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Heavy quarkonia in a non-extensive medium

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Abstract We study the non-extensive effects on the properties of heavy quarkonia in a quark-gluon plasma. For that purpose, we revisit the gluon self-energy and resummed gluon propagator in the presence of non-extensivity using the hard-thermal-loop resummation in real-time formalism. We find that the non-extensivity of medium results in distinct shifts on the Debye masses in the retarded/advanced and symmetric gluon self-energies. Based on the non-extensive modified resummed gluon propagator, we compute the dielectric permittivity of the medium, thereby deriving the in-medium heavy quark potential. We observe that the real part of the potential gets more screened as the non-extensive parameter q ($q \geq 1$) increases, reducing the binding energies of heavy quarkonia. Meanwhile, the non-extensivity of the medium enhances the magnitude of the imaginary part of the potential, causing a broadening in the decay widths of heavy quarkonia. Based on these observations, we further study the sensitivity of the melting temperatures of heavy quarkonia to non-extensive effects. Our results indicate that the non-extensivity of the medium can lower the melting temperature of heavy quarkonia, facilitating the dissociation of heavy quarkonia.

1 Introduction

Studying the properties of strongly interacting matter at high temperatures and/or high densities is a frontier topic in nuclear physics. Strongly interacting matter under such extreme conditions is expected to exhibit a deconfined state of quarks and gluons called quark-gluon plasma (QGP). The wealth of data harvested in the relativistic heavy ion collider (RHIC) at BNL and the large hadron collider (LHC) at CERN provide compelling evidence for the formation of QGP [1,2]. At sufficiently high temperatures, the QGP behaves as a weak coupling plasma ($T \gg \Lambda_{\rm QCD}$, $g \ll 1$, $g \equiv \sqrt{4\pi\alpha_s}$ is the quantum chromodynamics (QCD) gauge coupling), the hard-thermal loop (HTL) resummation is an essential theoretical method to describe the thermodynamics of QGP [3–12]. It allows for systematic computations of various quantities such as parton self-energies [13,14], in-medium complex heavy quark potential [15–19], heavy quark diffusion coefficients [19–22], dilepton production rate [23–25], and parton energy loss [26], to study the QGP properties.

Heavy quarkonia, which are heavy quark and its antiquark bound states together by almost static gluons, serve as important probes of QGP produced in high-energy nuclear collisions. Heavy quark potential, which describes the interaction between a quark and its antiquark in quarkonium states, is the starting point for a non-relativistic approach to access the properties of heavy quarkonia. In a vacuum, the heavy quark potential is well described by the Cornell potential, which includes Coulomb and confined potential [27]. In a deconfined QCD medium, the color Debye screening effect results in the dissociation of charmonium, which is one of the proposed signatures of QGP formation [28]. To describe the in-medium interactions between heavy quark and antiquark, various phenomenological in-medium heavy quark potential descriptions have been proposed. In-medium heavy quark potential is usually complex valued, whose real part is Debye-screened Coulomb potential or Yukawa potential, which reflects the color screening effect preventing the formation of quarkonium states, whereas the imaginary part gives rise to the thermal decay widths of heavy quarkonia, also triggering the quarkonium dissociation. The imaginary part of heavy quark potential or thermal decay width at least comes from two distinct mechanisms: gluondissociation (dissociation of quarkonium by absorbing a time-like gluon from thermal bath) or called the siglect to octet thermal breaking-up, as well as inelastic parton scattering mediated by space-like gluon (dissociation of quarkonium by scattering with gluons and light quarks in the medium) or called Landau damping mechanism [29]. When the medium temperature

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is larger than the binding energy, the Landau damping contribute to the thermal decay width is dominant [30], thus the problem of in-medium quarkonium dissociation in the presence of non-extensivity is traced back to how the non-extensive effects affect gluon self-energy.

