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TN-63-12
K.L. Brown
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FIRST- AND SECOND-ORDER ABERRATION COEFFICIENTS OF WEDGE MAGNETS

During the year 1958-59 while in France*, the occasion arose when it was necessary to calculate the second-order magnetic optical properties of several beam transport systems the author had proposed **for** use with the Orsay Linear Accelerator. Having extensively studied the available literature, it was evident that the existing mathematical techniques were totally inadequate to handle the problems at hand. The (2×2) matrix methods had been used extensively by the alternating gradient synchrotrons' designers at CERN and Brookhaven; and at the suggestion of W. Chinowsky of the Brookhaven National Laboratory, the momentum term was added to the matrix formalism by Sam Penner, thereby evolving the (3×3) matrix formalism as later reported by Penner in the Review of Scientific Instruments†. It occurred to the author (in late 1958) that the matrix techniques could be extended to include the second- and higher-order aberration terms. As a consequence, the basic second order matrices for uniform field wedge magnets as well as non-uniform (n-value gradient focusing wedge magnets were derived in collaboration with Roger Belbe and Paul Bounin of the Orsay Laboratory. Some of the results of this work was issued last year in TN-62-16, and it is the purpose of this technical note to tabulate further results relating to second-order beam transport matrix algebra. The notation used here for the first- and second-order terms will be that introduced by J.F. Streib in H.E.P.L. Report No. 104.

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†S. Penner, Calculations of Properties of Magnetic Deflection Systems, Rev. Sci Instr., Vol. 32, No. 2, 150-160, February, 1961.

Tabulated below are the baisc matrix elements of the various focusing elements commonly encountered in practice including the matrix for a free drift of the charged particles. The key matrix elements of M_ℓ for particles drifting a distance ℓ ~~is~~ are as follows:

$$\left(x|x_0\right) = 1 \qquad \left(\theta|x_0\right) = 0$$

$$\left(x|\theta_0\right) = \ell \qquad \left(\theta|\theta_0\right) = 1$$

where x is the displacement of a particle from the optic axis (normalized to the mean bending radius ρ_0) and θ is the angle the particle makes with respect to the optic axis. All second-order terms are zero. θ is considered positive if the ray is diverging from the optic axis and negative if the ray is converging toward the optic axis. The matrix M_α for a uniform field wedge magnet with normal entry and exit of the optic axis is as follows:

$$\gamma = \frac{\Delta p}{p} \qquad \alpha = \text{the bending angle}$$

$$\left(x|x_0\right) = \cos\alpha \qquad \left(\theta|x_0\right) = -\sin\alpha$$

$$\left(x|\theta_0\right) = \sin\alpha \qquad \left(\theta|\theta_0\right) = \cos\alpha$$

$$\left(x|\gamma\right) = 1 - \cos\alpha \qquad \left(\theta|\gamma\right) = \sin\alpha$$

$$\left(x|x_0^2\right) = -\frac{1}{2}\sin^2\alpha \qquad \left(\theta|x_0^2\right) = 0$$

$$\left(x|\theta_0^2\right) = \frac{\cos\alpha(1 - \cos\alpha)}{2} \qquad \left(\theta|\theta_0^2\right) = -\frac{\sin\alpha}{2}$$

$$\left(x|\gamma^2\right) = -\frac{\sin^2\alpha}{2} \qquad \left(\theta|\gamma^2\right) = -\sin\alpha$$

$$\left(\theta|x_0\theta_0\right) = 0$$

$$\left(x|x_0\theta_0\right) = \sin\alpha\cos\alpha \qquad \left(\theta|x_0\gamma\right) = \sin\alpha$$

$$\left(x|x_0\gamma\right) = \sin^2\alpha \qquad \left(\theta|\theta_0\gamma\right) = 0$$

