (see also [112, 113] for closely related potentials) and, in particular, [201–203] for preheating and [204] for oscillon dynamics in this context (see in particular [118] for gravitational radiation from preheating in oscillon models).

In short, our message is the following. Due to the typical instantonic modulations of the potential, first order phase-transition-like, violent dynamics may occur after the end of inflation and before reheating. This leads to additional gravitational waves, which are of course very different in frequency from those studied in the CMB. Thus, monodromy models may (in addition to or independently of their prediction of r) be established by future ground- or space-based interferometers. The final word would thus come from the new field of gravitational-wave astronomy [13] (for a recent review see e.g. [205]).

Of course, gravitational waves are a well-known signature of cosmic strings, including axionic strings. For recent work on the non-trival late-time dynamics of axionic models and gravitational waves which is closer in spirit to our proposal see, e.g. [117, 119–123]. Emission of gravitational waves from axionic couplings in inflationary setups has recently been considered in [97, 98] (see also [94, 95] for constraints). Independently of the gravitational wave signal, related dynamical phenomena may also occur in the dark matter context [206, 207].

Let us now explain the physics underlying our scenario in more detail. The basic building block of monodromy inflation is an axion with sub-planckian decay constant f. Non-perturbative effects induce the familiar $\cos(\phi/f)$-type potential. When the monodromy effect is included, this cosine-potential shows up in the form of modulations of the long-range, polynomial term. Even if the relative size of these modulations is small at large field values, where slow-roll inflation is realized, they can become dominant near the minimum of the polynomial potential. After inflation, the field oscillates with decreasing amplitude such that its motion eventually becomes confined to the vicinity of one of these local minima. However, due to field fluctuations, different local minima may be chosen in different regions of the same Hubble patch. In other words, the Universe is decomposed into phases.

Two comments are in order. First, the field fluctuations inducing the above phenomenon can have different origin. On the one hand, there are inflationary super-horizon fluctuations which have become classical at the time when they re-enter the horizon. On the other hand, the field is subject to an intrinsic quantum uncertainty at any given time. As we will see, this second type of uncertainty is, after parametric amplification [207], more likely to source the desired phase decomposition.

Second, while we for definiteness identify the monodromic axion with the inflaton, the phenomenon can occur also in different contexts. In particular, similar physics may arise in any non-inflationary axion model with a monodromy (the relaxion being one recent popular example [181], see also [108]). Moreover, even within the inflationary context, our proposal of 'dynamical phase decomposition' is not restricted to monodromy models. Indeed, models of 'aligned' or 'winding' inflation [31, 106, 159] can naturally exhibit short-range modulations on top of a long-range periodic potential. The Weak Gravity Conjecture for instantons may in fact demand such modulations [68, 69] and the simple string construction presented in chapter 3 may naturally provide them. As discussed in chapter 4, the Weak Gravity Conjecture for domain walls also constrains models based on monodromy. In particular, the size of the wiggles is bounded.

Let us now complete the discussion of the cosmological dynamics: After a phase decomposition has occurred, the regions with the lowest-lying populated minimum will expand. This is very similar to the way in which a strong first-order phase transition is completed through the collision of cosmic bubbles of true vacuum. However, in our case the transition occurs before reheating and very far from thermal equilibrium. It may thus be more appropriate to talk about 'dynamical phase decomposition' rather than about a phase transition in the usual sense. Nevertheless, the concept of bubble formation and collision is still appropriate in our setting.

Thermodynamic cosmological phase transitions have been widely explored in various contexts (see [114–116] for early seminal work and [124] and refs. therein for the case of the electroweak phase transition). In particular, it is well known that they source gravitational radiation (see [208–210] for recent reviews and e.g. [211] for radiation from other cosmological sources such as cosmic strings and preheating). We will rely on these results.

This chapter is structured as follows: section 5.2 provides the basic setup. More specifically, subsection 5.2.1 introduces our axion monodromy setting with dominant quadratic potential and a series of local minima, subsection 5.2.2 briefly discusses reheating, and subsection 5.2.3 explains the post-inflationary dynamics, which may be characterised by what we call a "dynamical phase decomposition". In section 5.3 we estimate the probability that such a decomposition occurs as a consequence of field fluctuations. In particular, we treat inflationary fluctuations in subsection 5.3.1 and the intrinsic quantum uncertainty in subsection 5.3.2. In section 5.4 we address the crucial issue of the possible enhancements of fluctuations due to background-field-oscillations in the modulated potential. Ultimately, in section 5.5 we estimate the spectrum and abundance of the gravitational radiation produced during the phase transition before we conclude in section 5.6. Additionally, we devote appendix B to a more detailed discussion of scalar field fluctuations after inflation. Appendix C provides some details of an inflection point model of inflation in which our gravitational wave signal may also arise.

5.2 Phases from axion monodromy

In this section we introduce a string-motivated scenario of the inflationary universe. It is based on the framework of Axion Monodromy Inflation, which we have already extensively discussed. We begin by explaining the basic features of the axion potential and the possibility of having coexisting populated axionic vacua.

5.2.1 Local minima in axion monodromy

The inflaton potential of a model of axion monodromy inflation contains an oscillatory term, which respects a discrete shift symmetry, and a polynomial term, which breaks it explicitly. In this work we will take the polynomial term to be quadratic, as this will be sufficient to demonstrate the effect we wish to study:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \Lambda^4 \cos\left(\frac{\phi}{f} + \gamma\right). \tag{5.1}$$

The potential (5.1), plotted in figure 5.1, can have local minima, whose existence depends on the values of the prefactor Λ^4, the so-called axion decay constant f and the inflaton mass m. Although we have in mind the specific case in which ϕ is the inflaton, many of our considerations apply to the case of a generic axion-like field with potential (5.1).[1]

We begin with an analysis of the classical evolution of ϕ. The equation of motion of ϕ reads:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{5.2}$$

where the prime denotes a derivative with respect to ϕ. For constant Hubble rate H and temporarily neglecting the cosine term in (5.1), the solution of (5.2) is

$$\phi \sim e^{-\frac{3}{2}Ht}\cos(mt). \tag{5.3}$$

Therefore, the amplitude of ϕ decreases. This conclusion remains valid even for time-varying H. In fact, since $H(t) \sim \rho_\phi^{1/2}/M_p$, the Hubble rate decreases as the amplitude of ϕ falls. Once the amplitude is sufficiently small, the cosine oscillations in (5.1) cannot be neglected any longer. Eventually, the field is caught in one of the cosine wells. One can observe that the field is more likely to get trapped in one of the lowest-lying minima. This can be understood as follows: the friction term in (5.2) becomes less and less relevant as $H(t)$ falls. This implies that the fractional energy loss per oscillation also decreases. Therefore, even though the field can in principle get stuck at any time, it is more likely to do so late in its evolution, when it is oscillating near the bottom of the well containing the lowest-lying minima.[2] For this reason, we will mostly focus on the last two wells.

The existence of different local minima implies that the universe is potentially decomposed into several phases. This happens if the field settles in different minima in different parts of the universe. Due to fluctuations, the scalar ϕ can end up in one or the other

[1] While we expect a similar qualitative behaviour the numerical results may depart significantly from the values we find here.

[2] The actual argument to show that wells at the bottom of the potential are more likely to host the inflaton requires more care. In section 5.4 we provide numerical examples that support this statement.

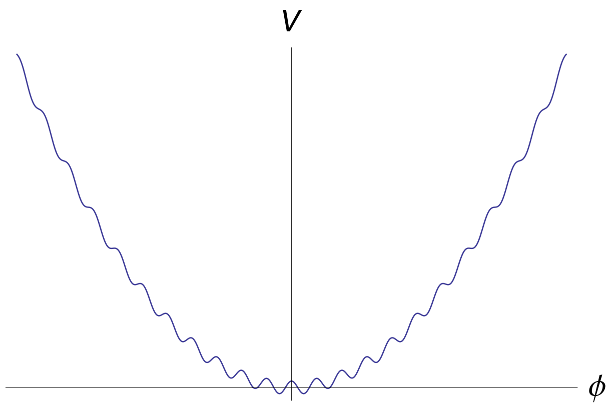


Figure 5.1: Monodromy potential, as in (5.1) with $\gamma = 0$. The parameter κ/π approximately measures the number of local minima. Here $\kappa \simeq 50$.

minimum in different regions of the same Hubble patch. In this chapter, we are interested in studying the conditions under which such a phase decomposition can take place.

If such a phenomenon occurs, eventually the field will settle in the state of lower energy as a consequence of the expansion of bubbles containing the true vacuum. This corresponds to a phase transition, which can in principle have strong cosmological signatures, above all the radiation of gravitational waves. We would like to provide an estimate for the spectrum of gravitational waves produced in such an event (see also [117,119–122] for related work, but in different contexts).

Before moving on to study the details of this scenario, let us determine the condition on the parameters Λ, f, m for the potential to exhibit local minima. In order to have local minima, the equation $V' = 0$ must have non-vanishing solutions. Let us, for a moment, simplify by setting $\gamma = 0$. We then have:

$$V' = 0 \qquad \Rightarrow \qquad m^2\phi = \frac{\Lambda^4}{f} \sin\left(\frac{\phi}{f}\right). \tag{5.4}$$

Graphically, it is clear that this equation has non-vanishing solutions only if

$$\kappa \equiv \frac{\Lambda^4}{f^2 m^2} \geq 1. \tag{5.5}$$

Here we have used $\gamma = 0$, but the equation remains parametrically valid even for $\gamma \neq 0$. Under this condition the potential has the form represented in figure 5.1. Practically however, as we have already remarked, we will focus only on the two lowest local minima, which are in general non-degenerate for $\gamma \neq 0$ (see figure 5.2).[3]

We are now ready to move on to a detailed discussion of phase decomposition in our scenario. However, before doing so, a few comments about reheating are in order.

5.2.2 Reheating

Typically, after inflation the Universe undergoes a so-called *reheating* phase, where the energy density in the inflaton sector is transferred to standard model degrees of freedom,

[3] The choice $\gamma = 0$ may lead to stable domain walls, which are generically problematic.

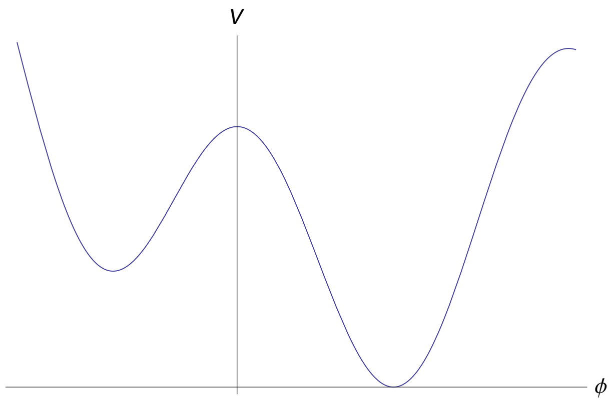


Figure 5.2: Non-degenerate two-well potential, as it can be obtained from (5.1) by focusing on the two wells closest to the origin.

and possibly to dark sectors. Our analysis of the dynamics of the inflaton after inflation needs to take this into account. We therefore first focus on the following question: does reheating happen before or after the inflaton field is caught in one of the cosine wells?

In order to answer this question, we need to specify the interactions of the inflaton with matter and/or radiation. Here let us consider the example of a Planck-suppressed modulus-like coupling to a scalar field χ. The largest decay rate, barring the possibility of parametric resonances, is obtained if χ enjoys a coupling $(1/M_p)\chi^2\Box\phi$, leading to

$$\Gamma_{\phi\to\chi\chi} \sim \frac{m_\phi^3}{M_P^2}. \tag{5.6}$$

This can arise if χ is a Higgs boson (see e.g. [125,126]). Decay rates to gauge bosons may also be of this type. The inflaton may also couple to scalar matter as $(1/M_P)\phi\bar{\chi}\Box\chi + c.c.$, but this leads to even smaller decay rates.

The strategy is now as follows: we assume that the field is trapped in one of the cosine wells, and compare the perturbative decay rate (5.6) with the Hubble rate $H^2 \sim \Lambda^4/M_P^2$, as we are assuming that the field is oscillating in the last wells. For the same reason, we should take $m_\phi^2 = V''|_{min} \simeq \Lambda^4/f^2$ in (5.6). As usual in cosmology, the condition for the decay to be efficient is $\Gamma \gtrsim H$. If we find $\Gamma < H$, then we will be consistent with our assumption that the field is first caught in one of the wells and decays only later. Now,

$$\Gamma_\phi < H \quad \Rightarrow \quad \kappa\left(\frac{m}{f}\right)\left(\frac{m}{M_p}\right) < 1. \tag{5.7}$$

The inequality (5.7) is easily satisfied in our setup, as in quadratic inflation $m \sim 10^{-5} M_P$ and f may be only slightly smaller than M_P, while $\kappa \gtrsim O(1)$. Therefore the field generically decays perturbatively only after getting caught in one of the cosine wells. In what follows, we will therefore not consider the decay of ϕ any longer.

5.2.3 Field oscillations and damping

We first focus on how the Hubble parameter changes after inflation. The energy density of the universe is a sum of three terms: $\rho = V_\phi + T_\phi - V_\lambda$, where V_λ is the energy density

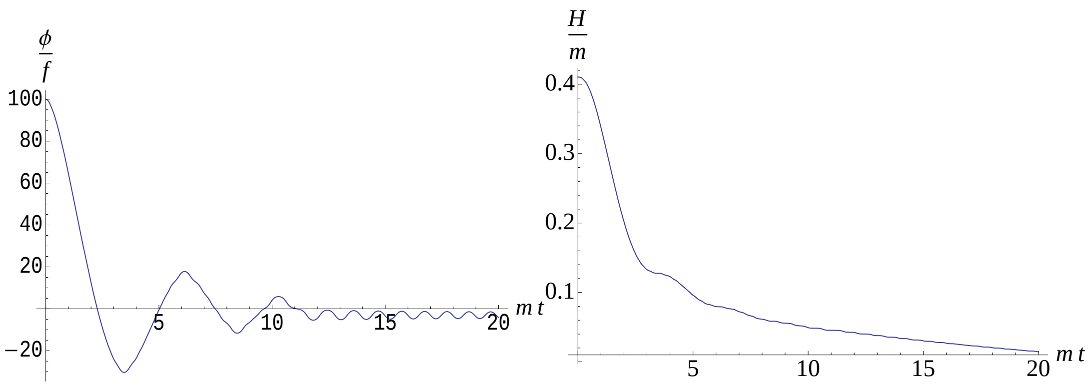


Figure 5.3: Evolution of: (a) the scalar field ϕ and (b) the Hubble rate H according to (5.9) and (5.8). V_ϕ is as in (5.1) and $V_\lambda = 0$. The initial condition is $\phi(t_0) = M_P$. Furthermore we have chosen $\kappa = 60, f = 10^{-2} M_P$. The scalar field oscillates around $\phi = 0$ over a wide field range crossing several wells, before getting caught in one of the local minima at $t \approx 11/m$.

due to the cosmological constant. We absorb V_λ in V_ϕ and define V_ϕ such that the global minimum has vanishing potential energy. The evolution of the Hubble parameter and of the scalar field ϕ is dictated by the Friedmann equation, together with the equation of motion of ϕ:

$$3H^2 = \frac{1}{2} m^2 \dot{\phi}^2 + V_\phi \tag{5.8}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'_\phi. \tag{5.9}$$

Since the energy density is decreasing due to friction, it is clear that also $H(t)$ will decrease with t. However, whenever $\dot{\phi} = 0$, ϕ is undamped and the energy density is constant. In consequence $H(t)$ is stationary as well. The typical behaviour of $\phi(t)$ and $H(t)$ is shown in figure 5.3.

Let us now examine the energy density in the inflaton field in more detail. Its amplitude decreases after each oscillation due to Hubble friction. At some point the energy density ρ_ϕ is comparable to the height of the last cosine wells. The field is then caught in one of the local minima, depending on the initial conditions. In the absence of spatial field inhomogeneities, the inflaton will populate only one of these two minima. This situation is shown in figure 5.4.

The conclusions can radically change in the presence of field fluctuations $\delta\phi(t, \mathbf{x})$. In this case, the field may end up in one or the other minimum in different regions belonging to the same Hubble patch. The lines drawn in figure 5.4 become bands of a certain width, corresponding to the uncertainty $\delta\rho$ in the energy density induced by $\delta\phi$ (see figure 5.5). Now suppose that, as a consequence of friction, the energy density has decreased to a value close to the height of the barrier separating the two last minima in figure 5.4. During the next oscillation, the field will start rolling inside one of the two wells, say the one on the left of figure 5.5. At the end of the oscillation, since its energy density is smaller than the height of the barrier separating the two minima, the field is very likely

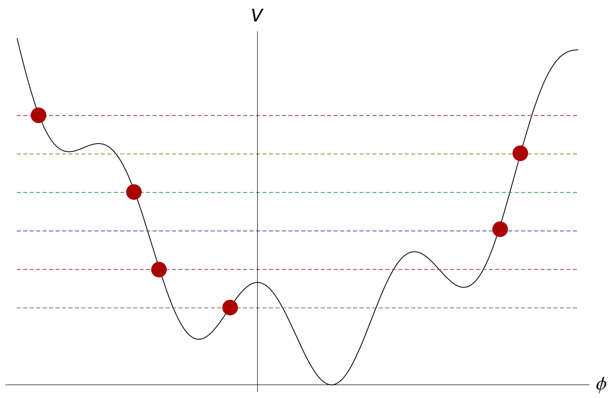


Figure 5.4: Monodromy potential, as in (5.1), with $\gamma \neq 0$. The axes are chosen such that the local maximum between the last two wells sits at $\phi = 0$ and the lowest minimum has $V = 0$. Here $\kappa \approx 12$. The turning points in the field trajectory are shown. The amplitude of the oscillations decreases as a consequence of Hubble friction.

to remain confined in the left well. However, due to the field fluctuations, there is a non-vanishing probability that the field actually reaches the other well on the right hand side and remains there in some regions of the Universe. If this is the case, at different point in the same Hubble patch the field lives in different minima. Therefore, the Universe decomposes into two phases. This is precisely the situation we are interested in.

Quantitatively, let us compute the energy density lost during one oscillation, as a consequence of Hubble friction. In general this is not an easy task given the complicated shape of the potential in figure 5.4. However, it is greatly simplified by focusing only on the last cosine wells. In this case, the loss of energy during a half oscillation inside a single well can be estimated by a quadratic approximation of the potential:

$$V_{approx} = \frac{1}{2} M^2 \phi^2, \tag{5.10}$$

with $M^2 = |V''|_{min}$, i.e. the curvature of the potential (5.1) around the minimum of the well, where $\cos(\phi/f) \approx -1$. We obtain

$$M^2 = m^2 + \frac{\Lambda^4}{f^2} = (1+\kappa) m^2 \tag{5.11}$$

since we are interested in the regime $\kappa > 1$, we can take $M^2 \approx \Lambda^4/f^2$. Immediately after inflation the Hubble rate evolves approximately as during matter domination, i.e.

$$H = \frac{2}{3t}; \qquad \rho \sim a^{-3} \sim t^{-2}, \tag{5.12}$$

so that the relative decrease in energy density in one half period $\Delta t \sim M^{-1}/2$ is given by

$$\frac{\Delta\rho}{\rho} \sim 2\frac{\Delta t}{t} = 3H\Delta t \sim \frac{3}{2}\frac{H}{M}, \tag{5.13}$$

where $\Delta\rho \equiv |\rho_f - \rho_i|$. We now focus on the last two wells, such that $\rho \sim \Lambda^4$. Using Friedmann's equation, $3H^2 M_P^2 = \rho$, and $M^2 \approx \frac{\Lambda^4}{f^2}$ we find

$$\Delta\rho \sim \rho \cdot \frac{\rho^{1/2}}{M_P M} \sim \Lambda^4 \frac{f}{M_P} = \kappa \frac{m^2 f^3}{M_P}. \tag{5.14}$$

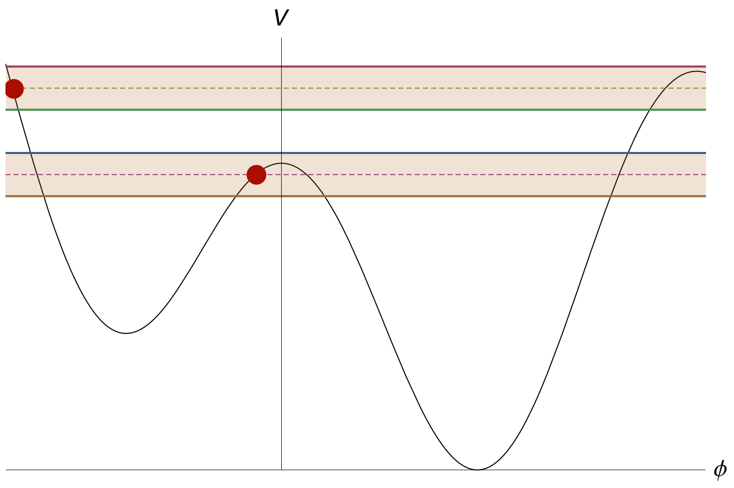


Figure 5.5: Same as figure 5.4, but now only the last two wells are shown. The width of the shaded bands represent the uncertainty of the scalar field energy density $\delta\rho$, due to field fluctuations. The distance between the two bands corresponds to the energy loss $\Delta\rho$ due to friction.

We can now quantitatively discuss the probability of having a phase decomposition. To this end we have to compare the decrease in energy density due to friction $\Delta\rho$ and the uncertainty due to field fluctuations $\delta\rho$. If we have $\delta\rho \sim \Delta\rho$, the field will populate more than one vacuum with $O(1)$ probability, as should be clear from figure 5.5. The term probability here refers to the exact choice of model parameters which, at that level of precision, appears arbitrary to the low-energy effective field theorist.

The task of the next section is therefore to present two possible origins of the fluctuations $\delta\rho$. These are respectively classical inflationary inhomogeneities and quantum uncertainties of the scalar field ϕ.

5.3 Fluctuations and phase decomposition

In this section we will analyse two sources of fluctuations of the inflaton field. In subsection 5.3.1 we focus on (classical) inflationary fluctuations. Those originate as sub-horizon size quantum fluctuations, but are then stretched to super-horizon size and become classical. After inflation, they re-enter the horizon and may lead to the phase decomposition described above. In subsection 5.3.2 we focus instead on the intrinsic quantum uncertainty which characterizes a quantum field at any given time. Independently of any inflationary pre-history of our field, this effect is present directly during the oscillatory stage and may also lead to phase decomposition. In fact, the estimates that we obtain in this section imply a small probability of phase decomposition. However, in section 5.4 we will see that the probability can be much larger because fluctuations may be enhanced after inflation.

5.3.1 Inflationary fluctuations

The evolution of inflationary fluctuations on sub- and super-horizon scales is well-known (see e.g. [212, 213]). For us the only crucial point is that, once a certain inflationary mode re-enters the horizon it behaves like a dark matter fluctuation, i.e. it is a decaying oscillation (see [214, 215] for a detailed study of scalar field fluctuations after inflation). From now on we focus on the amplitude of such an oscillation, which we denote by $\delta\phi_k^{inf}$. The background is denoted by ϕ_0 and satisfies the equation of motion (5.2).

The initial conditions on $\delta\phi_k^{inf}$ are determined by matching with the power spectrum of the gauge-invariant curvature perturbation $\mathcal{R}$. This quantity is conserved on superhorizon scales and is given by

$$\Delta^2_{\mathcal{R}}(k) = \frac{1}{8\pi^2}\left[\frac{1}{\epsilon}\frac{H^2}{M_P^2}\right]_{exit} , \qquad (5.15)$$

where the right hand side is evaluated at horizon exit, i.e. for $k \sim H$. In the slow-roll regime the quantity ϵ in (5.15) coincides with the familiar expression $\epsilon = \frac{1}{2}M_P^2(V'/V)^2$.

To determine the probability of phase decomposition we need to understand how field fluctuations $\delta\phi_k^{inf}$, once they re-enter the horizon, give rise to density fluctuations $\delta\rho_k^{inf}$. More specifically, we wish to determine $\delta\rho_k^{inf}$ at time t_Λ. This is the time at which the amplitude of the background ϕ_0 has decreased to values comparable to the width of the last wells, and it is given by

$$t_\Lambda \sim H_\Lambda^{-1} \sim \frac{1}{\kappa^{1/2}(f/M_P)m} . \qquad (5.16)$$

Let us now consider a mode which exits the horizon just before the end of inflation, i.e. when $H \sim m$. Such a mode re-enters the horizon shortly afterwards, at time $t_0 \sim m^{-1} < t_\Lambda$ and has therefore momentum $k(t_0) \sim m$. At t_0 we can determine the size of $\delta\phi_k^{inf}$ by matching with the curvature perturbation. However, the field fluctuation $\delta\phi_k^{inf}$ thus obtained will be out of phase with the oscillation of the background ϕ_0. It is maximal when $\dot{\phi}_0$ is maximal and vanishes when ϕ_0 is at a turning point. If this remained the case for the subsequent evolution until t_Λ, the fluctuation could not give rise to a phase decomposition. This can only occur if we have a sizable fluctuation at a turning point of ϕ_0.

However, we expect decoherence between $\delta\phi_k^{inf}$ and ϕ_0 after only a few oscillations. The reason is that ϕ_0 oscillates with frequency m while a mode with $k \sim m$ will oscillate with frequency $\sqrt{k^2+m^2} \sim \sqrt{2}m$. Thus, at some time after t_0 the field fluctuation $\delta\phi_k^{inf}$ will be an admixture of out-of-phase but also in-phase-oscillations w.r.t. to ϕ_0. It is exactly the in-phase-oscillations which do not vanish at turning points and it is these fluctuations which give rise to density perturbations $\delta\rho_k^{inf}$. In the following we will thus assume that once a mode enters the horizon, while initially out of phase with ϕ_0, it will give an $\mathcal{O}(1)$ contribution to an in-phase oscillation with corresponding $\delta\rho_k^{inf}$ after only a few periods.[4]

[4] As we cannot quantify this effect exactly, we will now drop exact numerical prefactors in all following expressions.

To estimate the size of fluctuation at time t_Λ given a fluctuation at t_0 we need to take the expansion of the universe into account. In particular, the energy density scales as

$$\rho \sim a^{-3} , \qquad \frac{\delta\rho}{\rho} \sim a \quad \Rightarrow \quad \delta\rho \sim a^{-2} . \tag{5.17}$$

Thus, for a density fluctuation with $k \sim m$ there is a dilution in the time span between t_0 and t_Λ.

Let us now determine the probability of phase decomposition due to a mode with $k \sim m$ at the time t_0 of horizon re-entry. As argued before, the field fluctuation will quickly give rise to a density fluctuation. Instead of taking the intermediate step via field fluctuations, let us match the density fluctuations directly to the curvature fluctuations at re-entry:

$$\Delta^2_{\mathcal{R}} \sim \frac{\Delta^2_{\delta\rho}}{\rho^2} \quad \Rightarrow \quad \delta\rho^{inf}_k(t_0) \sim \rho(t_0)\sqrt{\Delta^2_{\mathcal{R}}} \sim \frac{\rho(t_0)}{M_p}\left[\frac{H}{\epsilon^{1/2}}\right]_{exit} . \tag{5.18}$$

Now, using

$$\rho(t_0) \sim m^2 M_p^2 , \qquad \left[\frac{H}{\epsilon^{1/2}}\right]_{exit} \sim m , \qquad \frac{a^{-2}(t_\Lambda)}{a^{-2}(t_0)} \sim \kappa^{2/3}(f/M_p)^{4/3}, \tag{5.19}$$

where the second equation follows from $H \sim m, \epsilon \sim 1$ at the end of inflation, we obtain

$$\frac{\delta\rho^{inf}}{\Delta\rho} \sim \kappa^{-1/3}\left(\frac{m}{M_P}\right)\left(\frac{M_P}{f}\right)^{5/3} . \tag{5.20}$$

The above probability was derived for modes with $k \sim m$ at horizon re-entry. However, we are interested in the situations when the above probability is largest. Thus let us consider how this result is modified if we consider modes that re-enter the horizon later. Such modes have $k < m$ at re-entry and they spend less time inside the horizon before the moment t_Λ. Hence they would in principle give rise to a larger probability than (5.20). However, they cannot enter too late as they need to have enough time to decohere w.r.t. ϕ_0. A more detailed analysis would be needed to determine how late a mode can enter the horizon and nevertheless give rise to a sizable density fluctuation. Thus, (5.20) should be seen as a reasonable estimate. We shall comment more on the size of this probability at the end of subsection 5.3.2.

5.3.2 Quantum fluctuations

There is another potentially relevant source of fluctuations of the field ϕ. This is the intrinsic uncertainty due to the quantum nature of our scalar. It can be simply written as

$$\delta\phi^q_k \sim k. \tag{5.21}$$

In order to estimate the maximal effect of these fluctuations, let us consider the following setting: consider a scalar field ϕ with fluctuations $\delta\phi^q_k$ approaching the local maximum of some potential. This is basically as in figure 5.2, with the field approaching the maximum from the left side. We then ask the following question: what is the field distance from the local maximum which the background field has to reach such that its fluctuations can lift it over the potential barrier? The relevant energy scale around the maximum is $O(M)$, where M^2 is the curvature of the potential at the maximum. One can convince oneself that only fluctuations of the order $\delta\phi \sim M$ are relevant for overcoming the barrier: modes with $k \gg M$ have an effective non-tachyonic mass and are insensitive to the instability. In contrast, modes with $k \ll M$ are sensitive to the tachyonic instability, but their fluctuations are smaller than those of modes with $k \sim M$.