Due to the highly explosive nature of created matter in heavy-ion collisions, most studies of in-medium heavy quarkonia need to extend to more complex and realistic scenarios. Specifically, the momentum anisotropy due to rapid longitudinal expansion in early stage of heavy-ion collisions and the bulk viscous nature of fluid are embedded in the distribution function of thermal partons [31–33], to influence heavy quark potential [34–40]. On the other hand, there has been a widespread consensus that strong dynamical correlations, long-range color interactions, and microscopic memory effects can exist and cannot be neglected in high-energy collisions. Non-extensive statistical mechanism, first proposed by Tsallis [41], provides an essential theoretical framework to deal with these physical phenomena. In non-extensive statistics, a (real) non-extensive parameter q is introduced to incorporate the intrinsic fluctuating ambiance and measure the degree of non-extensivity in the system [42]. In recent years, the use of non-extensive statistics has gained prominence in high-energy physics. A successful application is the accurate fitting of transverse momentum (p_T) spectra of final state particles in high-energy collision experiments [42–53]. The measured p_T spectra of identified particles deviate significantly from the exponential form predicted by $q = 1$ (Boltzmann–Gibbs distribution) in the high p_T region; instead, they exhibit a power-law tail, which is better described by the non-extensive distribution at both low and high p_T. There have also been a considerable variety of issues in exploring the possible non-extensive effects on hydrodynamics [54–56], thermodynamics [57–59], transport coefficients [60,61], and the structure of phase transition [62,63], electromagnetic responses of the QGP [64] and so on. Given that the heavy quarkonia are sensitive to the nature of the medium around them, it is of great significance to study how the non-extensive effects of the medium are imprinted on the properties of heavy quarkonia.

The goal of this work is to study the sensitivity of heavy quarkonia to the non-extensive feature of QGP medium, starting from the non-extensive bare propagators for quarks and gluons in real-time quantum field theory. We revisit the retarded, advanced, and symmetric gluon self-energies and corresponding resummed propagators in the presence of non-extensivity using HTL resummation. Following the methodology in Refs. [65,66], we compute the color dielectric permittivity of the medium through non-extensive modified resummed gluon propagators, which is then used to obtain the in-medium heavy quark potential. We investigate the responses of the real and imaginary parts of heavy quark potential to non-extensive effects. Then, we use the real part of the modified potential in Schrödinger equation to derive the binding energies of heavy quarkonia, whereas the imaginary part is used to evaluate decay widths of heavy quarkonia. The melting temperature of heavy quarkonia in the non-extensive medium is also estimated.

The article is organized as follows. In Sect. 2, we provide the basic formalism, including the non-extensive distribution functions of quarks and gluons, the non-extensive real-time bare propagators of quarks and gluons, and the general expressions of HTL resummed gluon propagators to order $(q-1)$ in Keldysh presentation. In Sect. 3, we compute the non-extensive correction to the retarded, advanced, and symmetry (time-order) HTL gluon self-energies as well as the corresponding resummed gluon propagators. In Sect. 4, we apply the modified HTL resummed gluon propagators to derive the dielectric permittivity of QGP, then compute the static complex heavy quark potential, analyzing the effect of non-extensivity on its real and imaginary parts. In the appendix, we present some derivations of the HTL gluon self-energy in non-extensive statistics within the real-time formalism of finite temperature field theory.

2 Formalism

2.1 Non-extensive distribution functions of (anti)quarks and gluons

Following Refs. [67,68], the non-extensive versions of single-particle distribution functions for (anti)quarks and gluons are respectively given as

$$f^{\pm}_{q,F}(E^f) = \frac{1}{[\exp_q(\beta(E^f \mp \mu_f))]^q + 1},$$
$$f_{q,B}(k) = \frac{1}{[\exp_q(\beta E)]^q - 1}, \quad (1)$$

where the subscript "F" relates to quarks (superscript "+") and antiquarks (subscript "−"), and the subscript "B''" relates to gluons. μ_f stands for the chemical potential of f-th flavor quark, in this work we takes $\mu_u = \mu_d = \mu_s = \mu$. $\beta = 1/T$ is the inverse temperature of the system. The energy of f-th flavor (anti)quarks is given by $E^f = \sqrt{k^2 + m_f^2}$ with $k \equiv |\boldsymbol{k}|$, wherein m_f is current mass of f-th quark. From now on, we will take the massless quark limit ($m_f = 0$), then arrive at $E^f = k$. In Eq. (1), $\exp_q(x)$ is the non-extensive exponential. For $x \le 0$ and $q > 1$, $\exp_q(x)$ is given as