$$\left(x|\theta_0\gamma\right) = \sin\alpha(1 - \cos\alpha)$$

Often it is convenient to transform the reference planes of a magnet from the entrance and exit boundaries to the principal planes. This is accomplished by th transformation $\bar{M}_{\alpha} = M_{-\ell_2} M_{\alpha} M_{-\ell_1}$ where ℓ_1 and ℓ_2 are the distances from the magnet face to the principal planes. For the uniform field wedge magnet having normal entry and normal exit $\ell_1 = \ell_2 = \ell = \text{Tan}\,\frac{\alpha}{2}$ in which case, the calculation of $\tilde{M}_{\alpha}$ yields

$$(x|x_0) = 1 \qquad (\theta|x_0) = -\sin\alpha$$

$$(x|\theta_0) = 0 \qquad (\theta|\theta_0) = 1$$

$$(x|\gamma) = 0 \qquad (\theta|\gamma) = \sin\alpha$$

$$(x|x_0^2) = -\frac{1}{2}\sin^2\alpha \qquad (\theta|x_0^2) = 0$$

$$(x|\theta_0^2) = 0 \qquad (\theta|\theta_0^2) = -\frac{1}{2}\sin\alpha$$

$$(x|\gamma^2) = \frac{1}{2}(1-\cos\alpha)^2 \qquad (\theta|\gamma^2) = -\sin\alpha$$

$$(x|x_0\theta_0) = \sin\alpha \qquad (\theta|x_0\theta_0) = 0$$

$$(x|x_0\gamma) = \cos\alpha(1-\cos\alpha) \qquad (\theta|x_0\gamma) = \sin\alpha$$

$$(x|\theta_0\gamma) = \frac{(1-\cos\alpha)^2}{\sin\alpha} \qquad (\theta|\theta_0\gamma) = \cos\alpha - 1$$

The matrix M_R for a curved boundary at either the entrance or exit of a wedge magnet has the same form and is dependent upon the angle β between the optic axis and the normal to the face of the magnet (see Fig. 1). M_R consists of the unity matrix plus one off-diagonal term - namely,

$$(\theta|x_0^2) = \frac{1}{2R\cos^3\beta}$$

where R is the radius of curvature of the entrance or exit boundary as the case may be. This matrix as given is also valid for an n-type magnet.

The matrix for a rotated input face of a wedge magnet is as follows:

where β_1 = the angle the optic axis makes with respect to the normal to the input face of the magnet and β_1 is taken as positive as shown in Fig. 1.

$(x|x_0) = 1$

$(x|x_0^2) = -\frac{1}{2}\,\text{Tan}^2\,\beta_1$

$(\theta|x_0) = \text{Tan}\,\beta_1$

$(\theta|\theta_0) = 1$

$(\theta|x_0^2) = -n\,\text{Tan}\,\beta_1$

$(\theta|x_0\theta_0) = \text{Tan}^2\,\beta_1$

$(\theta|x\gamma) = -\,\text{Tan}\,\beta_1$

All other x and θ terms are zero. Note that M_{β_1} as given here is valid for an n-type gradient magnet as well as for a uniform **field** $n = 0$ wedge magnet. For a rotated output face, the result M_{β_2} is similar but not symmetrical with M_{β_1}. M_{β_2} is given by

$(x|x_0) = 1$

$(x|x_0^2) = \frac{1}{2}\,\text{Tan}^2\,\beta_2$

$(\theta|x_0) = \text{Tan}\,\beta_2$

$(\theta|\theta_0) = 1$

$(\theta|x_0^2) = -\,n\,\text{Tan}\,\beta_2 - \frac{1}{2}\,\text{Tan}^3\,\beta_2$

$(\theta|x_0\theta_0) = -\,\text{Tan}^2\,\beta_2$

$(\theta|x_0\gamma) = -\,\text{Tan}\,\beta_2$

all other elements are zero.

Given the basic matrix elements M_α, M_R, M_β it is now possible to derive any combination thereof, provided the order of multiplication is as follows:

$$M_T = M_{\beta_2} M_{R_2} M_\alpha M_{R_1} M_{\beta_1}$$

The result M_T is in agreement with TN-62-16* the results of which are reproduced below for convenience.

