Universality in Nuclear Few-body Dynamics Employing Resonating Group Method in $\not\pi$ EFT

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Introduction

The universality of few-body systems with two clusters with an identical number of particles is analyzed in an energy regime where the relative motion between the clusters is small in comparison to the excitation scale of each cluster [1]. The model-independent framework of pionless Effective Field Theory ($\not\pi$EFT) is employed via the computational framework of a Resonating Group Method (RGM) to extract possible universal signatures of observables in systems with three or more constituent particles. The underlying binding mechanisms of light nuclei ($A \geq 4$), attributing either to specific consequences of their substructure or emergent universal characteristics that follow from the certain separation of scales, is understood by this method. To achieve this goal, we began by first studying the behavior of dimer-dimer scattering, each consisting of two-component fermions and obtained the universal result that the dimer-dimer scattering length is 0.6 times the fermion-fermion scattering length, irrespective of the short-distance nature of the fermion-fermion interaction potential. As the next step, this study is extended to trimer-trimer and tetramer-tetramer scattering.

Theory

Two very-low-energy particles, represented by a field ψ, can be described by an effective Lagrangian

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) \psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}[(\psi\psi)^\dagger(\psi \overleftrightarrow{\nabla}^2 \psi) + H.c.] + ... \quad (1)$$

Where $\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ is the Galilei- invariant derivative and H.c. denotes the Hermitian conjugate. The "..." represents local operators with other combinations of derivatives, including relativistic corrections. C_0, C_2 represents low energy constants. For a two-cluster system, contact interaction potential is

$$\mathcal{V}_{AB} = C(\lambda) \sum_{i\epsilon A, j\epsilon B} \delta_\lambda^{(3)}(r_i - r_j) + D(\lambda) \sum_{i,j,k, i\epsilon A => jvk\epsilon B} \delta_\lambda^{(3)}(r_i - r_j) \times \delta_\lambda^{(3)}(r_i - r_k) \quad (2)$$

The singular nature of contact interactions requires regularization, which is performed by applying a local Gaussian regulator that suppresses momenta above an ultraviolet cutoff. Physical quantities predicted by the theory must be independent of the cutoff since it is not a physical quantity. This is achieved via renormalization, i.e., by fitting the values of the LECs to run with the cutoff so that a chosen set of physical observables is reproduced. Regularization for one Cartesian dimension

$$\delta^{(1)}(x) = \lim_{\lambda\to\infty} \sqrt{\frac{\lambda}{\pi}} e^{-\lambda x^2} \quad (3)$$

The regularised LO two-body potential is

$$V_2^{(0)} = \sum_{i<j} C(\lambda) g_\lambda(r_{ij}) \quad (4)$$

and three-body potential

$$V_3^{(0)} = \sum_{i<j<k} \sum_{cyc} D(\lambda) g_\lambda(r_{ij}) g_\lambda(r_{ik}) \quad (5)$$

where $g_\lambda(r) \propto exp(-\frac{\lambda^2 r^2}{4})$ is the chosen regulator, cyc denotes the cyclic sum, and $r_{ij} = |r_i - r_j|$ is a relative distance between nucleons i and j. As $\lambda \to \infty$, the contact

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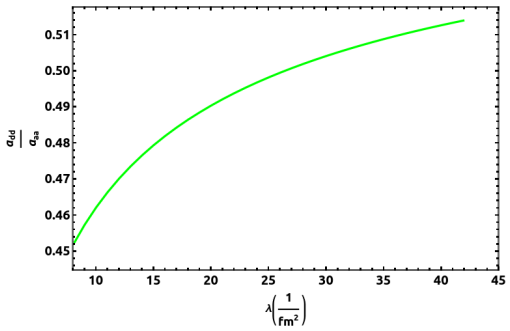

FIG. 1: Dependence of dimer-dimer scattering length upon Cut-off scale λ.

nature of the interaction is recovered.

The RGM method can be used to investigate the universality in the case of dimer-dimer scattering where the term 'dimer' alludes to a cluster having two atoms and it is the statistics, specifically Bose and Fermi dynamics, followed by the particles that choose the way of behaving of the dimer. Consider a framework with two dimers; every dimer has atoms of the same mass. Some possibilities of this kind of system can be represented as follows:

$$(ab) : (cd) \tag{6}$$

$$(ab) : (ab) \tag{7}$$

Where a,b,c, and d represent the atoms of different kinds.

Results and Discussion

After some straightforward calculations of the direct and exchange potentials as well as the exchange kernel of the RGM equation and taking two masses of the fermion mass same, the remarkable universality of the ratio between dimer-dimer and fermion-fermion attraction measured by scattering lengths $a_{dd}/a_{aa} >$ 0.6 – has been numerically discovered [2], where a_{dd} and a_{aa} are the scattering length between the two dimers and two atoms respectively as shown in fig 1.

Another kind of system to study dimer-dimer scattering can be considered 2-2 partition of four 4-flavour fermions i.e. (ab):(cd) where all particles are distinguishable. By all we know, such a system is scale-invariant, which means

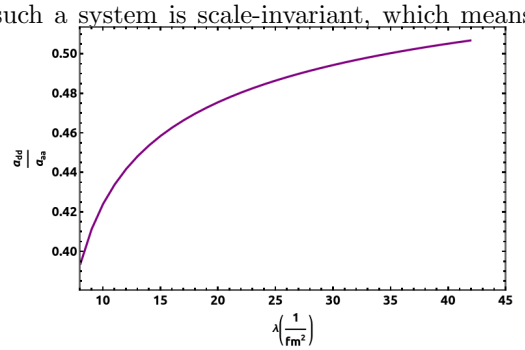

FIG. 2: Dependence of dimer-dimer scattering length upon Cut-off scale λ.

that its low-energy behavior is independent of λ and does not require the additional three-body renormalization constraint manifested in $D(\lambda)$. As there is no identical pair of particles, no permutation is possible for this system. The dependence of the ratio of dimer-dimer scattering length to atom-atom scattering length on lambda is shown in Fig 2. For higher cutoffs, this ratio is almost constant.

References

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