$$\exp_q(x) = [1 + (q-1)x]^{1/(q-1)}. \quad (2)$$

In phenomenological investigations concerning the application of non-extensive distribution in high-energy physics, the non-extensive parameter q is considered a free input parameter, and its typical value is generally required to be greater than 1 [42,47,69–73]. In the limit of $q \to 1$, $\exp_q(x) = \exp(x)$, Eq. (1) recovers the standard Fermi–Dirac and Bose–Einstein distributions, which are respectively presented as

$$f_F^{0\pm}(k) = \frac{1}{\exp(\beta(k \mp \mu)) + 1}, \quad f_B^0(k) = \frac{1}{\exp(\beta k) - 1}. \tag{3}$$

In the literature of high-energy physics, the typical value of q is considered as never being far from 1. For instance, the value of q is in the range 1.1–1.2 by fitting the charged-particle transverse momentum spectra measured in p-p collisions at $\sqrt{s} = 0.9$ TeV [73]. Furthermore, the thermodynamic consistency of the non-extensive distribution requires $1 < q < 4/3$ [59]. Therefore, in this work we focus on small deviations from the standard statistics ($q = 1$), and it is reasonable to expand Eq. (1) to leading order of $(q-1)$, yielding the following result,

$$f_{q,F}^{\pm}(k) = f_F^{0\pm}(k) + \delta f_{q,F,(1)}^{\pm}(k). \tag{4}$$

The correction term $f_{q,F,(1)}^{\pm}$ is a measure of the degree of non-extensivity of the system, and its specific form is given as

$$f_{q,F,(1)}^{\pm}(k) = \frac{[(k \mp \mu)^2 - 2(k \mp \mu)T](q-1)}{2T^2} \times f_F^{0\pm}(k)(1 - f_F^{0\pm}(k)). \tag{5}$$

It is worth noting that the linear expansion holds when k/T is not too large. The HTL approximation, which is based on the assumption that soft momenta of the order $k \sim gT$ and hard ones of the order $k \sim T$ can be distinguished in the weak coupling limit $g \ll 1$, satisfies this condition. For gluons, their non-extensive correction term of the distribution function is given as

$$f_{q,B,(1)}(k) = \frac{(k^2 - 2kT)(q-1)}{2T^2} f_B^0(k)(1 + f_B^0(k)). \tag{6}$$

2.2 Real-time bare propagators in the non-extensive statistics

Following Refs. [68,74], the real-time bare propagator for massless quarks within the non-extensive statistic at a finite chemical potential is a 2×2 matrix, which takes the following form,

$$iS(K) = \not{K} \begin{pmatrix} \frac{i}{K^2+i\epsilon} & 0 \\ 0 & \frac{-i}{K^2-i\epsilon} \end{pmatrix} - \not{K} 2\pi\delta(K^2) \begin{pmatrix} N(k_0) & N(k_0) - \Theta(-k_0) \\ N(k_0) - \Theta(k_0) & N(k_0) \end{pmatrix}, \tag{7}$$

with four-momentum $K = (k_0, \boldsymbol{k})$ and $N(k_0) = \Theta(k_0) f_{q,F}^{+}(k_0) + \Theta(-k_0) f_{q,F}^{-}(-k_0)$. $\Theta(x)$ is the Heaviside step function. In the limit $q \to 1$, we return to the standard real-time bare quark propagator.