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*Note: Page numbers that follow will be as for TN-62-16; they will then continue as TN-63-12, pp. 5, 6, etc.

TN 62-16
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SECOND ORDER ABERRATION COEFFICIENTS OF A MAGNET*

We have calculated the aberrations for a uniform field magnet with curved rotated faces and for an "n" magnet with perpendicular input and output. We have neglected the fringe field effect. In other words, these coefficients correspond to a theoretical magnet without fringe field. The parameters which appear in the formulas (α, β_1, β_2, R_1, R_2, n) are different from the corresponding parameters of the real magnet. The relation between these two magnets will depend on the fringe field configuration, and it is impossible to give some general rules to calculate the perturbation of the coefficient.

We give only the coefficients of the mid-plane trajectories.

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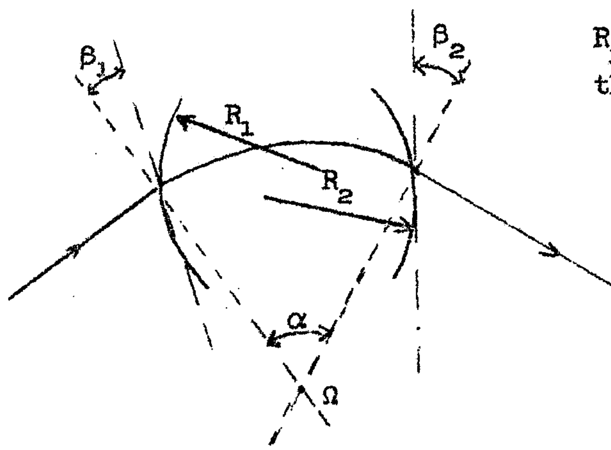
*This work was done at ORSAY (Laboratoire de l'Accélérateur Lineaire)

CONSTANT FIELD MAGNET

$M_{\alpha\beta_1\beta_2R_1R_2}$

$\rho_o = 1$

R_1, R_2, β_1 and β_2 are positive on the figure



$$\left(x\middle|x_o\right) = \cos\alpha + \sin\alpha\tan\beta_1$$

$$\left(x\middle|\theta_o\right) = \sin\alpha$$

$$\left(x\middle|\frac{\Delta p}{p_o}\right) = 1 - \cos\alpha$$

$$\left(x\middle|x_o^2\right) = -\frac{1}{2}\left(\sin\alpha - \cos\alpha\tan\beta_1\right)^2 + \frac{\tan^2\beta_2}{2}\left(\cos\alpha + \sin\alpha\tan\beta_1\right)^2 + \frac{\sin\alpha}{2R_1\cos^3\beta_1}$$

$$\left(x\middle|\theta_o x_o\right) = \frac{\sin 2\alpha}{2} + \tan\beta_1\left[\cos\alpha\,(1-\cos\alpha) + \sin\alpha\tan\beta_1\right]$$

$$+ \sin\alpha\tan^2\beta_2\left(\cos\alpha + \sin\alpha\tan\beta_1\right)$$

$$\left(x\middle|\theta_o^2\right) = \frac{\cos\alpha\,(1-\cos\alpha)}{2} + \frac{\sin^2\alpha\tan^2\beta_2}{2}$$

$$\left(x\middle|\left(\frac{\Delta p}{p_o}\right)^2\right) = -\frac{\sin^2\alpha}{2} + \frac{(1-\cos\alpha)^2}{2}\tan^2\beta_2$$

$$\left(x\middle|x_o \frac{\Delta p}{p_o}\right) = \sin\alpha \left(\sin\alpha - \cos\alpha \tan\beta_1\right) + \tan^2\beta_2 \,(1 - \cos\alpha)\left(\cos\alpha + \sin\alpha \tan\beta_1\right)$$

$$\left(x\middle|\theta_o \frac{\Delta p}{p_o}\right) = \frac{\sin\alpha\,(1 - \cos\alpha)}{\cos^2\beta_2}$$

