The role of redundancy in blind signal estimation for multiple gravitational wave detectors

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In this work, we extend our previous blind GW signal estimation method [1] to the case of multiple detectors and show that, with a full use of redundancy, it gives promising results, e.g. a faster decay of fluctuations than that expected from the central limit theorem. This method, whose design explicitly accounts for redundancy in multiple measurements, considerably improves the efficiency of signal extraction in a multi-detector network.

1. Introduction

Given that detector noise is several orders of magnitude larger than expected gravitational wave (GW) signals from black hole mergers, the analysis of LIGO's data is challenging. This is one reason why LIGO consists of two independent detectors separated by 3000 km. It is reasonable to divide the task into three relatively distinct parts. These include event detection, waveform extraction, and the identification of its physical origin. Here, we will be concerned with the two final steps in this process, focusing on black hole mergers like GW150914. Since thermal and seismic effects lead to substantial low frequency noise and since quantum noise is dominant at high frequencies, it is possible to reduce the effects of noise by band-pass filtering (or equivalent operations) to a restricted frequency range e.g., 30 − 300 Hz. LIGO extracts waveforms by comparing the data from the Livingston and Hanford detectors with (suitably time-shifted) elements of a bank of templates describing the merger of two black holes with various masses, spins and initial conditions. A satisfactory level of agreement would then lead to the acceptable conclusion that the data is *not inconsistent* with a black hole merger. An equally satisfactory conclusion would be that, *if* the event is due to a black hole merger, it has the parameters corresponding to the best-fit template. However, in our view, the far stronger conclusion that such agreement proves that the event actually *is* due to a black hole merger with the corresponding parameters is unwarranted.

Moreover, there are disturbing features in the noise associated with LIGO's events. Although this noise is complex, its two essential characteristics are clear and unambiguous. First, LIGO noise is neither stationary nor Gaussian.[2] In fact, both stationarity and Gaussianity are implicitly assumed in the Bayesian likelihood analysis as a diagonal approximation for the noise covariance matrices for both the Hanford and Livingston detectors.[3] Second, the LIGO "residuals" (defined as the cleaned data minus the best-fit template) for the two detectors should not be correlated. In fact, the residuals associated with GW150914 are strongly correlated when shifted by the same time lag as the template itself and considerably larger than the expected level of "accidental" correlations for LIGO data.[4,5] Since the Hanford (H) and Livingston (L) detectors are assumed to be completely independent except for a possible GW signal, this correlation in the residuals tells us that a template-based analysis does not provide a reliable description of the common signal seen by the H and L detectors.

Template-based analyses have a strong tendency to be self-fulfilling, this alone should be sufficient to emphasize the importance of maintaining a clean separation between the extraction of a signal and attempts to divine its physical origin.

In order to make optimal use of the independence of the H and L detectors and to quantify the uncertainties in previously extracted waveforms, we have constructed a template-free method that includes minimization of residual correlations in order to determine a "best common signal" for GW150914.[1] Since the residual for an individual detector is a complicated function of time (or frequency), there are many ways to realize the desired absence of correlations. This means that our algorithm yields a family of best common signals that enable us to estimate the probability, p, that the best common signal is a black hole merger template. For GW150914, we find $p = 0.008$. It is unclear whether this small probability is indicative of inadequacies in the gravitational wave templates, imperfect knowledge of the acceptance of the instruments, or of a completely different physical explanation — either astrophysical or terrestrial — of this event. As the amplitude of the signal becomes smaller with respect to the noise level, the relative width of the envelope of best common solutions obtained by this method grows and eventually covers the line of zero signal. The signal to noise ratio (SNR) for other possible black hole merger events is smaller than that found for GW150914, and we find that all of these subsequent events are consistent with a best common signal of precisely zero.