For the real-time bare propagator of gluons in the non-extensive statistics, its 2×2 matrix can be formulated as

$$iG(K) = \begin{pmatrix} \frac{i}{K^2+i\epsilon} & 0 \\ 0 & \frac{-i}{K^2-i\epsilon} \end{pmatrix} + 2\pi\delta(K^2) \begin{pmatrix} f_{q,B}(k_0) & f_{q,B}(k_0) + \Theta(-k_0) \\ f_{q,B}(k_0) + \Theta(k_0) & f_{q,B}(k_0) \end{pmatrix}. \tag{8}$$

The four components of the real-time bare propagator are not independent, which satisfy $D_{11} - D_{12} - D_{21} + D_{22} = 0$, where D_{ij} stands for S_{ij} or G_{ij}. It is more useful to write the bare propagators in terms of three independent components, i.e., the retarded (R), advanced (A), and symmetric (F) components, in Keldysh representation [75,76]. Accordingly, one gets

$$S_{R/A}(K) = S_{11}(K) - S_{12/21}(K) = \frac{\not{K}}{K^2 \pm i\,\mathrm{sgn}(k_0)\epsilon}, \tag{9}$$

$$S_F(K) = S_{11}(K) + S_{22}(K) = -2\pi i \not{K}[1 - 2N(k_0)]\delta(K^2), \tag{10}$$

where "±" in Eq. (9) represent retarded propagator and advanced propagator, respectively. Only the symmetric component of the bare propagator depends on the temperature, chemical potential, as well as non-extensive parameter. For the bare gluon propagator, three independent components in Keldysh representation take the following forms:

$$G_{R/A}(K) = G_{11}(K) - G_{12/21}(K) = \frac{1}{K^2 \pm i\,\mathrm{sgn}(k_0)\epsilon}, \tag{11}$$

$$G_F(K) = G_{11}(K) + G_{22}(K) = -2\pi i[1 + 2f_{q,B}(k_0)]\delta(K^2). \tag{12}$$

Using real-time formalism, the self-energy also becomes a 2×2 matrix and fulfills the relation $\Pi_{11}(K) + \Pi_{12}(K) + \Pi_{21}(K) + \Pi_{22}(K) = 0$. The three components of gluon self-energy in Keldysh representation are defined as [77]

$$\Pi_R(K) = \Pi_{11}(K) + \Pi_{12}(K), \tag{13}$$

$$\Pi_A(K) = \Pi_{11}(K) + \Pi_{21}(K), \tag{14}$$

$$\Pi_F(K) = \Pi_{11}(K) + \Pi_{22}(K). \quad (15)$$

2.3 Resummed gluon propagators in the presence of non-extensivity

Having the bare propagators and gluon self-energies, one can compute the resummed gluon propagator, which describes the propagation of a collective plasma mode. Here, we restrict ourselves to the system in Coulomb gauge,[1] where only the temporal components of the self-energies and bare or resummed propagators, such as Π_R^{00} and G_R^{00}, are considered. In the following, we will omit the superscript "00" of temporal components for simplification unless otherwise specified. Similar to the case in thermal field theory within the extensive quantum statistics, the resummed retarded/advanced gluon propagator within non-extensive statistics in the Coulomb gauge can also be determined from the following Dyson-Schwinger equation,

$$G^*_{R/A}(K) = G_{R/A}(K) + G_{R/A}(K)\Pi_{R/A}(K)G^*_{R/A}(K), \quad (16)$$

where $G_{R/A}(K) = \frac{1}{k^2}$ is the temporal component of bare retarded/advanced propagator. We use the superscript "$*$" here to label a resummed propagator. The resummed symmetric gluon propagator satisfies the following Dyson-Schwinger equation,

$$\begin{aligned} G^*_F(K) = &G_F(K) + G_R(K)\Pi_R(K)G^*_F(K) \\ &+ G_F(K)\Pi_A(K)G^*_A(K) \\ &+ G_R(K)\Pi_F(K)G^*_A(K). \end{aligned} \quad (17)$$

Using identity for bare symmetric gluon propagators in non-extensive statistics, $G_F(K) = (1 + 2f_{q,B}(k_0))\mathrm{sgn}(k_0)(G_R(K) - G_A(K))$, the solution to Eq. (17) takes the following form:

$$\begin{aligned} G^*_F(K) = &(1 + 2f_{q,B}(k_0))\mathrm{sgn}(k_0)(G^*_R(K) - G^*_A(K)) \\ &+ G^*_R(K)[\Pi_F(K) - \left(1 + 2f_{q,B}(k_0)\right)\mathrm{sgn}(k_0) \\ &\times \left(\Pi_R(K) - \Pi_A(K)\right)]G^*_A. \end{aligned} \quad (18)$$