Alternatively, we can understand this point by considering tunnelling. Hence we take a homogeneous scalar field approaching the maximum and study the conditions under which quantum tunnelling to the other side of the barrier becomes efficient. In order to answer this question, let us use the following standard tunnelling formulae: the tunnelling rate is given by e^{-S_0}, where S_0 is the action of critical bubble formation. When the thin wall approximation is applicable, this reads (see e.g. [64])

$$S_0 = \frac{27\pi^2\sigma^4}{2(\Delta V)^3} = \frac{27\pi^2\delta\phi^2}{2M^2}, \tag{5.22}$$

where $\sigma \sim M\delta\phi^2$ is the bubble wall tension and ΔV can be estimated with a quadratic approximation, $\Delta V \sim M^2\delta\phi^2$. However, in most of the cases the thin-wall calculation is not appropriate. Nevertheless, we still expect $S_0 \sim \delta\phi^2/M^2$, with a different prefactor.[5] According to (5.22) the tunnelling rate is unsuppressed when $\delta\phi \gtrsim 10^{-1}M$, which is the same condition we found with the previous approach up to a numerical prefactor. Given the uncertainty in the prefactor, for the time being we use the parametric dependence $\delta\phi \sim M$.

Following these two arguments the uncertainty in the energy density induced by such field fluctuations is

$$\delta\rho^q \sim M^2\delta\phi_k^{q2} \sim M^4. \tag{5.23}$$

If quantum fluctuations induce an energy gain which is larger than the loss due friction one expects phase decomposition. Therefore, we compare (5.23) with (5.14), using also $M^2 \simeq \Lambda^4/f^2$. We obtain

$$\frac{\delta\rho^q}{\Delta\rho} \sim \kappa\left(\frac{m}{M_P}\right)^2\left(\frac{M_P}{f}\right)^3. \tag{5.24}$$

Comparing (5.20) and (5.24) we conclude that the probability of a phase decomposition due to inflationary fluctuations is larger than the one due to quantum ones by at least a factor $(f/M_P\kappa)^{4/3}(M_P/m)$.

Let us now discuss the size of the probabilities (5.20), (5.24). In quadratic inflation, $m \sim 10^{-5}M_P$. For $\kappa \sim O(10)$ and $f \sim 10^{-13/5}M_p$ the probability (5.20) is of the order of 0.1. This implies that a phase decomposition is rather likely for these values of parameters. For the same choices, the probability (5.24) is only slightly smaller, i.e. $\mathcal{P}^q \sim 10^{-6/5}$. However, further numerical suppression is expected in (5.24). Therefore we conclude that in the regime $\kappa \sim O(10)$, $f \lesssim 0.3 \cdot 10^{-2}M_P$ phase decomposition is likely

[5] We have checked the behaviour for an inverted parabola. In the vicinity of the maximum this is a reasonable approximation, with a prefactor of the order of 10^4.

to happen as a consequence of inflationary fluctuations. If f grows above 10^{-2} phase decomposition quickly becomes improbable. Whether such small values of f are natural depends on the details of the model leading to (5.1). We will comment more on the size of f at the end of section 5.4.

Further, in chapter 4 we presented arguments based on the Weak Gravity Conjecture (WGC) which constrain the size of modulations of the potential in axion monodromy inflation (for an earlier somewhat different perspective see [108]). It is thus important to check whether the region of parameter space considered in this work is consistent with these bounds. Given a domain wall with tension T and charge e the electric WGC demands $T \lesssim eM_p$. In our case we have $T = \Lambda^2 f$ and $e = 2\pi m f$ (see subsection 4.2.1 for details). Using our definition $\Lambda^2 = \sqrt{\kappa} m f$ the WGC bound reads

$$\sqrt{\kappa} \lesssim \frac{M_p}{f} \ . \tag{5.25}$$

As a result, while κ is bounded from above, our preferred parameter region for phase decomposition ($\kappa \sim \mathcal{O}(10)$, $f \lesssim 10^{-2} M_p$) is consistent with the WGC. Interestingly, let us notice that the region of parameter space which is ruled out by the WGC is also constrained by current observations, which require $\Lambda^4/(m^2\phi^2) \lesssim (10^{-3} - 10^{-2})$ during inflation [90, 216], where $\phi \gtrsim M_P$.

To close this section let us make two important remarks. First, note that phase decomposition can still occur even if the probability is small. In this case we only expect very few bubbles per Hubble patch. The second comment concerns once again the distinction between classical and quantum fluctuations. These two sources can also be distinguished based on the length scale R at which their effect is strongest. As we explained, classical inhomogeneities are most relevant at $R \sim H_\Lambda^{-1} \sim M_p/\Lambda^2$ while the quantum effect is strongest for $R \sim M^{-1} \sim f/\Lambda^2$. Since $f < M_P$, the size R of the latter inhomogeneities is parametrically smaller than that of the inflationary ones.

5.4 Enhancement of fluctuations

Until now we have assumed that fluctuations, whether they are of classical or quantum nature, remain small during the evolution of the universe after inflation. The aim of this section is to discuss a possible enhancement of the fluctuations $\delta\phi_k$ due to the functional form of the potential (5.1). Before going into details, let us summarise the main result. In this section we are mainly interested in fluctuations with $k \sim m$ at time t_Λ. When the field oscillates at the bottom of the potential containing only the last few well, these fluctuations can be enhanced for certain values of f and κ. Crucially, these modes never exit the horizon during inflation, because at $t < t_\Lambda$ their wavelength is smaller than the Hubble radius. Therefore, these modes are never classicalised. Nevertheless, in this section we study their enhancement treating them as classical. We expect that our analysis will still capture the main effect. We provide a more detailed discussion at the end of this section.

A large growth of fluctuations can severely affect our conclusions. On the one hand, large fluctuations of the inflaton field may be desirable to some degree in our setup: the larger $\delta\phi_k$, the easier it is to cross the barrier between two local minima. Furthermore, if a mode with $k \gg m$ has a large amplitude, it may induce a phase decomposition independently of the modes with $k \sim m$ that we have studied in the previous section. If this is the case (5.20) underestimates the probability for phase decomposition.

On the other hand, large fluctuations with $k \gg m$ can lead to short-range violent dynamics rather than to the formation of well defined bubbles (which need a length scale $\gtrsim 1/M$). As we will describe in more detail in section 5.5, this can negatively affect the strength of the gravitational wave signal related to our setting.

For these reasons, it is crucial to assess if any large growth of fluctuations occurs in our setting. As we have already mentioned, we focus on the enhancement of classical fluctuations and comment later on the applicability of the results for quantum modes. Such an analysis involves solving the coupled equations of motion for the background field and for the fluctuation in an expanding background and in the presence of a non-zero gravitational field. In appendix B.3 we present a first step towards this goal by providing the relevant equations of motion.

Here we will perform a somewhat different, simplified analysis, assuming that the gravitational field is negligible for the following reason. As we describe in appendix B, the addition of a gravitational field leads to a dark matter-like growth of fluctuations, which is negligible compared to the exponential growth that we are seeking in this section. Let us rewrite the equations of motion in a way that is more suitable for a numerical analysis. Namely, we define $t' = mt$ and $\varphi_0 = \phi_0/f$. Then the linearised equations of motion without gravity read:

$$\varphi_0'' + \frac{2}{t'}\varphi_0' + \varphi_0 - \kappa \sin\left(\varphi_0\left(t'\right)\right) = 0 \tag{5.26}$$

$$\delta\phi_k'' + \frac{2}{t'}\delta\phi_k' + \left[1 + \frac{k^2}{m^2 a^2(t')} - \kappa\cos\left(\varphi_0\left(t'\right)\right)\right]\delta\phi_k\left(t'\right) = 0, \tag{5.27}$$

where 'prime' now denotes a derivative w.r.t. t' and where we have used $\frac{a'}{a} = \frac{2}{3t'}$ during matter domination.

In the absence of friction, the background solution φ_0 is periodic and the equation of motion for $\delta\phi_k$ is a Hill's equation. Solutions to such an equation exhibit a resonant behaviour for certain values of k [207, 217]. This is similar to the resonances encountered in the context of preheating (for a review see [218]). However, the inclusion of a time-dependent background as well as friction, may affect the growth of the solution. Generically, one expects that modes with $k \gtrsim m$ may experience exponential growth for certain values of f and κ. We investigated the behaviour of ϕ_0 and $\delta\phi$ numerically, for certain values of the parameters. In figure 5.6 the background ϕ_0 is plotted as a function of t for $f = 10^{-2}M_P, \kappa = 60$. For this parameter choice the field is caught near one of the local minima after only one oscillation.

For the same values of f and κ we study the evolution of the mode $\delta\phi_{5m}$. In particular, we distinguish two regimes. Firstly, we focus on the behaviour of $\delta\phi_{5m}$ until the time at which the field is caught in one of the local minima. This is the interesting regime for the mechanism of phase decomposition that we have presented in section 5.2.3. As shown in figure 5.7, the mode does not grow in this time interval. Note that our equations

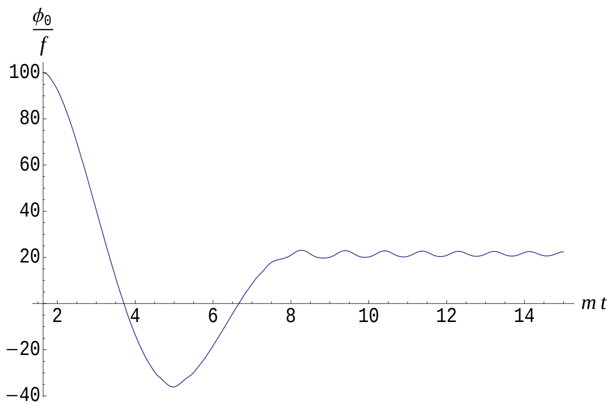


Figure 5.6: Evolution of the background $\phi_0\,(t')$, according to (5.26). The initial conditions $\phi_0\,(t'_i) = M_P$ and $t_i = 2\sqrt{2}/(\sqrt{3}m)$ are determined by violation of the slow roll condition. Furthermore $\phi'_0\,(t'_i) = 0$, Here $f = 10^{-2}M_P, \kappa = 60$. The field is caught in one of the cosine wells around $t = 8/m$.

are homogeneous in $\delta\phi_k$. Hence our numerical determination of the enhancement is not affected by the initial value of $\delta\phi_k$.

Secondly, we show the behaviour of the fluctuations after the field is caught around one of the local minima in figure 5.8. We observe that the mode grows after the background settles in one of the wells and decays again at late times. In particular, the field is caught around $t_1 \approx 8/m$ and by the time $t_2 \approx 13/m$ the size of its fluctuation $\delta\phi_{5m}$ has grown by roughly one order of magnitude with respect to its initial value $\delta\phi|_{\text{initial}} \approx 10^{-5}M_P$. [6] Meanwhile, the amplitude of the background field ϕ keeps decreasing between t_1 and t_2, as shown in figure 5.6: in particular $|\phi(t_1)| - |\phi(t_2)| \approx 1.5f$. The mild growth of $\delta\phi$ described in this particular example is therefore not likely to induce a late-time phase decomposition.

Other modes may nevertheless experience a stronger growth after the field settles in one of the local minimum. However, as our previous example shows, a fluctuation $\delta\phi \gtrsim f$ is generically needed to overcome the decay of the background amplitude. This possibility clearly cannot be studied by means of linearised equations of motion for $\delta\phi$. Therefore, in what follows we will focus purely on the growth of fluctuations before the field is caught in one of the cosine wells. We leave the interesting possibility of a late-time phase decomposition before reheating for future study.

Focusing on the first regime, the situation can be radically different from the previous example, depending on the values of the parameters appearing in the potential. In figure 5.9 we plot the background scalar field for $f = M_P/300, \kappa = 20$. The field gets stuck in one of the cosine wells after six oscillations. As a consequence of the longer time that the field spends oscillating across several cosine wells, fluctuations can now grow significantly. In figure 5.10, we plot the logarithm of the absolute value of $\delta\phi_k$, again for $k = 5m, f = M_P/300, \kappa = 20$. We see that the amplitude of this mode grows by three

[6] This growth is nevertheless small compared to the growth by several orders of magnitude that we find below for the case where the field performs several oscillations before being caught in one of the minima.

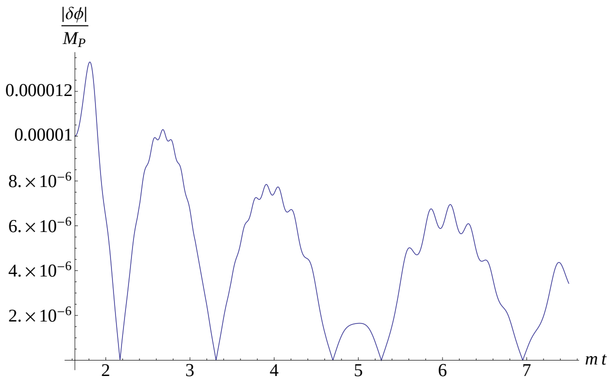


Figure 5.7: Evolution of the fluctuations $\delta\phi_k\,(t')$, according to (5.27). We have chosen: $k = 5m, \delta\phi'_k\,(t'_i) = 0$, and $t_i = 2\sqrt{2}/(\sqrt{3}m)$. Furthermore $f = 10^{-2}M_P, \kappa = 60$. After $t \approx 8/m$, ϕ_0 is stuck in one of the cosine wells.

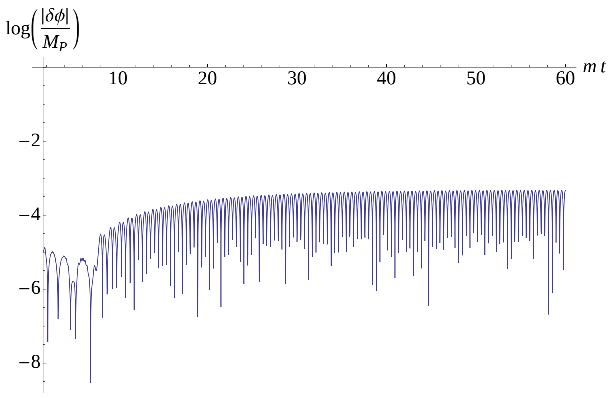


Figure 5.8: Logarithmic evolution of the fluctuations $\delta\phi_k\,(t')$, including the late-time regime, after the background field is caught in one of cosine wells. We have chosen: $k = 5m, \delta\phi'_k\,(t'_i) = 0$, and $t_i = 2\sqrt{2}/(\sqrt{3}m)$. Furthermore $f = 10^{-2}M_P, \kappa = 60$.

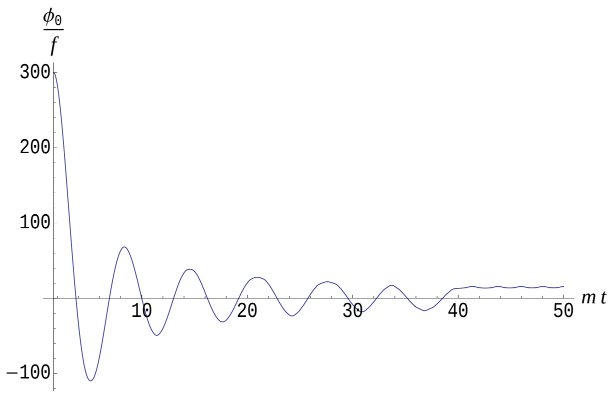


Figure 5.9: Evolution of the background $\phi_0\,(t')$, according to (5.26). The initial conditions $\phi_0\,(t'_i) = M_P$ and $t_i = 2\sqrt{2}/(\sqrt{3}m)$ are determined by violation of the slow roll condition. Furthermore $\phi'_0\,(t'_i) = 0$. Here $f = M_P/300, \kappa = 20$. The field is caught in one of the cosine wells around $t = 40/m$.

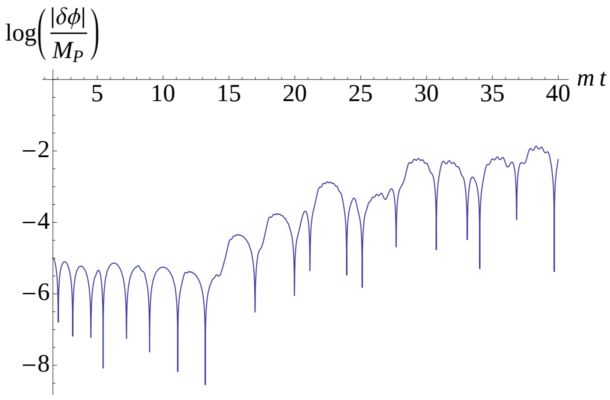


Figure 5.10: Logarithmic evolution of the absolute value of $\delta\phi_k\,(t')$, according to (5.27). We have chosen: $k = 5m$, $\delta\phi'_k\,(t_i) = 0$, and $t_i = 2\sqrt{2}/(\sqrt{3}m)$. Furthermore $f = M_P/300, \kappa = 20$. After $t \approx 40/m$, ϕ_0 is stuck in one of the cosine wells.

orders of magnitude before the background field settles in one of the local minima. In fact, such strong growth takes us out of the regime of validity of the linearized equation of motion (as $\delta\phi_k$ becomes comparable to f).

Recently, the growth of fluctuations in a potential with cosine modulations was studied in [207]. The authors argue that, for a relatively small Hubble scale and neglecting gravity, the fluctuations for $k \sim m$ can grow as e^{N_k}, where N_k is roughly given by

$$N_k \sim \frac{m}{H_{k\sim m}} F(\kappa). \tag{5.28}$$

Here $F(\kappa)$ is a function of the order up to a few whose value depends on the initial amplitude of ϕ. In our case, since $H_{k\sim m} \sim \Lambda^2/M_P \sim \kappa^{1/2} m(f/M_P)$, we conclude that fluctuations should grow with exponent:

$$N_{k\gtrsim m} \sim \frac{M_P}{f}, \tag{5.29}$$

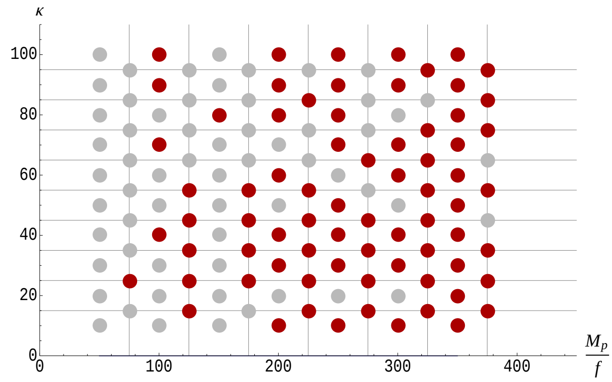


Figure 5.11: Grid showing for which values of κ and f resonance occurs. Grey points correspond to the case of no (or too small) enhancement. Red points correspond to large enhancement. By the latter we mean that fluctuations grow by at least two orders of magnitude before getting caught near a local minimum. The equations of motion are solved for $k = M = \kappa^{1/2} m$ at the time when $H \sim O(\Lambda^2/M_P)$.

where we have dropped any κ-dependence due to our ignorance regarding $F(\kappa)$. Our numerical examples, which take Hubble expansion into account explicitly, confirm that for small f such an exponential growth does happen for most values of $\kappa \gtrsim 10$. In contrast, we do not observe enhancement if the value of f is chosen too large.

Apart from this qualitative discussion, we are unfortunately unable to provide an analytical understanding of the dependence of the enhancement on the parameters κ and f. This is partly due to the fact that the phenomenon strongly depends on the precise minimum the field ultimately settles in. The latter question can only be addressed in a probabilistic approach, i.e. we can only say where the field is more *likely* to get trapped. Therefore, we do not have a precisely monotonic dependence of the enhancement in terms of f and κ.

In the absence of an analytical treatment, we performed a numerical search for enhanced fluctuations, focusing on modes with $k \gtrsim m$ at the time when $H \sim O(\Lambda^2/M_P)$. The results are reported in figure 5.11 in the form of a grid of points. Each point corresponds to a value of κ and f. We observe that in the region of interest fluctuations tend to be enhanced whenever $f \lesssim M_P/200$. Here, we define 'enhancement' as follows: we will refer to a mode to be enhanced if its original amplitude has grown by roughly two orders of magnitude before getting caught. The enhanced $\delta\phi_k$ is comparable to f and we are therefore at the boundary between the linear and non-linear regime. Interestingly, this boundary corresponds to parameter values such that the probability (5.20) is of the order 10^{-1}. However, note that the probability in (5.20) was determined for modes which will exhibit $k < m$ at t_Λ. While such fluctuations may experience growth, the generic expectation is that enhancement does not occur for modes with $k < m$.

Let us now briefly discuss enhancement of quantum fluctuations. This is in principle a complicated issue: we cannot use the classical equations of motions to analyse the behaviour of the quantum system. However, it is important to notice that, if quantum fluctuations are initially enhanced, they quickly become classical. Here by "classical" we mean that their occupation number becomes large, such that (5.27) can be used to study their evolution. Parametric resonance in quantum mechanics and quantum field theory has been studied analytically and numerically: the conclusion is that quantum modes do experience exponential growth (see e.g. [219, 220]). We expect that the same effect will occur also in the system analysed here. Therefore, our study of classical fluctuations should extend, at least partially, to quantum fluctuations. If more enhancement occurs in the quantum case, then phase decomposition is even more likely. We leave a more detailed study of this effect for future work.

Finally, let us summarise our findings concerning the probability of phase decomposition:

1. For $f \gtrsim 0.5 \cdot 10^{-2} M_P$ enhancement is generically not observed. The probabilities (5.20) and (5.24) are small, so that a phase decomposition is unlikely. Nevertheless, it is still possible that very few bubbles of tiny size containing the state of lower energy are nucleated. As we describe in the next section, observational signatures from such a situation may be quite strong. Classical inflationary fluctuations are the dominant cause of phase decomposition in this regime.

2. For $f \lesssim 0.5 \cdot 10^{-2} M_P$ we have the following situation. On the one hand, fluctuations with $k \sim m$ at $t = t_\Lambda$ are generically enhanced. These modes are genuinely quantum modes, since they never exited the horizon. In this region, the enhancement may be just large enough to give rise to a probability of phase decomposition of order $O(1)$. Furthermore, we observe numerically that, at fixed f and κ, modes with $k \gg m$ do not experience the same exponential growth. This will turn out to be a useful observation when examining the gravitational wave signal from the associated phase transition.

   On the other hand, according to (5.20) and (5.24), classical and quantum modes with $k \lesssim m$ can lead to a phase decomposition, even if they are not enhanced. Therefore we conclude that a phase decomposition is very likely to be induced. Assuming that our analysis of enhanced classical fluctuations extends to the quantum ones, the dominant cause of phase decomposition are quantum modes with $k \sim m$ at t_Λ.

3. For $f \ll 10^{-2} M_P$ fluctuations with $k \sim m$ are strongly enhanced. In this region phase decomposition is very likely to occur. However, it is hard to provide any description of such a highly non-linear regime. Classical and quantum fluctuations with $k \lesssim m$ are generically not enhanced, but would also lead to phase decomposition according to (5.20) and (5.24).

One more comment is in order before moving on to the phenomenological signatures of our setup. Phase decomposition happens generically for rather small axion decay constants. One may question whether such values of f are plausible in the spirit of axion monodromy. The answer depends very much on the framework in which monodromy is implemented. In a stringy setup it is a question of moduli stabilisation: e.g. in the Large Volume Scenario (LVS) [50, 51] decay constants are generically suppressed by the volume of the compactification manifold and are therefore naturally small.

5.5 Gravitational radiation from Phase Transitions

In the previous sections, we have described a mechanism which can potentially lead to phase decomposition in the early universe after inflation. In this section, we will assume that such phase decomposition indeed occurs. In the presence of different populated vacua, bubbles containing the state of lowest energy can form and expand. Their collisions are a very interesting and well-known source of gravitational radiation. This has been studied in detail in the literature in various contexts and regimes (see [209] and references therein).

The aim of this section is twofold. First of all, we would like to elucidate the peculiarities of our setup concerning the energy released into gravitational waves during the collision of bubbles. Rather than focusing on precise calculations, we will give a qualitative discussion and provide formulae analogous to the more familiar case of bubbles colliding in a relativistic plasma. In this case, there are three possible sources of gravitational radiation: the collision of bubble walls, sound waves in the plasma and its turbulent motion. The second goal of this section is to give estimates of the relic density and frequency of the gravitational wave signal which can be obtained in our setup.

5.5.1 Gravitational waves from bubble collision

The focus of this subsection is the collision of bubbles and the possible shocks in the fluid surrounding them. These phenomena are usually studied in the so-called *envelope* approximation [116]. The latter consists in assuming that the energy liberated in gravitational waves resides only in the bubble walls before the collision. Furthermore, it is assumed that only the uncollided region of those walls contributes to the production of gravitational waves, i.e. the interacting region is neglected. Such an approximation has been initially applied to the case of vacuum-to-vacuum transitions [115] and later to collisions in a radiation bath.

In a thermal phase transition, the energy released into gravitational waves depends on four parameters. First of all, there is the time scale of the phase transition δ^{-1} or, equivalently, the initial separation between two bubbles $d \sim \delta^{-1}$. Secondly, there is the ratio η of the vacuum energy density ϵ released in the transition to that of the thermal bath, i.e.

$$\eta \equiv \frac{\epsilon}{\rho^{\star}_{rad}}, \tag{5.30}$$

where $\star$ specifies that the quantity is evaluated at the time of completion of the phase transition. Thirdly, the efficiency factor λ characterizes the fraction of the energy density ϵ which is converted into the motion of the colliding walls. Finally, the bubble velocity v_b is not necessarily luminal, as the walls have to first displace the fluid around them. The energy released into gravitational waves of peak frequency is then given by [124]:

$$\frac{\rho_{GW}}{\rho_{tot}} \sim \theta \left(\frac{H_\star}{\delta}\right)^2 \lambda^2 \frac{\eta^2}{(1+\eta)^2} v_b^3, \tag{5.31}$$

where ρ_{tot} is the background energy density at completion of the phase transition. The parameters v_b and λ are actually expected to be functions of η, in such a way that for $\eta \sim O(1)$, also $\lambda, v_b \sim O(1)$.

In our case, bubbles collide before reheating, therefore there is no radiation bath around them. However, as we describe in appendix B, an oscillating scalar field corresponds to the presence of a matter fluid. Crucially, the time scale of the field oscillations is set by m^{-1}, and may be smaller than the time of collision. Therefore, oscillations of the scalar field cannot be generically neglected. Unfortunately, we do not have specific formulae for this case. Since we are interested only in an order of magnitude estimate for the spectrum of gravitational waves, it seems reasonable to extend (5.31) to our setup, with the obvious modification

$$\eta \equiv \frac{\epsilon}{\rho^{\star}_{matter}}. \tag{5.32}$$

Furthermore, we shall hide our ignorance about the dependence of λ and v_b on η by defining $\theta_0 = \theta\lambda^2 v_b^3$ and leave the determination of these parameters for future work. Therefore, we base our estimates on the following formula for the energy released in gravitational waves from the collision of bubbles and shocks in the matter fluid:

$$\frac{\rho_{GW}}{\rho_{tot}} \approx \theta_0 \left(\frac{H_\star}{\delta}\right)^2 \frac{\eta^2}{(1+\eta)^2}, \tag{5.33}$$

where in our case $\rho_{tot} \sim \Lambda^4$. In addition, the peak frequency of gravitational waves in the envelope approximation is given by

$$\omega_{peak} \simeq \sigma\delta, \tag{5.34}$$

where $\sigma \lesssim O(0.1)$ should be fixed numerically and includes effects due to subluminal bubble walls velocity.

The next task is to estimate δ and η. Let us start with the ratio $H_\star/\delta$. The energy density at the time of the phase transition corresponds to the height of the barrier separating the two minima, therefore $H_\star \sim \rho^{1/2}/M_P \sim \Lambda^2/M_P$. We expect the typical frequency of the phase transition to be set by the momentum k of the spatial inhomogeneities of the field ϕ. The phase transition can be induced by any mode which is present at $t = t_\Lambda$. The largest frequency that one can take is set by $k \sim M$, as we have already discussed in subsection 5.3.2. This corresponds to a scenario where phase decomposition is likely. However, bubble collisions are most violent when the field makes it over the barrier separating the two minima only very rarely. In this case there are only few bubbles per Hubble patch. This latter scenario gives the strongest signal as it corresponds roughly to

$$\frac{H_\star}{\delta} \sim O(1). \tag{5.35}$$

In order to understand how strong can the signal be in our setup, we assume (5.35) in what follows, but one should keep in mind that this is optimistic.