The fact that LIGO does not make use of the only relevant information available about noise properties (i.e., the fact that L and H residuals must be uncorrelated) also suggests that their analysis does not optimally exploit the benefits of redundancy that should accrue from two independent measurements of the same signal. Since the power of redundancy becomes clearer as the degree of redundancy is increased, our primary concern in this paper is to extend the results of Ref. 1 to the case of an arbitrary number of GW detectors. We emphasize that this approach is not merely a pedagogic exercise. There is considerable current discussion about adding additional detectors to the GW network. Since the effects an additional detector can have on the accuracy of signal attraction depends sensitively on the degree to which redundancy is exploited, we believe that the present work can be of practical relevance in deciding the extent to which such an investment is justified.

We begin in section 2 by presenting simple schematic models that illustrate the dramatic differences found when analyzing multi-detector events with or without consideration of the effects of redundancy. These models will provide a gauge of the extent to which redundancy is realized in practice. The extension of our earlier model for the blind estimation of the common signal to the case of many detectors will be described in section 3 along with the results of realistic simulations. Finally, section 4 contains a discussion based on these results.

2. The value of redundancy and its price

We all have an intuitive understanding of the important role that redundancy can play in the accurate determination of a signal transmitted in the presence of noise. When told an important telephone number in a noisy environment, our immediate reaction is to ask for it to be repeated. Hearing the same number twice greatly increases our confidence that it has been transmitted correctly. Clearly, redundancy is equally important in a scientific context, and it was surely one of the primary reasons that LIGO wished to have multiple GW detectors. In the analysis of GW170814, LIGO makes use of the Virgo detector as a consistency check for the results from Hanford and Livingston.[6] The additional information from Virgo improves the false-alarm rate compared to the two detector case. In general, the SNR of the LIGO/Virgo network is defined as a sum in quadrature of the individual detector SNRs, i.e.[7]

$$\rho^2_{\text{network}} = \sum_{i=1}^{M} \rho_i^2. \tag{1}$$

The precondition for this definition is ideal detector noise uncorrelated between detectors, with off-diagonal terms of the noise covariance matrix neglected. Furthermore, the addition of detector SNRs in this way fails to properly exploit redundancy, because the network SNR can be dominated by the SNR of a single detector. A high network SNR does not necessarily imply agreement between detectors.

Our aim in this section is to provide a better understanding of both the power of redundancy and the price that must be paid to obtain it.

We first consider a schematic but instructive example of redundancy in which there are two independent measurements of a common signal consisting of N pieces of data. In the absence of noise, the two measured signals will be identical to one another and to the true signal. Now, simulate noise by assuming that there is a probability, p, that any given piece of genuine data, d_i, has been replaced by noise n_i (with $n_i \neq d_i$). The probability that both of the measured signals are free of errors and therefore correct is evidently $(1-p)^{2N}$, which vanishes exponentially as the signal becomes more complex (i.e., $N \to \infty$). It is necessary, however, to consider the possibility of false positives, for which the signals are not correct in spite of being identical. To investigate this question, assume that the probability that two randomly drawn pieces of noise are identical is q. (E.g., If the n_i are randomly drawn digits between 0 and 9, $q = 1/9$.) The probability that the two signals will be identical (either genuinely or accidentally) is

$$\sum_{m=0}^{N} (1-p)^{2(N-m)} p^{2m} q^m \frac{N!}{(N-m)!m!} = \left[(1-p)^2 + p^2 q\right]^N \ . \tag{2}$$

Thus, the probability that identical results of the two measurements will actually be correct is

$$\left[\frac{(1-p)^2}{(1-p)^2 + p^2 q}\right]^N \ .$$

The extension of this problem to the case where there are M detectors is straightforward. In this case, all M detectors see the same event. The probability that all of the measured signals are free of errors (and therefore measured correctly) is evidently $(1-p)^{MN}$, which clearly decreases exponentially with increasing M. The generalization of Eq. 2 describing the probability that these M signals will be identical (but not necessarily correct) then becomes

$$P = \left[(1-p)^M + p^M q^{M-1}\right]^N \ , \tag{3}$$

and thus the probability that identical results for all M measurements will actually be the correct signal is

$$P = \left[\frac{(1-p)^M}{(1-p)^M + p^M q^{M-1}}\right]^N \ . \tag{4}$$

To illustrate this result, consider the transmission of an 8-digit telephone number in a noisy environment. For this case, assume $N = 8$, $p = 1/4$, and $q = 1/9$ and consider the cases of $M = 2$, 3, and 4 detectors. The probability that identical (but not necessarily correct) results will be obtained for all M measurements is approximately 0.01, 0.001, and 0.0001, respectively. The probabilities that these identical results will be false are 0.0935, 0.00365, and 0.000135, respectively. The exponential decrease of this error is evident and emphasizes the dramatic improvement in accuracy that results from M identical measurements of the same signal. Unfortunately, it also reminds us that the probability of actually obtaining M identical signals also decreases exponentially with M.

