Relic magnetic wormholes as possible source of toroidal magnetic fields in galaxies

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We present the hypothesis that some of ring galaxies were formed by relic magnetic torus - shaped wormholes. In the primordial plasma before the recombination magnetic fields of wormholes trap baryons whose energy is smaller than a threshold energy. They work as the Maxwell's demons collecting baryons from the nearest (horizon size) region and thus forming clumps of baryonic matter which have the same torus-like shapes as wormhole throats. Such clumps may serve as seeds for the formation of ring galaxies and smaller objects having the ring form. Upon the recombination torus-like clumps may decay and merge. Unlike galaxies, such objects may contain less or even no dark matter in halos. However, the most stringent feature of such objects is the presence of a large - scale toroidal magnetic field. We show that there are threshold values of magnetic fields which give the upper and lower boundary values for the baryon clumps in such protogalaxies.

Keywords: Wormholes; cosmic rays; ring galaxies; galactic magnetic fields.

1. Introduction

One of challenges of modern astrophysics is to finding possible traces of relic wormholes. It is expected that relic cosmological wormholes were created from virtual wormholes on quantum stage, or during the inflationary period of evolution of the Universe. Virtual wormholes exist at Planckian scales and they represent the most natural objects to compose spacetime foam picture.[1,2] Their existence is straightforwardly predicted by lattice quantum gravity models, e.g., see Refs.[3–6] and references therein.

Spherically symmetric relic wormholes are highly instable and collapse very rapidly. To be stable they require the presence of exotic matter at throats or an essential modification of general relativity.[7,8] This means that all primordial spherical wormholes have collapsed long ago and at present they cannot probably be distinguished from black holes. However less symmetric configurations can be made stable without exotic matter or any modification of GR. First rigorous example was presented in Ref.[9] It was demonstrated that in the open Friedmann model a stable wormhole can be obtained simply by the factorization of space over a discrete subgroup of the group of motions of space. To illustrate such a factorization one may consider flat plane in which one of coordinates, say x, becomes periodic $x = R\varphi/(2\pi)$ with the angle $0 < \varphi < 2\pi$. Then the space becomes a cylinder with the radius R. In the open model (on space of a constant negative curvature) any parallel geodesics ($\varphi = 0$ and $\varphi = 2\pi$) diverge and the distance between them changes $R(\ell)$, where ℓ is a parameter along the geodesic line. Therefore, if we move along the geodesic line from the point where the distance is the shortest R_{min}, the space opens out $R \to \infty$ and becomes unrestricted.[10] Here the wormhole configuration coincides with a cylinder on which the metric is specified to produce a constant negative curvature. The simplest 3D wormhole obtained by the factorization has also the topology of a cylinder, while its throat has the topology of the 2D- torus.[9,10] In flat space such a configuration corresponds to a wormhole whose throat has the shape of a doughnut. Such wormholes are not static but expand in agreement with the expansion rate of the Universe. It can be considered as having been frozen into space and, therefore, they are static in the co-moving coordinates. It is important that the factorization allows to get an arbitrary number of such wormholes in space.

In flat space the torus - like wormholes become dynamical objects and evolve.[9] Whether they are static, expand, or collapse, depends on surrounding matter and peculiar motions of throats. The shape of throats of such wormholes resembles a doughnut and is characterized by two radii R_w and r_w. In the limit $R_w \gg r_w$ it can be approximately described by the cylindrical (axial) configuration. It turns out that static and stationary cylindrical wormhole solutions do exist and it was found that asymptotically flat wormhole configurations do not require exotic matter violating the weak energy condition.[11,12] This shows that such objects, as doughnut - shaped wormholes, have all chances to be observed in astrophysical systems. We point out that the investigation of the evolution of doughnut-shaped wormholes in flat space is a rather complex problem.