The estimate of η is less straightforward, at least conceptually. If we adopt the envelope approximation, then we need to compute the vacuum energy density released in the phase transition. This is simply the difference ϵ between the energy density of the two minima in figure 5.12. Using a quadratic approximation, we find

$$\epsilon \sim m^2\Delta\phi^2 \sim m^2 f^2, \tag{5.36}$$

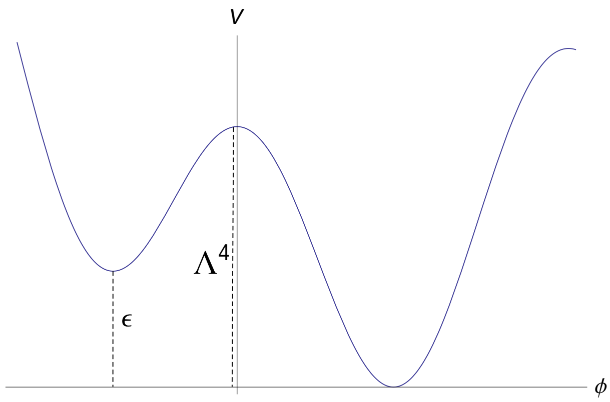


Figure 5.12: Two-well potential. In the picture ϵ is the energy difference between the two minima, while Λ^4 is the value of the potential at the local maximum.

where $\Delta\phi \sim f$ is the approximate field separation between the two minima. The energy in the matter fluid $\rho^{\star}_{matter}$ is roughly given by the height of the deepest well, i.e. $\rho^{\star}_{matter} \simeq \Lambda^4$. This is because at completion of the phase transition the oscillations of the scalar field span almost the whole well. Therefore, in the envelope approximation we obtain

$$\eta \sim \kappa^{-1}. \tag{5.37}$$

As we have mentioned, deviations from this simple picture may arise in our case. On the one hand, a certain fraction of the energy of the walls might for example be dissipated into the matter fluid. In this case, only a fraction of ϵ would lead to production of gravitational waves. This effect might be captured by the efficiency prefactor λ.[7] On the other hand, the energy released into the fluid while the bubbles expand and collide might also contribute to the production of gravitational waves. Namely, this energy might be converted into bulk motion of the fluid. In this case, the energy released in gravitational radiation should be larger than ϵ, and could possibly be as large as Λ^4. This effect is captured by studying the fluid as a source of gravitational waves. We comment very briefly on this topic in the next subsection.

5.5.2 Gravitational waves from the matter fluid

In analogy with the case of radiation there are at least two effects which can further contribute to the total energy released in gravitational radiation during the phase transition. Here we just provide the formulae given in [209] for the thermal case, keeping in mind that they may not straightforwardly extend to our setup:

1. **Sound waves in the fluid**: this arises because a certain fraction λ_v of the energy of the walls is converted after the collision into motion of the fluid (and is only later dissipated). In the case of radiation this gives a contribution

$$\frac{\rho_{GW,sw}}{\rho_{tot}} \sim \theta_{sw} \left(\frac{H_\star}{\delta}\right) \lambda_v^2 \left(\frac{\eta^2}{(1+\eta)^2}\right) \tag{5.38}$$

[7] Let us also notice that bubble walls may also be generically crossed by the fluid. We neglect this effect in our discussion.

The prefactor θ_{sw} is expected to be smaller than θ in (5.31).

2. **Turbulence in the fluid**: one expects further contributions as a certain fraction λ_{turb} of the energy of the walls is converted into turbulence. In the case of radiation one obtains:
$$\frac{\rho_{GW,turb}}{\rho_{tot}} \sim \theta_{turb} \left(\frac{H_\star}{\delta}\right) \lambda_{turb}^{3/2} \left(\frac{\eta^{3/2}}{(1+\eta)^{3/2}}\right). \tag{5.39}$$
The prefactor is expected to be larger than θ in (5.31). Note, that for these two mechanism the dependence on H/δ is only linear.

In certain regimes, these two effects may be larger than the one due to bubble collisions and shocks in the fluid. However, they are not fully understood, even in the case of radiation. Therefore, in the next subsection we will neglect them, and obtain only a lower bound on the relic abundance of gravitational waves. This should still be useful to understand the approximate size and frequency of the signal. Nevertheless, the reader should keep in mind that there are other possible contributions even beyond the ones mentioned in this subsection (see e.g. [221] for recent progress).

5.5.3 Frequency and signal strength of gravitational waves

In order to compute the relic abundance and frequency of gravitational waves emitted during the phase transition, we need to know the equation of state of the background energy density from the end of the phase transition to today. Assuming standard evolution after reheating the behaviour of the scale factor until today is essentially fixed[8]. Furthermore, the inflaton field generically behaves as non-relativistic matter after inflation. It remains to be addressed whether deviations from the equation of state $w \approx 0$ might occur immediately after the phase transition, before reheating.

Due to the very large release of energy during the collision of bubble walls, it is conceivable that the fluid initially behaves relativistically. This would correspond to an early phase of radiation domination, i.e. $w = 1/3$, in a similar fashion to some *preheating* scenarios. Eventually, the fluid cools down and its non-relativistic behaviour is restored. Depending on the reheating temperature, this may or may not happen before the inflaton decays. If the system were in a thermal ensemble, the fluid would behave non-relativistically after $T \sim m_\phi \sim M$. For the time being, we allow for a general equation of state parameter w after the phase transition and before $T \sim M$.

Therefore the background energy density at reheating is given by

$$\rho_{RH} = \Lambda^4 \left(\frac{a_\star}{a_{NR}}\right)^{3(1+w)} \left(\frac{a_{NR}}{a_{RH}}\right)^3, \tag{5.40}$$

where from now on the subscript NR denotes that a certain quantity is evaluated at the

[8]It is characterized by the effective number of degrees of freedom.

time when the fluid becomes non-relativistic. Let us define the prefactors

$$\nu_w \equiv \left(\frac{a_\star}{a_{NR}}\right) \sim \left(\frac{\rho_{NR}}{\rho_\star}\right)^{\frac{1}{3(1+w)}}, \tag{5.41}$$

$$\nu_{nr} \equiv \left(\frac{a_{NR}}{a_{RH}}\right) \sim \left(\frac{\rho_{RH}}{\rho_{NR}}\right)^{1/3}. \tag{5.42}$$

The prefactor ν_w quantifies the duration of a period of matter domination before reheating, while ν_{nr} quantifies the duration of an early epoch of matter domination. Obviously $0 < \nu_w, \nu_{nr} \leq 1$.

The energy density in gravitational waves scales as a^{-4}. According to (5.33), at reheating we have

$$\rho_{GW}(t_{RH}) \approx \Lambda^4 \nu_w^4 \nu_{nr}^4 \left[\theta_0 \left(\frac{H_\star}{\delta}\right)^2 \frac{\eta^2}{(1+\eta)^2}\right] = \nu_w^{-3(w-1/3)} \cdot \nu_{nr} \left[\theta_0 \left(\frac{H_\star}{\delta}\right)^2 \frac{\eta^2}{(1+\eta)^2}\right] \rho_{RH}, \tag{5.43}$$

where in the last step we have used (5.40). The relic energy density of gravitational waves is then

$$\rho_{GW}(t_0) \approx \nu_w^{-3(w-1/3)} \cdot \nu_{nr} \left[\theta_0 \left(\frac{H_\star}{\delta}\right)^2 \frac{\eta^2}{(1+\eta)^2}\right] \left(\frac{a_{RH}}{a_0}\right)^4 \rho_{RH}, \tag{5.44}$$

where t_0 is the current age of the Universe. The ratio a_{RH}/a_0 can be determined by imposing entropy conservation. Furthermore, ρ_{RH} can be computed using the standard formula for the energy density of radiation

$$\rho_{RH} = \frac{\pi^2 g_\star(T_{RH})}{30} T_{RH}^4. \tag{5.45}$$

Finally, the density parameter today $\Omega_{GW}(t_0) \equiv \frac{\rho_{GW(t_0)}}{\rho_{crit}} = \rho_{GW(t_0)} \frac{8\pi G_N}{3H_0^2}$ today is given by

$$\Omega_{GW}(t_0) \simeq \frac{10^{-5} \nu_w^{-3(w-1/3)} \nu_{nr}}{h^2} \cdot \theta_0 \left[\frac{10^2}{g_*(T_{RH})}\right]^{1/3} \cdot \left[\frac{H_\star}{\delta}\right]^2 \cdot \frac{\eta^2}{(1+\eta)^2}, \tag{5.46}$$

where $h \equiv H_0/(100 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1})$.

Let us now estimate the peak frequency of the emitted radiation. For this quantity the only relevant parameter is δ. Frequencies scale as $\sim a^{-1}$, therefore we have:

$$\omega_0 \sim \omega_{peak} \left(\frac{a_\star}{a_{NR}}\right) \left(\frac{a_{NR}}{a_{RH}}\right) \left(\frac{a_{RH}}{a_0}\right) \sim \omega_{peak} \cdot \nu_w \cdot \nu_{nr} \cdot \left(\frac{a_{RH}}{a_0}\right). \tag{5.47}$$

By combining (5.47), (5.34), $H^2 \sim \rho/M_P^2$ and (5.45) one obtains

$$\omega_0 \sim 10^8 \text{Hz} \cdot \sigma \cdot \nu_w \cdot \nu_{nr} \left(\frac{\delta}{H_*}\right) \left(\frac{g_*(T_{RH})}{10^2}\right)^{1/6} \left[\frac{T_{RH}}{10^{15}\text{GeV}}\right]. \tag{5.48}$$

We have therefore determined the relevant parameters of the emitted radiation as function of δ and η. Now we can plug in (5.35) and (5.37), to obtain final formulae. Then we have:

$$\Omega_{GW}(t_0)h^2 \simeq 10^{-5}\nu_w^{-3(w-\frac{1}{3})} \cdot \nu_{nr} \cdot \theta_0 \left[\frac{10^2}{g_*(T_{RH})}\right]^{1/3} \kappa^{-2} \tag{5.49}$$

$$\omega_0 \simeq 10^8\text{Hz} \cdot \sigma \cdot \nu_w \cdot \nu_{nr} \cdot \left(\frac{g_*(T_{RH})}{10^2}\right)^{1/6} \left[\frac{T_{RH}}{10^{15}\text{GeV}}\right]. \tag{5.50}$$

In the envelope approximation, it is also possible to compute the full spectrum of the gravitational radiation emitted from the collision of the bubble walls. This reads [209]:

$$\Omega_{GW}(t_0)h^2 \simeq 10^{-5}\nu_w^{-3(w-1/3)}\nu_{nr} \cdot \theta_0 \cdot \kappa^{-2} \left[\frac{10^2}{g_*(T_{RH})}\right]^{1/3} S_{env}(\omega) \tag{5.51}$$

$$\text{with} \quad S_{env}(\omega) = \frac{3.8(\omega/\omega_0)^{2.8}}{1+2.8(\omega/\omega_0)^{3.8}}.$$

In order to estimate the maximal possible size of our signal, let us now specify to the case in which the energy present after the bubble collisions is converted into radiation. This corresponds to setting $w = 1/3$ in (5.51). At $T \sim M$ the inflaton goes back to a non-relativistic behaviour. The prefactors ν_w and ν_{nr} can now be explicitly computed, using $\rho_{NR} \sim T_{NR}^4 \sim M^4, \rho_{RH} \sim T_{RH}^4$. We obtain

$$\nu_w = \left(\frac{\rho_{NR}}{\rho_\star}\right)^{1/4} \sim \frac{M}{\Lambda} \sim \frac{\kappa^{1/4}m^{1/2}}{f^{1/2}} \tag{5.52}$$

$$\nu_{nr} = \left(\frac{\rho_{RH}}{\rho_{NR}}\right)^{1/3} \sim \left(\frac{T_{RH}}{m\kappa^{1/2}}\right)^{4/3}. \tag{5.53}$$

From (5.53), it is clear that the largest signal is obtained for $T_{RH} \sim T_{NR} \sim M$. In this case $\nu_{nr} \sim 1$ and the signal is completely unsuppressed. For completeness, let us mention that the largest suppression of the signal occurs for $w \approx 0$. In this case, as should be clear from (5.40), the background energy density scales as matter from completion of the phase transition until reheating.

In figure 5.13 we plot the spectrum (5.51) for three different choices of parameters, using $w = 1/3$. We fix $m \simeq 10^{-5}M_P$ as required by observations. We have chosen parameters in such a way as to maximize the overlap with sensitivity regions of current and future space- and ground-based detectors, which are bounded by dashed lines in the plot. We have also fixed $\theta_0 = 10^{-2}, \sigma = 10^{-1}$. Our plot provides examples of the wide range of frequencies that can be obtained in our setting, simply varying the axion decay constant, the reheating temperature and the number of local minima. Interestingly, for reasonable choices of parameters the signals are in the reach of future detectors.

5.6 Summary and Conclusions

In this chapter we investigated the production of gravitational waves from post-inflationary dynamics in models of Axion Monodromy inflation. We expect such phenomena to

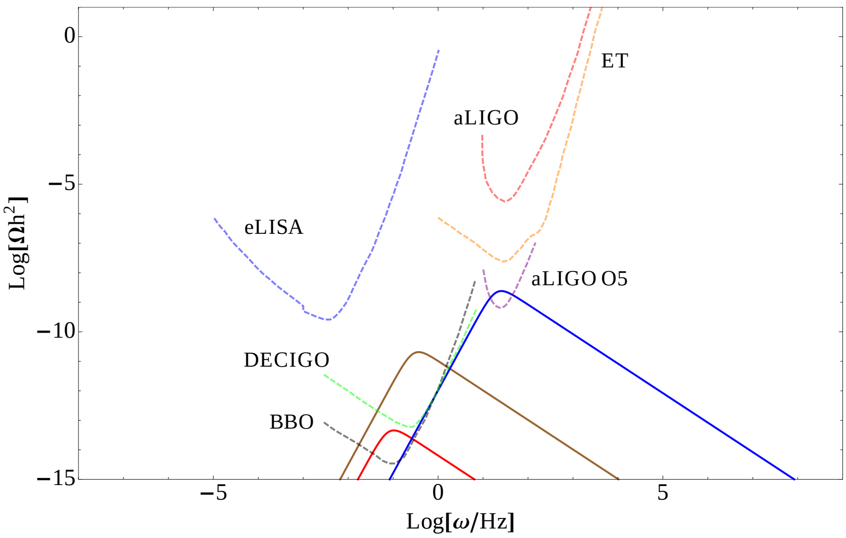


Figure 5.13: Gravitational wave spectra as in (5.51) with $w = 1/3$. The inflaton mass is fixed to $m \sim 10^{-5} M_P$. Spectra are shown as solid lines for different values of κ, f and T_{RH}: the blue curve is obtained for $\kappa = 5, f = 0.1 M_P, T_{RH} \sim 10^{12}$ GeV; the brown curve for $\kappa = 10, f = 0.01 M_P, T_{RH} \sim 10^{11}$ GeV; the red one for $\kappa = 70, f = 0.001 M_P, T_{RH} \sim 10^{11}$ GeV. We have also taken $w = 1/3, \theta_0 = 10^{-2}, \sigma = 10^{-1}$ in (5.51). For the values of the reheating temperature considered here, we have $g_*(T_{RH}) \sim 10^2$. Sensitivity curves of some ground- and space-based interferometers are shown for comparison as dashed curves (data taken from [222]).

also occur for generic axionic fields with potentials with sizable modulations, albeit the numerical results may differ significantly in these cases.

The main observation is that in models of axion monodromy inflation, the inflaton potential consists of a monotonic polynomial with superimposed cosine-modulations. While these modulations have to be small to allow for successful inflation, they tend to dominate near the bottom of the potential. In fact, these cosine 'wiggles' can be large enough such that the potential exhibits a series of local minima.

After inflation ends, the inflaton is exploring this wiggly part of the potential. As a consequence of Hubble friction, the rolling axion can get stuck in one of the wells. This may happen well before the inflaton reheats the standard model degrees of freedom. Since the energy density in the axion field decreases together with H, the field is more likely to get caught in one of the last minima. We therefore focused on a two-well setting, which is obtained by "zooming" into the full monodromy potential.

Taking fluctuations of the inflaton field into account, the inflaton field does not necessarily get caught in one unique local minimum in the entire Hubble patch. If field fluctuations are sufficiently large, a phase decomposition occurs such that at least two different vacua are populated after inflation. The probability of this occurring is given by $\mathcal{P} \sim \delta\rho/\Delta\rho$, where $\delta\rho$ is the uncertainty in the axion energy density induced by the field fluctuations and $\Delta\rho$ is the frictional loss of energy in one oscillation.

We can distinguish two sources of field fluctuations which may lead to a phase decomposition. Firstly, there are the classical inhomogeneities naturally inherited from inflation. Secondly, there are the intrinsic quantum uncertainties characterising any quantum field. These two sources are essentially indistinguishable at very early and late times. However they are in principle of different size in the intermediate regime that we are interested in. In particular, we have found that inflationary fluctuations are more likely to induce a phase decomposition: the probability that they do so is $\mathcal{P} \sim \kappa^{-1/3}(m/M_P)(M_P/f)^{5/3}$. Here m is the mass of the inflaton-axion, f the axion decay constant which defines the axion periodicity $2\pi f$ and κ/π roughly counts the number of local minima of the potential. Therefore, we observe that a phase decomposition is likely for $\kappa \sim O(10)$ and $f \lesssim 0.3 \cdot 10^{-2} M_P$. The probability that quantum fluctuations induce a phase decomposition is smaller by a factor $(f/(M_P\kappa))^{-4/3} m/M_P$.

Furthermore, due to the oscillatory term in the axion potential, fluctuations can experience exponential growth. This effect arises generically for modes with $k \sim m$ at the time when the field is rolling over the last wells. These fluctuations never exited the horizon, therefore they are effectively quantum modes. We extended the study of enhancement in [207] to the case with varying H, but still neglecting the gravitational field in the equations of motion. Numerically and using a classical approximation, we observe the existence of a region of parameter space where a phase decomposition is likely to occur. This happens roughly in the same regime where inflationary classical fluctuations are also likely to induce a phase decomposition. However, given the exponential enhancement of quantum modes, the latter are more likely to be the dominant cause of phase decomposition. Enhanced quantum modes quickly become classical, therefore we expect our main results to hold even after a more detailed analysis, which we leave for future work. For larger values of f modes are not enhanced. Phase decomposition, although unlikely, might still occur as a result of quantum or classical fluctuations. For smaller decay constants fluctuations are very strongly enhanced. Phase decomposition occurs but it is hard to understand the physics in such a highly non-linear regime.

If a phase decomposition occurs, bubbles containing minima of lowest energy expand. Collisions of these bubbles source gravitational waves. We estimated the energy density and frequency of the emitted radiation in terms of the axion parameters, in the envelope approximation. Furthermore, we note that the matter fluid associated to the oscillating inflaton may also radiate gravitational waves. This is similar to the case of a thermal phase transition. The spectrum of the emitted radiation can peak in a wide range of frequencies (from mHz to GHz), depending on the reheating temperature and on the time of the phase transition. In this sense, our source is similar to other post-inflationary phenomena, such as preheating and cosmic strings. However, it is interesting to observe that the frequency may be lowered in our case since a matter dominated phase can follow the phase transition. The spectrum is at least partially in the ballpark of future space- and ground-based detectors. Thus, we can hope that axion monodromy may one day be investigated by means of gravitational wave astronomy.

Chapter 6

Dark Radiation in the Large Volume Scenario

Reheating can be importantly affected by degrees of freedom belonging to hidden sectors. In particular, the existence of Dark Radiation can be observationally investigated in the CMB power spectrum, which depends on N_{eff}, the effective number of relativistic species at CMB time. In this chapter, we analyse the implications of the current observational bound on N_{eff} for a popular string-derived framework of physics beyond the Standard Model, the Large Volume Scenario (LVS) [50]. The latter is indeed characterised by the existence of at least one Dark Radiation candidate, associated to the volume of the compactification manifold. A basic review of the LVS is provided in subsection 1.1.4.

This chapter is based on the publication [129].

6.1 Introduction

In Part I and in the Intermezzo of this thesis we studied certain consequences of String Theory and more in general of Quantum Gravity for Large Field Inflation. In the previous chapter we focused instead on a phenomenological signature of the pre-reheating epoch after inflation. The relation to Quantum Gravity in this latter case is indirect, in that it is mediated by axion monodromy, which is a string-motivated model of LFI. In this chapter we focus on the effects of certain hidden sectors on reheating. The relation to Quantum Gravity is again indirect: hidden sectors arise naturally in string-derived scenarios beyond the Standard Model, in particular in the Large Volume Scenario (LVS).

Observationally, two quantities may help shed light on them. On one hand, hidden matter fields would contribute to the observed abundance of Dark Matter. On the other hand, hidden radiation would affect the value of N_{eff}, i.e. the effective number of relativistic species at CMB temperature, photons excluded. In the Standard Model, only the three species of neutrinos are light enough to contribute to N_{eff} at CMB temperatures. More precisely, one has $N_{eff,SM} = 3.046$. Any deviation ΔN_{eff} from this value should be considered as evidence for Dark Radiation (DR). Practically, this quantity can be inferred from the CMB temperature map. The most recent measurement from the Planck collaboration is consistent with the Standard Model prediction, though a small amount of Dark Radiation cannot be completely excluded: $N_{eff} = 3.04 \pm 0.18$ [2] (*Planck* TT,TE,EE+lowP+BAO, 68% C.L.). Slightly less stringent constraints are obtained if only the *Planck*TT+lowP+BAO data are considered, i.e. $N_{eff} = 3.15 \pm 0.23$ (68% C.L.) [2].

It is intuitively clear that this observation represents a potential clash with one of the few generic predictions of String Theory. Namely, string compactifications are characterised by a plenitude of very light fields. The lightest among them may contribute to N_{eff}. The impact of a measurement of N_{eff} in agreement with the Standard Model to a very high precision is therefore potentially dramatic for String Theory and in general for physics beyond the Standard Model.

In this chapter, we assess concretely the consequences of the current measurement of N_{eff}. We focus on the framework of moduli stabilisation known as the Large Volume Scenario (LVS) [50]. In models which make use of the LVS to stabilise moduli, the lightest modulus τ_b is responsible for reheating. Crucially, it also decays to hidden sectors, in particular to a very light axion-like particle [125–128], which we will denote by a. The latter is light enough to be relativistic at CMB temperature. The current measurement of N_{eff} constrains the abundance of this axion-like particle, thereby bounding the decay rate of τ_b into a compared to the decay rate into SM degrees of freedom. In particular, we have:

$$\Delta N_{eff} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{\frac{1}{3}} \frac{\rho_{DR}}{\rho_{SM}}\bigg|_{T=T_d} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{\frac{1}{3}} \frac{\Gamma_{\tau_b \to DR}}{\Gamma_{\tau_b \to SM}} \ . \tag{6.1}$$

Here T_d is the decay temperature of the modulus τ_b, which effectively coincide with the reheating temperature. The effective number of particle species at temperature T is denoted by $g_\star(T_d)$. In the LVS, the decay rates into a and into Standard Model fields depends on important features of the compactification, such as the amount of Higgs-like fields. It is therefore possible to find important constraints on compactifications setups based on the LVS.

This chapter is structured as follows: In section 6.2 we review the existing constraints on a particular realisation of the LVS which allows for TeV-scale soft terms. This also gives us the opportunity to collect some relevant formulae. In section 6.3 we list possibilities to suppress the amount of DR, focusing on the decay of the lightest modulus into gauge bosons. In section 6.4 we focus on one such possibility, where the bulk modulus reheats the SM indirectly, via gauge bosons belonging to an approximate global symmetry of the SM. In section 6.5 we briefly discuss two possibilities to evade the DR bound beyond the sequestered LVS, in setups with high-scale supersymmetry. Finally, we summarise the content of the chapter in section 6.6.

6.2 Dark Radiation in the Large Volume Scenario

The aim of this section is to briefly review predictions for Dark Radiation in the one popular set of realisations of the LVS: the so-called *sequestered* scenario (cf. [125, 126] for description of the setup, [125–127, 223] for predictions of DR).

In the sequestered LVS, the visible sector lives on stack of D3-branes at a singularity. This choice is motivated by the resulting supersymmetric spectrum in the visible sector: while the superpartners acquire TeV-scale masses, the gravitino and the moduli remain heavy enough to evade the Cosmological Constant Problem (CMP) [45–47]. In particular, at lowest order in string perturbation theory the following hierarchy of masses arises:

$$m_{1/2} \sim m_{soft} \sim \frac{M_P}{\mathcal{V}^2} \ll m_{3/2} \sim \frac{M_P}{\mathcal{V}^{3/2}} \,, \tag{6.2}$$

Since all the scales are set by $\mathcal{V}$, we will discuss the prediction for DR in terms of $\mathcal{V}$. Notice that TeV-scale soft terms are achieved for $\mathcal{V} \approx 3 \cdot 10^7$ (here we measure the volume in string units).

As it is customary in the LVS, let us consider compactification of 10D string theory on a Swiss-Cheese CY with volume given by:

$$\mathcal{V} = \alpha \left(\tau_b^{3/2} - \sum_i \gamma_i \tau_{s,i}^{3/2} \right) \,. \tag{6.3}$$

Here τ_b and $\tau_{i,s}$ are the real components of the Kähler moduli $T_b = \tau_b + i a_b$ and $T_{i,s} = \tau_{i,s} + i a_{i,s}$, describing the volume of the bulk and of the small cycles of the CY respectively. Thanks to an interplay of non-perturbative effects and α' corrections [50], the volume of the compactification as well as at least one of the $\tau_{i,s}$ are stabilised. Crucially, the minimum of the F-term scalar potential for the Kähler moduli occurs at an exponentially large value of $\mathcal{V}$.

We will be mostly concerned about the decay rate of the real part of the bulk modulus τ_b into its axionic partner a_b. While the former is parametrically as heavy as the gravitino, the latter is for all purposes massless:

$$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \,, \qquad m_{a_b} \sim M_P \; e^{-2\pi \mathcal{V}^{2/3}} \,. \tag{6.4}$$

The decay rates are determined once the Kähler potential and the gauge kinetic functions of the $\mathcal{N} = 1$ effective theory are specified. In particular, we have:

$$K = -3\ln\left(T_b + \bar{T}_b - \frac{1}{3}\left[C^i\bar{C}^i + H_u\bar{H}_u + H_d\bar{H}_d + \{zH_uH_d + \text{h.c.}\}\right]\right) + \dots \tag{6.5}$$

$$= -3\ln(T_b + \bar{T}_b) + \frac{C^i\bar{C}^i}{T_b + \bar{T}_b} + \frac{H_u\bar{H}_u + H_d\bar{H}_d}{T_b + \bar{T}_b} + \frac{zH_uH_d + \text{h.c.}}{T_b + \bar{T}_b} + \dots ,$$

$$f_a = S + h_{a,k}T_{s_a,k} , \tag{6.6}$$

where C^i are chiral matter superfields, H_u and H_d are the MSSM Higgs doublets and $T_{s_a,k}$ are blow-up modes. As long as the Giudice-Masiero coupling $z \sim \mathcal{O}(1)$, decay into the visible sector is dominated by $\tau_b \to H_uH_d$.[1] In particular, using (6.5) one finds:

$$\text{Dark Radiation:} \qquad \Gamma_{\tau_b \to a_b a_b} = \frac{1}{48\pi}\frac{m_{\tau_b}^3}{M_P^2} , \tag{6.7}$$

$$\text{Visible Sector:} \qquad \Gamma_{\tau_b \to H_u H_d} = \frac{2z^2}{48\pi}\frac{m_{\tau_b}^3}{M_P^2} . \tag{6.8}$$

If we extend the field content beyond the MSSM, we can allow for a generic number n_H of Higgs doublets. According to (6.1), (6.7), (6.8), we then find

$$\Delta N_{eff} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{\frac{1}{3}}\frac{\Gamma_{\tau_b \to DR}}{\Gamma_{\tau_b \to SM}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{\frac{1}{3}}\frac{1}{n_H z^2} . \tag{6.9}$$

The reheating temperature is given by $T_d \sim \sqrt{\Gamma_{\tau_b} M_P} \simeq \mathcal{O}(0.1)M_P/\mathcal{V}^{-9/4}$. For TeV scale SUSY, we find $T_d \lesssim 1$ GeV. The effective number of particle species at this temperature is g_* is $g_* = 247/4$. The observational constraint $\Delta N_{eff} < 0.17$ (from *Planck* TT,TE,EE+lowP+BAO) requires: $n_H > 20$ for $z = 1$ and $z > 3$ for only one pair of Higgs doublets. Alternatively, for $\Delta N_{eff} < 0.33$ (from*Planck* TT+lowP+BAO) one needs $n_H > 10$ for $z = 1$ and again $z > 2$ for only one pair of Higgs doublets. Allowing for soft terms $m_{soft} \gtrsim 10$ TeV, which may be more natural given the current absence of supersymmetry at the LHC, relaxes these constraints only very mildly.

It is therefore clear that the current measurement of N_{eff} already leads to dramatic constraints on the sequestered LVS.[2] In particular, it remains an open question whether $z > 1$ occurs naturally in the string landscape. Notice that $z = 1$ can be derived from a shift symmetry in the Higgs sector, therefore it may be considered a motivated natural value for z.[3]

6.3 Reheating through gauge bosons

The aim of this section is to investigate different possibilities beyond the sequestered LVS to improve the rather unsatisfactory situation presented in the previous section.

[1] See e.g. [129] for a review of decays into other visible sector fields

[2] See however [224], where the authors argue that string loop corrections to the Kähler potential can relax the constraints presented in this section.

[3] In type IIB/F-theory such a symmetry can arise if the Higgs is contained in brane deformation moduli [225–228]. However, in this case the decay channel $\tau_b \to H_uH_d$ is closed, because the Kähler metric is independent of Kähler moduli.