The preceding example might seem to indicate that redundant measurement of a given signal will lead to signal detection and extraction with a confidence that grows to 1 exponentially with the redundancy M. We now wish to consider a second example to illustrate that this is not necessarily the case. To this end, consider M independent measurements each of which for simplicity consists of a common signal, s, and an independent realization of N pieces of random noise.

The data obtained at detector k is thus $d^{(k)} = s + n^{(k)}$.[a] We wish to extract s from this data without making unwarranted assumptions about the noise such as stationarity and/or Gaussianity. Given this strong constraint, the only assumption that can be made is that the measurements in the various detectors are genuinely independent and that there are therefore no correlations in their noise realizations To be concrete, we will imagine that cross-correlations are given as the Pearson cross-correlation to be adopted below.[b]

In these circumstances, it might seem natural to approximate the best common signal as the average record

$$\mathcal{S} = \frac{1}{M}\sum_{k=1}^{M} d^{(k)} = s + \frac{1}{M}\sum_{k=1}^{M} r^{(k)} \ . \tag{5}$$

Given this guess, we can re-express the individual data strings as $d^{(k)} = \mathcal{S} + \rho^{(k)}$ where the residuals are given as

$$\rho^{(k)} = \frac{1}{M}\left[(M-1)r^{(k)} - \sum_{j\neq k} r^{(j)}\right] \ . \tag{6}$$

For sufficiently large N, it is reasonable to make the approximation that the cross-correlators $C(s, r^{(k)}) = 0$ for all k and $C(r^{(j)}, r^{(k)}) = 0$ for all $j \neq k$. This leads to the result that

$$C(s,\mathcal{S}) = \frac{1}{\sqrt{1+1/M}} \approx 1 - \frac{1}{2M} \quad \text{and} \quad C(\rho^{(j)},\rho^{(k)}) = -\frac{1}{M-1} \ . \tag{7}$$

It is true that $\mathcal{S}$ converges to the exact result s and that the correlations between the residuals, $\rho^{(k)}$, vanish as expected in the limit of large M. Unfortunately, these convergence rates, which are an elementary consequence of the central limit theorem, are far too slow to be useful. The fact that all of the $M(M-1)/2$ correlators between the residuals have the same value of $-1/(M-1)$ is also unphysical. Thus, the assumption that the signal can be approximated by Eq. 5 is unjustified. For the case $M = 2$, the cross-correlator $C(s,\mathcal{S})$ has the unsatisfactorily small value of $\sqrt{2/3}$, and there is a perfect anti-correlation between the residuals with $C(\rho_1, \rho_2) = -1$. As noted in our earlier work[4,8], similarly large and unphysical correlations in the residuals determined by LIGO for GW150914[9] suggest the existence of problems with the corresponding GW signal.[c]

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[a] Note that this is the model used for Bayesian analysis of the LIGO events.[3]

[b] For two vector records of length N, we first shift the records so that each has average value zero and rescale them so that the scalar product of each vector with itself is 1. The Pearson cross-correlation is then the scalar product of these shifted and rescaled vectors and will have a value between -1 and $+1$.