The possibility to directly observe wormholes attracts the more increasing attention, e.g., see Refs.[13–15] At first glance the most promising are collective effects produced by a distribution of wormholes in space. However, our previous investigation have shown that observational effects of a distribution of wormholes are very well hidden under analogous effects produced by ordinary matter e.g., see Refs.[16,17] The only exclusion may be the noise (stochastic background) produced by the scattering of emitted by binaries gravitational waves on wormholes.[18]

In general a single wormhole produces much less noticeable effects (lensing, cosmic ray scattering, etc.). However, wormholes may possess non-trivial magnetic fields as vacuum solutions. In this case possible imprints of wormholes in the present picture of the Universe may be rather considerable. In particular, when such a magnetic wormhole gets close to a galaxy, it starts to work as an accelerator of charged particles[10,19] which is capable of explaining the origin of high-energy cosmic-ray particles.[20] The idea that the observed cosmic rays require magnetic fields for their creation was first suggested by Fermi.[21] In voids a relic magnetic wormhole works simply as a generator of synchrotron radiation and can be detected via the magnetic field.[22,23] The primordial magnetic fields[24] in turn may form small-scale non-linear clumps of baryonic matter[25,26] and as it was recently discussed in Ref.[27] they may allow to solve the existing Hubble tension. In other words, magnetic wormholes should leave a clear imprint on the sky. In particular, relic magnetic wormholes may play also the key role in formation of ring type baryonic structures, analogous to the ring galaxies without involving dark matter.[28] Moreover, such wormholes may also form ring - type structures even in the present epoch which may explain the origin of the recently found unexpected class of astronomical ring-type objects.[29]

2. Vacuum magnetic fields of wormholes

Nontrivial topology of space which contains a static wormhole allows us to get additional nontrivial solutions of static vacuum Maxwell equations.[30] This means that already in the absence of real sources (charged particles, electric currents) space may possess nontrivial quasi-static magnetic and electric fields. The physical mechanism is rather clear, during the quantum period of the Universe when the wormhole forms, it may capture some portion of closed magnetic or electric lines. Such lines cannot simply leave the wormhole. Electric fields perform work and decay very rapidly in the primordial plasma. Magnetic fields do not perform work and may survive till the present days. For exact solutions which involve magnetic fields and wormholes see e.g.,[31,32]

In this section for the sake of simplicity we assume that sufficiently far from the wormhole entrances the space is flat. Generalization to the curved spacetime and consideration of magnetic fields in the Friedmann model can be found, e.g., in.[33] Consider first the simplest genus $n = 0$ wormhole. Then the space-time metric can be taken as

$$dl^2 = c^2dt^2 - h^2(r)\left(dx^2 + dy^2 + dz^2\right). \tag{1}$$

We shall use the Ellis–Bronnikov massless wormhole[34,35] when the scale function is simply $h = 1 + \frac{R^2}{r^2}$. This model of a wormhole possesses two asymptotic Euclidean spaces E_+ as $r \gg R$ and E_- as $r \ll R$. In the region $r \ll R$ the transformation $\widetilde{r} = R^2/r$ reduces the above metric to the standard Euclidean form. The Maxwell equations for magnetic field take the standard form

$$rot\mathbf{B} = div\mathbf{B} = 0, \tag{2}$$

where $B_i = \varepsilon_{ijk}F^{jk}$. These equations possess a nontrivial solution in the form

$$\mathbf{B} = -\frac{Q}{r^2 h}\mathbf{n} \tag{3}$$

with an arbitrary constant value Q, where $\mathbf{n} = \mathbf{r}/r$ is the unit vector. Indeed, vacuum magnetic field can be expressed via the magnetic scalar potential $\mathbf{B} = -\nabla\phi$ which obeys the equation

$$\Delta\phi = 0. \tag{4}$$

In the spherically symmetric case this equation reduces to

$$\frac{1}{r^2h^3}\partial_r r^2 h \partial_r \phi = 0 \tag{5}$$

and has the solution as

$$\phi = \int_0^r \frac{Q}{r^2 h} dr + \phi_0. \tag{6}$$

This defines the magnetic field

$$\mathbf{B} = -\nabla\phi = -\frac{Q}{r^2 h}\nabla r = -\frac{Q}{r^2 h}\mathbf{n}. \tag{7}$$

This solution works for any spherically symmetric wormhole with an arbitrary scale function $h(r)$ in (1).

In the region E_+ $(r \gg R)$ it describes the field of the magnetic charge Q homogeneously distributed over the sphere $r = R$ (Coulomb law). In the region E_- $(r \ll R)$ the transformation $\widetilde{r} = R^2/r$ interchanges the inner and outer regions of the sphere $r = R$ and we get the same field with the magnetic charge $-Q$.