In order to relax the constraints on z and n_H we need to boost the decay rate of τ_b into visible sector fields. There are in principle three possibilities to do so:

- **Matter scalars**: let us consider one such matter field C. The relevant term in the Kähler potential is of the form $K \supset \tau_b C\bar{C}$. The decay rate is therefore given by $\Gamma \sim \frac{m_{soft}^2 m_{\tau_b}}{M_P^2}$. Clearly, this rate is comparable to (6.8) only if the scalar mass is close to threshold, i.e. $m_{soft} \lesssim m_{\tau_b}/2$. While the precise determination of m_{soft} requires knowledge of yet undetermined corrections to the Kähler potential, the hierarchy (6.2) seems difficult to evade in the sequestered LVS. In setups with high-scale supersymmetry breaking, one typically has $m_{soft} \gg m_{\tau_b}$ [51, 229, 230], therefore the decay to matter scalars is kinematically forbidden. As a consequence, we will not discuss the decay to matter scalars any longer.

- **Matter fermions**: the decay rate into matter fermions is chirality suppressed. This is a model independent statement [125] and the reason why we will not consider this decay channel any longer.

- **Gauge bosons**: let us consider one such gauge boson with field strength $F_{\mu\nu}$ coupled to τ_b via the interaction term $\tau_b F_{\mu\nu}F^{\mu\nu}$. The latter induces a decay rate:

$$\Gamma_{\tau_b \to AA} \sim \frac{1}{96}\frac{m_{\tau_b^3}}{M_P^2}. \tag{6.10}$$

  Notice that this rate is parametrically comparable to the decay rate into axions (6.7) or Higgs fields (6.8). However, in the sequestered LVS τ_b does not couple at tree level to visible sector gauge bosons. At loop level, the decay channel is open but obviously suppressed with respect to the decay into Dark Radiation. Crucially, the decay into gauge bosons can occur at tree level only if the associated gauge kinetic function depends on the modulus τ_b.

The most optimistic channels among the ones we have listed is the one into gauge bosons. We can envision to possibilities to overcome the difficulties of the sequestered LVS:

1 **Coupling to visible sector gauge fields**: a tree-level decay into (MS)SM gauge bosons can occur if the visible sector arises from D7-branes wrapping 4-cycles in the geometric regime. This general setup, referred to as *non-sequestered* (in the sense of supersymmetry breaking) LVS, is characterised by $m_{soft} \sim m_{3/2}$. Notice that $m_{3/2} \gtrsim \mathcal{O}(10)$ TeV to evade the CMP, therefore this setup is crucially accompanied by high-scale supersymmetry.[4] This possibility may then be particularly motivated by the absence of supersymmetric partners at the LHC at the time of writing.

2 **Coupling to hidden sector gauge fields**: alternatively, one can remain in the framework of the sequestered LVS with the possibility of low-scale supersymmetry. In this case, we can introduce so-called *flavour branes* [233]. The gauge theories living on these branes do not belong to the SM gauge group. Nevertheless, the

[4]For example, this situation arises in F-theory GUTs (for reviews see e.g. [231, 232]) with high-scale SUSY [226].

SM fields can be charged under such gauge symmetries. Crucially, the associated gauge fields can be massive if the gauge symmetry is broken at some sub-stringy scale. They then serve as intermediate states during reheating: namely, the bulk modulus decays to hidden gauge bosons, which subsequently decay to the MS(SM). Similarly to the non-sequestered case, flavour branes are 7-branes wrapping cycles in the geometric regime.

In the next section, we will analyse the second option in detail. We will then discuss more briefly some models in the framework of the non-sequestered LVS, mainly aiming at the predictions for the amount of Dark Radiation in those setups.

6.4 Dark Radiation in the LVS with flavour branes

Here we will examine the situation where the lightest modulus reheats the visible sector fields via intermediate states. We will argue that gauge bosons arising from the world-volume theory of so-called flavour branes are ideal candidates for such intermediaries.

Flavour branes are (stacks of) 7-branes in the geometric regime going through the singularity at which the SM is geometrically engineered. They are known since the very early days of 'model building at a singularity' [234] and can also be viewed as a tool for generating (approximate) global flavour symmetries of the SM.[5] Flavour branes wrap bulk cycles such that for a large bulk volume the gauge theory on their worldvolume is extremely weakly coupled. There will be visible sector states charged under the flavour brane gauge group. This gauge theory has to be spontaneously broken such that, at low scales, a global symmetry of visible sector states emerges. For state-of-the-art string model building employing flavour branes see [233].

The setup which we are considering in this case is as follows. The Calabi-Yau exhibits a large bulk cycle and a small blow-up cycle giving rise to a non-perturbative effect. These cycles are stabilised by the standard LVS procedure. The visible sector is realised by D3-branes at a singularity as in the sequestered case. However, in addition there are flavour branes, which wrap the bulk cycle but also intersect the singularity. A globally consistent realisation of such a setup in Calabi-Yau orientifolds is described in [233]. As we model the visible sector by D3-branes at a singularity, supersymmetry breaking is sequestered and gravity-mediated soft terms are suppressed w.r.t. the gravitino mass: $m_{soft} \sim M_P/\mathcal{V}^2 \sim m_{3/2}/\mathcal{V}$. However, flavour branes may affect these soft terms.

Reheating in LVS models with flavour branes

We now review the important steps in the cosmological history of the universe, which lead to the reheating of the SM in our setup.

1. As in the previous scenarios, the energy density of the universe after inflation is dominated by the lightest modulus, which is the volume modulus τ_b in LVS models[6].

[5] Approximate global symmetries in string theory can also arise from approximate isometries of the compactification space. See [35] for more details.

[6] We do not consider fibred Calabi-Yau manifolds here, where the lightest modulus can be given by a mode orthogonal to the volume.

In the following, we wish to reheat the visible sector fields via gauge bosons A_μ on flavour branes. If this scenario is to reheat the visible sector more efficiently (and thus lead to a lower ΔN_{eff}) than in the sequestered setup without flavour branes, the decay of τ_b into pairs of A_μ should be the dominant decay channel of τ_b. In the following we will proceed under this assumption. The decay rate $\Gamma_{\tau_b \to A_\mu A_\mu}$ can be determined from the tree-level interaction between τ_b and A_μ captured by the supersymmetric Lagrangian term $f_{fl} W_\alpha W^\alpha$, where the gauge kinetic function for flavour branes wrapping a bulk cycle is given by $f_{fl} = T_b$.[7] The resulting decay rate is given by

$$\Gamma_{\tau_b \to A_\mu A_\mu} = \frac{N_f}{96\pi} \frac{m_{\tau_b}^3}{M_P^2} \,, \tag{6.11}$$

where N_f is the number of generators of the flavour brane gauge theory. After τ_b has decayed the energy density of the universe is then dominated by the gauge bosons A_μ and the axions.

2. The subsequent evolution of the universe then crucially depends on the mass m_A of the flavour brane gauge bosons. Hence we will now examine the bounds on m_A.

   (a) The upper bound on the flavour brane gauge boson mass is given by $m_A = m_{\tau_b}/2$, as A_μ are then produced at threshold. To determine the subsequent development of the universe, we determine the decay rate of A_μ into SM particles. In particular, there are SM fermions which are charged under the flavour brane gauge group and the decay rate of A_μ into these fermions is given by

   $$\Gamma_{A_\mu \to f\bar{f}} \sim \alpha_f m_A \,, \tag{6.12}$$

   where $\alpha_f \sim 1/\tau_b \sim \mathcal{V}^{-2/3}$. One can now easily check that for a wide range of masses m_A below threshold the flavour brane gauge bosons decay into SM fields as soon as they are produced by decays of τ_b:

   $$\Gamma_{A_\mu \to f\bar{f}} \sim \alpha'_f m_{A'} \sim \mathcal{V}^{-2/3} m_{A'} \,, \tag{6.13}$$

   $$\Gamma_{\tau_b \to A_\mu A_\mu} = \frac{N_f}{96\pi} \frac{m_{\tau_b}^3}{M_P^2} \sim \mathcal{V}^{-3} m_{\tau_b} \,. \tag{6.14}$$

   It follows that $\Gamma_{A_\mu \to f\bar{f}} > \Gamma_{\tau_b \to A_\mu A_\mu}$ if $m_A > \mathcal{V}^{-7/3} m_{\tau_b}$. The flavour brane gauge bosons then decay into SM fields instantaneously.

   (b) If $m_A \lesssim \mathcal{V}^{-7/3} m_{\tau_b}$, the flavour brane gauge bosons will not decay instantaneously, but form a population of highly relativistic particles carrying a significant fraction of the energy density of the universe. In the end these particles still have to reheat the SM. An interesting question for the following evolution of the universe is whether the flavour brane gauge bosons become non-relativistic before they decay into SM degrees of freedom. If they become non-relativistic, their energy density will scale as matter with time, while any

[7] The decay rate $\Gamma_{\tau_b \to A_\mu A_\mu}$ depends on $\mathrm{Re}(f_{fl}) = \tau_b \sim \mathcal{V}^{2/3}$. While there are corrections to the gauge kinetic function due to fluxes such that $f_{fl} = T_b + hS$, these corrections are negligible in here, as $\mathcal{V} \gg 1$.

DR produced by the decay of τ_b earlier will scale as radiation. Consequently, the fraction of the energy density in A_μ over the energy density in DR would grow. As the flavour brane gauge bosons would eventually decay into SM fields, the relic abundance of SM fields would be enhanced with respect to the relic abundance of the axionic Dark Radiation. Correspondingly ΔN_{eff} could be further suppressed.

However, one can show that the population of A_μ will always remain relativistic until they decay. Initially the population of flavour brane gauge bosons is relativistic with energy density $\rho_A = \frac{\pi^2}{30} g_{*,A}(T)T^4$. If the temperature falls to $T \sim m_A$ the gauge bosons become non-relativistic. One can now check that T_d at which the gauge bosons decay into SM fields is always higher than m_A. To determine T_d we note that A_μ will decay when $\Gamma_{A_\mu \to f\bar{f}} = H$. As the gauge bosons are highly relativistic initially, we need to correct the decay rate into SM fermions by multiplying by a time-dilation factor for relativistic particles. This can be justified *a posteriori*, as we will show that A_μ stay relativistic until they decay. The decay rate (6.12) is modified as

$$\Gamma_{A_\mu \to f\bar{f}} \underset{\text{rel.}}{\sim} \alpha_f \frac{m_A^2}{T} , \tag{6.15}$$

The decay temperature T_d can then be determined using the following equations:

$$3H^2 M_P^2 = \rho_A = \frac{\pi^2}{30} g_{*,A}(T_d) T_d^4 , \quad H = \Gamma_{A_\mu \to f\bar{f}} \underset{\text{rel.}}{\sim} \alpha_f \frac{m_A^2}{T_d} , \tag{6.16}$$

leading to

$$T_d = \left(\frac{90}{\pi^2 g_{*,A}} \right)^{\frac{1}{6}} \left(\frac{M_P}{m_A} \alpha_f \right)^{\frac{1}{3}} m_A . \tag{6.17}$$

We recall that for the gauge bosons not to decay instantly when produced their mass had to be small: $m_A \lesssim \mathcal{V}^{-7/3} m_{\tau_b} \sim \mathcal{V}^{-23/6} M_P$. It then follows that $T_d > m_A$ and flavour brane gauge bosons always remain relativistic.

(c) While the upper bound on the gauge boson mass m_A is set by the kinematics of the decay of τ_b we want to examine whether there is a cosmological lower bound on m_A. In particular, we will require that when reheating the SM through the decay of flavour brane gauge bosons, the reheating temperature of the SM is $T_{SM} \gtrsim \mathcal{O}(1)$ MeV to allow for standard BBN. To determine the decay temperature of the SM we recall that A_μ will decay into SM fields when $\Gamma_{A_\mu \to f\bar{f}} = H$. We have

$$3H^2 M_P^2 = \rho_{SM} = \frac{\pi^2}{30} g_{*,SM}(T_{d,SM}) T_{d,SM}^4 , \tag{6.18}$$

$$H = \Gamma_{A_\mu \to f\bar{f}} \underset{\text{rel.}}{\sim} \alpha_f \frac{m_A^2}{T_{d,A}} . \tag{6.19}$$

To relate $T_{d,A}$ to $T_{d,SM}$ we recall that the comoving entropy density $s = g_* a^3 T^3$ is conserved when A_μ decays:

$$T_{d,A} = \left(\frac{g_{*,SM}(T_{d,SM})}{g_{*,A}(T_{d,A})} \right)^{\frac{1}{3}} T_{d,SM} \ . \tag{6.20}$$

Putting (6.18) and (6.19) together one finds

$$m_A = \left(\frac{\pi^2 g_{*,SM}}{90} \right)^{\frac{1}{3}} \left(\frac{g_{*,SM}}{g_{*,A}} \right)^{\frac{1}{6}} \alpha_f^{-1/2} \sqrt{\frac{T_{d,SM}}{M_P}} T_d \ . \tag{6.21}$$

Standard BBN requires $T_d \gtrsim \mathcal{O}(1)$ MeV and thus we find the following lower bound on m_A:

$$m_A \gtrsim \mathcal{V}^{1/3} \sqrt{\frac{1 \text{ MeV}}{M_P}} \text{ MeV} \ . \tag{6.22}$$

Given that we require $\mathcal{V} \lesssim 10^{14}$ to evade the Cosmological Moduli Problem, it is clear that the cosmological lower bound on m_A is very low. As a result, our setup can successfully reheat the SM – for a wide range of masses from threshold to the lower limit shown above.

While constraints from reheating allow very light weakly coupled vector bosons there will be further constraints on the parameter space of such particles from collider experiments and precision measurements.

Beyond cosmological constraints, there are also consistency conditions on the string construction. The setup of visible sector and flavour branes has to satisfy local tadpole cancellation conditions. In addition, the local D-brane charges of the flavour branes at the intersection locus with the visible sector have to originate from restrictions of charges of globally well-defined D7-branes. While these consistency conditions do not determine a unique setup of allowed flavour branes, they constrain the number of flavour branes allowed given a particular visible sector [233].

Predictions for Dark Radiation

Here we determine the decay rates of the lightest modulus into Dark Radiation and Standard Model fields. The lightest modulus is the bulk volume modulus as in the sequestered case. The rate of decays of τ_b into its associated axion can be determined from $K = -3\ln(T_b + \bar{T}_b)$ and gives the familiar result obtained before (6.7).

As argued before, decays of τ_b into gauge bosons on flavour branes lead to a direct reheating of the Standard Model. As flavour branes wrap bulk cycles, there is a tree-level coupling between τ_b and the gauge bosons on the flavour brane through the kinetic term $f_{fl} W_\alpha W^\alpha$. For flavour branes on the bulk cycle $f_{fl} = T_b$, where we ignore any flux-induced corrections. Alternatively, we can locate flavour branes on other large cycles τ_i which intersect the visible sector. The ratio τ_i/τ_b is then stabilised by D-terms leading to $f_{fl} = T_i = cT_b$ with $c \sim \mathcal{O}(1)$.

Last, there can also be direct decays of the volume modulus into visible sector matter fields. As described in section 6.2 the dominating decay channel is given by the interaction

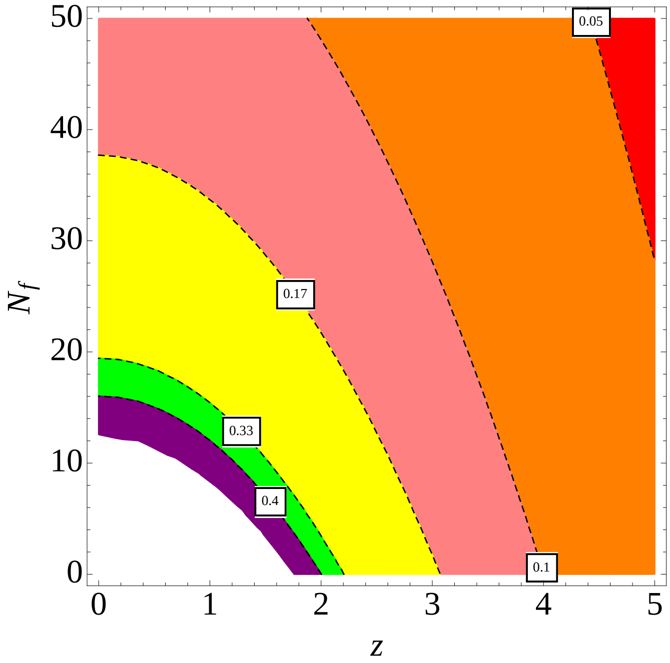


Figure 6.1: Contour plot of ΔN_{eff} vs. z and the number of gauge bosons N_f of the gauge theory living on the stack of flavour branes wrapping the bulk cycle. The plot was produced for $g_* = 75.75$, corresponding to a reheating temperature $T_d \simeq 1$ GeV.

of τ_b with Higgs fields, which arises from the Giudice-Masiero term $(zH_uH_d + \text{h.c.})$ in the Kähler potential. In contrast to the non-sequestered setups studied before, here both Higgs scalars are light enough to be produced by decays of τ_b leading to a decay rate (6.8).

Overall, we find the following decay rates for the volume modulus:

Decays into DR:
$$\Gamma_{\tau_b \to a_b, a_b} = \frac{1}{48\pi} \frac{m_{\tau_b}^3}{M_P^2} , \tag{6.23}$$

Decays into SM:
$$\Gamma_{\tau_b \to A_\mu^{flavour} A_\mu^{flavour}} = \frac{N_f}{96\pi} \frac{m_{\tau_b}^3}{M_P^2} , \tag{6.24}$$

$$\Gamma_{\tau_b \to H_u H_d} = \frac{2z^2}{48\pi} \frac{m_{\tau_b}^3}{M_P^2} , \tag{6.25}$$

where N_f is the number of generators of the flavour brane gauge group.

Thus we find the following expression for the effective number of neutrino species:

$$\Delta N_{eff} = \frac{43}{7} \left(\frac{10.75}{g_*(T_d)} \right)^{\frac{1}{3}} \frac{1}{\frac{N_f}{2} + 2z^2} . \tag{6.26}$$

We plot ΔN_{eff} as a function of z and N_f, with $g_* = 75.75$, in figure 6.1. One finds that the bound $\Delta N_{eff} < 0.17$ can be achieved without any restrictions on the Higgs sector as long as $N_f > 38$. For $z = 1$ the DR bound $\Delta N_{eff} < 0.17$ requires at least $N_f > 35$ gauge bosons. For $\Delta N_{eff} < 0.33$ one needs $N_f > 19$ for $z = 0$ and $N_f > 17$ for $z = 1$. Thus for a number of flavour branes as small as 10 current bounds on DR can be easily met.

ΔN_{eff}	N_f	
	$z = 0$	$z = 1$
< 0.40	> 16	> 12
< 0.33	> 19	> 17
< 0.17	> 38	> 35
< 0.10	> 65	> 61
< 0.05	> 128	> 124

Table 6.1: Minimum number of gauge bosons N_f on flavour branes needed to evade upper bound on ΔN_{eff} for $z = 0$ and $z = 1$

If bounds on ΔN_{eff} become more restrictive, we have the following options. If we do not wish to impose constraints on the Higgs sector of the model, we require a larger number of generators on flavour branes. A table of the minimum numbers of gauge bosons needed for fixed z given an upper bound on ΔN_{eff} is shown in 6.1. However, conditions for globally consistent models restrict the maximum number of flavour branes for a given visible sector [233] and thus place an upper limit on the allowed number of gauge bosons. The exact constraints on the numbers of flavour branes will depend on the details of the individual model. While we cannot be more specific it follows that lower bounds on ΔN_{eff} cannot necessarily be evaded by simply introducing more flavour branes.

6.5 Dark Radiation in the non-sequestered LVS

In the previous section we studied how to boost the decay rate into SM fields by coupling the bulk modulus τ_b to gauge bosons which do not belong to the SM gauge group. In this section instead we will focus on setups where τ_b decays at tree level to visible sector gauge fields. As we already mentioned in section 6.3, this can be achieved by realising the visible sectors on $D7$-branes wrapping 4-cycles in the geometric regime.

We will discuss three options to stabilise such 4-cycles: through a flux-induced D-term potential, through string loop corrections and through non-perturbative effects. Here, we do not aim at a detailed description of the necessary model building, which can be found in [129]. Rather, our aim is to derive predictions for the amount of Dark Radiation in these setups. Comparison with results from other realisations of non-sequestered LVS [235], [236] can be found in [129] and in [164].

6.5.1 Stabilisation through D-terms

Let us consider a CY X, exhibiting several 4-cycles as well as 2-cycles. We use standard notations τ_i and t_i for the volume moduli of the 4- and 2-cycles respectively. In this subsection we wish to stabilise all but two 4-cycles in a geometric regime using a D-term potential. The remaining two 4-cycles are then identified with the "small" cycle τ_s and the bulk cycle τ_b stabilised à la LVS. In particular, we will focus on geometries which lead to the usual Swiss-Cheese-type volume:

$$\mathcal{V} = \alpha(\tau_b^{3/2} - \gamma\tau_s^{3/2}). \tag{6.27}$$

A D-term potential can arise from a anomalous $U(1)$ symmetries living on $D7$-branes wrapping the 4-cycles of X. In this case, it takes the form:

$$V_D = \sum_i \frac{g_i^2}{2}\left(\sum_j c_{ij}|\phi_j|^2 - \xi_i\right)^2 . \tag{6.28}$$

Here ϕ_j are open string states charged under the $U(1)$ symmetry and ξ_i are FI-terms, given by

$$\xi_i = \frac{1}{4\pi\mathcal{V}} q_{ij} t^j, \tag{6.29}$$

where q_{ij} are the charges of the Kähler superfields T_i under the anomalous $U(1)$ [237,238]. Notice the double sum in (6.28): the index i runs over all 7-branes, while j specifies a given open string state. Stabilisation proceeds by imposing $\xi_i = 0$ for all i.[8]

We now briefly outline the strategy to stabilise the visible sector cycle, whose volume modulus will be denoted by τ_a, in terms of the bulk cycle. In particular, we will be able to achieve $\tau_a = c_a\tau_b$, where c_a is a numerical prefactor.

1 First, we fix all but two 2-cycles by imposing $\xi_i = 0$ for all i. In order to do so, we assume that one 2-cycle t^1 does not appear in the FI terms (6.29) and is left unstabilised at this point. Then the D-term condition generically leaves one other 2-cycle t^2 unfixed.

2 Correspondingly, two 4-cycles are left unfixed. This can be shown by means of the following important relation between 4- and 2-cycles, valid for CY threefolds:

$$\tau_i = \frac{\partial\mathcal{V}}{\partial t^i} = \frac{1}{2}k_{ijk}t^j t^k, \tag{6.30}$$

where k_{ijk} are the triple intersection numbers of X. By an appropriate choice of geometries, one can ensure w.l.o.g. that all the 4-cycles, except for τ_s are fixed in terms of the bulk cycle τ_b. In particular, $\tau_a = c_a\tau_b$, as desired.

A complete description of our stabilisation procedure is described in [129] (see also [238] for an explicit example of such a construction). The crucial feature of this model is the desired coupling between the bulk modulus and the SM gauge bosons, which now arises from the supersymmetric Lagrangian of the visible sector gauge theory:

$$\mathcal{L} \supset \int \mathrm{d}^2\theta\ T_a W_\alpha W^\alpha = \int \mathrm{d}^2\theta\ c_a T_b W_\alpha W^\alpha = c_a\tau_b F_{\mu\nu}F^{\mu\nu} + c_a a_b F_{\mu\nu}\tilde{F}^{\mu\nu} . \tag{6.31}$$

As usual, a_b is almost massless and therefore contributes to N_{eff}. Let us now pause to make a comment on the value of the coefficient c_a. Notice that the VEV of τ_a has to be fixed small in order to produce the Standard Model gauge coupling, i.e. $\alpha_{SM}^{-1} = \langle\tau_a\rangle \approx 25$ (neglecting flux contributions so far). Since in the LVS $\langle\tau_b\rangle \gg 1$, a potentially severe tuning of c_a is required.

[8]That is, we do not appeal to VEVs of charged fields.

Predictions for Dark Radiation

We now compute the relevant decay rates and obtain prediction for Dark Radiation in this setup. The Kähler potential (see [239], [226] and [129] for more details) and gauge kinetic functions appropriate for our setup are

$$\begin{aligned} K = & -3\ln(T_b+\bar{T}_b)+ \qquad (6.32)\\ & +\frac{(T_a+\bar{T}_a)^{1/2}}{T_b+\bar{T}_b}\left(H_u\bar{H}_u+H_d\bar{H}_d\right)+\frac{(T_a+\bar{T}_a)^{1/2}}{T_b+\bar{T}_b}\left(zH_uH_d+\text{h.c.}\right)+\dots\ , \\ f_a = & \ T_a+hS\ . \qquad (6.33)\end{aligned}$$

As shown above, D-term stabilisation imposes $T_a = c_a T_b$. The decay of τ_b into the heavy Higgs is kinematically forbidden, because $m_{soft} \gg m_{\tau_b}$ in the non-sequestered LVS. Therefore the remaining decay channels with their associated rates are:

Decays into DR: $$\Gamma_{\tau_b\to a_b a_b} = \frac{1}{48\pi}\frac{m_{\tau_b}^3}{M_P^2}\ , \qquad (6.34)$$

Decays into SM: $$\Gamma_{\tau_b\to hh} = \frac{z^2}{96\pi}\frac{\sin^2(2\beta)}{2}\frac{m_{\tau_b}^3}{M_P^2}\ , \qquad (6.35)$$

$$\Gamma_{\tau_b\to AA} = \frac{N_g}{96\pi}\,\gamma^2\,\frac{m_{\tau_b}^3}{M_P^2}\ . \qquad (6.36)$$

Here N_g is the number of gauge bosons, $\tan\beta$ is the ratio of Higgs VEVs and

$$\gamma \equiv \frac{\tau_a}{\tau_a + h\ \text{Re}(S)}\ . \qquad (6.37)$$

Notice that h quantifies the effect of fluxes on the gauge kinetic function f_a. Finally, using (6.34), (6.35) and (6.36) we obtain:

$$\Delta N_{\text{eff}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{1/3}\frac{\Gamma_{\tau_b\to DR}}{\Gamma_{\tau_b\to SM}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{1/3}\frac{1}{\frac{\sin^2(2\beta)}{4}z^2+\frac{N_g}{2}\gamma^2}\ . \qquad (6.38)$$

We will consider the minimal (MS)SM field content, with $N_g = 12$ and only one light Higgs. Furthermore, motivated by high-scale supersymmetry, we take $\sin(2\beta) = 1$ [225, 240–242]. Notice that this choice also minimises ΔN_{eff}, which exhibits a mild dependence on g_*. In table 6.2 we provide examples of values of g_* with the corresponding values of T_d and m_{τ_b}.

Predictions for Dark Radiation are shown in figure 6.2 (a) and (b) for $g_* = 10.75$ and $g_* = 106.75$ respectively, as a function of the parameters z and γ. The crucial difference with respect to the situation presented in section 6.2 is that the current observational bound $\Delta N_{eff} < 0.17$ ($\Delta N_{eff} < 0.33$) can be satisfied with $z \lesssim 1$, as long as $\gamma \gtrsim 2.4$ ($\gamma \gtrsim 1.7$) for $g_* = 10.75$ or $\gamma \gtrsim 1.6$ ($\gamma \gtrsim 1.2$) for $g_* = 106.75$. According to (6.37) the most stringent bound requires a mild fine-tuning (to 1 part in 2) between τ_b and $h\text{Re}(S)$. Future observations may increase the delicacy of this cancellation as shown in figure 6.2.

m_{τ_b} [GeV]	T_d	g_*
$5 \cdot 10^5$	50 MeV	10.75
$2 \cdot 10^6$	300 MeV	61.75
10^7	3 GeV	75.75
$\geq 2 \cdot 10^8$	$\geq$ 200 GeV	106.75

Table 6.2: Number of degrees of freedom corresponding to different reheating temperatures obtained for different masses of the modulus τ_b. The reheating temperature is determined as $T_d = (1 - B_a)^{1/4}(\pi^2 g_*/90)^{-1/4}\sqrt{\Gamma_{\tau_b,total}M_P}$, where B_a is the branching ratio for decays of τ_b into axions. The values of T_d are obtained by considering the very conservative branching ratio $B_a \simeq 0.1$.