In the presence of small non-extensivity, we only consider the contributions at leading order in $(q - 1)$. Consequently, the temporal components of the resummed gluon propagator and gluon self energies can be expanded as:

$$G^*_{R/A/F}(K) \approx G^*_{R/A/F,(0)}(K) + G^*_{R/A/F,(1)}(K), \quad (19)$$

$$\Pi_{R/A/F}(K) \approx \Pi_{R/A/F,(0)}(K) + \Pi_{R/A/F,(1)}(K). \quad (20)$$

The temporal component of resummed retarded/advanced/symmetric propagator to the order of $(q - 1)^0$, denoted as $G^*_{R/A/F,(0)}(K)$, which satisfies the relation, i.e., $G^*_{R/A,(0)}(K) = G_{R/A}(K)+G_{R/A}(K)\Pi_{R/A,(0)}(K)G^*_{R/A,(0)}(K)$. For the linear term of order $(q - 1)$ in Eq. (19), its expression is presented as,

$$\begin{aligned} G^*_{R/A,(1)}(K) = &G_{R/A}(K)\Pi_{R/A,(1)}(K)G^*_{R/A,(0)}(K) \\ &+ G_{R/A}(K)\Pi_{R/A,(0)}(K)G^*_{R/A,(1)}(K). \end{aligned} \quad (21)$$

Finally, we can get

$$G^*_{R/A,(0)}(K) = \frac{1}{G^{-1}_{R/A}(K) - \Pi_{R/A,(0)}(K)}, \quad (22)$$

$$G^*_{R/A,(1)}(K) = \frac{\Pi_{R/A,(1)}}{\left(G^{-1}_{R/A}(K) - \Pi_{R/A,(0)}(K)\right)^2}. \quad (23)$$

In the absence of non-extensivity, the term in the square brackets of Eq. (18) vanishes as a consequence of the Kubo-Martin-Schwinger boundary condition:

$$G^*_{F,(0)}(K) = (1 + 2f^0_B(k_0))\mathrm{sgn}(k_0)[G^*_{R,(0)}(K) - G^*_{A,(0)}(K)], \quad (24)$$

which is free of possible pinch problems and reflects the dissipation-fluctuation theorem. The non-extensive correction term of the temporal component of the resummed symmetric gluon propagator at the leading order in $(q - 1)$ is given by

$$\begin{aligned} &G^*_{F,(1)}(K) \\ &= (1 + 2f^0_B(k_0))\mathrm{sgn}(k_0)\left[G^*_{R,(1)}(K) - G^*_{A,(1)}(K)\right] \\ &\quad + 2f_{q,B,(1)}(k_0)\mathrm{sgn}(k_0)\left[G^*_{R,(0)}(K) - G^*_{A,(0)}(K)\right] \\ &\quad + G^*_{R,(0)}(K)\Big\{\Pi_{F,(1)}(K) \\ &\quad - (1 + 2f^0_B(k_0))\mathrm{sgn}(k_0)\left[\Pi_{R,(1)}(K) - \Pi_{A,(1)}(K)\right] \\ &\quad - 2f_{q,B,(1)}(k_0)\mathrm{sgn}(k_0) \\ &\quad \times \left[\Pi_{R,(0)}(K) - \Pi_{A,(0)}(K)\right]\Big\}G^*_{A,(0)}(K). \end{aligned} \quad (25)$$

To obtain the definite expressions of these resummed gluon propagators, we need to first compute the HTL gluon self-energy, which will be addressed in detail in the following section.

[1] Throughout this paper, we use the Coulomb gauge, which is convenient for later applications. Since the final results for physical quantities are gauge-independent using the HTL resummation, we can choose any gauge.