In the case when both entrances are in the same space this transforms to the dipole field. Let the positions of the two spheres are $\mathbf{x}_+$ and $\mathbf{x}_-$ then the field can be taken as ($h_\pm = h(r_\pm) \sim 1$ as $r_\pm = |\mathbf{x} - \mathbf{x}_\pm| \gg R$)

$$\mathbf{B}(\mathbf{x}) = \frac{Q\,(\mathbf{x}-\mathbf{x}_+)}{h_+\,|\mathbf{x}-\mathbf{x}_+|^3} - \frac{Q\,(\mathbf{x}-\mathbf{x}_-)}{h_-\,|\mathbf{x}-\mathbf{x}_-|^3}. \tag{8}$$

Consider now the genus $n = 1$ wormhole. In the flat space such a wormhole can be constructed by means of cutting two solid tori and gluing along their surfaces. In this case we have two different new kinds of solutions. First is obtained by placing an arbitrary magnetic charge density $\rho(x)$ in the internal region of one torus and the opposite density $-\rho(x)$ in the internal region of the second torus. Then we solve the system

$$rot\mathbf{B} = 0, \quad div\mathbf{B} = 4\pi\rho. \tag{9}$$

Recall that internal regions of tori correspond to fictitious points, while $\rho(x) = 0$ as x lies outside the tori. Therefore, such a system coincides exactly with (2). In this case the exact form of the magnetic field is rather complicated. However averaging over orientations of the tori we restore the spherical symmetry of the wormhole and get exactly the same solution as (8). In this sense for the sake of simplicity we may always restrict to the spherically symmetric wormholes (i.e., consider $n = 0$ wormhole as the first order approximation).

Solutions of the second kind can be obtained by placing an arbitrary current density $\mathbf{j}(x)$ within the surface of torus/throat and solving the system

$$rot\mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \quad div\mathbf{B} = 0. \tag{10}$$

Again here the non-vanishing current density corresponds only to fictitious points, while in physical regions of space such a system represents the same system (2).

The system (10) corresponds to the field generated by an electric loop which produces the toroidal magnetic field. Such solutions cannot be reduced to the spherically symmetric case, since upon the averaging over orientations of the loop the field vanishes.

In conclusion of this section we point out that the new two classes of vacuum solutions reflect the topological non-triviality of space. Indeed, according to the Stocks theorem the system (2) implies $\oint \mathbf{Bdl} = 0$ for any loop which can be pulled to a point. In the case of a non-trivial topology of space there appear new classes of loops Γ_a which cannot be contracted to a point and, therefore, to fix the unique solution we have to fix additional boundary data $\oint_{\Gamma_a} \mathbf{Bdl} = C_a$. In general $C_a \neq 0$. In the case of genus $n = 0$ wormhole there is only one such a non-trivial loop which goes through the wormhole throat. In the case of genus $n = 1$ wormhole we have already two such loops, one goes through the throat (it corresponds to the system (9)) and one additional crosses the torus (the system (10)).

3. Wormhole as an accelerator

In galaxies charged particles undergo an acceleration when interacting with galactic radiation. For definiteness we shall speak of electrons. Indeed, by means of the Compton scattering photons transmit part of their momentum to electrons. Upon the scattering the momentum obtained by the electron from the incident photon is

$$\mathbf{\Delta p} = \mathbf{p}' - \mathbf{p} = \frac{h\nu}{c}\left(\mathbf{n} - \frac{\nu'}{\nu}\mathbf{n}'\right) \tag{11}$$

where $\mathbf{n}$ and $\mathbf{n}'$ are the direction of the photon before and after scattering and

$$\frac{\nu'}{\nu} = \frac{\left(1 - \frac{c}{E}\mathbf{pn}\right)}{\left(1 + \frac{h\nu}{E}\left(1 - \mathbf{nn}'\right) - \frac{c}{E}\mathbf{pn}'\right)}. \tag{12}$$