$$\left(\theta\middle|x_o\right) = -\sin\alpha + \cos\alpha\left(\tan\beta_1 + \tan\beta_2\right) + \sin\alpha\tan\beta_1\tan\beta_2$$

$$\left(\theta\middle|\theta_o\right) = \cos\alpha + \sin\alpha\tan\beta_2$$

$$\left(\theta\middle|\frac{\Delta p}{p_o}\right) = \sin\alpha + (1 - \cos\alpha)\tan\beta_2$$

$$\left(\theta\middle|x_o^2\right) = -\frac{\tan\beta_2}{2}\left[-\sin\alpha + \cos\alpha\left(\tan\beta_1 + \tan\beta_2\right) + \sin\alpha\tan\beta_1\tan\beta_2\right]^2 + \frac{\cos\alpha + \sin\alpha\tan\beta_2}{2R_1\cos^3\beta_1} + \frac{\left(\cos\alpha + \sin\alpha\tan\beta_1\right)^2}{2R_2\cos^3\beta_2}$$

$$\left(\theta\middle|\theta_o x_o\right) = \left[-\sin\alpha + \cos\alpha\left(\tan\beta_1 + \tan\beta_2\right) + \sin\alpha\tan\beta_1\tan\beta_2\right]\left[\tan\beta_1 - \tan\beta_2\left(\cos\alpha + \sin\alpha\tan\beta_2\right)\right] + \frac{\sin\alpha\left(\cos\alpha + \sin\alpha\tan\beta_1\right)}{R_2\cos^3\beta_2}$$

$$\left(\theta\middle|\theta_o^2\right) = -\frac{\sin\alpha}{2} + \frac{\cos\alpha}{2}\tan\beta_2 - \frac{\tan\beta_2}{2}\left(\cos\alpha + \sin\alpha\tan\beta_2\right)^2 + \frac{\sin^2\alpha}{2R_2\cos^3\beta_2}$$

$$\left(\theta\middle|\left(\frac{\Delta p}{p_o}\right)^2\right) = -\left[\sin\alpha + (1 - \cos\alpha)\tan\beta_2\right] - \frac{\tan\beta_2}{2}\left[\sin\alpha + (1 - \cos\alpha)\tan\beta_2\right]^2 + \frac{(1 - \cos\alpha)^2}{2R_2\cos^3\beta_2}$$

$$\left(\theta \middle| x_o \frac{\Delta p}{p_o}\right) = - \left[- \sin\alpha + \cos\alpha \left(\tan\beta_1 + \tan\beta_2\right) + \sin\alpha \tan\beta_1 \tan\beta_2\right]$$

$$\left[1 + \sin\alpha \tan\beta_2 + (1 - \cos\alpha) \tan^2\beta_2\right] + \frac{(1 - \cos\alpha)\left(\cos\alpha + \sin\alpha \tan\beta_1\right)}{R_2 \cos^3\beta_2}$$

$$\left(\theta \middle| \theta_o \frac{\Delta p}{p_o}\right) = - \tan\beta_2 \left[\cos\alpha + \sin\alpha \tan\beta_2\right] \left[\sin\alpha + (1 - \cos\alpha) \tan\beta_2\right] + \frac{\sin\alpha\,(1 - \cos\alpha)}{R_2 \cos^3\beta_2}$$