3 Non-extensive correction to gluon self-energy and resummed gluon propagator

In the HTL approximation, the one-loop contributions from $N_f = 3$ quarks and $2N_c = 6$ gluons to the temporal component of the retarded gluon self-energy, denoted as Π_R, within the framework of non-extensive statistics are given as (for a detailed derivation, see Appendix A),

$$\Pi_R(Q) = \frac{g^2}{(2\pi)^3}\int kdk\frac{d\Omega_k}{2}\Big(N_f f^+_{q,F}(k) + N_f f^-_{q,F}(k) + 2N_c f_{q,B}(k)\Big)\left[\frac{1-x^2}{[x+(\omega+i\epsilon)/\widetilde{q}]^2} + \frac{1-x^2}{[-x+(\omega+i\epsilon)/\widetilde{q}]^2}\right], \tag{26}$$

where $Q = (\omega, \widetilde{\boldsymbol{q}})$ denotes the external four-momentum in the one-loop diagram and is a soft scale. The differential solid angle is given by $d\Omega_k = \sin\theta d\theta d\phi = dx d\phi$, where $x = \boldsymbol{k}\cdot\widetilde{\boldsymbol{q}}/(k\widetilde{q})$ with $\widetilde{q} \equiv |\widetilde{\boldsymbol{q}}|$. The variable x ranges from -1 to 1. As q approaches 1, $f^{\pm}_{q,F}(k)$ and $f_{q,B}(k)$ recover to $f^{0\pm}_F(k)$ and $f^0_B(k)$, respectively. If the distributions are angular-independent, the square bracket term in Eq. (26) after the integration over Ω_k arrives at

$$\int d\Omega_k\left[\frac{1-x^2}{[x+(\omega+i\epsilon)/\widetilde{q}]^2} + \frac{1-x^2}{[-x+(\omega+i\epsilon)/\widetilde{q}]^2}\right] = 16\pi\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right). \tag{27}$$

Subsequently, to the order of $(q-1)^0$, Eq. (26) is computed as

$$\Pi_{R,(0)}(Q) = m^2_{D,R}\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right), \tag{28}$$

which is retarded gluon self-energy within the standard quantum statistics. In Eq. (28), $m_{D,R}$ presents the usual retarded Debye mass, and is given by

$$m^2_{D,R} = m^2_D = \frac{g^2T^2}{6}\left[N_f\left(1+\frac{3\alpha^2}{\pi^2}\right) + 2N_c\right], \tag{29}$$

where $\alpha = \mu/T$. In the space-like region where $\omega^2 < \widetilde{q}^2$, Eq. (28) has an imaginary part, and the bracket term has the following structure

$$\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right) = \frac{\omega}{2\widetilde{q}}\left[\ln\left|\frac{\omega+\widetilde{q}}{\omega-\widetilde{q}}\right| - i\pi\Theta(\widetilde{q}^2-\omega^2)\right] - 1. \tag{30}$$

In the presence of non-extensivity, specifically the small value of $(q-1)$, the temporal component of retarded gluon self-energy is modified as $\Pi_R(Q) = \Pi_{R,(0)}(Q) + \Pi_{R,(1)}(Q)$, where $\Pi_{R,(1)}(Q)$ represents the non-extensive correction to $\Pi_R(Q)$ in the leading order of $(q-1)$. Here, one-loop contributions from quarks and gluons to $\Pi_{R,(1)}(Q)$, are computed as follows:

$$\Pi_{R,(1)}(Q) = \frac{g^2}{\pi^2}\int kdk\Big(N_f f^+_{q,F,(1)}(k) + N_f f^-_{q,F,(1)}(k) + 2N_c f_{q,B,(1)}(k)\Big)\times\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right) \tag{31}$$

$$= m^2_{D,R,(1)}\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right), \tag{32}$$

where $m_{D,R,(1)}$ denotes the non-extensive correction term of the retarded Debye mass. By combining Eqs. (28) and (32), the temporal component of total retarded gluon self-energy, including non-extensive correction, can be expressed as

$$\Pi_R(Q) = \widetilde{m}^2_{D,R}\left(\frac{\omega}{2\widetilde{q}}\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon} - 1\right). \tag{33}$$