Upon averaging over possible orientations of $\mathbf{n}'$ we get for the average momentum transmitted from the incident photon

$$\mathbf{\Delta p} = \frac{1}{c}\beta_\nu(\mathbf{p})h\nu\mathbf{n}. \tag{13}$$

Here the spectral coefficient $\beta_\nu(\mathbf{p}) = 1 - \left\langle \frac{\nu'}{\nu}\mathbf{nn}' \right\rangle$ is given by

$$\beta_\nu(\mathbf{p}) = 1 - \frac{1}{4\pi}\int\left(\frac{\nu'}{\nu}\mathbf{nn}'\right)d\Omega', \tag{14}$$

where $\frac{\nu'}{\nu}$ is determined by (12). In the case when $\mathbf{p} = p\mathbf{n}$, it reduces to the form

$$\beta_\nu = 1 + \frac{1}{2x}\left[2 - \left(1 + \frac{1}{x}\right)\ln\left(1 + 2x\right)\right],$$

where $x = \left(h\nu + cp\right)\left(E + cp\right)/m^2c^4$. Now multiplying (13) on the number density of photons with the frequency ν and on the cross section we get the spectral force which accelerates the electron in the form

$$\mathbf{f}_\nu = \frac{\Delta \mathbf{p}}{\Delta t} = c\sigma_T N_\nu \frac{1}{c}\beta_\nu(p) h\nu \mathbf{n} = \frac{\sigma_T \beta_\nu(p)}{c}\mathbf{P}_\nu \tag{15}$$

where $\mathbf{P}_\nu$ is the spectral component of the Poynting vector $\mathbf{P}_\nu = \frac{c}{4\pi}\mathbf{E}_\nu \times \mathbf{B}_\nu$ and σ_T is the Thomson cross section. The total force is given by $\mathbf{F} = \int \mathbf{f}_\nu d\nu$.

It is important that the force is determined by the Poynting's vector $\mathbf{P}_\nu$. In a quite (quasi-stationary or steady state) galaxy both the Poynting's vector and the force have the potential character, i.e., they can be presented as $\mathbf{P}_\nu = -\nabla \Psi_\nu$. Non-stationary processes in active galactic nuclei may produce some additional acceleration, e.g., the stochastic Fermi acceleration, etc., which we do not discus here. For the quasi-stationary galaxy the Poynting's theorem gives the discontinuity equation

$$div \mathbf{P}_\nu = \ell_\nu \tag{16}$$

where $\ell_\nu(x)$ is the spectral density of sources of radiation (stars, hot gas, dust, etc.) or the radiative capability of a unite volume in the galaxy. If the topology is simple, then sufficiently far from the galaxy we get

$$\mathbf{P}_\nu = \frac{M_\nu}{4\pi r^2}\mathbf{l}, \tag{17}$$

where $\mathbf{l} = \mathbf{r}/r$, $\mathbf{r}$ is the distance from the center of the galaxy, and M_ν is the total spectral energy emitted by the galaxy in the unit time. Observations show that the intergalactic medium possesses a magnetic field.[22, 23] Therefore, the electron may have a closed trajectory. It is easy to verify that the total energy obtained by the electron from the galactic radiation during the cycle is exactly zero, i.e., $\oint \mathbf{f}_\nu \mathbf{dl} \equiv 0$. This means that in the case when topology is simple, the only possible mechanism of the electron acceleration relates to high-energy non-stationary processes (jets, shock waves, supernovae explosions, active galactic nuclei, etc.).

The situation changes when the galaxy is accompanied with a wormhole. In the presence of the wormhole the Poynting's field also admits non-trivial solutions of (16). For the sake of simplicity we consider the spherically symmetric (genus $n = 0$) wormhole. For a more general wormhole the rough picture remains the same, at least from the qualitative standpoint. We point out that the genus $n \geq 1$ wormholes are more preferred from the astrophysical standpoint, since they may work as accelerators even in the case when a wormhole is not traversable (e.g., when the length of the trajectories which go through the throat are too big).