N MAGNET (perpendicular input and output)

$$B = B_o\left(1 - nx + \beta x^2\right) \qquad \alpha' = \sqrt{1 - n}\,\alpha \qquad \rho_o = 1$$

$$\left(x\middle|x_o\right) = \cos\alpha' \qquad \left(\theta\middle|x_o\right) = -\sqrt{1 - n}\,\sin\alpha'$$

$$\left(x\middle|\theta_o\right) = \frac{\sin\alpha'}{\sqrt{1 - n}} \qquad \left(\theta\middle|\theta_o\right) = \cos\alpha'$$

$$\left(x\middle|\frac{\Delta\rho}{p_o}\right) = \frac{1 - \cos\alpha'}{1 - n} \qquad \left(\theta\middle|\frac{\Delta\rho}{p_o}\right) = \frac{\sin\alpha'}{\sqrt{1 - n}}$$

$$\left(x\middle|x_o^2\right) = \frac{3n - 2\beta - 1}{4(1 - n)} - \frac{n - \beta}{3(1 - n)}\cos\alpha' - \frac{5n - 2\beta - 3}{12(1 - n)}\cos 2\alpha'$$

$$\left(x\middle|\theta_o x_o\right) = \frac{2}{3}\,\frac{n - \beta}{3(1 - n)^{3/2}}\sin\alpha' - \frac{5n - 2\beta - 3}{6(1 - n)^{3/2}}\sin 2\alpha'$$

$$\left(x\middle|\theta_o^2\right) \doteq \frac{3n - 2\beta - 1}{4(1 - n)^2} - \frac{7n - 4\beta - 3}{6(1 - n)^2}\cos\alpha' + \frac{5n - 2\beta - 3}{12(1 - n)^2}\cos 2\alpha'$$

$$\left(x\middle|\left(\frac{\Delta\rho}{p_o}\right)^2\right) = \frac{7n - 6\beta - 1}{4(1 - n)^3} - \frac{4}{3}\,\frac{n - \beta}{(1 - n)^3}\cos\alpha' - \frac{5n - 2\beta - 3}{12(1 - n)^3}\cos 2\alpha' - \frac{n^2 + n - 2\beta}{2(1 - n)^3}\alpha'\sin\alpha'$$

$$\left(x\middle|x_o\frac{\Delta\rho}{p_o}\right) = \frac{-3n + 2\beta + 1}{2(1 - n)^2} + \frac{2}{3}\,\frac{n - \beta}{(1 - n)^2}\cos\alpha' + \frac{5n - 2\beta - 3}{6(1 - n)^2}\cos 2\alpha' + \frac{n^2 + n - 2\beta}{2(1 - n)^2}\alpha'\sin\alpha'$$

$$\left(x\middle|\theta_o\frac{\Delta\rho}{p_o}\right) = \frac{3n^2 - 7n - 2\beta + 6}{6(1 - n)^{5/2}}\sin\alpha' + \frac{5n - 2\beta - 3}{6(1 - n)^{5/2}}\sin 2\alpha' - \frac{n^2 + n - 2\beta}{2(1 - n)^{5/2}}\alpha'\cos\alpha'$$

The matrix elements of a non-uniform field wedge magnet having normal entry and normal exit may be derived directly from the Streib formalism[1] or by generalizing the results of Ikegami[2] to include all momenta particles. Again, we normalize all dimensions to the central orbit ρ_0, i.e., let $\rho_0 = 1$. As before x is the radial deviation of a ray from the central orbit as projected on the magnetic midplane and θ is the corresponding angle of the ray with respect to the optic axis. y and φ are the corresponding projections on the vertical midplane. The magnetic field expansions are assumed to have the form

$$H_y(x,y) = H_0\left[1 - nx + \beta x^2 + \left(\frac{n}{2} - \beta\right) y^2 + \ldots\right]$$

$$H_x(x,y) = H_0\left[- ny + 2\beta xy + \ldots\right]$$

consistent with Maxwell's equation to second-order in x and y. It should be noted that the field expansion used here is generally consistent with the available literature but differs with that adopted by Streib[1].