The bulk axion as Dark Matter

So far we have considered the axionic partner of the bulk modulus τ_b as Dark Radiation. However, a natural the supersymmetric interaction (6.31) includes a topological coupling between a_b and the SM gauge fields. In particular, a_b couples to QCD gluons. Thus, after moduli stabilisation, non-perturbative QCD effects will generate a potential for a_b, which essentially takes the rôle of a QCD axion. The consequences are as follows: a_b is produced both as Dark Radiation via the decay of τ_b and as Dark Matter through the axion misalignment mechanism. In particular, the abundance of axion Dark Matter is determined by the coupling of a_b to QCD gluons. More specifically, the full effective lagrangian for a_b is (see e.g. [243, 244]):

$$\mathcal{L} \supset K_{T_b\bar{T}_b}\partial_\mu a_b \partial^\mu a_b + \frac{c_a \tau_b}{4\pi}F_{\mu\nu}F^{\mu\nu} + \frac{c_a a_b}{4\pi}F_{\mu\nu}\tilde{F}^{\mu\nu} \ . \tag{6.39}$$

The axion decay constant can be found by canonically normalisinga_b and A_μ. The coupling between a_b and A_μ then reads

$$\mathcal{L} \supset \frac{g^2}{32\pi^2}\frac{c_a a_b}{f_{a_b}}F_{\mu\nu}\tilde{F}^{\mu\nu} \ , \tag{6.40}$$

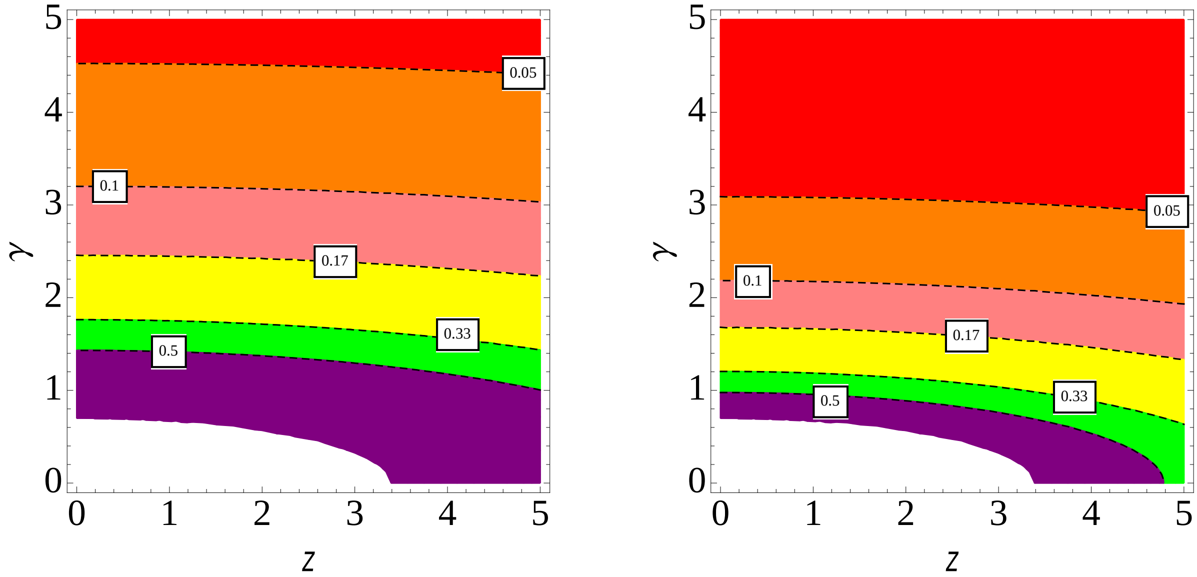
where $g^{-2} = c_a\tau_b/4\pi$ and $f_{a_b} = \sqrt{K_{T_b}\bar{T}_b}/2\pi$. Observationally, the effective decay constant f_a/c_a should lie in the window:

$$10^9\text{GeV} < \frac{f_{a_b}}{c_a} < 10^{12}\text{GeV}, \tag{6.41}$$

to avoid overcooling of stars and overproduction of dark matter [245]. The upper bound in (6.41) can be evaded if the PQ symmetry is broken before inflation. In this case indeed the axion relic density is given by (see e.g. [245]):

$$\Omega_a h^2 \sim 3 \times 10^3 \left(\frac{f_a/c_a}{10^{16}\text{GeV}}\right)^{7/6} \theta_i^2 \ , \tag{6.42}$$

where $\theta_i = \frac{a_{b,initial}}{f_{a_b/c_a}} \in [-\pi, \pi)$. Imposing $\Omega_a \lesssim \Omega_{DM}h^2 = 0.1199$ [2] we obtain the upper bound of (6.41) for $\theta_i \sim \mathcal{O}(1)$. Alternatively, by tuning the initial misalignment angle θ_i



(a) (b)

Figure 6.2: Contour plot of ΔN_{eff} vs. z and γ (defined in (6.37)), with (a) $g_* = 10.75$ and (b) $g_* = 106.75$. While the predictions are valid for $\sin(2\beta) = 1$, they can be reinterpreted for general values of $\sin(2\beta)$. To this end we define an effective parameter $\tilde{z}^2 = \sin^2(2\beta) z^2$ and relabel $z \to \tilde{z}$ on the horizontal axis.

we can allow $f_a > 10^{12}$ GeV.[9] Notice that (6.42) is valid only if the axion starts oscillating before the QCD phase transition. This requires $f_a \lesssim 10^{16}$ GeV.

Unfortunately, we indeed have to require such a tuning of θ_a. Indeed the effective axion decay constant in our setup will violate the upper bound in(6.41), as the following computation shows:

$$\frac{f_{a_b}}{c_a} = \frac{\sqrt{K_{T_b \bar{T}_b}} M_P}{2\pi c_a} = \frac{\sqrt{3} M_P}{4\pi c_a \tau_b} \sim 10^{16}\ \text{GeV} \quad \text{for} \quad c_a \tau_b = \alpha_{vis}^{-1} \sim 25\ . \tag{6.43}$$

Therefore, we need to impose $\theta_i \sim 10^{-2}$ to evade overproduction of DM.

6.5.2 Further constructions

In this subsection we will briefly discuss two other options to stabilise the visible sector cycle in the non-sequestered LVS.

Stabilisation by string loop corrections

As an alternative to stabilisation by D-terms, we can employ string loop effects. The advantage with respect to the case discussed in the previous section is that the flux tuning required to have a small c_a can be avoided. Stabilisation by string loop effects

[9]See [246] for an anthropic justification of such a tuning.

is better understood for fibred Calabi-Yau manifolds, whose volume is generically of the form

$$\mathcal{V} = \alpha\Big(\sqrt{\tau_1}\tau_2 - \gamma_{np}\tau_{np}^{3/2}\Big). \tag{6.44}$$

The overall volume $\mathcal{V}$ and the modulus τ_{np} are stabilised à la LVS, such that $\mathcal{V} \approx \sqrt{\tau_1}\tau_2$. The remaining flat direction χ, corresponding to simultaneous changes in τ_1 and τ_2 that leave the overall volume invariant, will be stabilised by string loop corrections as in [247]. The moduli χ and $\mathcal{V}$ can have comparable masses. Here we will consider the simpler case when τ_1 is not fixed too small and χ is the lightest modulus, so that $\mathcal{V}$ can be integrated out.

We realise the visible sector by means of D7 branes on the fiber τ_1. As discussed in [129] and [238, 248], this choice leads to the so-called "anisotropic limit" $\tau_2 \gg \tau_1 \gg \tau_{np}$.[10] As in the previous section, the following coupling between τ_1 and the visible sector gauge fields arises:

$$\mathcal{L} \supset \tau_1 F_{\mu\nu}F^{\mu\nu} + a_1 F_{\mu\nu}F^{\mu\nu}. \tag{6.45}$$

In this setup there are at least (if only one cycle is stabilised via string loop effects) two light axions to which χ can decay: the axionic partners of τ_1 and τ_2 respectively. After integrating out $\mathcal{V}$, we have $\tau_2 = \alpha^{-1}\mathcal{V}\tau_1^{-1/2}$ and χ is defined by canonically normalising τ_1 (see also [249])

$$\tau_1 = e^{\frac{2}{\sqrt{3}}\chi}. \tag{6.46}$$

The effective Lagrangian for the canonically normalised χ is

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}e^{-\frac{4}{\sqrt{3}}\chi}\partial_\mu a_1'\partial^\mu a_1' + \frac{1}{2}e^{\frac{2}{\sqrt{3}}\chi}\partial_\mu a_2'\partial^\mu a_2' \ . \tag{6.47}$$

The Kähler potential for the Higgs fields and the gauge kinetic function are given by

$$K \supset \frac{(T_1+\bar{T}_1)^{1/2}}{\mathcal{V}^{2/3}}\Big(H_u\bar{H}_u + H_d\bar{H}_d\Big) + \frac{(T_1+\bar{T}_1)^{1/2}}{\mathcal{V}^{2/3}}\left(zH_uH_d + \text{h.c.}\right) + \ldots \tag{6.48}$$

$$f_{vis} = T_1 + hS \ . \tag{6.49}$$

The modulus χ decays to the light Higgs, gauge fields and Dark Radiation due to (6.46), (6.48), (6.49) and (6.47). The corresponding rates are

Decays into DR:

$$\Gamma_{\chi\to a_1,a_1} = \frac{1}{24\pi}\frac{m_\chi^3}{M_P^2} \tag{6.50}$$

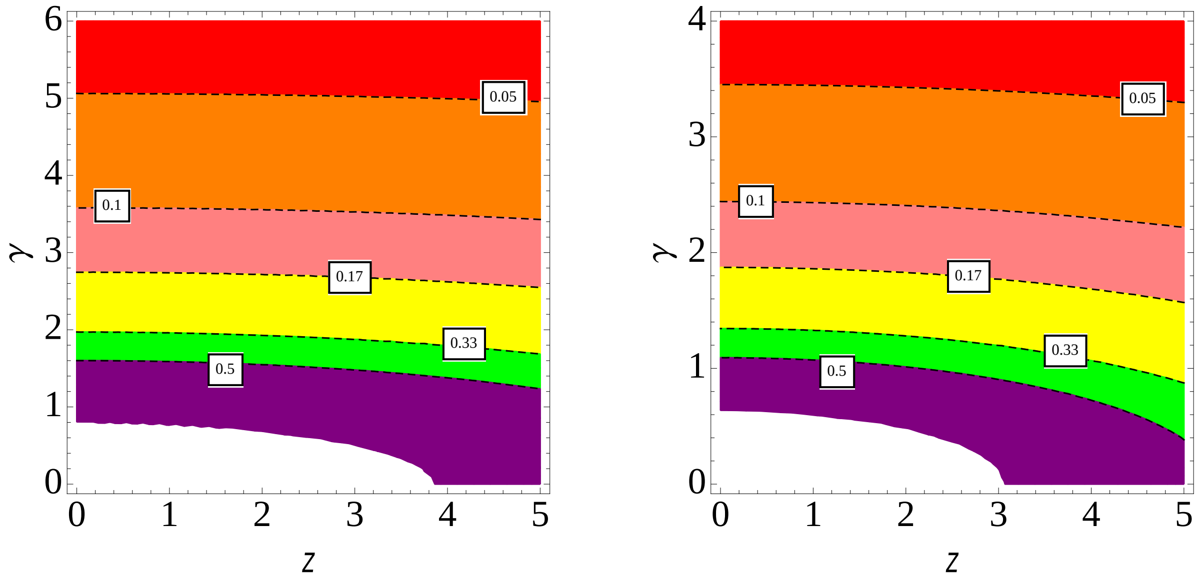
$$\Gamma_{\chi\to a_2,a_2} = \frac{1}{96\pi}\frac{m_\chi^3}{M_P^2} \tag{6.51}$$

Decays into SM:

$$\Gamma_{\chi\to h_1,h_1} = \frac{z^2\sin^2(2\beta)}{96\pi}\frac{m_\chi^3}{M_P^2} \tag{6.52}$$

$$\Gamma_{\chi\to A_1,A_1} = \frac{N_g}{48\pi}\gamma^2\frac{m_\chi^3}{M_P^2}, \tag{6.53}$$

[10] Alternatively, as discussed in [129], one can wrap the $D7$ branes on another cycle τ_a, whose size is then set by τ_1 via D-terms, as in the previous section.



(a) (b)

Figure 6.3: Contour plot of ΔN_{eff} vs. z and γ (defined in (6.37)) for an LVS model with cycles stabilised by string loop corrections and D-terms, for (a) $g_* = 10.75$ and (b) $g_* = 106.75$. As before, the plots have been produced choosing $\sin(2\beta) = 1$, but they can be reinterpreted for general values of $\sin(2\beta)$. To this end we define an effective parameter $\tilde{z}^2 = \sin^2(2\beta) z^2$ and relabel $z \to \tilde{z}$ on the horizontal axis.

where γ is defined as in (6.37). Finally, for $N_g = 12$ and $\sin(2\beta) = 1$, ΔN_{eff} is given by

$$\Delta N_{\rm eff} = \frac{43}{7} \left(\frac{10.75}{g_*(T_d)} \right)^{1/3} \frac{\Gamma_{\chi \to DR}}{\Gamma_{\chi \to SM}} = \frac{43}{7} \left(\frac{10.75}{g_*(T_d)} \right)^{1/3} \frac{5}{z^2 + 24\gamma^2} \,. \tag{6.54}$$

Predictions for Dark Radiation according to (6.54) are shown in figure 6.3 (a) and (b) for $g_* = 10.75$ and $g_* = 106.75$ respectively. The situation is very similar to the case of stabilisation by D-terms. In order to satisfy the current observational bound $\Delta N_{eff} < 0.17$, we need $\gamma > 2.7(1.8)$ for $g_* = 10.75(106.75)$ and $z = 0$. For $\Delta N_{eff} < 0.33$, we need $\gamma > 1.9(1.3)$ for $g_* = 10.75(106.75)$. Therefore once again a mild cancellation between τ_1 and $h\mathrm{Re}(S)$ is required. Furthermore, due to the topological coupling to QCD in (6.45), the axion a_1 will be produced also as Dark Matter. As in the previous section, a tuning of the initial misalignment angle is required to avoid overproduction of Dark Matter. To summarise, stabilisation via string loop effects is useful to avoid some potentially severe tunings which characterise stabilisation via D-terms. Predictions for Dark Radiation are instead essentially unchanged with respect to the previous section.

Stabilisation by non-perturbative effects

In principle there exists yet another option to stabilise the visible sector cycle: that is, to use non-perturbative terms in the superpotential (see [250], [251] for a discussion of possible complications associated to this procedure). Sources of such terms can be e.g.

D-brane instantons or gaugino condensates . If the non-perturbative effects depend on the visible sector modulus τ_{vis}, minimisation of the F-term scalar potential fixes the latter in terms of τ_b, thereby providing the desired coupling between the bulk modulus and the visible sector gauge bosons. However, this construction is more heavily constrained by the bound on Dark Radiation than the other options presented in this chapter. This option is explored in [129] and reviewed in [164].

6.6 Summary and Conclusions

Current CMB bounds on the presence of Dark Radiation (DR) can have an important impact on physics Beyond the Standard Model. In this chapter, we focused on the implications on the framework of moduli stabilisation in string compactifications, known as the Large Volume Scenario (LVS). Some amount of Dark Radiation is a generic prediction of this setup.

More specifically, the lightest modulus τ_b, associated to the volume of the bulk cycle of the compactification manifold, is generically responsible for reheating of SM degrees of freedom. Crucially, it can decay to its axionic partner a_b, which is almost massless and therefore constitutes Dark Radiation. Thus the deviation ΔN_{eff} from the Standard Model effective number of relativistic species at CMB temperature, $N_{eff} = 3.046$, is proportional to the ratio between the rates of the decays of τ_b into a_b and into SM particles.

We started by reviewing constraints on the so-called sequestered LVS, where the visible sector is realised by $D3$ branes at a singularity. This setup is phenomenologically interesting because it allows for TeV-scale superpartners. The modulus τ_b reheats the SM by decaying mainly to Higgs doublets. The coupling between τ_b and the Higgs fields is set by a Giudice-Masiero term with coefficient z. The current bound $\Delta N_{eff} < 0.17$ then requires: to extend the MSSM field content by allowing for $n_H > 20$ Higgs doublets if $z = 1$; alternatively $z > 3$ for $n_H = 2$. For $\Delta N_{eff} < 0.33$ one needs $n_H > 10$ if $z = 1$ and $z > 2$ for $n_H = 2$. While these values of n_H and z may be realisable in this framework, it is not clear whether they occur naturally in the string landscape.

Obviously, the amount of DR can be suppressed by boosting the decay rate of τ_b into SM particles. In particular, we showed that the decay rate into gauge bosons is parametrically comparable to the decay into DR and Higgs fields. In this chapter we proposed the following options to alleviate the constraints on the LVS via decay into gauge bosons:

- **Couple the bulk modulus to non-SM gauge bosons**: this can be done in the sequestered LVS by introducing *flavour branes*. These are stacks of 7-branes in the geometric regime which wrap the bulk cycle and cross the singularity where the SM leaves. The gauge symmetry associated to the flavour branes is spontaneously broken and the corresponding gauge bosons become massive. The bulk modulus can decay into these gauge bosons if they are light enough. The latter then decay into SM particles and reheating takes place. The most stringent bound on DR with the MSSM field content in the visible sector is satisfied for $N_f = 35 - 38$ flavour gauge bosons.

- **Couple the bulk modulus to SM gauge bosons**: in the sequestered case such a coupling is loop-suppressed. We therefore considered non-sequestered constructions, where the visible sector arises from 7-branes wrapping a cycle in a geometric regime. We argued that this setup requires high-scale supersymmetry. While the bulk cycle and another small cycle of the compactification manifold are stabilised à la LVS, we examined two possibilities to stabilise the visible sector cycle, with volume modulus τ_{vis}:

  1 **D-terms**: in this case the visible sector cycle is stabilised by means of a D-term potential induced by fluxes on the D7-branes. The gauge kinetic function of the visible sector gauge theory is given by $f_{vis} \supset \tau_{vis} + h\text{Re}(S)$, where S is the axio-dilaton and h is a flux-dependent parameter. Stabilisation fixes $\tau_{vis} = c\tau_b$. Therefore the bulk modulus is coupled to visible sector gauge bosons. The most stringent DR bound can be evaded if we allow for a very mild fine-tuning between τ_{vis} and $h\text{Re}S$ to 1 part in 2-3. However, two further tunings are required. Firstly, c has to be taken small to reproduce the correct SM gauge coupling: this amounts to a potentially severe tuning of fluxes, since $\tau_b \gg \tau_{vis}$. Secondly, in this setup a_b couples topologically to QCD and takes the rôle of the QCD axion. Therefore it is also produced as Dark Matter by the misalignment mechanism. In order to avoid DM overproduction, the initial misalignment angle has to be tuned $\theta_i \sim 10^{-2}$.

  2 **String-loop corrections**: in this case the visible sector cycle is stabilised by string loop effects. In fibred CY threefolds with LVS stabilisation, there can be a flat direction which is lighter than the bulk volume. The visible sector cycle modulus τ_{vis} can be then fixed by string-loop effects in terms of this light modulus. Therefore, the latter couples to visible sector gauge bosons. The constraints imposed by the DR bound are very similar to the case of D-term stabilisation, but the tuning of c is avoided.

Yet another possibility, which we did not discuss in this chapter, is to consider the decay of the bulk modulus into non-MSSM singlets, mediated by Giudice-Masiero couplings.

The constructions that we proposed in this chapter all exhibits a certain amount of complexity (large number of gauge bosons for flavour branes, tunings of parameters for the non-sequestered LVS). This can in turn be interpreted as a proof of the constraining power of the measurement of N_{eff} on string constructions based on the LVS.

Conclusions

Summary and outlook

In this thesis we explored consequences of quantum gravity and string-motivated scenarios of physics beyond the Standard Model for large field inflation and reheating. Concerning inflation, we used both compactifications of Type IIB String Theory and effective field theory arguments to assess the effects of UV degrees of freedom on the flatness of the inflationary trajectory. With regard to reheating, we investigated exotic signatures of models of axion monodromy inflation and CMB constraints on stringy dark sectors.

We began with an introductory chapter, which set the stage for the work presented in the rest of the thesis. In particular, we described the most important features of Type IIB string compactifications, with special attention to geometric moduli and their stabilisation. We also reviewed the statement of the Weak Gravity Conjecture (WGC) and its applications to models of axion inflation.

Part I was devoted to the effects of stringy degrees of freedom on Large Field Inflation (LFI). The general framework is that of compactifications of 10D Type IIB String Theory on six-dimensional Calabi-Yau orientifolds. More specifically, the focus is on models where inflation is realised in the complex structure moduli sector of 4D string-derived $\mathcal{N} = 1$ supergravities with F-term potential, in the spirit of [101–103]. The axionic (i.e. imaginary) parts of these moduli enjoy shift-symmetries at the level of the Kähler potential, whenever the corresponding real partners (saxions) are fixed at Large Complex Structure (LCS). We examined two sources of limitations on transplanckian field displacements in this setup: namely, backreaction of other geometric moduli and non-perturbative corrections to the shift-symmetric geometry.

In order to study backreaction, in chapter 2 we identified the inflaton with a complex structure axion $y \equiv \text{Im}(u)$ in the LCS regime. We considered the F-term scalar potential generated by a linear flux-induced superpotential $W \supset a(z)u$, where z are other complex structure moduli, which do not need to be in the LCS regime. Rather than as a fully realistic model of inflation, this construction should be considered as a setup where to assess the feasibility of transplanckian displacements. We noticed the necessity of tuning both $a(z)$ and its derivatives $\partial_z a$ small to ensure that the shift symmetry is only weakly broken. We noticed that there exist two options to tune the coefficients of the quadratic term of the inflaton potential. We then investigated the effects of backreaction of the other complex structure moduli on the inflationary trajectory, by expanding the potential at leading order in the moduli displacements δz and in a set of fine-tuned small parameters ϵ. In both tuning scenarios we found a transplanckian region in field space ($\Delta y \sim \mathcal{O}(10)M_P$ can be easily accommodated in both cases) where the displacements remain small, in particular $\delta z \sim \epsilon^2$ or $\delta z \sim \epsilon$ depending on the specific choice of tuning. Nonetheless, in this regime they induce a non-negligible correction to the inflationary potential. In particular, the backreacted potential is still quadratic but the inflaton mass is smaller than its original value. Moreover, local minima with positive energy density can appear in the post-inflationary region of the potential, which may thus also offer new possibilities for uplifting to de Sitter space. Kähler moduli backreaction was also briefly studied in one of the two tuning scenarios. In this latter case, it seems possible to achieve a flat direction, but the detailed phenomenology of the resulting inflationary model might be different from the quadratic case.

In chapter 3 we considered a setup with two complex structure axions. By means of a flux-induced F-term scalar potential, we fixed a flat trajectory which winds multiple times around the compact field space of the two axions, á la Kim-Nilles-Peloso (KNP) [31]. Deviations from the LCS regime manifest themselves as exponentially suppressed, instanton-like, oscillations in the Kähler potential and in the superpotential. They generate a scalar potential on the flat winding trajectory, which is of cosine type. Its periodicity can be made transplanckian (in particular larger than $\approx 7M_P$, as required by observations [15]) by an appropriate choice of fluxes, but is limited by tadpole cancellation.

Perhaps the most interesting aspect of our proposal is that it realises a well-known loophole of the WGC. More precisely, in our model there are two type of instanton-like oscillations, associated to the original two complex structure axion. We assumed that moduli can be stabilised such that the term with a short, subplanckian, periodicity is negligible and does do not endanger the flatness of the inflationary trajectory. However, it is precisely this term that satisfies the WGC, which is thus overall fulfilled only in its *mild version*: i.e. not by the "lightest" instanton (i.e. the one with smallest action). In this sense, our model suggests that only the mild form of the conjecture is respected by ST.

Overall, in Part I we did not find fundamental obstructions to large field inflation: backreaction and exponentially suppressed deviations from the LCS do not spoil the flatness of the inflationary potential in the models that we considered. This is true across a limited but transplanckian field range. Nevertheless, we also found that our model of axion monodromy inflation requires a potentially very severe, albeit realisable, tuning of parameters. We believe that this cumbersomeness concerns, at least partially, other realisations of axion monodromy as well.

The WGC was the main focus of chapter 4, where we applied it to models of axion monodromy inflation. We proposed an effective description of its potential (in the case in which monodromy induces a mass term) in terms of a 3-form gauge potential coupled to domain walls. The latter separate the axionic wells which are characteristic of this class of models. We showed that an upper bound on the inflationary field range can be obtained by means of the magnetic WGC applied to domain walls and 3-form gauge fields: $\phi \lesssim m^{-2/3} f^{1/3} M_P^{4/3}$. While conceptually important, this bound can accommodate transplanckian inflation for not too small axion decay constants, since $m \sim 10^{-5} M_P$. Though obtained following a different strategy, our result agrees with the analysis of [108]. Our extension of the magnetic WGC to domain walls in 4D is justified from a stringy perspective, as extensively discussed.

We supplemented the qualitative justification with a 10D computation which provides a strategy to generalise the WGC for particle and gauge fields to any $(p+1)$-form gauge theory with electrically charged p-branes. Essentially, in ST 4D particles and gauge fields descend from dimensional reduction of p-form potentials on certain cycles of the compactification manifold. Thus we showed that the standard WGC translates into a bound on the volumes and intersections of such cycles. In formulae:

$$\frac{V_X^{1/2}|q^\Sigma|}{V^\Sigma} \geq A_d,$$

where V_X is the volume of the compactification manifold, V^Σ is the volume of the cycle Σ on which dimensional reduction is performed, $|q^\Sigma|$ is the norm of the harmonic form related to Σ using the metric on X, and A_d is a $O(1)$ number given explicitly in subsection 4.3.2. The conjecture can thus be automatically extended to any object of the 4D theory descending from dimensional reduction on the same cycles. Moreover, the charge-to-tension ratio of any p-dimensional object wrapped on some q-cycle Σ of a CY is always given by the left hand side of the inequality above. Thus the conjecture can be extended to any object descending from dimensional reduction of some gauge potential on a given cycle. In particular, the numerical prefactor A_d does not depend on p and q. Our purely geometric approach improves on previous strategies to generalise the WGC, which rely on string dualities [69].

As in the case of Part I, the conclusions of this Intermezzo are also positive: we did not find fatal constraints on axion monodromy inflation from the WGC.

Part II of this thesis was devoted to the post-inflationary phase of certain string-derived setups. In chapter 5 we showed that after axion monodromy inflation the inflaton might populate more than one local minimum of the potential, as a consequence of field fluctuations. We determined analytically the probability that inflationary fluctuations induce a phase decomposition, as a function of the number of local minima κ and the axion parameters m, f: $\mathcal{P} \sim \kappa^{-1/3}(m/M_P)(M_P/f)^{5/3}$. Thus, for $\kappa \sim \mathcal{O}(10), m \sim 10^{-5}M_P$ a dynamical phase decomposition is likely to occur if $f \lesssim 0.3 \cdot 10^{-2}M_P$. Quantum uncertainties of the inflaton field (among them, modes with $k \gtrsim m$ which never exited the horizon during inflation) are also present after inflation and their effects were investigated both analytically and numerically, by means of the linearised inflaton equations of motion. The probability that they induce a phase decomposition is smaller by a factor $(f/(M_P\kappa))^{-4/3}m/M_P$. However, we showed numerically that modes with $k \gtrsim m$ can exhibit a resonant enhancement and are thus likely to be the dominant cause of phase decomposition for approximately the same values of parameters above. For larger values of f, a phase decomposition is less likely but the following phase transition is more violent. We computed the spectrum of the gravitational radiation emitted in the violent collision of cosmic bubbles containing the true vacuum, following the phase decomposition. We found that the signal can peak in a wide range of frequencies (mHz to GHz). If the collision is followed by phases of radiation and matter domination before reheating, the spectrum can at least partially lie in the ballpark of future space- and ground-based interferometers.

Chapter 6 investigated the tension between the prediction for dark radiation in the Large Volume Scenario (LVS) and the latest Planck data on N_{eff}. The current observational bound on ΔN_{eff} heavily constraints the most popular realisations of the LVS, since the latter is characterised by an essentially massless axion produced copiously during reheating [125]. Therefore, we proposed several strategies to boost the decay rate of the bulk LVS modulus into SM fields. Among them, the most promising involves the addition of dark massive gauge bosons living on flavour branes, to which the bulk modulus can decay. We showed that, when these flavour bosons couple to the SM, the current constraints from Planck can be evaded for $\lesssim \mathcal{O}(10)$ flavour branes (more precisely, $N_f \gtrsim 35$ gauge bosons are needed to satisfy the most stringent observational bound). We also proposed other more tuned possibilities where the bulk modulus couples directly to SM gauge bosons, under the requirement of high-scale supersymmetry. We consider our results an example of how cosmological observations can already strongly constrain known setups of moduli stabilisation in ST.

Given the diverse content of this thesis, there are multiple directions in which more progress can (should) be made. Concerning more phenomenological perspectives, we plan to understand whether the gravitational wave signal that was described in chapter 5 can be associated with other peculiar signatures of axion monodromy. In this regard, a possibility may be offered by oscillations in the CMB power spectrum [93]. A correlation between these two signatures would allow to select axion monodromy in case of new CMB and interferometric observations.

Our analysis of gravitational waves signals during reheating might also be applied to other models of axion inflation, in particular to scenarios with two or more "aligned" fields. In this regard, it might be especially interesting to study the relation between the requirements of the WGC and the possibility of phase decomposition. We believe that an improvement of our analysis of resonant quantum fluctuations might also reveal new possibilities for exotic signatures from the (p)reheating epoch.

Concerning Large Field Inflation, recently the authors of [252] have pointed out that effective theories of axion monodromy inflation are affected by higher-dimensional operators which are not suppressed in the limit of weak breaking of the shift symmetry, for large field displacements. Starting from this observation, they cast serious doubts on models of relaxion monodromy [108]. It is a very interesting task to understand the implications of their analysis for inflationary models, such as the ones investigated in this thesis.

The refinement and the applications of the WGC are currently widely discussed in the community. In particular, in chapter 4 we considered an effective description of axion monodromy inflation by means of one 3-form field. Yet another effective theory of axion monodromy exists, also using a 3-form field to describe the quadratic part of the potential. However, following the strategy of [111], another 3-form can be considered to effectively describe also the oscillatory part of the potential. It thus seems that a complete effective description of axion monodromy may be obtained by means of more than one 3-form field. In the near future, we thus plan to understand whether such a formulation of axion monodromy can be useful, with particular attention to further constraints which may come from the generalised multi-field electric WGC.