Indeed, the scattering of the radiation on the wormhole (e.g., see[16, 17]) produces an additional field in the dipole form

$$\delta \mathbf{P}_\nu = \frac{\delta M_\nu}{4\pi r_-^2}\mathbf{n}_- - \frac{\delta M_\nu}{4\pi r_+^2}\mathbf{n}_+ \tag{18}$$

where $\mathbf{n}_\pm = \mathbf{r}_\pm / r_\pm$, $\mathbf{r}_\pm = \mathbf{r} - \mathbf{x}_\pm$, $\mathbf{x}_\pm$ are positions of the wormhole entrances (we assume that $r_+ \ll r_-$), $\delta M_\nu \simeq \frac{M_\nu \pi R^2}{4\pi r_+^2}$ is the portion of the spectral energy absorbed by the closest entrance into the wormhole throat and R is the radius of the throat. If the wormhole possesses a magnetic field in the form (8), it forms a magnetic trap for the electron, while the Poynting's vector field $\delta\mathbf{P}_\nu$ in form (18) forms the accelerating force which acts exactly along the magnetic lines. On every cycle the electron will gain the energy from radiation $A = \int A_\nu d\nu > 0$, where $A_\nu = \oint \delta\mathbf{f}_\nu \mathbf{dl}$, whose exact value depends on the length of the trajectory of the electron and on all the rest parameters of the wormhole (distance to the galaxy, throat size, etc.). Some part of this energy will be spent on the synchrotron radiation of electrons and, therefore, there is a competition between the acceleration produced by the galactic radiation and loss of energy on the reradiation. The reradiation can be accounted for by adding the standard force of the radiation friction.

4. Magnetic wormholes as baryon traps

Consider a single wormhole whose throat has the shape of a torus (the genus - 1 wormhole by the classification suggested in Ref.[10]). In the presence of such a wormhole Maxwell's equations possess two additional classes of non-trivial vacuum solutions. Indeed, according to the Stocks theorem the system of vacuum Maxwell equations implies $\oint \mathbf{Bdl} = 0$ for any loop which can be pulled to a point (where $\mathbf{B}$ is the magnetic field). In the case of a non-trivial topology of space[a] there appear new classes of loops Γ_a which cannot be contracted to a point and, therefore, to fix the unique solution we have to fix additional boundary data $\oint_{\Gamma_a} \mathbf{Bdl} = \frac{4\pi}{c} I_a$ and in general $I_a \neq 0$. The constants I_a depend only on time and they can be viewed as fictitious currents[b] which intersect the loops Γ_a. In the case of genus $n = 0$ (spherical) wormhole there is only one such a non-trivial loop which goes through the wormhole throat. In the case of genus $n = 1$ wormhole (doughnut - shaped throat) we have already two such loops, one goes through the throat and one additional goes through the hole in the center of the doughnut and surrounds the throat.

The first class produces the magnetic field of a wormhole which can be described by magnetic poles placed in two different entrances into the throat. The two entrances have opposite magnetic poles. If the distance between the entrances is big enough the resulting field is very weak and when crossing such a field high-energy charged particles only slightly change the direction of propagation. The field can be strong only very close to the throat entrances. However, since charged particles can

[a] The simplest scheme to get a general genus - n wormhole can be described as follows. We take a couple of equal spheres with n handles in space, remove the internal regions (insides of the spheres), and glue along their surfaces (the so-called Hegor diagrams). If we take two simple spheres without handles the resulting space corresponds to a spherical wormhole (throat is the sphere). The sphere with a handle is the torus. A couple of toruses corresponds to the doughnut shaped wormhole, etc.

[b] We point out that the currents are fictitious, for from the point of view of the Hegor diagrams they take place in inner regions of spheres which are removed, i.e., in fictitious regions.

freely propagate along the field lines (which are roughly orthogonal to entrances), the particles captured by the field are distributed in the whole region between the entrances. Such fields have the long-range character and can be used to explain the origin of long-correlated magnetic fields in voids[22, 23] and, more generally, of primordial magnetic fields.[24]

The situation changes when the wormhole possesses also the field of the second class. The second class corresponds to the field produced by a single loop of a current (the loop of the corresponding fictitious current goes inside of the surface of the doughnut). In this case the field lines of force repeat the shape of the entrance (the shape of a doughnut) which roughly corresponds to the fields observed in spiral galaxies.[36–38] It is necessary to point out that the magnetic fields in the galaxies cannot exactly repeat the shape of a doughnut. There are a number of well-known processes in galaxies which naturally lead to the generation of galactic magnetic fields[37] and the actual field does not reduce to such a simple model. It is known that the pitch angle (between the toroidal field and radial one) is usually between 4 and 17 degrees.