The resulting matrix elements for the radial motion are then as follows:

$$\alpha' = (1-n)^{1/2}\alpha \qquad \alpha'' = n^{1/2}\alpha \qquad \gamma = \frac{\Delta p}{p}$$

$$\left(x|x_0\right) = \cos\alpha'$$

$$\left(x|\theta_0\right) = (1-n)^{-1/2}\sin\alpha'$$

$$\left(x|\gamma\right) = (1-n)^{-1}(1-\cos\alpha')$$

$$\left(x|x_0^2\right) = \frac{3n-2\beta-1}{4(1-n)} - \frac{(n-\beta)}{3(1-n)}\cos\alpha' - \frac{(5n-2\beta-3)}{12(1-n)}\cos 2\alpha'$$

$$\left(x|\theta_0^2\right) = \frac{3n-2\beta-1}{4(1-n)^2} - \frac{7n-4\beta-3}{6(1-n)^2}\cos\alpha' + \frac{5n-2\beta-3}{12(1-n)^2}\cos 2\alpha'$$

$$\left(x|\gamma^2\right) = \frac{7n-6\beta-1}{4(1-n)^3} - \frac{4(n-\beta)}{3(1-n)^3}\cos\alpha' - \frac{5n-2\beta-3}{12(1-n)^3}\cos 2\alpha' - \frac{n^2+n-2\beta}{2(1-n)^3}\alpha'\sin\alpha'$$

$$\left(x|x_0\theta_0\right) = \frac{2(n-\beta)}{3(1-n)^{3/2}}\sin\alpha' - \frac{5n-2\beta-3}{6(1-n)^{3/2}}\sin 2\alpha'$$

$$\left(x|x_0\gamma\right) = \frac{1-3n+2\beta}{2(1-n)^2} + \frac{2(n-\beta)}{3(1-n)^2}\cos\alpha' + \frac{5n-2\beta-3}{6(1-n)^2}\cos 2\alpha' + \frac{n^2+n-2\beta}{2(1-n)^2}\alpha'\sin\alpha'$$

$$\left(x|\theta_0\gamma\right) = \frac{3n^2-7n-2\beta+6}{6(1-n)^{5/2}}\sin\alpha' + \frac{5n-2\beta-3}{6(1-n)^{5/2}}\sin 2\alpha' - \frac{n^2+n-2\beta}{2(1-n)^{5/2}}\alpha'\cos\alpha'$$

$$\left(x|y_0^2\right) = -\frac{n-\beta}{2(1-n)} + \frac{1}{2}\left[\frac{n-\beta}{1-n} + \frac{\beta}{5n-1}\right]\cos\alpha' - \frac{1}{2}\left(\frac{\beta}{5n-1}\right)\cos 2\alpha''$$

$$\left(x|y_0\varphi_0\right) = \frac{\beta}{5n-1}\left[\frac{2}{(1-n)^{1/2}}\sin\alpha' - \frac{1}{n}\sin 2\alpha''\right]$$

$$\left(x|\varphi_0^2\right) = -\frac{1}{2n}\left[\frac{n-\beta}{1-n} - \left(\frac{n-\beta}{1-n} - \frac{\beta}{5n-1}\right)\cos\alpha' - \frac{\beta}{5n-1}\cos 2\alpha''\right]$$

The matrix elements for θ are given by noting that

$$\theta = \frac{\partial x}{\partial(\rho\alpha)} = \frac{1}{\rho}\frac{\partial x}{\partial\alpha}$$

but $\rho = 1+x$ since we have normalized to $\rho_0 = 1$

Hence $\theta = \frac{\partial x}{\partial \alpha} - x\theta$ yields the **first- and second-order matrix elements for** θ **as follows:**

$$\left(\theta|x_0\right) = -(1-n)^{1/2}\sin\alpha'$$

$$\left(\theta|\theta_0\right) = \cos\alpha'$$

$$\left(\theta|\gamma\right) = (1-n)^{-1/2}\sin\alpha'$$

$$\left(\theta|x_0^2\right) = \frac{n-\beta}{3(1-n)^{1/2}}\left[\sin\alpha' + \sin 2\alpha'\right]$$

$$\left(\theta|\theta_0^2\right) = \frac{7n-4\beta-3}{6(1-n)^{3/2}}\sin\alpha' - \frac{n-\beta}{3(1-n)^{3/2}}\sin 2\alpha'$$