[c] The presence of correlations in the Hanford and Livingston residuals determined in original template-based analysis of GW150914 raises questions about this analysis. It is important for others to confirm the existence of these correlations. This can be done using the publicly available data, as described on our webpage:
http://www.nbi.ku.dk/gravitational-waves/residual_correlations_notebook.html

The two examples presented in this section can serve as a measure of the extent to which the benefits of redundancy have been realized by a given method of signal extraction. When redundancy is not exploited, we find that the extracted signal converges to the true signal with an error that vanishes slowly (i.e., like $1/\sqrt{M}$) as the degree of redundancy increases. In contrast, a maximal implementation of redundancy leads to an exponentially decreasing error rate as a function of M. This test will be applied in practice in the following sections of this paper. It should be noted, however, that this increased confidence level has a relatively high price. As we have seen, the probability that an event will pass the redundancy test also vanishes exponentially with M. Thus, if event rates are too low, it may be impossible to realize fully the benefits of redundancy. We stress that the examples here are highly schematic and are intended to illustrate the general fact that the benefit of multiple independent measurements depends sensitively on the way these measurements are analyzed. They do not tell us to how the benefits of redundancy can be maximally realized in the case of GW data. We consider one such approach in the following section.

3. Application of the blind estimation method to multiple GW detectors

3.1. *Basis of the blind estimation method: cross-correlation and Fisher transformation*

We briefly review our previous work on blind estimation.[1] The strain signal detected by LIGO in the i-th detector is assumed to be

$$X_i(t) = a_i \cdot h(t, \Delta\tau_i, \Delta\theta_i) + N_i(t), \tag{8}$$

where $X_i(t)$ is the total strain data, $N_i(t)$ is the noise, and $a_i \cdot h(t, \Delta\tau_i, \Delta\theta_i)$ is the gravitational wave signal with given amplitude a_i, time lag $\Delta\tau_i$ and phase shift $\Delta\theta_i$, which contain both contributions from projection and detector acceptance. As mentioned in Ref. 1, we pre-match detector data to roughly remove the contributions of $\Delta\tau_i$ and $\Delta\theta_i$ (this can be done precisely with, e.g., an EM-counterpart), and then the equation becomes

$$X_i(t) = a_i \cdot h(t) + N_i(t). \tag{9}$$

For a blind estimation, we also need to further derive the residual noise, for which we consider two data sets X_1 and X_2 of length N (e.g. cleaned strain data from two independent detectors) which contain a common signal A. The amplitude of A could potentially be different in each detector, either due to projection or detector acceptances. For convenience, we assume that X_1, X_2, and A have been shifted to have zero average values and normalized to have variance unity. Then the Pearson cross-correlation coefficient of two such vectors, $C_{X_1\,X_2}$, is simply the inner product $S_{X_1\,X_2}$ given by

$$S_{X_1 X_2} = \frac{1}{N-1} \sum_{k=1}^{N} X_1(k) \cdot X_2(k). \tag{10}$$

The residuals are defined as

$$R_i = X_i - A \cdot \frac{S_{AX_i}}{S_{AA}} = X_i - A \cdot S_{AX_i}. \tag{11}$$

As mentioned above, the amplitude of the term A (i.e. S_{AX}/S_{AA}) can be different for two decectors. By construction, the correlations of both R_1 and R_2 with A are zero.

The criterion for determining the blind estimate of A is to maximize the C_{AX_i} while simultaneously minimizing the cross-correlation between the residuals $C_{R_1R_2}$. Note that the residuals R_1 and R_2 are not automatically normalized.

For familiarity and simplicity, we obtain approximate Gaussianity of the resulting correlations by using the Fisher transformation:[10]

$$Z_{XY} = \frac{1}{2} \log\left(\frac{1 + C_{XY}}{1 - C_{XY}}\right). \tag{12}$$

3.2. *Extension of the likelihood approach*

In our previous work,[1] a blind GW-template estimation was done by considering the likelihood that a given initial guess, A, is the common signal observed by two detectors (X_1 and X_2) as

$$\log(L) = Z_{AX_1}^2 + Z_{AX_2}^2 - kZ_{R_1R_2}^2, \tag{13}$$

where Z_{AX_1} and Z_{AX_2} represent the similarity between A and the detector data as measured by the Pearson cross correlation, $Z_{R_1R_2}$ represents the similarity between the residuals from the two detectors, and k is a constant factor determining the relative weight of the two contributions. The likelihood function is designed to be maximized at higher $Z_{AX_1}^2$ and $Z_{AX_2}^2$ and lower $Z_{R_1R_2}^2$. The initial guess A is then improved by a random walk approach until the likelihood reaches an oscillatory region (see Fig. 2 of Ref. 1). The oscillatory region is used to estimate the range of fluctuation for each pixel as shown in Figs. 4 and 5 of Ref. 1.