The most strong field is close to the entrance and particles captured by the field remain always near the entrance. In the primordial plasma before the recombination such a wormhole traps all baryons[c] propagating near it and thus forms a primeval structure of a galaxy (a protogalaxy). Such a scheme works only for wormholes which possess sufficiently strong magnetic fields, since high-energy baryons cannot be captured by the wormhole. The mean energy of baryons is determined by the temperature which depends on the redshift. The intensity of the magnetic field also depends on the redshift. While baryons are relativistic particles the threshold value of the fictitious (or equivalent) current does not depend on time.

Indeed, only particles below the threshold energy are captured by the wormhole magnetic field[10] which is given by

$$E = 3kT < E_{th} = eBR_w,$$

where e is the electron charge and R_w is the biggest radius of the doughnut - shaped throat of the wormhole. The energy of relativistic baryons behaves with the redshift as $T = T_\gamma(1+z)$, where T_γ is the present day temperature of CMB radiation. The intensity of the magnetic field can be estimated by the value in the hole of the doughnut. We take it as

$$B = \frac{\kappa I}{cR_w},$$

where $\kappa = 2\pi$ in the center of the doughnut hole, while close to the throat (surface of the doughnut), where the field reaches the maximum value, in the approximation $r_w/R_w \ll 1$ we get the estimate $\kappa \simeq 2R_w/r_w$. The field depends on the parameter I

[c]We present here estimates for baryons only, since leptons are much lighter than baryons and in the primordial plasma leptons simply follow baryons (bounded by the Coulomb potential).

(which is the fictitious or equivalent current) and the big radius R_w of the doughnut which also depends on the redshift as $R_w = R_0/(1+z)$.

The constant I behaves with the redshift as $I = I_0(1+z)$. Indeed the invariant characteristics is the number of magnetic lines captured by the throat (which go through the internal hole of the doughnut). This gives $\int_S \mathbf{Bds} = const = \Phi$, where S an arbitrary surface whose boundary contour γ (dual to Γ) lays on the surface of the throat and cannot be contracted to a point, so that all magnetic lines intersect S only once. Taking the minimal surface S we find $\Phi \sim \frac{2\pi^2}{c} R_w I$ which gives the behavior $I \sim 1/R_w \sim 1/a$, where $a = a_0/(1+z)$ is the scale factor of the Universe. We point out that the same dependence on the redshift z follows from the fact that the energy density of the magnetic field $\rho_B \sim B^2/4\pi$ decreases with the scale factor as $\rho_B \sim 1/a^4$. This gives the threshold value for the equivalent current which defines the intensity of the magnetic field as

$$I_0 > I_{th} = \frac{ce}{\kappa r_\gamma} \sim \frac{3.2}{\kappa} \times 10^{-5} A, \tag{19}$$

where $r_\gamma = \frac{e^2}{3kT_\gamma}$. It is convenient to express r_γ as follows $r_\gamma = r_p(1+z_r)$, where $r_p = \frac{e^2}{m_p c^2}$ is the classical radius of the proton and $1+z_r = \frac{m_p c^2}{3kT_\gamma} \sim 10^{12}$ is the redshift at which baryons become relativistic particles. It is curious that the threshold value I_{th} is extremely small. It does not depend on the absolute size of the wormhole (which is given by the big radius R_w) but only on the ratio of the wormhole radii $\kappa = 2R_w/r_w$. All wormholes with the present day values $I_0 > I_{th}$ strongly interact with baryons. We may say that they are frozen into baryons and, therefore, peculiar motions of baryons repeat peculiar motions of such wormholes. Wormholes with smaller magnetic fields $I_0 < I_{th}$ slightly interact with baryons and can be considered as free objects. They may participate in independent from baryons motions. As we shall see on the early stage of the evolution of the Universe, before the recombination, they cannot capture baryons and therefore do not form an enhancement in the baryon density. On latter stages upon reheating they may capture charged particles and form cosmic rays and compact sources of synchrotron radiation. The relativistic stage for baryons finishes at the redshift z_r upon which (e.g., for $z < z_r$) baryons become non-relativistic and the above consideration brakes.

At redshifts $z_r > z > z_{rec}$ baryons are non-relativistic, while upon the recombination z_{rec} baryons form neutral Hydrogen atoms and do not interact with magnetic fields of all wormholes. They again start to interact with wormholes only at the epoch of re-ionization when first stars have fired.