$$\left(\theta|\gamma^2\right) = \left[\frac{-3n^2+11n-2\beta-6}{6(1-n)^{5/2}}\right]\sin\alpha' + \frac{n-\beta}{3(1-n)^{5/2}}\sin 2\alpha' - \frac{n^2+n-2\beta}{2(1-n)^{5/2}}\alpha'\cos\alpha'$$

$$\left(\theta|x_0\theta_0\right) = \frac{2(n-\beta)}{3(1-n)}\left[\cos\alpha' - \cos 2\alpha'\right]$$

$$\left(\theta|x_0\gamma\right) = \frac{1}{6(1-n)^{3/2}}\left[3n^2-7n-2\beta+6\right]\sin\alpha' - \frac{2(n-\beta)}{3(1-n)^{3/2}}\sin 2\alpha' + \frac{n^2+n-2\beta}{2(1-n)^{3/2}}\alpha'\cos\alpha'$$

$$+ \frac{n^{1/2}\beta}{5n-1} \sin 2\alpha''$$

$$\left(\theta | y_0 \varphi_0\right) = \frac{2\beta}{5n-1} \cos \alpha' - \frac{2\beta \cos 2\alpha''}{(5n-1)n^{1/2}}$$

$$\left(\theta | \varphi_0^2\right) = \left[\frac{n-\beta}{(1-n)^{1/2}} - \frac{\beta(1-n)^{1/2}}{(5n-1)}\right] \sin \alpha' + \frac{2\beta n^{1/2}}{5n-1} \sin 2\alpha''$$

All other terms are zero.

The first-order coefficients in the y plane are as follows:

$$\left(y | y_0\right) = \cos \alpha'' \qquad \left(\varphi | y_0\right) = - n^{1/2} \sin \alpha''$$

$$\left(y | \varphi_0\right) = \frac{1}{n^{1/2}} \sin \alpha'' \qquad \left(\varphi | \varphi_0\right) = \cos \alpha''$$

The second-order coefficients in the y plane have not been calculated since all of the applications that I have encountered have not required this result. However, this may readily be done from the Streib results, and again, noting that

$$\varphi = \frac{\partial y}{\partial(\rho\alpha)}$$

or $$\varphi = \frac{\partial y}{\partial \alpha} - \varphi x$$

Occasionally, it is necessary to represent the trajectory through a magnet that bends the particles in an opposite sense to that shown in Fig. 1. It is easy to see that this is equivalent to changing the sign of the variables x and θ as the particle enters and exits from the inverted magnet. This coordinate transformation M_I is given by

$$(x|x_0) = -1 \qquad (\theta|x_0) = 0$$

$$(x|\theta_0) = 0 \qquad (\theta|\theta_0) = -1$$

Then the matrix of the inverted magnet is

$$M_{-\alpha} = M_I M_\alpha M_I$$

the net result being to change the sign of the

$(x|\gamma)$, $(x|x_0^2)$, $(x|\theta_0^2)$, $(x|\gamma^2)$, $(x|x_0\theta_0)$ and the

$(\theta|\gamma)$, $(\theta|x_0^2)$, $(\theta|\theta_0^2)$, $(\theta|\gamma^2)$, $(\theta|x_0\theta_0)$ terms in the M_α matrix to generate the $M_{-\alpha}$ matrix. Note that all of the above information applies to the magnetic midplane orbits only unless otherwise indicated.

REFERENCES

1. F. Streib, Design Considerations for Magnetic Spectrometers, HEPL, No. 104, Stanford University, Stanford, California, November 1960.

2. H. Ikegami, Rev.Sci.Instr. 29, p. 943(1958).

ACKNOWLEDGMENT

The assistance of Harold Butler and Sam Howry in double checking the derivations is hereby acknowledged. In particular, the original M_{β_2} result was in error and has since been corrected. The results of TN-62-16 have, therefore, been verified by an independent technique to that used in the original derivation in 1959.