From a stringy perspective, it would be interesting to understand explicitly how the well-known incompatibility between global symmetries and the string worldsheet theory [34] translates in the language of compactifications.

Axion monodromy inflation (or similar scenarios [194]) may represent the appropriate framework to investigate which mechanism sets the initial conditions for inflation. In the paradigm of the string landscape, this should be a tunnelling process from a metastable vacuum to a region where slow-roll inflation can be realised (see e.g. [54]). We plan to study concrete realisations of such tunnelling in a setup of string compactifications.

Overall, we believe that more work needs to be done to extract stronger constraints on transplanckian trajectories both from string compactifications and at the level of EFT. However, while there is currently consensus that very large field displacements (such as the ones needed in the mechanism of relaxation of the electroweak scale [181]) are heavily constrained by quantum gravity, it is not clear that similarly strong constraints can be imposed on slightly transplanckian inflationary field ranges. From the point of view of EFT expectations about Quantum Gravity, $\mathcal{O}(1)$ (and even larger) uncertainties are inherent in these arguments and may after all allow for phenomenologically relevant scenarios of LFI.

Future observation of CMB polarisation may favour models with less generic potentials and possibly remove the necessity of transplanckian field displacements. Nevertheless, the study of the interplay between UV degrees of freedom and effective field theories of elementary scalars (inflation, electroweak symmetry breaking) still promises to guide our journey towards an understanding of the physics of the Planck scale.

Appendices

Appendix A

Differential forms and complex geometry

In this appendix we collect the basic definitions concerning differential forms and generalised p-form gauge theories, which are essential ingredients of string compactifications. We do not aim at mathematical rigour. An excellent introduction to these topics can be found in e.g. [253]. Here we follow closely the presentation given in [135] and [136].

A.1 Differential forms

A *differential form* ω of rank p, or a *p-form*, is a totally antisymmetric covariant tensor with p indices. The vector space of p-forms on a manifold X is denoted by $T_p(X)$. A basis of $T_p(M)$ is provided by

$$dx^1 \wedge \cdots \wedge dx^p = \sum_{P \in S_p} \mathrm{sgn}(P) dx^{\mu_{P(1)}} \otimes \dots dx^{\mu_{P(p)}}, \tag{A.1}$$

where P is a permutation belonging to the symmetric group of rank p, S_p. Therefore, a p-form can be expanded in the basis (A.1), as follows

$$\omega = \frac{1}{p!} \omega_{\mu_1 \dots \mu_p} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}. \tag{A.2}$$

The *wedge product* between two differential forms ω_p and η_q results in a $(p+q)$-form with components

$$(\omega_p \wedge \eta_q)_{\mu_1 \dots \mu_{p+q}} = \frac{(p+q)!}{p!q!} \omega_{[\mu_1 \dots \mu_p} \eta_{\mu_{p+1} \dots \mu_{p+q}]}, \tag{A.3}$$

where $[\dots]$ denotes antisymmetrisation (including the factorial prefactor). Notice that $\omega_p \wedge \eta_q = (-1)^{pq} \eta_q \wedge \omega_p$.

The exterior derivative of ω_p is the $(p+1)$-form defined by

$$\eta_{p+1} = d\omega_p = \frac{1}{p!} \partial_{\mu_1} \eta_{\mu_1 \dots \mu_p} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{p+1}}. \tag{A.4}$$

Notice that $d^2 = 0$ and $d(\omega_p \wedge \eta_q) = d\omega_p \wedge \eta_q + (-1)^p \eta_q \wedge \omega_p$. A p-form can be integrated on an orientable p-dimensional manifold, using its decomposition (A.2). In particular, Stokes's theorem holds, i.e.

$$\int_{\Sigma_{p+1}} d\omega_p = \int_{\partial\Sigma_p} \omega_p, \tag{A.5}$$

where Σ_{p+1} is a manifold whose boundary is the $(p-1)$-manifold $\partial\Sigma_{p+1}$.

The *Hodge star* operator $\star$ on a d dimensional manifold X is defined via its action on a p-form

$$\star\,\omega_p = \frac{1}{p!}\epsilon^{\nu_1\ldots\nu_p}_{\mu_1\ldots\mu_{d-p}}\omega_{\nu_1\ldots\nu_p}, \tag{A.6}$$

where $\epsilon_{\mu_1\ldots\mu_n}$ is the totally anti-symmetric Levi-Civita tensor (i.e. its non-vanishing components are equal to $\pm g^{1/2}$, not to ± 1). Notice that $\star\star\omega_p = (-1)^{p(d-p)}$ on a Riemannian manifold of dimension d, while $\star\star\,\omega_p = (-1)^{p(d-p)+1}$ on a Lorentzian manifold. The Hodge star can be used to define a symmetric inner product in the space of p-forms, as follows:

$$(\omega_p, \eta_p) \equiv \int_X \omega_p \wedge \star\eta_q = \frac{1}{p!}\int d^d x\sqrt{|g|}\omega_{\mu_1\ldots\mu_p}\eta^{\mu_1\ldots\mu_p}. \tag{A.7}$$

We can now define the adjoint operator $d^\dagger$ as follows: $(d\omega_p, \eta_{p+1}) = (\omega_p, d^\dagger\eta_{p+1})$. The Laplace-Beltrami operator is then defined as $\Delta = dd^\dagger + d^\dagger d$.

A.2 Homology and cohomology

Let us begin by introducing de Rham cohomology. We say that a p-form is *closed* if $d\omega_p = 0$ and *exact* if $\omega_p = d\eta_{p-1}$. Obviously an exact form is also closed. Let $Z^p(X), B^p(X)$ be the sets of closed and exact p-forms on X respectively. Then the cohomology group $H^p(X)$ is defined as the quotient of Z^p by B^p, i.e.

$$H^p(X) = Z^p(X)/B^p(X). \tag{A.8}$$

In other words, the elements of $H^p(X)$ are the equivalence classes of closed forms. The equivalence relation is obviously $\omega_p \simeq \omega_p + d\eta_{p-1}$ and the class is denoted by $[\omega_p]$. The *Betti number* b_p is the dimensions of H^p. According to the *Hodge decomposition theorem*, every form ω_p on a compact orientable Riemannian manifold X can be written as

$$\omega_p = h_p + d\eta_{p-1} + d^\dagger\gamma_{p+1}. \tag{A.9}$$

Therefore every equivalence class $[\omega_p]$ has a unique harmonic representative h_p.

Now let us move to homology. On a differentiable connected manifold X, a real *p-chain* A is the linear combination of p-dimensional oriented submanifolds M_i, i.e. $A = \sum_i a_i A_i$, where $a_i \in \mathbb{R}$. A *p-cycle* is a p-chain without boundaries. Consider the operation ∂ of taking the boundaries (with the induced orientation) of a p-chain. Then for a p-cycle M, $\partial M = 0$. We can then define the set $Z_p(X), B_p(X)$ of p-chains which are closed ($\partial M = 0$) and exact ($M = \partial N$) respectively. The *homology* group $H_p(X)$ is defined as

$$H_p(X) = Z_p(X)/B_p(X), \tag{A.10}$$

and its elements are the equivalence classes $[M_p]$ defined by the relation $M_p \simeq M_p + \partial N_{p+1}$. Furthermore the following relation in a d-dimensional manifold holds: $H_p(X) \simeq H_{d-p}(X)$.

We are finally ready to introduce the duality between $H_p(X)$ and $H^p(X)$. The duality map is provided by the inner product between $M_p \in H_p$ and $\omega_p \in H^p$:

$$(M_p, \omega_p) \equiv \int_{M_p} \omega_p. \tag{A.11}$$

Notice that the inner product is independent of the particular element of an equivalence class, thanks to Stokes's theorem. The product (A.11) is sometimes referred to as the *period* of ω_p over the cycle M_p. De Rham's theorem states precisely that H^p and H_p are dual vector spaces on a compact manifold and that they are finite dimensional. In particular, given a basis $\{[M_i]\}$ of $H_p(X)$, there exists a basis $\{[\omega_i]\}$ of $H^p(X)$ such that

$$\int_{M_i} \omega_j = \delta_{ij}. \tag{A.12}$$

This formula is particularly useful when performing dimensional reduction.

Furthermore, given a p-cycle M_p on a compact manifold X of dimension d, there exist a $d-p$-form $\omega_{d-p} \in H^{d-p}(X)$ such that, for any p-form $\eta_p \in H^p(X)$

$$\int_{M_p} \eta_p = \int_X \omega_{d-p} \wedge \eta_p. \tag{A.13}$$

Then we say that ω_{d-p} is the *Poincaré dual* of M_p.

A.3 Complex manifolds

Here we review the extension of the concepts introduced in the previous subsections to complex manifolds.

A *complex* manifold N is a real manifold of even dimension $2n$ that locally looks like $\mathbb{C}^n$. In more technical words, its atlas $\{U_\alpha, \phi_\alpha\}$ is made of charts $\phi_\alpha : U_\alpha \to \mathbb{C}^n$. Furthermore, the transition maps $\tau_{\alpha,\beta} = \phi_\beta \circ \phi_\alpha^{-1}$ are holomorphic. It is thus natural to introduce complex coordinates $z^n = x^n + iy^n, \bar{z} = x^n - iy^n, n = 0, \ldots, n-1$ on N.

If A, B are tangent vectors of N as a real manifold, the *complexified tangent space* at a point p of N is defined as: $T^pN^{\mathbb{C}} = \{A + iB | A, B \in T_pN\}$. A basis of $T^pN^{\mathbb{C}}$ is obviously provided by $\partial/\partial z^n, \partial/\partial \bar{z}^n$. The complexified cotangent space $\Omega_p^{\mathbb{C}}$, is similarly defined starting from its real counterpart T_p. A basis of $\Omega_p^{\mathbb{C}}$ is thus $dz^n, d\bar{z}^n$. We will often drop the $\mathbb{C}$. The generalisation to forms of higher degree is straightforward. In particular, a generic element of $\Omega^{(p,q)}(N)$, i.e. a (p,q) form can be written as:

$$\omega = \frac{1}{p!q!} \omega_{i_1 \ldots i_p, \bar{j}_1 \ldots \bar{j}_q} dz^{i_1} \wedge \cdots \wedge dz^{i_p} \wedge d\bar{z}^{\bar{i}_1} \wedge d\bar{z}^{\bar{i}_q}. \tag{A.14}$$

In particular, we say that a (p,q)-form has p holomorphic indices and q anti-holomorphic indices.

Let us now discuss cohomology. Consider the *Dolbeault operators*

$$\partial : \Omega^{(p,q)} \to \Omega^{(p+1,q)} \tag{A.15}$$

$$\bar{\partial} : \Omega^{(p,q)} \to \Omega^{(p,q+1)}, \tag{A.16}$$

which act as the standard exterior derivative on the holomorphic and anti-holomorphic indices respectively. Then the action of the exterior derivative d on a (p,q)-form is defined by $d = \partial + \bar{\partial}$. The concept of *cohomology* can now be straightforwardly generalised to complex differential forms. In particular, we say that a (p,q)-form is $\bar{\partial}$-closed if $\bar{\partial}\omega = 0$ and $\bar{\partial}$-exact if $\omega = \bar{\partial}\eta$, with $\eta \in \Omega^{p,q-1}$. The set of (p,q)-forms that are $\bar{\partial}$-closed but not $\bar{\partial}$-exact is called the Dolbeault cohomology group $H^{(p,q)}(N,\mathbb{C})$. The complex dimensions of these groups are called *Hodge numbers*, i.e. $h^{p,q} = \mathrm{dim} H^{(p,q)}(N,\mathbb{C})$.

The *Hodge-*$\bar{\star}$ operator is defined in analogy with the real case, i.e. $\star\Omega^{p,q}(N) \to \Omega^{n-q,n-p}(N)$. An inner product on $\Omega^{p,q}(N)$ is provided by:

$$(\omega,\eta) \equiv \int_N \omega \wedge \star\bar{\eta}, \tag{A.17}$$

with $\omega, \eta \in \Omega^{p,q}(N)$. The adjoints $\partial^\dagger$ and $\bar{\partial}^\dagger$ of the Dolbeault operators are defined with respect to the inner product (A.17). The laplacians $\Delta_\partial, \Delta_{\overline{\partial}}$ are thus defined as $\Delta_\partial = \partial\partial^\dagger + \partial^\dagger\partial$, $\Delta_\partial = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}$. The set of (p,q)-forms that are harmonic with respect to $\Delta_{\bar{\partial}}$, i.e. such that $\Delta_{\bar{\partial}}\omega = 0$, is denoted by $\mathcal{H}^{p,q}(N)$. The Hodge theorem then states that

$$H^{p,q}(N) \cong \mathcal{H}^{p,q}(N). \tag{A.18}$$

De Rham's theorem automatically extends to complex manifolds, seen as real manifolds, and complex differential forms, expanded in their real components.

Now let us provide definitions which are relevant for Calabi-Yau (CY) manifolds. An *almost complex structure* $\mathcal{J}$ on N is a $(1,1)$ tensor $\mathcal{J} : T(N) \to T(N)$, such that $\mathcal{J}^2 = -\mathbb{1}$. In particular, in local coordinates we have

$$\mathcal{J}_p^n = i\delta_p^n, \mathcal{J}_{\bar{p}}^{\bar{n}} = -i\delta_{\bar{p}}^{\bar{n}}. \tag{A.19}$$

A Hermitian metric is a real, positive definite, covariant tensor field

$$g = g_{i\bar{j}} dz^i \otimes d\bar{z}^{\bar{j}} + g_{\bar{i}j} d\bar{z}^{\bar{i}} \otimes dz^j. \tag{A.20}$$

A complex manifold equipped with an Hermitian metric is called an *Hermitian manifold*. The *Kähler form* of an Hermitian manifold with hermitian metric g is the $(1,1)$-form defined by:

$$J = i g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}. \tag{A.21}$$

A *Kähler manifold* is an Hermitian manifold with closed Kähler form, i.e. $dJ = 0$. The hodge numbers of a Kähler manifold are not all independent. In particular $h^{p,q} = h^{q,p}$ and $h^{p,q} = h^{n-p,n-q}$. The Betti numbers b_k are defined as $b_k = \sum_{p+q=k} h^{p,q}$. The Euler characteristic is given in terms of the Betti numbers by

$$\chi(N) = \sum_{p,q} (-1)^{p+q} h^{p,q}. \tag{A.22}$$

The Ricci form on a Kähler manifold is defined as: $\mathcal{R} = i R_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$, where $R_{i\bar{j}}$ is the Ricci tensor obtained from the hermitian metric g. Crucially, on a Kähler manifold $\mathcal{R}$ is closed but not necessarily exact: it is thus the representative of a cohomology class, which is referred to as *first Chern class*

$$c_1 \equiv \frac{1}{2\pi} [\mathcal{R}]. \tag{A.23}$$

A *Calabi-Yau (CY) manifold* is a Kähler manifold with vanishing first Chern class. Equivalently, a CY is a Kähler manifold which admits a Ricci-flat hermitian metric. There exists a unique $(n, 0)$ form on a CY of complex dimension n. Locally at a point p, it can be written as

$$\Omega_p = f(z)\epsilon_{i_1\dots i_n} dz^{i_1} \wedge \cdots \wedge dz^{i_n}, \tag{A.24}$$

where $f(z)$ is a holomorphic function, $\epsilon_{i_1\dots i_n} = \pm 1$ and $z^{i_1}, \dots, z^{i_n}$ are local coordinates. Therefore, in a CY manifold $h^{3,0} = h^{0,3} = 1$. The forms $\Omega \wedge \bar{\Omega} \sim J \wedge J \wedge J$ are proportional to the volume form of the CY manifold.

The only non-vanishing hodge numbers on a CY 3-fold are $h^{3,0} = h^{0,3} = h^{0,0} = h^{3,3} = 1, h^{1,1}$ and $h^{2,1} = h^{1,2}$. Thus the Euler characteristic of a CY 3-fold is simply given by $\chi = 2(h^{1,1} - h^{2,1})$. Furthermore, in a CY of complex dimension n, $h^{r,0} = 0$, for all $r < n$.

A.4 P-form gauge theories

Here we briefly recap the basics of generalised gauge theories, following [135]. In the language of differential forms, standard electrodynamics is described by a 1-form gauge potential A_1. The associated field strength is the two-form defined by $F_2 = dA_1$ and is obviously invariant under the gauge transformation $A_1 \to A_1 + d\alpha$, where α is a 0-form.

Now consider a $(p + 1)$-form potential A_{p+1} and define its field strength and its associated gauge transformation as

$$F_{p+2} = dA_{p+1}, \quad A_{p+1} \to A_{p+1} + d\alpha_p. \tag{A.25}$$

In terms of differential forms, the kinetic term of such gauge field can be written as

$$S \supset -\frac{1}{2} \int d^d x F_{p+2} \wedge \star F_{p+2} = -\frac{1}{2} \int d^d x \frac{\sqrt{-g}}{(p+2)!} F_{\mu_1\dots\mu_{p+1}} F^{\mu_1\dots\mu_{p+2}}. \tag{A.26}$$

We also use the notation

$$|F_{p+2}|^2 \equiv \frac{1}{(p+2)!} F_{\mu_1\dots\mu_{p+1}} F^{\mu_1\dots\mu_{p+2}}. \tag{A.27}$$

The equations of motion derived from (A.26) and the Bianchi identity in the absence of sources are

$$d \star F_{p+2} = 0, \quad dF_{p+2} = 0. \tag{A.28}$$

Notice that this set of equations is invariant under the dualisation $F_{d-(p+2)} = \star F_{p+2}$. Therefore the theories of A_{p+1} and B_{d-p-3} defined by $dB_{d-p-3} = F_{d-p-2}$ are equivalent. In particular, B_{d-p-3} has the same number of dynamical degrees of freedom as A_{p+1}. This is the reason why only potentials up to F_5 are included in the Type IIB action (1.1). A peculiar case is that of a $d/2$-form field strength in $d = 2$ mod 4 dimensions: in this case $F_{d/2}$ is either self- or anti-self-dual, i.e. $F_{d/2} = \pm \star F_{d/2}$. However, this constraint cannot be implemented at the level of the action, which would then be vanishing. Thus, self (anti) duality is to be imposed directly on the equations of motions. For instance, this is the case for the field strength F_5 in Type IIB ST.

A $(p+1)$-form potential couples to p-dimensional objects (p-branes). In particular, a 1-form gauge potential couples to a particle in 4D. We define the one-dimensional *worldline* of the particle as its path in space, i.e. the curve whose points are the locations of the particle at each moment in time. We can parametrise this curve by means of e.g. the proper time τ, such that the 4D coordinates at each instant are given by $x^\mu \equiv x^\mu(\tau)$.

Similarly, the same can be done to describe the history of a p-brane. Let $\xi^i, i = 0, \ldots, p$ be the coordinates of its $(p+1)$-dimensional *worldvolume* $\mathcal{W}_{p+1}$, and $X^\mu \equiv X^\mu(\xi^i), \mu = 0, \ldots, d-1$ the embedding of such worldvolume in d-dimensional spacetime. Then the coupling between a $(p+1)$-form gauge field and a p-brane is described by the action

$$S \supset e_p \int_{\mathcal{W}_{p+1}} A_{p+1} = \frac{e_p}{(p+1)!} \int d^{p+1}\xi A_{\mu_0 \ldots \mu_p} \left[\frac{\partial X^{\mu_0}}{\partial \xi^{i_0}} \cdots \frac{\partial X^{\mu_p}}{\partial \xi^{i_p}} \right] \epsilon^{i_0 \ldots i_p}, \tag{A.29}$$

where $\epsilon^{i_0 \ldots i_p}$ is the Levi-Civita tensor defined in Sec. A.1. Notice that the coupling e_p is dimensionful, in particular $[e_p] = [E]^{(p+2)-d/2}$.

Appendix B

Scalar field fluctuations after inflation

In this appendix, we provide a detailed analysis of the evolution of scalar field fluctuations after inflation. We begin by deriving the Klein-Gordon equations for a scalar field and its fluctuations with potential (4.2), including the gravitational field. We then focus on the simple case of a purely quadratic potential. This gives us the opportunity to review why scalar field fluctuations behave like dark matter perturbations. We then provide equations to study the fluctuations in the full potential containing the 'wiggles'.

B.1 Equations of motion

Let us begin with the equations of motion of a scalar field in the post-inflationary universe. The starting point is the Klein-Gordon equation:

$$\frac{1}{\sqrt{-g}}\partial_\mu \left[g^{\mu\nu}\sqrt{-g}\partial_\nu\phi\right] + V^{'}(\phi) = 0. \tag{B.1}$$

The metric appearing in (B.1) is the perturbed FRW metric (here we follow [212]):

$$ds^2 = a^2(\tau)\left[(1+2A)d\tau^2 - 2B_i dx^i d\tau - (\delta_{ij} + h_{ij})dx^i dx^j\right], \tag{B.2}$$

where $B_i = \partial_i B + \hat{B}_i$ and the hat denotes divergenceless vectors and traceless tensors. In particular, we can consistently focus only on scalar modes, since the vectors can be gauged away and the tensors are not sourced by $\delta\phi$. We therefore have

$$h_{ij}^{scalar} = 2C\delta_{ij} + 2\partial_{<i}\partial_{j>}E, \tag{B.3}$$

where $\partial_{<i}\partial_{j>}E \equiv \left[\partial_i\partial_j - (1/3)\delta_{ij}\nabla^2\right]E$. Perturbations defined by $\phi(t,\mathbf{x}) = \phi_0(t) + \delta\phi(t,\mathbf{x})$ and (B.2) are not gauge invariant. However, the following quantities are gauge invariant:

$$\begin{aligned}
\Psi &\equiv A + \mathcal{H}(B - E^{'}) + (B - E^{'})^{'} \\
\Phi &\equiv -C - \mathcal{H}(B - E^{'}) + \frac{1}{3}\nabla^2 E \\
\overline{\delta\phi} &\equiv \delta\phi - \phi_0^{'}(B - E^{'}).
\end{aligned}$$

In what follows we will perform our computation in the *Newtonian gauge*, defined by:

$$B = E = 0, C = -\Phi. \tag{B.4}$$

In the latter, the perturbed metric reads:

$$ds^2 = [(1 + 2\Psi)dt^2 - (1 - 2\Phi)a^2\delta_{ij}dx^i dx^j]. \tag{B.5}$$

The components Φ and Ψ are related by the perturbed Einstein equations. In the absence of off-diagonal terms in the spatial components of the perturbed stress-energy tensor the Einstein equations impose $\Phi = \Psi$. With this constraint, the metric (B.5) provides the newtonian limit of general relativity.

We are now in a position to write down the Klein-Gordon equation (B.1), expanding the scalar field as $\phi = \phi_0 + \delta\phi$ and keeping only the leading order terms in the perturbed quantities $\delta\phi, \Phi$. Then the background ϕ_0 obeys:

$$\ddot{\phi}_0 + 3\frac{\dot{a}}{a}\dot{\phi}_0 + V'(\phi_0) = 0, \tag{B.6}$$

while the inhomogeneous $\delta\phi(t, \mathbf{x})$ satisfies:

$$\ddot{\delta\phi} + 3\frac{\dot{a}}{a}\dot{\delta\phi} + \left(V''(\phi_0) - \frac{\nabla^2}{a^2}\right)\delta\phi - 4\dot{\phi}_0\dot{\Phi} + 2V'(\phi_0)\Phi = 0. \tag{B.7}$$

Furthermore, there are three Einstein equations relating the gravitational field Φ to the fluctuation $\delta\phi$:

$$\dot{\Phi} + \frac{\dot{a}}{a}\Phi = 4\pi G\dot{\phi}_0\delta\phi \tag{B.8}$$

$$\frac{\Delta\Phi}{a^2} - 3\frac{\dot{a}}{a}\dot{\Phi} - 3\frac{\ddot{a}^2}{a^2} = 4\pi G\delta\rho \tag{B.9}$$

$$\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + (2H^2 + H^2)\Phi = 4\pi G\left(\dot{\phi}_0\dot{\delta\phi} + V'(\phi_0)\right)\delta\phi, \tag{B.10}$$

where $\delta\rho = \dot{\phi}_0\dot{\delta\phi} + V'(\phi_0)$ is the perturbed energy density. The Einstein equations (B.8) and (B.9) lead to a Poisson equation with a gauge invariant energy density on the right hand side. In total, we have five equations for three quantities and therefore it is enough to consider only one of the Einstein equations.

B.2 Scalar field as Dark Matter

In order to understand why perturbations of a scalar field behave like dark matter fluctuations, we now focus on the case of a purely quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$. In this case it can be shown that fluctuations obeying (B.7) behave like an ideal pressureless fluid [214,254]. The strategy is as follows [254]: by using a WKB ansatz for ϕ_0 and $\delta\phi$ one can define three fluid quantities for a scalar field: energy density, velocity potential and pressure. One then performs an expansion of (B.7) in powers of (m^{-1}). The leading order in this expansion corresponds to the Euler equation for a perfect pressureless fluid. The subleading order gives the continuity equation. This establishes a dictionary between scalar field and fluid quantities. More precisely, the correspondence involves field quantities which are averaged over one period $T \sim m^{-1}$. One can then use the standard fluid approach to prove that scalar field energy fluctuations grow like a on subhorizon scales.

We now briefly review the approach of [254]: the aim is to provide equations which can be extended to include non-linearities. The following analysis is valid in the regime $m \gg H, k/a \gg H, k^2/(a^2H) \ll m$. The first two conditions are satisfied in our setup. However, this is not necessarily the case for the third one. Indeed, as explained in Sec. 5.3, we are mainly interested in modes with $k \gtrsim m$ at horizon re-entry, as these are most efficiently leading to a phase decomposition. However, due to the expansion of the universe $k^2/(a^2H)$ decreases as $a^{-1/2}$. Therefore, even if the condition $k^2/(a^2H) \ll m$ is originally not satisfied, it rapidly becomes fulfilled.

The starting point of the analysis is a WKB ansatz for the background and for the fluctuations (see also [255, 256] for related discussions):

$$\phi_0 = u(t)\cos(mt) + w(t)\sin(mt) \tag{B.11}$$

$$\delta\phi = B\,(t, \mathbf{x})\sin(mt) + A\,(t, \mathbf{x})\cos(mt). \tag{B.12}$$

The crucial point is that u, w, B and A vary on time scales which are much longer than m^{-1}. Using (B.11) in (B.6) and the Friedmann equation, we find $u \sim (mt)^{-1}$. One can then check that w represents a subleading correction, of $O(m^{-2})$. Assuming $\Phi \sim O(m^0)$, the choice consistent with Einstein equations is $B \sim O(m^0)$ and $A \sim O(m^{-1})$. At leading order in m^{-1}, i.e. at $O(m^1)$, (B.12) reads:

$$\dot{B} + \frac{3}{2}\frac{\dot{a}}{a}B + mu\Phi = 0. \tag{B.13}$$

Let us now write down the relevant fluid quantities. In analogy with the stress-energy tensor of a perfect fluid, they are defined as:

$$\delta\rho = \overline{\delta T_{00}} \tag{B.14}$$

$$v_i = -\frac{1}{\overline{(\rho + p)}}\overline{\delta T_i^0} \tag{B.15}$$

$$\delta p = -\overline{\delta T_j^i}, \tag{B.16}$$

where the overlines denote an average over a period m^{-1}. In what follows we will use the velocity potential v, defined as $v_i = \partial_i v$. Computing explicitly the components of the perturbed stress energy tensor, using (B.11),(B.12),(B.13) one finds the leading order expressions for the fluid quantities (B.14):

$$\delta\rho = m^2\,[uA + wB] \tag{B.17}$$

$$v = \frac{B}{mua} \tag{B.18}$$

$$\delta p = 0. \tag{B.19}$$

As expected, we see that the effective pressure of the fluctuations vanishes at leading order.

A perfect pressureless newtonian fluid in an expanding background is subject to the familiar continuity and Euler equations (see e.g. [212]):

$$\dot{\delta} = -\frac{1}{a}\nabla \cdot \mathbf{v} \tag{B.20}$$

$$\dot{\mathbf{v}} + H\mathbf{v} = -\frac{1}{a}\nabla\Phi, \tag{B.21}$$

where $\delta \equiv \delta\rho/\rho$ and $v_i \equiv \partial_i v$. According to (B.18), $v \sim B$. We see therefore that (B.13) closely resembles the Euler equation (B.21). In fact, it can be shown that (B.13), rewritten in terms of v, is equivalent to the relativistic generalisation of (B.21).

The next-to-leading order of (B.12), i.e. the $O(m^0)$, is obtained by taking into account that both the scale factor and the gravitational potential have oscillatory subleading terms, $\Phi_{osc} \sim O(m^{-1})$ and $a_{osc} \sim O(m^{-2})$. The details can be found in [254]. Here we report only the final result:

$$\ddot{B} + 3\frac{\dot{a}}{a}\dot{B} - \frac{\nabla^2}{a^2}B - 2m\dot{A} - 3m\frac{\dot{a}}{a}A + 4mu\dot{\Phi} + 2m^2 w\Phi - \frac{3\pi}{2}Gm^2u^2B = 0. \tag{B.22}$$

Since A, B and Φ vary on time scales which are larger than m^{-1}, the first two terms in (B.22) are subleading compared to the other terms in the equation. The last two terms come from the subleading parts of the background and the scale factor. Neglecting them, we see that (B.22) closely resembles (B.20). More precisely, it can be shown that the full (B.22), rewritten in terms of v and $\delta\rho$, reproduces the relativistic generalisation of (B.20).