The above method was initially designed for the GW150914 event for which there were only two detectors in the network. A natural extension of the method is to apply it to multiple detectors, so the likelihood function becomes:

$$\log(L) = \sum_{i=1}^{M} (Z_{AX_i})^2 - k \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} Z_{R_iR_j}, \tag{14}$$

where M is the number of detectors, X_i is the data from the i-th detector, A is the blind estimate of the signal, and R_i is the residual after removing this estimate from the i-th detector. With this modified likelihood function, the blind estimation method presented in Ref. 1 can be extended to the case of multiple detectors.

Also note that this kind of likelihood approach is different from a likelihood approach that assumes either a known covariance matrix or a known theoretical model, or even both. Here, neither the covariance matrix nor the theoretical model is assumed, and one starts only from basic ideas about "correlated signals and uncorrelated noise". Thus the likelihood approach here is totally blind, i.e. it makes minimal assumptions. This is unlike template-free methods currently in use by LIGO, such as BayesWave and oLIB, which begin with the assumption of a stationary Gaussian noise model.[11,12] Finally, we note that if there is any reliable additional information, such as a known correlation between two of the detectors, or an especially low SNR in one of the detectors, then such information can also be added to Eq. 14 by changing the weights of the corresponding terms.

3.3. *Test and results*

To test the performance of the blind estimation method for the case of multiple detectors, we run a simulation as follows: We select the GW150914 waveform template as the input "real signal", and inject it into genuine strain data taken $2, 3, \ldots, M+1$ seconds after the GW150914 event to simulate the data from multiple detectors. Here, we are mainly interested in the trend of how the error of estimation decreases with increasing number of detectors. Thus, for convenience, we assume identical projections and similar noise levels for all detectors (see also Sec 3.1 of Ref. 1). In practice, when multiple detectors have different projections and signal-to-noise ratios, the overall performance will become worse. For simplicity, we will neglect these concerns here.

For comparison, we adopt a reference estimator of the common signal that is simply the average of the data from the individual detectors and further assume that the detector noise is Gaussian. The estimated error will then scale like $1/\sqrt{M}$. We shall compare the performance of our method with this reference.

As mentioned above, the range of fluctuations is an immediate result from the oscillations of the likelihood function caused by chance correlations. In the simulation here, we select the 10th and 90th percentiles of the fluctuation range and use their difference as a measure of the range of fluctuation for each pixel. This quantity is averaged over the entire time range of the event as the final estimator of the uncertainty of blind estimation:

$$E_M = \langle S_{90}(t) - S_{10}(t) \rangle_M \,. \tag{15}$$

The result of the simulation described above is given in Fig. 1 where it is apparent that the blind estimation (black) performs better than the $1/\sqrt{M}$ reference (red). This result is not surprising because the $1/\sqrt{M}$ reference is obtained by simple averaging without consideration of the residual correlations. In other words, an estimation method that considers both the correlations between signal and data (first term in Eq. 14) and the correlations between residuals (second term in Eq. 14) will certainly give better results than simple averaging.

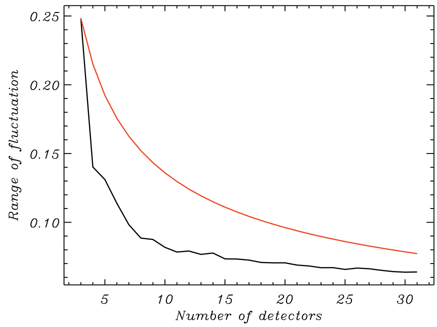

Fig. 1. The range of fluctuation defined by Eq. 15. The horizontal axis shows the number of detectors, and the vertical axis shows the estimator of uncertainty, E_M, calculated using the convention that strain data is shifted to have zero mean and normalized to have variance unity. The black line is the performance of the multi-detector blind estimation method, and the red line is the expectation of the central limit theorem that scales as $1/\sqrt{M}$ given by simple averaging, which has been normalized to the black line at $M = 3$.