On the stage $z_r > z > z_{rec}$ for every particular wormhole it is possible to define the critical redshift z_0 when it captures baryons (for $z > z_0$). The critical value z_0 can be estimated as follows. The mean energy of baryons is $m_p V^2/2 = 3kT/2$ with $T = T_\gamma(1+z)$. Then the critical redshift can be found from the inequality $r_B = \frac{V}{\omega_B} = \frac{V m_p c}{eB} < r_d$, where r_B and ω_B are Larmor radius and frequency respectively for protons, while r_d denotes the width of the baryon cloud around the wormhole throat. The width r_d should be smaller, than the proton diffusion length and it defines the critical value of the magnetic field. This gives

$$B = \frac{\kappa I}{cR_w} > \frac{m_p c}{e r_d}\sqrt{\frac{3kT}{m_p}},$$

or equivalently

$$I_0(1+z) > \frac{\alpha e}{\kappa r_p}\sqrt{\frac{3kT}{m_p}} = \alpha I_{th}(1+z_r)\sqrt{\frac{(1+z)}{(1+z_r)}},$$

where the ratio $\alpha = \frac{R_w}{r_d} \gg 1$. This defines the critical redshift z_0 at which the wormhole starts to trap baryons as

$$(1+z) > (1+z_0) = \alpha^2 \frac{I_{th}^2}{I_0^2}(1+z_r).$$

The relation between z_0 and I_0 can be rewritten as

$$I_0^2 = \frac{(1+z_r)}{(1+z_0)}\alpha^2 I_{th}^2. \tag{20}$$

When $z_0 = z_r$ we get $I_0 = \alpha I_{th}$ and if we take $\alpha = 1$, this will give the absolute threshold of the field. We point out that the value $\alpha = 1$ may have sense only for sufficiently small wormholes with the radius $R_w \lesssim 10^{-5} R_{gal}$, e.g., see estimates in the next section. At redshifts $z > z_r$ we get into the epoch where baryons are relativistic particles and wormholes with $I_0 < I_{th}$ do not bound baryons at all. There is one more critical value I_{rec} which corresponds to $z_0 = z_{rec}$.

$$I_{rec}^2 = \alpha^2 I_{th}^2 \frac{(1+z_r)}{(1+z_{rec})} \gg I_{th}^2. \tag{21}$$

All fields with $I_0 > I_{rec}$ are strong enough to capture baryons during the whole evolution $z > z_{rec}$.

5. The number of baryons in traps

The efficiency of wormhole traps can be described by the number of baryons collected. The number of baryons collected around magnetic wormholes depends on the proton diffusion length $\ell(z)$, the big radius a wormhole throat $R_w(z)$, and the radius of the baryon cloud $r_{cl}(z) \ll R_w$. For estimates one may use $r_{cl} \sim r_w$. This number can be estimated as the increase of the effective volume of the torus-shaped throat

$$\Delta N = < n_b > \left(V(R_w+\ell, r_{cl}+\ell) - V(R_w, r_{cl})\right),$$

where $< n_b >$ is the mean density of baryons and $V = 2\pi^2 R_w r_{cl}^2$ is the throat volume. We define the parameter $\delta_b = \Delta N/(V < n_b >)$ which depends on the position in space. Close to wormhole throats $\delta_b > 0$, while sufficiently far from the wormhole $\delta_b < 0$ since baryons from those regions have captured by the wormhole.

The value $<\delta_b^2>= b$, where brackets define the averaging over the space, relates to the baryon clumping factor $b = (<n_b^2> - <n_b>^2)/<n_b>^2$, e.g.,[25, 26]

In the case $\ell(z) \ll r_{cl}(z)$ we get

$$\delta_b(z) \sim \frac{\ell(z)}{R_w(z)} + \frac{2\ell(z)}{r_{cl}(z)} \ll 1. \tag{22}$$

In the intermediate case $r_{cl}(z) \ll \ell(z) \ll R_w(z)$ we find the estimate

$$\delta_b(z) \sim \frac{\ell^2(z)}{r_{cl}^2(z)} \left(1 + \frac{\ell(z)}{R_w(z)}\right) \gg 1. \tag{23}$$