The discussion of this section therefore proves that scalar field fluctuations behave like an ideal pressureless fluid after inflation. Having established this equivalence, one can show that $\delta\rho/\rho \sim a$ on subhorizon scales, using (B.20), (B.21) and the Poisson equation for the gravitational field (see e.g. [212]).

B.3 Equation including non-linearities

The next goal is to write down the analogue of (B.13) and (B.22) for the axion monodromy potential (4.2). This amounts to including non-linearities in the fluid equations. It turns out that in this case the scalar field perturbations do not behave like a pressureless fluid. Rather, the pressure is small but non-vanishing, and correspondingly the fluid is characterised by a small sound speed. In our setup we have $\kappa \gtrsim O(1)$, therefore $\Lambda^4/f^2 \sim O(m^2)$ and the $O(m^2)$ expansion of (B.7) is not necessarily trivially satisfied. One would therefore have to assign a certain order to the new terms involving the fast oscillations with argument ϕ_0/f, such that a consistent solution to Einstein equations and the equations of motion can be found.

In this subsection, we take a different approach. Namely, we derive two equations from (B.7) by splitting the sine and cosine oscillations. We then obtain equations that involve terms of different orders in m^{-1}. The three relevant orders are $O(m^2), O(m^1), O(m^0)$. At these orders there is no need to consider higher harmonics in mt, therefore we only keep terms in (B.7) that are either non-oscillating or oscillating with frequency m^{-1}. The result of such a computation is the following system of equations for A and B:

$$\begin{aligned}&\ddot{B} - 2m\dot{A} + 3\frac{\dot{a}}{a}B - B\frac{\Lambda^4}{f^2}\cos\left(\frac{\phi_0}{f}\right) - \frac{\nabla^2}{a^2}B + 4\dot{\Phi}mu - 4\left[-\frac{\dot{w}}{2} + m^2\pi G u^2 B\right] \\ &+ 2\left[m^2 w - \frac{\Lambda^4}{f}\sin\left(\frac{\phi_0}{f}\right)\right]\Phi - \frac{3}{4}m\pi G A\dot{u}^2 - 2m^3\pi G w u B = 0.\end{aligned} \tag{B.23}$$

$$\begin{aligned}&\ddot{A} + 2m\dot{B} + 3\frac{\dot{a}}{a}A - A\frac{\Lambda^4}{f^2}\cos\left(\frac{\phi_0}{f}\right) - \frac{\nabla^2}{a^2}A - 4\dot{\Phi}mw - 4\left[\frac{\dot{u}}{2} + m^2\pi G u w B\right] \\ &+ 2\left[m^2 u - \frac{\Lambda^4}{f}\sin\left(\frac{\phi_0}{f}\right)\right]\Phi - \frac{3}{4}\pi G m B\dot{u}^2 + 2m^3\pi G u^2 B = 0.\end{aligned} \tag{B.24}$$

The next step to analyse these equations is to expand A and B in Fourier modes in k. Notice that (B.23) and (B.24) contain oscillatory terms with argument ϕ_0/f. Those terms are the cause of potential resonant behaviour of the modes B_k and A_k. Solutions to (B.23), (B.24) should be sought numerically. While we leave a detailed study for future work, let us notice that before performing this numerical study one has to properly average and Fourier expand the functions $\cos(\phi_0/f), \sin(\phi_0/f)$.

Appendix C

Tuned small field inflation

In this appendix we would like to briefly address an issue related to our setup. In principle, while oscillating along the full potential (4.2), the inflaton may come close to a local maximum, without being able to actually reach it as a consequence of friction. In this case, the field motion may satisfy the slow roll conditions, therefore leading to a further inflationary phase. If the local maximum is flat enough, we might be in a scenario which closely resemble that of inflation at an inflection point (see [134] and references therein, see also [92], [257], [112, 113] for related previous work). If inflation is viable in this setup, then current observational constraints on m may be relaxed. Indeed in this case the axion mass would not necessarily be proportional to the inflationary power spectrum. This may turn out to be particularly relevant for the signatures that we described in this paper, as the probability of having a phase decomposition is suppressed by the smallness of m. Let us also remark that the potentials that we will consider in this Appendix may be relevant also for non-monodromic scenarios with more than one axion, such as alignment and winding models [31, 106, 159].

Let us then start by assuming that the inflaton slowly approaches a stationary inflection point ϕ_* of the potential. We first estimate how many e-foldings would be generated as the field comes close to ϕ_*. In this Appendix we work in Planck units, i.e. we set $M_p \equiv 1$.

Around an inflection point, the potential can be approximated as

$$V = V_0 \left(1 + \gamma \frac{\phi^3}{3}\right). \tag{C.1}$$

The slow roll parameters are therefore given by

$$\epsilon = \frac{1}{2}\gamma^2 \phi^4, \quad \eta = 2\gamma\phi. \tag{C.2}$$

The number of e-foldings can be estimated as:

$$\mathcal{N} = \int \frac{d\phi}{\sqrt{2\epsilon}} = \frac{1}{\gamma} \int_{\phi_e}^{\phi_*} \frac{d\phi}{\phi^2} \tag{C.3}$$

$$\simeq \frac{1}{\gamma\phi_*} \simeq \frac{2}{|\eta|}. \tag{C.4}$$

The latter equation may represent a conflict with observations: indeed in this model ϵ is very small, i.e. $\epsilon \ll 10^{-2}$, therefore we need $\eta \sim 10^{-2}$ to satisfy the observational constraint on $n_s - 1 = 2\epsilon - 6\eta \simeq 3 \cdot 10^{-2}$. From (C.3), we obtain $\mathcal{N} \sim 10^2$. Therefore it seems that in this simple case we obtain too many e-foldings.

However it is very unlikely that the field gets caught around such an inflection point. Indeed, the latter is the point after which there are no more local minima of the potential (4.2), and therefore sits far from the minimum of the parabola. As we have already remarked, the field is more likely to get caught in one of the minima at the bottom of the potential. Therefore, we now consider a more generic stationary point and modify the potential (C.1) by including also a small quadratic contribution.

We focus on the following expansion around a stationary point ϕ_0:

$$V = V_0 \left[1 + \frac{\alpha}{2}(\phi - \phi_0)^2 + \frac{\beta}{3!}(\phi - \phi_0)^3\right], \tag{C.5}$$

Here we take α and β to be positive without loss of generality. We want to assess the feasibility of such an inflationary potential (see [257] for the case in which a linear term is included, instead of a quadratic term). We take $\alpha \ll \beta$, and focus on small field ranges, so that $|\phi - \phi_0| < 1$ all along the inflationary trajectory.

The slow roll parameters are:

$$\begin{aligned}\epsilon &\equiv \frac{1}{2}\left[\frac{V'}{V}\right]^2 \simeq \frac{1}{2}\left[\alpha(\phi - \phi_0) + \frac{\beta}{2}(\phi - \phi_0)^2\right]^2 \\ \eta &\equiv \frac{V''}{V} \simeq \alpha + \beta(\phi - \phi_0) \Rightarrow \phi - \phi_0 = \frac{\eta - \alpha}{\beta}.\end{aligned} \tag{C.6}$$

The number of efoldings is given by:

$$\mathcal{N}_\star = \int_{\phi_e}^{\phi_\star} \frac{d\phi}{\sqrt{2\epsilon}} = \frac{1}{\alpha}\ln\left[\frac{(\eta_\star - \alpha)(\alpha + \frac{\eta_e - \alpha}{2})}{(\eta_e - \alpha)(\alpha + \frac{\eta_\star - \alpha}{2})}\right], \tag{C.7}$$

where $\phi_\star$ is the field value at the beginning of inflation. It is convenient to express $\eta_\star$ in terms of $\mathcal{N}_\star$, by inverting (C.7):

$$\eta_\star = \frac{e^{\alpha\mathcal{N}_\star}\alpha(\eta_e - \alpha)}{2\left[\alpha + \frac{\eta_e - \alpha}{2}(1 - e^{\alpha\mathcal{N}_\star})\right]} \simeq -\frac{\eta_e}{-2 + \mathcal{N}_\star\eta_e} - \frac{\alpha}{2}, \tag{C.8}$$

where in the last step we have kept only the first order in α. Similarly, we can express $\epsilon_\star$ in terms of $\eta_\star$, then in terms of $\mathcal{N}_\star$ by means of (C.8):

$$\epsilon_\star \simeq \frac{\eta_e^3}{4\beta^2(-2 + \eta_e\mathcal{N}_\star)^3}\left[\alpha + \frac{\eta_e}{2(-2 + \eta_e\mathcal{N}_\star)}\right]. \tag{C.9}$$

The end of inflation is determined either by the condition $\epsilon_e = 1$ or by $\eta_e = -1$. In our case, we find:

$$\begin{aligned}\epsilon_e = 1 &\Rightarrow |\phi_e^\epsilon - \phi_0| = \frac{2^{3/4}}{\beta^{1/2}} + \frac{\alpha}{\beta} \\ \eta_e = -1 &\Rightarrow |\phi_e^\eta - \phi_0| = \frac{\alpha}{\beta},\end{aligned} \tag{C.10}$$

therefore, under the assumption $\alpha \ll \beta$, $\eta_e = -1$ determines the end of inflation. Notice also that the ratio α/β sets precisely the inflationary range. Now we can write down the spectral index and the tensor-to-scalar ratio in terms of $\alpha, \beta, \mathcal{N}_\star$:

$$n_s = 1 - 6\epsilon_\star + 2\eta_\star \simeq 1 - \frac{1}{2+\mathcal{N}_\star}\left[2 + \frac{3}{4\beta^2(2+\mathcal{N}_\star)^3}\right] - \alpha\left[1 + \frac{2}{3\beta^2(2+\mathcal{N}_\star)^3}\right] \tag{C.11}$$

$$r = 16\epsilon_\star \simeq \frac{4}{\beta^2(2+\mathcal{N}_\star)^3}\left[\alpha + \frac{1}{2(2+\mathcal{N}_\star)}\right]. \tag{C.12}$$

We are therefore ready to study the parameter space of the model described by (C.5). First, we fix $\mathcal{N}_\star = 50$ and plot the constraints on the parameters α and β obtained by imposing the observed values for n_s and r: $n_s = 0.9652 \pm 0.0047$ at 68% C.L., $r < 0.10$ at 95% C.L. (*Planck* TT,TE,EE+lowP) [15].[1] We then repeat the analysis for $\mathcal{N}_\star = 60$.

We present the results in Fig. C.1 and C.2. We see that a model based on (C.5) is still a viable candidate to explain the CMB observables.

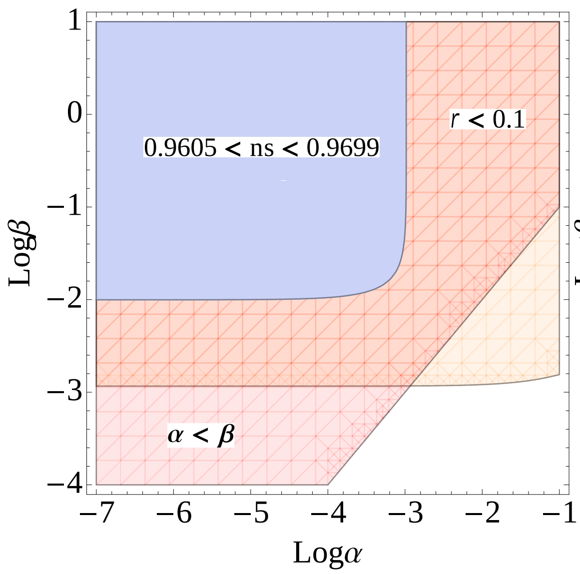


Figure C.1: Constraints on the parameters α and β for $\mathcal{N}_\star = 50$.

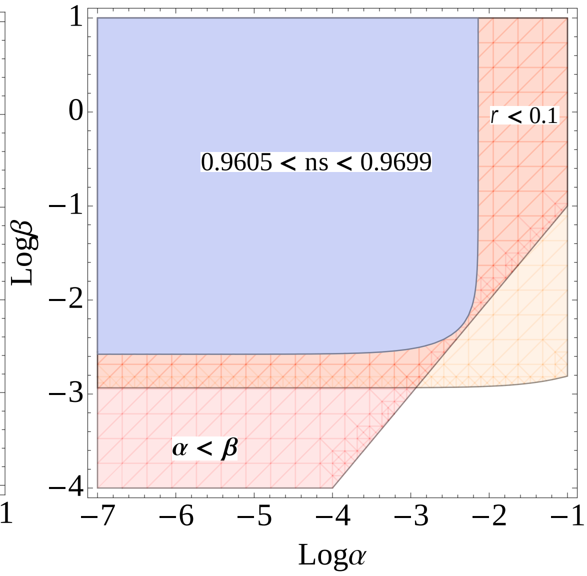


Figure C.2: Constraints on the parameters α and β for $\mathcal{N}_\star = 60$.

[1]Here we are using the bounds on n_s and r from Planck only, i.e. not combined with data from BICEP2/Keck-Array.

List of Publications

1 F. Rompineve, *Weak Scale Baryogenesis in a Supersymmetric Scenario with R-parity violation*, JHEP **1408** (2014) 014, *arXiv:1310.0840.*

2 A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Dark Radiation predictions from general Large Volume Scenarios*, JHEP **1409** (2014) 140, *arXiv:1403.6810.*

3 A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Tuning and Backreaction in F-term Axion Monodromy Inflation*, Nucl. Phys. B **894** (2015) 456, *arXiv:1411.2032.*

4 A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?*, Phys. Lett. B **748** (2015) 455, *arXiv:1503.07912.*

5 A. Hebecker, F. Rompineve and A. Westphal, *Axion Monodromy and the Weak Gravity Conjecture*, JHEP **1604** (2016) 157, *arXiv:1512.03768.*

6 A. Hebecker, J. Jaeckel, F. Rompineve and L. T. Witkowski, *Gravitational Waves from Axion Monodromy*, JCAP **1611** (2016) no.11, 003, *arXiv:1606.07812.*

7 M. Farina, D. Pappadopulo, F. Rompineve and A. Tesi, *The photo-philic QCD axion*, JHEP **1701** (2017) 095, *arXiv:1611.09855.*

This thesis is based on the publications 2, 3, 4, 5 and 6 of the list above. These publications are reference in this thesis as [129], [130], [131], [132] and [133] respectively.

Bibliography

[1] WMAP collaboration, G. Hinshaw et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, *Astrophys. J. Suppl.* **208** (2013) 19, [arXiv:1212.5226].

[2] Planck collaboration, P. A. R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13, [arXiv:1502.01589].

[3] Supernova Search Team collaboration, A. G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009–1038, [arXiv:astro-ph/9805201].

[4] Supernova Cosmology Project collaboration, S. Perlmutter et al., *Measurements of Omega and Lambda from 42 high redshift supernovae*, *Astrophys. J.* **517** (1999) 565–586, [arXiv:astro-ph/9812133].

[5] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev.* **D23** (1981) 347–356.

[6] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett.* **B91** (1980) 99–102.

[7] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, *Phys. Lett.* **B108** (1982) 389–393.

[8] A. Albrecht and P. J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, *Phys. Rev. Lett.* **48** (1982) 1220–1223.

[9] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuations and a Nonsingular Universe*, *JETP Lett.* **33** (1981) 532–535.

[10] S. W. Hawking, *The Development of Irregularities in a Single Bubble Inflationary Universe*, *Phys. Lett.* **B115** (1982) 295.

[11] A. A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys. Lett.* **B117** (1982) 175–178.

[12] A. H. Guth and S. Y. Pi, *Fluctuations in the New Inflationary Universe*, *Phys. Rev. Lett.* **49** (1982) 1110–1113.

[13] Virgo, LIGO Scientific collaboration, B. P. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, *Phys. Rev. Lett.* **116** (2016) 061102, [arXiv:1602.03837].

[14] M. Zaldarriaga, *The polarization of the cosmic microwave background*, arXiv:astro-ph/0305272.

[15] Planck collaboration, P. A. R. Ade et al., *Planck 2015 results. XX. Constraints on inflation*, *Astron. Astrophys.* **594** (2016) A20, [arXiv:1502.02114].

[16] P. Creminelli, D. L. López Nacir, M. Simonović, G. Trevisan and M. Zaldarriaga, *Detecting Primordial B-Modes after Planck*, *JCAP* **1511** (2015) 031, [arXiv:1502.01983].

[17] D. H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, *Phys. Rev. Lett.* **78** (1997) 1861–1863, [arXiv:hep-ph/9606387].

[18] D. Lust, *Strings at the LHC*, *Fortsch. Phys.* **58** (2010) 833–838.

[19] A. D. Linde, *Chaotic Inflation*, *Phys. Lett.* **B129** (1983) 177–181.

[20] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, *String Axiverse*, *Phys. Rev.* **D81** (2010) 123530, [arXiv:0905.4720].

[21] P. J. Steinhardt and M. S. Turner, *A Prescription for Successful New Inflation*, *Phys. Rev.* **D29** (1984) 2162–2171.

[22] G. F. Giudice, *Naturally Speaking: The Naturalness Criterion and Physics at the LHC*, arXiv:0801.2562.

[23] R. D. Peccei and H. R. Quinn, *CP Conservation in the Presence of Instantons*, *Phys. Rev. Lett.* **38** (1977) 1440–1443.

[24] R. D. Peccei and H. R. Quinn, *Constraints Imposed by CP Conservation in the Presence of Instantons*, *Phys. Rev.* **D16** (1977) 1791–1797.

[25] S. Weinberg, *A New Light Boson?*, *Phys. Rev. Lett.* **40** (1978) 223–226.

[26] F. Wilczek, *Problem of Strong p and t Invariance in the Presence of Instantons*, *Phys. Rev. Lett.* **40** (1978) 279–282.

[27] J. Preskill, M. B. Wise and F. Wilczek, *Cosmology of the Invisible Axion*, *Phys. Lett.* **B120** (1983) 127–132.

[28] L. F. Abbott and P. Sikivie, *A Cosmological Bound on the Invisible Axion*, *Phys. Lett.* **B120** (1983) 133–136.

[29] M. Dine and W. Fischler, *The Not So Harmless Axion*, *Phys. Lett.* **B120** (1983) 137–141.

[30] K. Freese, J. A. Frieman and A. V. Olinto, *Natural inflation with pseudo - Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233–3236.

[31] J. E. Kim, H. P. Nilles and M. Peloso, *Completing natural inflation*, *JCAP* **0501** (2005) 005, [arXiv:hep-ph/0409138].

[32] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, *N-flation*, *JCAP* **0808** (2008) 003, [arXiv:hep-th/0507205].

[33] T. Banks and N. Seiberg, *Symmetries and Strings in Field Theory and Gravity*, *Phys. Rev.* **D83** (2011) 084019, [arXiv:1011.5120].

[34] T. Banks and L. J. Dixon, *Constraints on String Vacua with Space-Time Supersymmetry*, *Nucl. Phys.* **B307** (1988) 93–108.

[35] C. P. Burgess, J. P. Conlon, L.-Y. Hung, C. H. Kom, A. Maharana and F. Quevedo, *Continuous Global Symmetries and Hyperweak Interactions in String Compactifications*, *JHEP* **07** (2008) 073, [arXiv:0805.4037].

[36] M. Montero, A. M. Uranga and I. Valenzuela, *Transplanckian axions!?*, *JHEP* **08** (2015) 032, [arXiv:1503.03886].

[37] A. Hebecker, P. Mangat, S. Theisen and L. T. Witkowski, *Can Gravitational Instantons Really Constrain Axion Inflation?*, arXiv:1607.06814.

[38] T. W. Grimm, *The Effective action of type II Calabi-Yau orientifolds*, *Fortsch. Phys.* **53** (2005) 1179–1271, [arXiv:hep-th/0507153].

[39] M. J. Duff, B. E. W. Nilsson and C. N. Pope, *Kaluza-Klein Supergravity*, *Phys. Rept.* **130** (1986) 1–142.

[40] J. Polchinski, *Dirichlet Branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724–4727, [arXiv:hep-th/9510017].

[41] L. Randall and R. Sundrum, *A Large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370–3373, [arXiv:hep-ph/9905221].

[42] L. Randall and R. Sundrum, *An Alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690–4693, [arXiv:hep-th/9906064].

[43] R. Sundrum, *Tasi 2004 lectures: To the fifth dimension and back*, arXiv:hep-th/0508134.

[44] R. Rattazzi, *Cargese lectures on extra-dimensions*, arXiv:hep-ph/0607055.

[45] G. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, *Cosmological Problems for the Polonyi Potential*, *Phys.Lett.* **B131** (1983) 59.

[46] T. Banks, D. B. Kaplan and A. E. Nelson, *Cosmological implications of dynamical supersymmetry breaking*, *Phys.Rev.* **D49** (1994) 779–787, [arXiv:hep-ph/9308292].

[47] B. de Carlos, J. Casas, F. Quevedo and E. Roulet, *Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings*, *Phys.Lett.* **B318** (1993) 447–456, [arXiv:hep-ph/9308325].

[48] S. B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys.Rev.* **D66** (2002) 106006, [arXiv:hep-th/0105097].

[49] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D68** (2003) 046005, [arXiv:hep-th/0301240].

[50] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **0503** (2005) 007, [arXiv:hep-th/0502058].

[51] J. P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **0508** (2005) 007, [arXiv:hep-th/0505076].

[52] R. Bousso and J. Polchinski, *Quantization of four form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006, [arXiv:hep-th/0004134].

[53] L. Susskind, *The Anthropic landscape of string theory*, arXiv:hep-th/0302219.

[54] B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, *Observational consequences of a landscape*, *JHEP* **03** (2006) 039, [arXiv:hep-th/0505232].

[55] P. Svrcek and E. Witten, *Axions In String Theory*, *JHEP* **06** (2006) 051, [arXiv:hep-th/0605206].

[56] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, *On the possibility of large axion decay constants*, *JCAP* **0306** (2003) 001, [arXiv:hep-th/0303252].

[57] E. Silverstein and A. Westphal, *Monodromy in the CMB: Gravity Waves and String Inflation*, *Phys.Rev.* **D78** (2008) 106003, [arXiv:0803.3085].

[58] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, *Phys.Rev.* **D82** (2010) 046003, [arXiv:0808.0706].

[59] P. Candelas, P. S. Green and T. Hubsch, *Rolling Among Calabi-Yau Vacua*, *Nucl. Phys.* **B330** (1990) 49.

[60] P. Candelas and X. de la Ossa, *Moduli Space of Calabi-Yau Manifolds*, *Nucl. Phys.* **B355** (1991) 455–481.

[61] S. Hosono, A. Klemm, S. Theisen and S.-T. Yau, *Mirror symmetry, mirror map and applications to complete intersection Calabi-Yau spaces*, *Nucl.Phys.* **B433** (1995) 501–554, [arXiv:hep-th/9406055].

[62] I. García-Etxebarria, T. W. Grimm and I. Valenzuela, *Special Points of Inflation in Flux Compactifications*, *Nucl. Phys.* **B899** (2015) 414–443, [arXiv:1412.5537].

[63] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The String landscape, black holes and gravity as the weakest force*, *JHEP* **06** (2007) 060, [arXiv:hep-th/0601001].

[64] S. R. Coleman, *Aspects of Symmetry, Selected Erice Lectures.* Cambridge University Press, 1985, 10.1017/CBO9780511565045.

[65] C. Cheung and G. N. Remmen, *Naturalness and the Weak Gravity Conjecture*, *Phys. Rev. Lett.* **113** (2014) 051601, [arXiv:1402.2287].

[66] T. Rudelius, *On the Possibility of Large Axion Moduli Spaces*, *JCAP* **1504** (2015) 049, [arXiv:1409.5793].

[67] A. de la Fuente, P. Saraswat and R. Sundrum, *Natural Inflation and Quantum Gravity*, *Phys. Rev. Lett.* **114** (2015) 151303, [arXiv:1412.3457].

[68] T. Rudelius, *Constraints on Axion Inflation from the Weak Gravity Conjecture*, *JCAP* **1509** (2015) 020, [arXiv:1503.00795].

[69] J. Brown, W. Cottrell, G. Shiu and P. Soler, *Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation*, *JHEP* **10** (2015) 023, [arXiv:1503.04783].

[70] T. C. Bachlechner, C. Long and L. McAllister, *Planckian Axions and the Weak Gravity Conjecture*, *JHEP* **01** (2016) 091, [arXiv:1503.07853].

[71] J. Brown, W. Cottrell, G. Shiu and P. Soler, *On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture*, *JHEP* **04** (2016) 017, [arXiv:1504.00659].

[72] D. Junghans, *Large-Field Inflation with Multiple Axions and the Weak Gravity Conjecture*, *JHEP* **02** (2016) 128, [arXiv:1504.03566].

[73] B. Heidenreich, M. Reece and T. Rudelius, *Weak Gravity Strongly Constrains Large-Field Axion Inflation*, *JHEP* **12** (2015) 108, [arXiv:1506.03447].

[74] E. Palti, *On Natural Inflation and Moduli Stabilisation in String Theory*, *JHEP* **10** (2015) 188, [arXiv:1508.00009].

[75] B. Heidenreich, M. Reece and T. Rudelius, *Sharpening the Weak Gravity Conjecture with Dimensional Reduction*, *JHEP* **02** (2016) 140, [arXiv:1509.06374].

[76] K. Kooner, S. Parameswaran and I. Zavala, *Warping the Weak Gravity Conjecture*, *Phys. Lett.* **B759** (2016) 402–409, [arXiv:1509.07049].

[77] D. Andriot, *A no-go theorem for monodromy inflation*, *JCAP* **1603** (2016) 025, [arXiv:1510.02005].

[78] B. Heidenreich, M. Reece and T. Rudelius, *Evidence for a Lattice Weak Gravity Conjecture*, arXiv:1606.08437.

[79] P. Saraswat, *The Weak Gravity Conjecture and Effective Field Theory*, *Phys. Rev.* **D95** (2017) 025013, [arXiv:1608.06951].

[80] D. Klaewer and E. Palti, *Super-Planckian Spatial Field Variations and Quantum Gravity*, *JHEP* **01** (2017) 088, [arXiv:1610.00010].

[81] H. Ooguri and C. Vafa, *Non-supersymmetric AdS and the Swampland*, arXiv:1610.01533.

[82] B. Freivogel and M. Kleban, *Vacua Morghulis*, arXiv:1610.04564.

[83] W. Cottrell, G. Shiu and P. Soler, *Weak Gravity Conjecture and Extremal Black Holes*, arXiv:1611.06270.

[84] A. Hebecker, P. Henkenjohann and L. T. Witkowski, *What is the Magnetic Weak Gravity Conjecture for Axions?*, arXiv:1701.06553.

[85] S. W. Hawking, *Gravitational radiation from colliding black holes*, *Phys. Rev. Lett.* **26** (1971) 1344–1346.

[86] J. D. Bekenstein, *Black holes and the second law*, *Lett. Nuovo Cim.* **4** (1972) 737–740.

[87] R. Bousso, *A Covariant entropy conjecture*, *JHEP* **07** (1999) 004, [arXiv:hep-th/9905177].

[88] J. P. Conlon, *Quantum Gravity Constraints on Inflation*, *JCAP* **1209** (2012) 019, [arXiv:1203.5476].

[89] N. Kaloper, M. Kleban, A. Lawrence and M. S. Sloth, *Large Field Inflation and Gravitational Entropy*, *Phys. Rev.* **D93** (2016) 043510, [arXiv:1511.05119].

[90] H. Peiris, R. Easther and R. Flauger, *Constraining Monodromy Inflation*, *JCAP* **1309** (2013) 018, [arXiv:1303.2616].

[91] R. Easther and R. Flauger, *Planck Constraints on Monodromy Inflation*, *JCAP* **1402** (2014) 037, [arXiv:1308.3736].

[92] T. Kobayashi, O. Seto and Y. Yamaguchi, *Axion monodromy inflation with sinusoidal corrections*, *PTEP* **2014** (2014) 103E01, [arXiv:1404.5518].

[93] R. Flauger, L. McAllister, E. Silverstein and A. Westphal, *Drifting Oscillations in Axion Monodromy*, arXiv:1412.1814.

[94] R. Z. Ferreira and M. S. Sloth, *Universal Constraints on Axions from Inflation*, *JHEP* **12** (2014) 139, [arXiv:1409.5799].

[95] R. Z. Ferreira, J. Ganc, J. Noreña and M. S. Sloth, *On the validity of the perturbative description of axions during inflation*, *JCAP* **1604** (2016) 039, [arXiv:1512.06116].

[96] R. Flauger, M. Mirbabayi, L. Senatore and E. Silverstein, *Productive Interactions: heavy particles and non-Gaussianity*, arXiv:1606.00513.

[97] V. Domcke, M. Pieroni and P. Binétruy, *Primordial gravitational waves for universality classes of pseudoscalar inflation*, *JCAP* **1606** (2016) 031, [arXiv:1603.01287].

[98] M. Peloso, L. Sorbo and C. Unal, *Rolling axions during inflation: perturbativity and signatures*, *JCAP* **1609** (2016) 001, [arXiv:1606.00459].