We also show the input signal and the range of error obtained from blind estimations for the extreme example of 32 detectors in Fig. 2. We include, for test purposes, a slight variation of the above method in which the first term in Eq. 14, $\propto Z^2_{AX_i}$, is dropped. One can see that even without this seemingly essential term, one still gets unbiased estimations of the "real signal". Only the error of estimation is larger.

The error of the estimated common signal can also be evaluated by comparison to the real signal $h(t)$. Three cases are considered: the simple average of multiple detectors, $B_1(t)$; the average of the blind estimations, $B_2(t)$; and the average of blind estimations calculated only using the residual correlation terms in Eq. 14, $B_3(t)$. For each of the three cases, we determine the deviation from the real signal $h(t)$ as

$$\begin{aligned} \delta_1 &= B_1(t) - h(t) \\ \delta_2 &= B_2(t) - h(t) \\ \delta_3 &= B_3(t) - h(t), \end{aligned} \tag{16}$$

and calculate the standard deviations as σ_1, σ_2 and σ_3 respectively. For a given number of detectors, we calculate two ratios

$$\begin{aligned} r_{12} &= \sigma_1/\sigma_2 \\ r_{13} &= \sigma_1/\sigma_3, \end{aligned} \tag{17}$$

which are defined such that larger values correspond to better performance than simply averaging. Our blind estimation method is expected to give smaller uncertainties than simple averaging, thus we should see $r_{12} > 1$. Larger values of r_{12} indicate better results given by the blind estimation. On the other hand, we expect

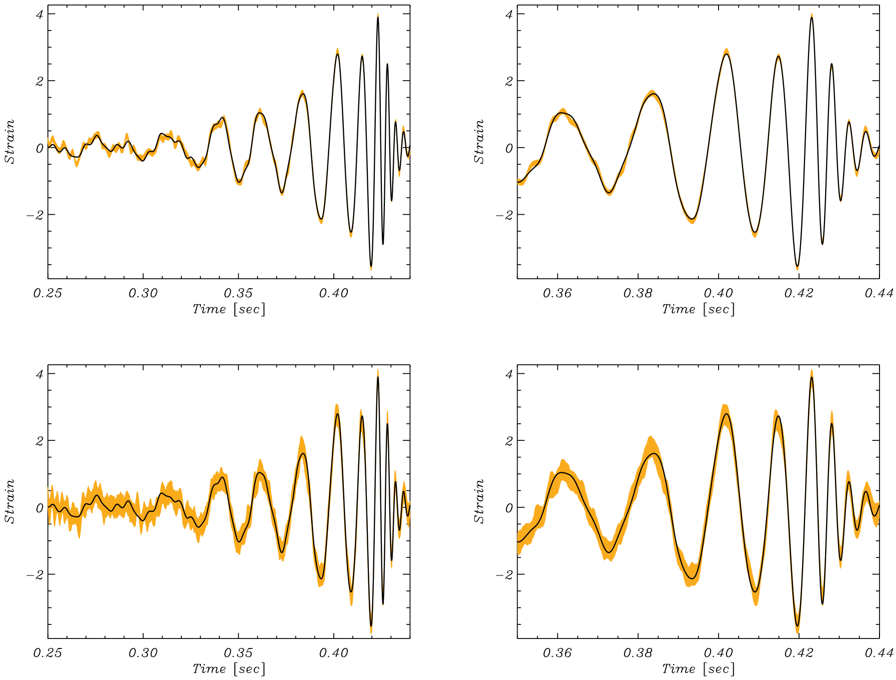

Fig. 2. The real input template (black) and the 10%–90% range of uncertainty (see Eq. 15) given by the blind estimation for the case of 32 detectors. Upper: using the whole likelihood, as in Eq. 14. Lower: same as the upper panels but only the residual terms are used. Left: the full time range. Right: only the second half.

that a blind estimation only using the correlations of the residuals is worse than simple averaging, but it should still be a reasonable estimation, thus r_{13} should be less than 1 but not much lower.

