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# Mathematical modeling of mass generation via confinement of relativistic particles

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**Abstract.** A simple mathematical model is presented for the generation of mass via the spontaneous confinement of elementary particles, e.g. electrons, neutrinos or quarks, in bound rotational states with relativistic particle velocities. In this simple mechanism, which is operative both in chemical and in physical systems, the new mass which is created corresponds to the kinetic energy of the trapped particles. It is shown that the ratio of the new mass created to the initial mass is very small for chemical systems but can be very high in the case of gravitational confinement of neutrinos in rotational states with relativistic particle velocities.

## 1. Introduction

The mechanism of mass generation is a central problem of interest in modern particle physics. Among the various proposed mechanisms, the Higgs boson mechanism [1] has attracted significant attention during the last few decades [2]. It postulates that matter obtains mass by interacting with a field, known as Higgs field [2].

In this work we describe a simple mechanism of mass generation via the confinement of elementary particles, e.g. electrons, neutrinos or quarks, in bound rotational states. It is shown that the ratio of new mass created to initial mass can be quite high for relativistic particle velocities inside the bound state.

## 2. Mathematical modeling

We examine the changes induced in the total energy (Hamiltonian) of a system as well as to its relativistic energy and rest mass for two cases: First when a H atom is formed via the Coulombic attraction of a proton and an electron and second when a rotational state corresponding to a baryon (e.g. a neutron) is formed via the relativistic gravitational attraction of three neutrinos [3, 4, 5].

The total energy, i.e. the Hamiltonian, $\mathcal{H}$, of a system consisting of $N$ particles (i=1,2,..., N) is the sum of the total relativistic energy, $E$, and of the potential energy of the system, $V$:

$$\mathcal{H} = E + V. \tag{1}$$

The relativistic energy, $E$, of the system is the sum of its rest energy, $\sum m_{o,i}c^2$, plus its kinetic energy $T$, which equals $\sum(\gamma_i - 1)m_{o,i}c^2$, i.e.

$$E = \sum_{i=1}^{N} m_{o,i}c^2 + \sum_{i=1}^{N}(\gamma_i - 1)m_{o,i}c^2. \tag{2}$$

### 2.1. *H atom*

Thus in the case of a H atom, using the Bohr model and neglecting the (rather small) kinetic energy of the proton, we can write with good accuracy:

$$E = m_p c^2 + m_e c^2 + (\gamma_e - 1) m_e c^2 \tag{3}$$

$$\mathcal{H} = m_p c^2 + m_e c^2 + (\gamma_e - 1) m_e c^2 + V_e, \tag{4}$$

where $V_e$ is the electrostatic energy of the electron. Since, $\mathtt{v} = \alpha c$, thus $\mathtt{v} \ll c$, the kinetic energy term reduces to $(1/2)m_e \mathtt{v}^2 = (1/2)\alpha^2 m_e c^2$, where $\alpha$ $(= e^2/\epsilon c\hbar = 1/137.036)$ is the fine structure constant. For the circular orbit of the Bohr model it is $V_e = -2T$, thus one obtains:

$$E = m_p c^2 + m_e c^2 + (1/2)\alpha^2 m_e c^2 \tag{5}$$

$$\mathcal{H} = m_p c^2 + m_e c^2 - (1/2)\alpha^2 m_e c^2. \tag{6}$$

In the initial state (a proton and an electron at rest and "infinite" distance), both $E$ and $\mathcal{H}$ equal $(m_p + m_e)c^2$, thus upon formation of the bound H atom state it is

$$\Delta E = (1/2)\alpha^2 m_e c^2 (= T) \tag{7}$$

$$\Delta \mathcal{H} = -(1/2)\alpha^2 m_e c^2 = -13.6 eV. \tag{8}$$

Thus while the system Hamiltonian has decreased by 13.6 $eV$, and therefore the formation of the H atom occurs spontaneously and is exoergic, i.e. energy is being released, at the same time the relativistic energy of the system, $\Delta E$, has increased by the Bohr energy $(1/2)\alpha^2 m_e c^2$, which actually equals $T$, i.e. the kinetic energy of the electron. Consequently for a laboratory observer who cannot detect the circular motion, the apparent *rest* energy of the system has increased by $(1/2)\alpha^2 m_e c^2$. This implies that the rest mass of the system has increased by $(1/2)\alpha^2 m_e$ and that therefore accounting also for the values of $m_e$ and $m_p$, the ratio of the masses in the final and initial state is 1.0000000145.