[99] C. Vafa, *Evidence for F theory*, *Nucl. Phys.* **B469** (1996) 403–418, [arXiv:hep-th/9602022].

[100] X. Dong, B. Horn, E. Silverstein and A. Westphal, *Simple exercises to flatten your potential*, *Phys. Rev.* **D84** (2011) 026011, [arXiv:1011.4521].

[101] A. Hebecker, S. C. Kraus and L. T. Witkowski, *D7-Brane Chaotic Inflation*, *Phys.Lett.* **B737** (2014) 16–22, [arXiv:1404.3711].

[102] F. Marchesano, G. Shiu and A. M. Uranga, *F-term Axion Monodromy Inflation*, *JHEP* **09** (2014) 184, [arXiv:1404.3040].

[103] R. Blumenhagen and E. Plauschinn, *Towards Universal Axion Inflation and Reheating in String Theory*, *Phys.Lett.* **B736** (2014) 482–487, [arXiv:1404.3542].

[104] D. R. Morrison, *Picard-Fuchs equations and mirror maps for hypersurfaces*, arXiv:hep-th/9111025.

[105] S. Hosono, A. Klemm and S. Theisen, *Lectures on mirror symmetry*, *Lect. Notes Phys.* **436** (1994) 235–280, [arXiv:hep-th/9403096].

[106] M. Berg, E. Pajer and S. Sjors, *Dante's Inferno*, *Phys.Rev.* **D81** (2010) 103535, [arXiv:0912.1341].

[107] I. Ben-Dayan, F. G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, *Phys.Rev.Lett.* **113** (2014) 261301, [arXiv:1404.7773].

[108] L. E. Ibanez, M. Montero, A. Uranga and I. Valenzuela, *Relaxion Monodromy and the Weak Gravity Conjecture*, *JHEP* **04** (2016) 020, [arXiv:1512.00025].

[109] N. Kaloper and L. Sorbo, *A Natural Framework for Chaotic Inflation*, *Phys. Rev. Lett.* **102** (2009) 121301, [arXiv:0811.1989].

[110] N. Kaloper, A. Lawrence and L. Sorbo, *An Ignoble Approach to Large Field Inflation*, *JCAP* **1103** (2011) 023, [arXiv:1101.0026].

[111] G. Dvali, *Three-form gauging of axion symmetries and gravity*, arXiv:hep-th/0507215.

[112] S. Parameswaran, G. Tasinato and I. Zavala, *Subleading Effects and the Field Range in Axion Inflation*, *JCAP* **1604** (2016) 008, [arXiv:1602.02812].

[113] K. Kadota, T. Kobayashi, A. Oikawa, N. Omoto, H. Otsuka and T. H. Tatsuishi, *Small field axion inflation with sub-Planckian decay constant*, *JCAP* **1610** (2016) 013, [arXiv:1606.03219].

[114] E. Witten, *Cosmic Separation of Phases*, *Phys. Rev.* **D30** (1984) 272–285.

[115] A. Kosowsky, M. S. Turner and R. Watkins, *Gravitational waves from first order cosmological phase transitions*, *Phys. Rev. Lett.* **69** (1992) 2026–2029.

[116] A. Kosowsky, M. S. Turner and R. Watkins, *Gravitational radiation from colliding vacuum bubbles*, *Phys. Rev.* **D45** (1992) 4514–4535.

[117] T. Hiramatsu, M. Kawasaki, K. Saikawa and T. Sekiguchi, *Axion cosmology with long-lived domain walls*, *JCAP* **1301** (2013) 001, [arXiv:1207.3166].

[118] S.-Y. Zhou, E. J. Copeland, R. Easther, H. Finkel, Z.-G. Mou and P. M. Saffin, *Gravitational Waves from Oscillon Preheating*, *JHEP* **10** (2013) 026, [arXiv:1304.6094].

[119] R. Daido, N. Kitajima and F. Takahashi, *Axion domain wall baryogenesis*, *JCAP* **1507** (2015) 046, [arXiv:1504.07917].

[120] R. Daido, N. Kitajima and F. Takahashi, *Domain Wall Formation from Level Crossing in the Axiverse*, *Phys. Rev.* **D92** (2015) 063512, [arXiv:1505.07670].

[121] T. Higaki, K. S. Jeong, N. Kitajima and F. Takahashi, *Quality of the Peccei-Quinn symmetry in the Aligned QCD Axion and Cosmological Implications*, *JHEP* **06** (2016) 150, [arXiv:1603.02090].

[122] T. Higaki, K. S. Jeong, N. Kitajima, T. Sekiguchi and F. Takahashi, *Topological Defects and nano-Hz Gravitational Waves in Aligned Axion Models*, *JHEP* **08** (2016) 044, [arXiv:1606.05552].

[123] A. Ashoorioon, *Exit from Inflation with a First-Order Phase Transition and a Gravitational Wave Blast*, *Phys. Lett.* **B747** (2015) 446–453, [arXiv:1502.00556].

[124] C. Grojean and G. Servant, *Gravitational Waves from Phase Transitions at the Electroweak Scale and Beyond*, *Phys. Rev.* **D75** (2007) 043507, [arXiv:hep-ph/0607107].

[125] M. Cicoli, J. P. Conlon and F. Quevedo, *Dark Radiation in LARGE Volume Models*, *Phys.Rev.* **D87** (2013) 043520, [arXiv:1208.3562].

[126] T. Higaki and F. Takahashi, *Dark Radiation and Dark Matter in Large Volume Compactifications*, *JHEP* **1211** (2012) 125, [arXiv:1208.3563].

[127] T. Higaki, K. Nakayama and F. Takahashi, *Moduli-Induced Axion Problem*, *JHEP* **1307** (2013) 005, [arXiv:1304.7987].

[128] R. Allahverdi, M. Cicoli, B. Dutta and K. Sinha, *Correlation between Dark Matter and Dark Radiation in String Compactifications*, *JCAP* **1410** (2014) 002, [arXiv:1401.4364].

[129] A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Dark Radiation predictions from general Large Volume Scenarios*, *JHEP* **09** (2014) 140, [arXiv:1403.6810].

[130] A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Tuning and Backreaction in F-term Axion Monodromy Inflation*, *Nucl. Phys.* **B894** (2015) 456–495, [arXiv:1411.2032].

[131] A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, *Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?*, *Phys. Lett.* **B748** (2015) 455–462, [arXiv:1503.07912].

[132] A. Hebecker, F. Rompineve and A. Westphal, *Axion Monodromy and the Weak Gravity Conjecture*, *JHEP* **04** (2016) 157, [arXiv:1512.03768].

[133] A. Hebecker, J. Jaeckel, F. Rompineve and L. T. Witkowski, *Gravitational Waves from Axion Monodromy*, *JCAP* **1611** (2016) 003, [arXiv:1606.07812].

[134] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge University Press, 2015.

[135] J. Polchinski, *String Theory, Volume 1 and Volume 2,* , *Cambridge University Press* (1998) .

[136] R. Blumenhagen, D. Lüst and S. Theisen, *Basic concepts of string theory*. Theoretical and Mathematical Physics. Springer, Heidelberg, Germany, 2013, 10.1007/978-3-642-29497-6.

[137] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest and A. Van Proeyen, *New formulations of D = 10 supersymmetry and D8 - O8 domain walls*, *Class. Quant. Grav.* **18** (2001) 3359–3382, [arXiv:hep-th/0103233].

[138] M. Fukuma, T. Oota and H. Tanaka, *Comments on T dualities of Ramond-Ramond potentials on tori*, *Prog. Theor. Phys.* **103** (2000) 425–446, [arXiv:hep-th/9907132].

[139] C. V. Johnson, *D-brane primer*, arXiv:hep-th/0007170.

[140] M. Dine, N. Seiberg, X. G. Wen and E. Witten, *Nonperturbative Effects on the String World Sheet*, *Nucl. Phys.* **B278** (1986) 769–789.

[141] T. W. Grimm and J. Louis, *The Effective action of N = 1 Calabi-Yau orientifolds*, *Nucl. Phys.* **B699** (2004) 387–426, [arXiv:hep-th/0403067].

[142] J. P. Conlon, *Moduli Stabilisation and Applications in IIB String Theory*, *Fortsch. Phys.* **55** (2007) 287–422, [arXiv:hep-th/0611039].

[143] T. Weigand, *Lectures on F-theory compactifications and model building*, *Class. Quant. Grav.* **27** (2010) 214004, [arXiv:1009.3497].

[144] F. Denef, *Les Houches Lectures on Constructing String Vacua*, arXiv:0803.1194.

[145] M. Grana, *Flux compactifications in string theory: A Comprehensive review*, *Phys. Rept.* **423** (2006) 91–158, [arXiv:hep-th/0509003].

[146] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and alpha-prime corrections to flux induced potentials*, *JHEP* **06** (2002) 060, [arXiv:hep-th/0204254].

[147] M. A. Shifman and A. I. Vainshtein, *On Gluino Condensation in Supersymmetric Gauge Theories. SU(N) and O(N) Groups*, *Nucl. Phys.* **B296** (1988) 445.

[148] E. Witten, *Nonperturbative superpotentials in string theory*, *Nucl. Phys.* **B474** (1996) 343–360, [arXiv:hep-th/9604030].

[149] A. Hebecker, S. C. Kraus, M. Kuntzler, D. Lust and T. Weigand, *Fluxbranes: Moduli Stabilisation and Inflation*, *JHEP* **01** (2013) 095, [arXiv:1207.2766].

[150] C. Vafa, *The String landscape and the swampland*, arXiv:hep-th/0509212.

[151] H. Ooguri and C. Vafa, *On the Geometry of the String Landscape and the Swampland*, *Nucl. Phys.* **B766** (2007) 21–33, [arXiv:hep-th/0605264].

[152] M. R. Douglas and Z. Lu, *Finiteness of volume of moduli spaces*, arXiv:hep-th/0509224.

[153] Y. Aharonov, A. Casher and S. Nussinov, *The Unitarity Puzzle and Planck Mass Stable Particles*, *Phys. Lett.* **B191** (1987) 51.

[154] L. Susskind, *Trouble for remnants*, arXiv:hep-th/9501106.

[155] T. Banks, M. Johnson and A. Shomer, *A Note on Gauge Theories Coupled to Gravity*, *JHEP* **09** (2006) 049, [arXiv:hep-th/0606277].

[156] M. Montero, G. Shiu and P. Soler, *The Weak Gravity Conjecture in three dimensions*, *JHEP* **10** (2016) 159, [arXiv:1606.08438].

[157] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, *Extra natural inflation*, *Phys. Rev. Lett.* **90** (2003) 221302, [arXiv:hep-th/0301218].

[158] C. Long, L. McAllister and P. McGuirk, *Aligned Natural Inflation in String Theory*, *Phys. Rev.* **D90** (2014) 023501, [arXiv:1404.7852].

[159] I. Ben-Dayan, F. G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, *Phys. Rev. Lett.* **113** (2014) 261301, [arXiv:1404.7773].

[160] Y. Bai and B. A. Stefanek, *Natural millicharged inflation*, *Phys. Rev.* **D91** (2015) 096012, [arXiv:1405.6720].

[161] L. McAllister, E. Silverstein, A. Westphal and T. Wrase, *The Powers of Monodromy*, *JHEP* **09** (2014) 123, [arXiv:1405.3652].

[162] R. Blumenhagen, D. Herschmann and E. Plauschinn, *The Challenge of Realizing F-term Axion Monodromy Inflation in String Theory*, *JHEP* **1501** (2015) 007, [arXiv:1409.7075].

[163] H. Hayashi, R. Matsuda and T. Watari, *Issues in Complex Structure Moduli Inflation*, arXiv:1410.7522.

[164] P. Mangat, *PhD thesis*, *to appear (2017)* (University of Heidelberg) .

[165] M. Arends, A. Hebecker, K. Heimpel, S. C. Kraus, D. Lust et al., *D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation*, *Fortsch.Phys.* **62** (2014) 647–702, [arXiv:1405.0283].

[166] A. Mazumdar and P. Shukla, *Model independent bounds on tensor modes and stringy parameters from CMB*, arXiv:1411.4636.

[167] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo and R. Valandro, *D-Branes at del Pezzo Singularities: Global Embedding and Moduli Stabilisation*, *JHEP* **09** (2012) 019, [arXiv:1206.5237].

[168] L. Aparicio, M. Cicoli, S. Krippendorf, A. Maharana, F. Muia and F. Quevedo, *Sequestered de Sitter String Scenarios: Soft-terms*, *JHEP* **11** (2014) 071, [arXiv:1409.1931].

[169] F. Denef and M. R. Douglas, *Distributions of flux vacua*, *JHEP* **0405** (2004) 072, [arXiv:hep-th/0404116].

[170] T. W. Grimm, J. Keitel, R. Savelli and M. Weissenbacher, *From M-theory higher curvature terms to α' corrections in F-theory*, *Nucl. Phys.* **B903** (2016) 325–359, [arXiv:1312.1376].

[171] D. Junghans and G. Shiu, *Brane curvature corrections to the $\mathcal{N} = 1$ type II/F-theory effective action*, *JHEP* **03** (2015) 107, [arXiv:1407.0019].

[172] L. E. Ibanez, F. Marchesano and I. Valenzuela, *Higgs-otic Inflation and String Theory*, *JHEP* **1501** (2015) 128, [arXiv:1411.5380].

[173] H. Abe, T. Kobayashi and H. Otsuka, *Natural inflation with and without modulations in type IIB string theory*, *JHEP* **04** (2015) 160, [arXiv:1411.4768].

[174] Y. Honma and M. Manabe, *Exact Kahler Potential for Calabi-Yau Fourfolds*, *JHEP* **1305** (2013) 102, [arXiv:1302.3760].

[175] R. Kappl, S. Krippendorf and H. P. Nilles, *Aligned Natural Inflation: Monodromies of two Axions*, *Phys.Lett.* **B737** (2014) 124–128, [arXiv:1404.7127].

[176] C. Long, L. McAllister and P. McGuirk, *Aligned Natural Inflation in String Theory*, *Phys.Rev.* **D90** (2014) 023501, [arXiv:1404.7852].

[177] T. Ali, S. S. Haque and V. Jejjala, *Natural Inflation from Near Alignment in Heterotic String Theory*, *Phys. Rev.* **D91** (2015) 083516, [arXiv:1410.4660].

[178] G. Shiu, W. Staessens and F. Ye, *Widening the Axion Window via Kinetic and Stückelberg Mixings*, *Phys. Rev. Lett.* **115** (2015) 181601, [arXiv:1503.01015].

[179] G. Shiu, W. Staessens and F. Ye, *Large Field Inflation from Axion Mixing*, *JHEP* **06** (2015) 026, [arXiv:1503.02965].

[180] W. Buchmuller, E. Dudas, L. Heurtier, A. Westphal, C. Wieck and M. W. Winkler, *Challenges for Large-Field Inflation and Moduli Stabilization*, *JHEP* **04** (2015) 058, [arXiv:1501.05812].

[181] P. W. Graham, D. E. Kaplan and S. Rajendran, *Cosmological Relaxation of the Electroweak Scale*, *Phys. Rev. Lett.* **115** (2015) 221801, [arXiv:1504.07551].

[182] J. R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs and G. Servant, *Cosmological Higgs-Axion Interplay for a Naturally Small Electroweak Scale*, *Phys. Rev. Lett.* **115** (2015) 251803, [arXiv:1506.09217].

[183] E. Hardy, *Electroweak relaxation from finite temperature*, *JHEP* **11** (2015) 077, [arXiv:1507.07525].

[184] S. P. Patil and P. Schwaller, *Relaxing the Electroweak Scale: the Role of Broken dS Symmetry*, *JHEP* **02** (2016) 077, [arXiv:1507.08649].

[185] J. Jaeckel, V. M. Mehta and L. T. Witkowski, *Musings on cosmological relaxation and the hierarchy problem*, *Phys. Rev.* **D93** (2016) 063522, [arXiv:1508.03321].

[186] R. S. Gupta, Z. Komargodski, G. Perez and L. Ubaldi, *Is the Relaxion an Axion?*, *JHEP* **02** (2016) 166, [arXiv:1509.00047].

[187] B. Batell, G. F. Giudice and M. McCullough, *Natural Heavy Supersymmetry*, *JHEP* **12** (2015) 162, [arXiv:1509.00834].

[188] K. Choi and S. H. Im, *Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry*, *JHEP* **01** (2016) 149, [arXiv:1511.00132].

[189] D. E. Kaplan and R. Rattazzi, *Large field excursions and approximate discrete symmetries from a clockwork axion*, *Phys. Rev.* **D93** (2016) 085007, [arXiv:1511.01827].

[190] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister and A. Murugan, *On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes*, *JHEP* **11** (2006) 031, [arXiv:hep-th/0607050].

[191] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, *Oscillations in the CMB from Axion Monodromy Inflation*, *JCAP* **1006** (2010) 009, [arXiv:0907.2916].

[192] J. D. Brown and C. Teitelboim, *Dynamical Neutralization of the Cosmological Constant*, *Phys. Lett.* **B195** (1987) 177–182.

[193] J. D. Brown and C. Teitelboim, *Neutralization of the Cosmological Constant by Membrane Creation*, *Nucl. Phys.* **B297** (1988) 787–836.

[194] G. D'Amico, R. Gobbetti, M. Kleban and M. Schillo, *Unwinding Inflation*, *JCAP* **1303** (2013) 004, [arXiv:1211.4589].

[195] S. R. Coleman and F. De Luccia, *Gravitational Effects on and of Vacuum Decay*, *Phys. Rev.* **D21** (1980) 3305.

[196] S. B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D66** (2002) 106006, [arXiv:hep-th/0105097].

[197] X. Chen, R. Easther and E. A. Lim, *Generation and Characterization of Large Non-Gaussianities in Single Field Inflation*, *JCAP* **0804** (2008) 010, [arXiv:0801.3295].

[198] Planck collaboration, P. A. R. Ade et al., *Planck 2015 results. XVII. Constraints on primordial non-Gaussianity*, *Astron. Astrophys.* **594** (2016) A17, [arXiv:1502.01592].

[199] BICEP2, Planck collaboration, P. A. R. Ade et al., *Joint Analysis of BICEP2/Keck Array and Planck Data*, *Phys. Rev. Lett.* **114** (2015) 101301, [arXiv:1502.00612].

[200] A. Ashoorioon, K. Freese and J. T. Liu, *Slow nucleation rates in Chain Inflation with QCD Axions or Monodromy*, *Phys. Rev.* **D79** (2009) 067302, [arXiv:0810.0228].

[201] H. Bazrafshan Moghaddam and R. Brandenberger, *Preheating with Fractional Powers*, *Mod. Phys. Lett.* **A31** (2016) 1650217, [arXiv:1502.06135].

[202] P. Adshead, J. T. Giblin, T. R. Scully and E. I. Sfakianakis, *Gauge-preheating and the end of axion inflation*, *JCAP* **1512** (2015) 034, [arXiv:1502.06506].

[203] E. McDonough, H. Bazrafshan Moghaddam and R. H. Brandenberger, *Preheating and Entropy Perturbations in Axion Monodromy Inflation*, *JCAP* **1605** (2016) 012, [arXiv:1601.07749].

[204] M. A. Amin, R. Easther, H. Finkel, R. Flauger and M. P. Hertzberg, *Oscillons After Inflation*, *Phys. Rev. Lett.* **108** (2012) 241302, [arXiv:1106.3335].

[205] D. Blair et al., *Gravitational wave astronomy: the current status*, *Sci. China Phys. Mech. Astron.* **58** (2015) 120402, [arXiv:1602.02872].

[206] G. D'Amico, T. Hamill and N. Kaloper, *Quantum Field Theory of Interacting Dark Matter/Dark Energy: Dark Monodromies*, *Phys. Rev.* **D94** (2016) 103526, [arXiv:1605.00996].

[207] J. Jaeckel, V. M. Mehta and L. T. Witkowski, *Monodromy Dark Matter*, *JCAP* **1701** (2017) 036, [arXiv:1605.01367].

[208] M. Hindmarsh, S. Huber, K. Rummukainen and D. Weir, *Gravitational waves from cosmological first order phase transitions*, *PoS* **LATTICE2015** (2016) 233, [arXiv:1511.04527].

[209] C. Caprini et al., *Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions*, *JCAP* **1604** (2016) 001, [arXiv:1512.06239].

[210] C. Guzzetti, M., N. Bartolo, M. Liguori and S. Matarrese, *Gravitational waves from inflation*, *Riv. Nuovo Cim.* **39** (2016) 399–495, [arXiv:1605.01615].

[211] W. Buchmüller, V. Domcke, K. Kamada and K. Schmitz, *The Gravitational Wave Spectrum from Cosmological $B-L$ Breaking*, *JCAP* **1310** (2013) 003, [arXiv:1305.3392].

[212] D. Baumann, *Cosmology, Part III Lecture Notes.* http://www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf.

[213] V. Mukhanov, *Physical Foundations of Cosmology.* Cambridge University Press, Oxford, 2005.

[214] B. Ratra, *Expressions for linearized perturbations in a massive scalar field dominated cosmological model*, *Phys. Rev.* **D44** (1991) 352–364.

[215] J.-c. Hwang, *Roles of a coherent scalar field on the evolution of cosmic structures*, *Phys. Lett.* **B401** (1997) 241–246, [arXiv:astro-ph/9610042].

[216] P. D. Meerburg and D. N. Spergel, *Searching for oscillations in the primordial power spectrum. II. Constraints from Planck data*, *Phys. Rev.* **D89** (2014) 063537, [arXiv:1308.3705].

[217] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, *Cambridge University Press* (1996) .

[218] L. Kofman, A. D. Linde and A. A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev.* **D56** (1997) 3258–3295, [arXiv:hep-ph/9704452].

[219] S. Weigert, *Quantum Parametric Resonance*, *J.Phys.* **A35** (2002) 4669, [arXiv:quant-ph/0106138].

[220] J. Berges and J. Serreau, *Parametric resonance in quantum field theory*, *Phys. Rev. Lett.* **91** (2003) 111601, [arXiv:hep-ph/0208070].

[221] G. Barenboim and W.-I. Park, *Gravitational waves from first order phase transitions as a probe of an early matter domination era and its inverse problem*, *Phys. Lett.* **B759** (2016) 430–438, [arXiv:1605.03781].

[222] C. J. Moore, R. H. Cole and C. P. L. Berry, *Gravitational-wave sensitivity curves*, *Class. Quant. Grav.* **32** (2015) 015014, [arXiv:1408.0740].

[223] S. Angus, J. P. Conlon, U. Haisch and A. J. Powell, *Loop corrections to ΔN_{eff} in large volume models*, *JHEP* **1312** (2013) 061, [arXiv:1305.4128].

[224] M. Cicoli and F. Muia, *General Analysis of Dark Radiation in Sequestered String Models*, *JHEP* **12** (2015) 152, [arXiv:1511.05447].

[225] A. Hebecker, A. K. Knochel and T. Weigand, *A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations*, *JHEP* **1206** (2012) 093, [arXiv:1204.2551].

[226] L. E. Ibanez, F. Marchesano, D. Regalado and I. Valenzuela, *The Intermediate Scale MSSM, the Higgs Mass and F-theory Unification*, *JHEP* **1207** (2012) 195, [arXiv:1206.2655].

[227] L. E. Ibanez and I. Valenzuela, *The Higgs Mass as a Signature of Heavy SUSY*, *JHEP* **1305** (2013) 064, [arXiv:1301.5167].

[228] A. Hebecker, A. K. Knochel and T. Weigand, *The Higgs mass from a String-Theoretic Perspective*, *Nucl.Phys.* **B874** (2013) 1–35, [arXiv:1304.2767].

[229] J. P. Conlon, S. S. Abdussalam, F. Quevedo and K. Suruliz, *Soft SUSY Breaking Terms for Chiral Matter in IIB String Compactifications*, *JHEP* **0701** (2007) 032, [arXiv:hep-th/0610129].

[230] K. Choi, H. P. Nilles, C. S. Shin and M. Trapletti, *Sparticle Spectrum of Large Volume Compactification*, *JHEP* **1102** (2011) 047, [arXiv:1011.0999].

[231] A. Maharana and E. Palti, *Models of Particle Physics from Type IIB String Theory and F-theory: A Review*, *Int.J.Mod.Phys.* **A28** (2013) 1330005, [arXiv:1212.0555].

[232] T. Weigand, *Lectures on F-theory compactifications and model building*, *Class.Quant.Grav.* **27** (2010) 214004, [arXiv:1009.3497].

[233] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo and R. Valandro, *D3/D7 Branes at Singularities: Constraints from Global Embedding and Moduli Stabilisation*, *JHEP* **1307** (2013) 150, [arXiv:1304.0022].

[234] G. Aldazabal, L. E. Ibanez, F. Quevedo and A. Uranga, *D-branes at singularities: A Bottom up approach to the string embedding of the standard model*, *JHEP* **0008** (2000) 002, [arXiv:hep-th/0005067].

[235] R. Blumenhagen, S. Moster and E. Plauschinn, *String GUT Scenarios with Stabilised Moduli*, *Phys.Rev.* **D78** (2008) 066008, [arXiv:0806.2667].

[236] T. Higaki, K. Kamada and F. Takahashi, *Higgs, Moduli Problem, Baryogenesis and Large Volume Compactifications*, *JHEP* **1209** (2012) 043, [arXiv:1207.2771].

[237] M. Haack, D. Krefl, D. Lust, A. Van Proeyen and M. Zagermann, *Gaugino Condensates and D-terms from D7-branes*, *JHEP* **0701** (2007) 078, [arXiv:hep-th/0609211].

[238] M. Cicoli, C. Mayrhofer and R. Valandro, *Moduli Stabilisation for Chiral Global Models*, *JHEP* **1202** (2012) 062, [arXiv:1110.3333].

[239] J. P. Conlon, D. Cremades and F. Quevedo, *Kahler potentials of chiral matter fields for Calabi-Yau string compactifications*, *JHEP* **0701** (2007) 022, [arXiv:hep-th/0609180].

[240] L. J. Hall and Y. Nomura, *A Finely-Predicted Higgs Boson Mass from A Finely-Tuned Weak Scale*, *JHEP* **1003** (2010) 076, [arXiv:0910.2235].

[241] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice et al., *Higgs mass and vacuum stability in the Standard Model at NNLO*, *JHEP* **1208** (2012) 098, [arXiv:1205.6497].

[242] A. Delgado, M. Garcia and M. Quiros, *Electroweak and supersymmetry breaking from the Higgs boson discovery*, *Phys. Rev.* **D90** (2014) 015016, [arXiv:1312.3235].

[243] J. P. Conlon, *The QCD axion and moduli stabilisation*, *JHEP* **0605** (2006) 078, [arXiv:hep-th/0602233].

[244] M. Cicoli, M. Goodsell and A. Ringwald, *The type IIB string axiverse and its low-energy phenomenology*, *JHEP* **1210** (2012) 146, [arXiv:1206.0819].

[245] P. Sikivie, *Axion Cosmology*, *Lect.Notes Phys.* **741** (2008) 19–50, [arXiv:astro-ph/0610440].

[246] M. P. Hertzberg, M. Tegmark and F. Wilczek, *Axion Cosmology and the Energy Scale of Inflation*, *Phys.Rev.* **D78** (2008) 083507, [arXiv:0807.1726].

[247] M. Cicoli, J. P. Conlon and F. Quevedo, *General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation*, *JHEP* **0810** (2008) 105, [arXiv:0805.1029].

[248] M. Cicoli, C. Burgess and F. Quevedo, *Fibre Inflation: Observable Gravity Waves from IIB String Compactifications*, *JCAP* **0903** (2009) 013, [arXiv:0808.0691].

[249] C. Burgess, M. Cicoli, M. Gomez-Reino, F. Quevedo, G. Tasinato et al., *Non-standard primordial fluctuations and nongaussianity in string inflation*, *JHEP* **1008** (2010) 045, [arXiv:1005.4840].

[250] R. Blumenhagen, S. Moster and E. Plauschinn, *Moduli Stabilisation versus Chirality for MSSM like Type IIB Orientifolds*, *JHEP* **0801** (2008) 058, [arXiv:0711.3389].

[251] T. W. Grimm, M. Kerstan, E. Palti and T. Weigand, *On Fluxed Instantons and Moduli Stabilisation in IIB Orientifolds and F-theory*, *Phys.Rev.* **D84** (2011) 066001, [arXiv:1105.3193].

[252] L. McAllister, P. Schwaller, G. Servant, J. Stout and A. Westphal, *Runaway Relaxion Monodromy*, arXiv:1610.05320.

[253] M. Nakahara, *Geometry, topology and physics*, *Bristol, UK: Hilger (1990) 505 p. (Graduate student series in physics)* (1990) .

[254] D. S. Gorbunov and V. A. Rubakov, *Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory.* Hackensack, USA: World Scientific (2011) 489 p, 2011, 10.1142/7874.

[255] L. A. Urena-Lopez, *Nonrelativistic approach for cosmological scalar field dark matter*, *Phys. Rev.* **D90** (2014) 027306, [arXiv:1310.8601].

[256] E. Braaten, A. Mohapatra and H. Zhang, *Nonrelativistic Effective Field Theory for Axions*, *Phys. Rev.* **D94** (2016) 076004, [arXiv:1604.00669].

[257] S.-M. Choi and H. M. Lee, *Inflection point inflation and reheating*, *Eur. Phys. J.* **C76** (2016) 303, [arXiv:1601.05979].