In Fig. 3, we show r_{12} and r_{13} as functions of the number of detectors. The ratio r_{12} lies around 1.2–1.4, indicating that simple averaging gives a 20%–40% larger error than blind estimation. Also, r_{13} is about 0.8–0.9, indicating that, even from only regarding the residual–residual terms of the likelihood, one can still get a reasonably good estimation of the real input signal. Therefore, Figs. 2 and 3 suggest that to only compare template to data and ignore residuals is an inefficient use of experimental resources. We also see from Fig. 3 that, if the number of detectors is less than 10, then the blind estimation method is significantly better than simple averaging. With increasing M, however, the improvement of the method slows. This is possibly due to the finite record length. Since in the near future, the number of GW detectors will not exceed 10, a blind estimation method such as this one is especially important.

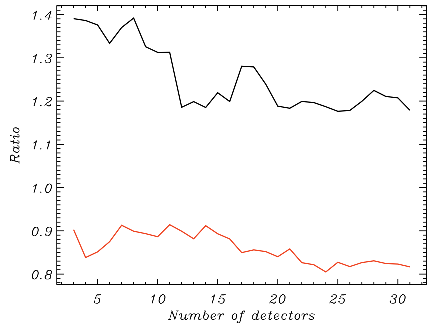

Fig. 3. The ratios r_{12} (black) and r_{13} (red) as functions of the number of detectors. $r_{12} > 1$ means our blind estimation gives lower estimation error than a simple averaging, while r_{13} close to 1 means even without using the template–data term, one can still get a reasonably good estimation of the potential common signal using the same blind estimation method.

4. Discussions

The aim of the present paper is two-fold. On the one hand, an exploration of the role of redundancy in the analysis of LIGO data can help better to understand the reliability of events already observed. On the other, it can offer some guidance regarding the most fruitful way to analyze data in the coming multi-detector era of gravitational wave science. It is our firm conviction that template-based analysis alone is in principle circular and thus fundamentally flawed. Clearly, a bank that only contains templates for black hole mergers can never detect anything other than black hole mergers. The best that one can hope for is to claim that an event is *not inconsistent* with black hole merger and then make the best case possible that other possible origins — terrestrial as well as astrophysical — can be excluded. Rather, we are convinced that data analysis should begin with the template-free extraction of a best common signal that can later be compared with specific physical models. This conviction led to the development of the blind signal estimation initially presented in Ref. 1 and extended here. This method enabled us to determine the probability that the best common signal could be described by a gravitational wave template. Unfortunately, this probability is remarkably low. The probability that the common signal is LIGO's original published template was found to be 4×10^{-6}. The best GW template was found to have substantially higher masses (38 and 48 solar masses) and high spins (0.96 and -0.85, respectively). While better than the published template, the probability that it was the best common signal still had the unacceptably small value of 0.008. It should be noted that GW150914 is by far the strongest event seen. A similar template-free analysis of all other putative gravitational wave events is consistent with a common signal of zero.

Gravitational wave signals are characteristically much smaller than measured data indicating that the noise is much stronger than the signal. Thus, a reliable blind analysis of GW data requires detailed knowledge of the origin and nature of detector noise. Unfortunately, it is generally acknowledged that this noise is neither Gaussian nor stationary. In order to make a reliable detection of a GW signal, it is essential that there be no correlation between the residuals observed at individual detectors. This obvious requirement lies at the heart of the justification for incurring the expense of performing redundant measurements of a given signal with two or more independent detectors. We have thus provided simple schematic examples to show that the accuracy of detection can be improved exponentially with an increasing degree of redundancy. This stands in sharp contrast to the far slower convergence expected from a simple average of the measured signals. These results are supported by simulations based on a realistic gravitational wave form and real LIGO noise data. These results indicate the important role played by the residual correlations in the data analysis. Indeed, it is possible to obtain a reasonably good estimation of the injected signal by using only these terms. In summary, these results provide a strong reminder of the importance of exploiting redundancy in the analysis of both present and future gravitational wave data and suggest that this can be accomplished by including the suppresion of residual correlations in the construction of a satisfactory likelihood function.

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