Thus the rest energy increase is equal to the kinetic energy of the electron (or of the proton-electron pair) in the confined H atom state (Fig. 1, left).

### 2.2. *Neutrino confinement*

Similar is the situation when a composite particle is formed via the gravitational confinement of a number (e.g. two or three [3, 4]) of light particles such as neutrinos [3, 4]. The bound rotational states resulting from the confinement of three highly relativistic neutrinos have been found recently to have, surprisingly, many of the properties of baryons, including their masses [3, 4, 5].

Table 1 summarizes the recent Bohr-type model for the gravitational confinement of three relativistic neutrinos and compares it with the Bohr model for the H atom [3, 4, 5]. The only difference is that the elestrostatic attraction of the H atom model is replaced in the three neutrino model by the gravitational attraction, accounting for special relativity and for the equivalence principle [3, 4, 5]. The justification for using Newton's universal gravitational law rather than general relativity has been discussed [5].

For the initial state, i.e. three non-interacting neutrinos at rest, it is $E_o = \mathcal{H}_o = 3m_o c^2$ where $m_o$ is the rest mass of each neutrino. Upon formation of the bound state it is

$$E = 3m_o c^2 + 3(\gamma - 1)m_o c^2 = 3\gamma m_o c^2 \tag{9}$$

$$\mathcal{H} = 3\gamma m_o c^2 + V_G = 3\gamma m_o c^2 - 5\gamma m_o c^2 = -2\gamma m_o c^2, \tag{10}$$

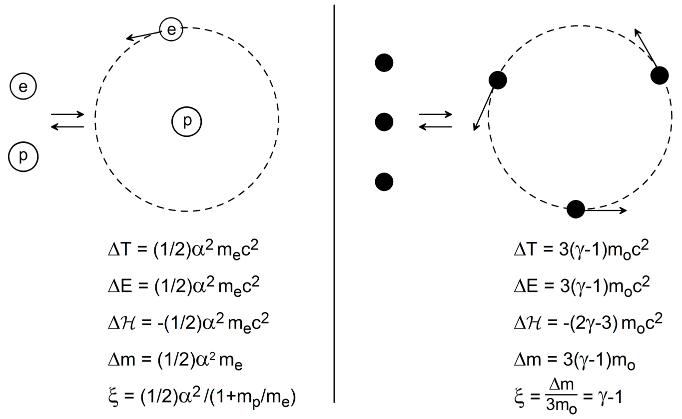


**Figure 1.** Apparent mass generation mechanism during formation of a H atom from a proton and an electron (left) and of a neutral baryon (e.g. a neutron) from three neutrinos (right).

where in (10) we have expressed the gravitational potential energy, $V_G$, using refs. [3] and [4].

Thus again it is $\Delta\mathcal{H} < 0$, i.e. the reaction is exoergic and occurs spontaneously, but at the same time it is $\Delta E > 0$, i.e. the relativistic energy of the system increases. Consequently, for a laboratory observer, the apparent rest energy increase is $\Delta E = 3(\gamma - 1)m_o c^2$ [3, 4, 5] and thus again the rest energy increase equals the kinetic energy of the three rotating particles (Fig. 1, right). However in this case, the ratio of the masses of the final and initial states is very high. It equals $\gamma_n = m_n/3m_o (= 7.163 \cdot 10^9)$ where $\gamma_n$ is the Lorentz factor corresponding to the rotational speed of the three neutrinos and $m_n$ is the neutron mass. This is computed from the following expression obtained by solving the two equations of the Bohr-type rotating neutrino model of Table 1 [3, 4, 5]:

$$\gamma_n = 3^{1/12} m_{Pl}^{1/3} / m_o^{1/3} (= 7.163 \cdot 10^9). \tag{11}$$

In this expression $m_{Pl} (= (\hbar c/G)^{1/2})$ is the Planck mass and $m_o (= 0.051 \pm 0.01\ eV/c^2)$ [6] is the neutrino mass. Exact agreement of the computed neutron mass with experiment is obtained for $m_o = 0.043723\ eV/c^2$ [3, 4, 5]. Thus in this case apparent rest energy and apparent rest mass are generated simply by the action of gravity, which causes particle confinement in a high kinetic energy, thus high $\gamma$ value state. The two Bohr models for electrostatic and for gravitational confinement are compared in Table 1 and also in Figure 1. In both cases the rest energy increase and concomitant rest mass increase corresponds to the kinetic energy of the confined particles. It should be noted that this very simple mass generation mechanism does not attempt to compete with the Higgs mechanism, since it is obvious that some initial mass is necessary for the present mechanism to become operative.

One can define a parameter, $\xi$, as the ratio of new mass created to initial mass, i.e.

$$\xi = \frac{new\ mass\ created}{initial\ mass}. \tag{12}$$

Thus in the case of the H atom it is:

$$\xi = \frac{(1/2) m_e \alpha^2}{m_p + m_e} = 1.45 \cdot 10^{-8}, \tag{13}$$

while for the case of the neutron it is:

$$\xi = \frac{3(\gamma_n - 1) m_o}{3 m_o} = \gamma_n - 1 \approx 7.163 \cdot 10^9. \tag{14}$$

**Table 1.** Comparison of the Bohr model for the H atom and of the Bohr type model for the neutron [4, 5]

| **Bohr model for the H atom** | **Bohr model for the neutron** |
|---|---|
| Electron as *particle*<br>$m_e \frac{v^2}{R} = \frac{e^2}{\varepsilon R^2}$<br>↑ Newton's 2nd law ↑ Coulomb law | Neutrino as *particle*<br>$\gamma m_o \frac{v^2}{R} = \frac{G m_o^2 \gamma^6}{\sqrt{3} R^2}$<br>↑ Relativistic equation of motion for circular motion ↑ Newton's gravitational law accounting for special relativity $(m_i = \gamma^3 m_o)$ and for equivalence principle $(m_g = m_i)$ |
| Electron as *wave*<br>$\frac{\hbar}{m_e v} = R$<br>De Broglie (for n=1) | Neutrino as *wave*<br>$\frac{\hbar}{m_o v} \approx \frac{\hbar}{m_o c} = R$<br>De Broglie (for n=1) |
| Results<br>$v = \alpha c \;;\; \gamma \approx 1$<br>$R = \frac{\hbar}{\alpha m_e c} = 0.5292 \cdot 10^{-10} m$<br>$V_e = -27.2 \text{ eV}$<br>$T = 13.6 \text{ eV}$<br>$\mathcal{H} = T + V_e = -13.6 \text{ eV}$<br>$\Delta(RE) \equiv \Delta mc^2 = T = (1/2)\alpha^2 m_e c^2 = 13.6 \text{ eV}$ | Results (with $m_o$=0.043723 eV/$c^2$)<br>$v \approx c \;;\; \gamma = 3^{1/12}(\hbar c/G)^{1/6}/m_o^{1/3} (= 7.163 \cdot 10^9)$<br>$R = \frac{\hbar}{\gamma m_o c} = 0.630 \cdot 10^{-15} m$<br>$V_g = -(5/3) m_n c^2 = -1565.9 \text{ MeV}$<br>$T = m_n c^2 = 939.565 \text{ MeV}$<br>$\mathcal{H} = T + V_G = -(2/3) m_n c^2 = -626.4 \text{ MeV}$<br>$\Delta(RE) = \Delta mc^2 = T = m_n c^2 = 939.565 \text{ MeV}$<br>Per particle: $T_p = 313.2 \text{ MeV}$; $\mathcal{H}_p = -208.8 \text{ MeV}$ |

This process of mass generation via particle confinement is simple and different from other proposed mass generation schemes [1, 2, 7, 8, 9]. It is similar in its conclusions with the QCD results of Dürr et al. [7] who showed that even if the quark masses vanished, the baryon mass would not change much, a phenomenon sometimes called "mass without mass" [8, 9].

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