And in the case $\ell(z) \gg R_w(z)$ the estimate has the order

$$\delta_b(z) \sim \frac{\ell^3(z)}{R_w(z) r_{cl}^2(z)} \gg 1. \tag{24}$$

On the stage $z > z_r$ protons are relativistic particles, plasma is degenerate, and the length of the proton propagation can be estimated by the value of the horizon size $\ell(z) \sim l_h = c/H(z)$. At the redshift $z = z_r$ it is extremely small and has the order $\ell(z_r) \sim (7 \div 8) \times 10^{-15} pc$. Consider a wormhole throat with the big radius $R_w(0) \sim 15kpc$ which corresponds to a galaxy size,[41] while the radius of the baryon cloud has the order $r_{cl} \sim 0.2R_w$. Then we find $R_w(z_r) \sim 15 \times 10^{-9} pc \gg \ell(z_r)$ and from (22) we get $\delta_b(z_r) \sim (0.5 \div 0.6) \times 10^{-5}$. Such a value is too small to form a galaxy without additional means (e.g., a dark matter clump).

At the recombination $z_{rec} = 1100$ the proton diffusion length has the co-moving value of the order $\ell(z_{rec}) \sim 0.4 \div 1pc$, while $R_w(z_{rec}) \sim 13.6pc$ and we still may consider $\ell(z_{rec}) \ll R_w$ and $\ell \lesssim r_{cl}$. Therefore we again may use (22) and find $\delta_b(z) \sim 0.32 \div 0.8$. Such a big value shows that the respective protogalaxy forms immediately after the recombination and it is already in the non-linear regime. We should expect that wormholes with such strong clumps of baryons depart the Hubble expansion very soon and form rather small objects. Smaller wormholes form too strong inhomogeneities before recombination and probably collapse to blackholes. We point out that this may give a new mechanism of blackhole formation with huge masses.

Galaxies are observed up to $z \simeq 11$ (e.g., the most distant GN-z11 is observed at $z = 11.09$[39]). Therefore, to be consistent with the present day size of a typical ring galaxy the wormhole radius should be at least two orders bigger $R_w(0) \lesssim 1Mpc$, which gives already $R_w(z_{rec}) \lesssim 900pc$ and $\delta_b(z_{rec}) \sim 4,8 \times 10^{-3}$. Such a clump departs the Hubble expansion already at $z \sim 100$ and gives a ring of the order $R \sim 10Kpc$.

6. Conclusions

In conclusion we point out two important facts. First one is that the wormholes whose size $R(z_{rec})$ exceeds the value $\ell(z_{rec})$ more than on the factor 10^3 do not form a sufficient enhancement in the baryon number density and, therefore, cannot form ring galaxies. They however may form ring-type structures in the future, e.g. see the recently reported findings of unexpected class of astronomical ring-type objects in.[29]

The second fact is that upon the recombination $z < z_{rec}$ the doughnut-shaped wormholes do not interact with baryons and evolve. Therefore, they may leave the ring clump formed. They either expand or collapse forming a magnetized black hole in the middle. If the wormhole collapses, it should also draw some portion of the baryon clump and form a bulge in the center of the ring. In this case it may form the ideal symmetric structure similar to the Hoag's object. Additional rotational perturbations of clumps may however lead to irregular structures. If the wormhole expands further, the center part of the ring remains to be empty. We may expect that in such a case the mean value of the magnetic field within the ring retains. We recall that different active processes in galaxies generate magnetic fields which in general have a turbulent character. Therefore, by the measuring the mean flow of the magnetic field in a ring galaxy one may at least estimate the present day value of the big radius $R = \frac{\kappa I}{cB}$ of such a wormhole and estimate its present day position. This may be used in the direct search for wormhole traces in the Universe.

At first glance the most direct indication on the possible role of relic magnetic wormholes in the formation of some ring galaxies should be the discrepancy between the observed amount of dark matter in such galaxies and the predictions of the standard theory.[40,42,43] However there exist some extensions of general relativity which are capable of reproducing dark matter effects in galaxies without dark matter particles, e.g., see[44,45] and references therein. Therefore, we think that the only rigorous indication on the presence of relic magnetic wormholes should be large-scale toroidal magnetic fields.

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