Here $\widetilde{m}^2_{D,R} = m^2_D + m^2_{D,R,(1)}$ represents the non-extensive modified retarded Debye mass, and the total correction term $m^2_{D,R,(1)}$ is written as

$$m^2_{D,R,(1)} = \frac{g^2T^2}{6}\frac{q-1}{2}\left[2N_c a^{\text{gluon}}_R + N_f\left(1+\frac{3\alpha^2}{\pi^2}\right)a^{\text{quark}}_R\right], \tag{34}$$

where the dimensionless quantities a^{quark}_R and a^{gluon}_R are respectively defined as

$$a^{\text{quark}}_R = \frac{2}{(q-1)}\frac{\sum_{b=\pm}\int kdk f^b_{q,F,(1)}(k)}{\sum\limits_{b=\pm}\int kdk f^{0b}_F(k)},\quad a^{\text{gluon}}_R = \frac{2}{(q-1)}\frac{\int kdk f_{q,B,(1)}(k)}{\int kdk f^0_B(k)}, \tag{35}$$

and their explicit forms are respectively given as

$$a^{\text{quark}}_R = -\frac{36}{\pi^2+3\alpha^2}\left[\text{Li}_3(-e^{\alpha}) + \text{Li}_3(-e^{-\alpha})\right] + \frac{24}{\pi^2+3\alpha^2}\left[\text{Li}_2(-e^{\alpha}) + \text{Li}_2(-e^{-\alpha})\right] + \frac{24}{\pi^2+3\alpha^2}\alpha\left[\text{Li}_2(-e^{\alpha}) - \text{Li}_2(-e^{-\alpha})\right] + \frac{6}{\pi^2+3\alpha^2}\alpha^2\left[\ln(1+e^{\alpha}) + \ln(1+e^{-\alpha})\right]$$

$$+\frac{12}{\pi^2+3\alpha^2}\alpha\left[\ln(1+e^{\alpha})-\ln(1+e^{-\alpha})\right], \quad (36)$$

$$a_{R/A}^{\text{gluon}}=\frac{36}{\pi^2}\zeta(3)-4, \quad (37)$$

with $\text{Li}_n(x)$ being the polylogarithm functions. Finally, in the absence of chemical potential, the non-extensive modified retarded Debye mass is given as

$$\widetilde{m}_{D,R}^2=\frac{g^2T^2}{6}2N_c\left[1+(q-1)\left(\frac{18\zeta(3)}{\pi^2}-2\right)\right] + \frac{g^2T^2}{6}N_f\left[1+(q-1)\left(\frac{27\zeta(3)}{\pi^2}-2\right)\right]. \quad (38)$$

In the HTL approximation, the one-loop contributions from N_f quarks and $2N_c$ gluons to the temporal component of the symmetric gluon self-energy, denoted as Π_F, in non-extensive statistics are computed as (for a detailed derivation, see Appendix A),

$$\Pi_F(Q)=-ig^2\int\frac{dkk^2}{2\pi}\left[\sum_{b=\pm}N_f f_{q,F}^b(k)(1-f_{q,F}^b(k)) + 2N_c f_{q,B}(k)(1+f_{q,B}(k))\right]\frac{2}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2). \quad (39)$$

Upon substitution of the distributions in Eq. (39) with $f_F^{0\pm}(k)$ and $f_B^0(k)$, Eq. (39) to order $(q-1)^0$ is expressed as

$$\Pi_{F,(0)}(Q)=-2\pi i m_{D,F}^2\frac{T}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2), \quad (40)$$

where $m_{D,F}=m_D$ is the usual symmetric Debye mass in the standard quantum statistics. Similar to the retarded gluon self-energy, considering the small non-extensivity, the temporal component of total symmetric gluon self-energy is expressed as $\Pi_F(Q)=\Pi_{F,(0)}(Q)+\Pi_{F,(1)}(Q)$, where $\Pi_{F,(1)}(Q)$ represents the non-extensive correction to $\Pi_F(Q)$ in the leading order of $(q-1)$. The one-loop contribution from quarks and gluons to $\Pi_{F,(1)}(Q)$, is computed as follows:

$$\begin{aligned}\Pi_{F,(1)}(Q) &= -ig^2\int\frac{dkk^2}{2\pi}\left[\sum_{b=\pm}N_f f_{q,F,(1)}^b(k)(1-2f_F^{0b}(k)) + 2N_c f_{q,B,(1)}(k)(1+2f_B^0(k))\right]\frac{2}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2) \quad (41)\\ &= -2\pi i m_{D,F,(1)}^2\frac{T}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2), \quad (42)\end{aligned}$$

where $m_{D,F,(1)}$ denotes the non-extensive correction term of the symmetric Debye mass. Finally, the temporal component of total symmetric gluon self-energy, including the effect of non-extensivity, is written as

$$\Pi_F(Q)=-2\pi i\widetilde{m}_{D,F}^2\frac{T}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2). \quad (43)$$

Here, $\widetilde{m}_{D,F}^2=m_D^2+m_{D,F,(1)}^2$ represents the non-extensive modified symmetric Debye mass and the associated non-extensive correction term to order $(q-1)^1$ is given by

$$m_{D,F,(1)}^2=\frac{g^2T^2}{6}\frac{q-1}{2}\times\left[2N_c a_F^{\text{gluon}}+N_f\left(1+\frac{3\alpha^2}{3}\right)a_F^{\text{quark}}\right], \quad (44)$$

where the dimensionless quantities a_F^{quark} and a_F^{gluon} are respectively defined as

$$a_F^{\text{quark}}=\frac{2}{q-1}\frac{\sum_{b=\pm}\int dk\,k^2 f_{q,F,(1)}^b(k)(1-2f_F^{0b}(k))}{\sum_{b=\pm}\int dk\,k^2 f_F^{0b}(k)(1-f_F^{0b}(k))}, \quad (45)$$

$$a_F^{\text{gluon}}=\frac{2}{q-1}\frac{\int dk\,k^2 f_{q,B,(1)}(k)(1+2f_B^0(k))}{\int dk\,k^2 f_B^0(k)(1+f_B^0(k))}, \quad (46)$$

and their explicit forms are

$$\begin{aligned}a_F^{\text{quark}} =& -\frac{72}{\pi^2+3\alpha^2}\left[\text{Li}_3(-e^{-\alpha})+\text{Li}_3(-e^{\alpha})\right]\\ &+\frac{36}{\pi^2+3\alpha^2}\left[\text{Li}_2(-e^{-\alpha})+\text{Li}_2(-e^{\alpha})\right]\\ &-\frac{36\alpha}{\pi^2+3\alpha^2}\left[\text{Li}_2(-e^{-\alpha})-\text{Li}_2(-e^{\alpha})\right]\\ &+\frac{6\alpha^2}{\pi^2+3\alpha^2}\left[\ln(1+e^{\alpha})+\ln(1+e^{-\alpha})\right]\\ &+\frac{12\alpha}{\pi^2+3\alpha^2}\left[\ln(1+e^{\alpha})-\ln(1+e^{-\alpha})\right], \quad (47)\end{aligned}$$

$$a_F^{\text{gluon}}=\frac{72}{\pi^2}\zeta(3)-6. \quad (48)$$

Finally, the symmetric Debye mass, including the small non-extensive correction, is expressed as

$$\widetilde{m}_{D,F}^2=\frac{g^2T^2}{6}\left[2N_c\left[1+\left(\frac{54\zeta(3)}{\pi^2}-3\right)(q-1)\right] + N_f\left[1+\left(\frac{36\zeta(3)}{\pi^2}-3\right)(q-1)\right]\right]. \quad (49)$$

Inserting Eq. (28) into Eq. (22), the temporal component of HTL resummed retarded gluon propagator to the order $(q-1)^0$, i.e., $G_{R,(0)}^*$, can be determined, and in the static limit $(\omega\to 0)$, it is written as

$$\lim_{\omega\to 0} G^*_{R,(0)}(Q) = \frac{1}{\widetilde{q}^2+m_D^2} - i\frac{m_D^2}{2\widetilde{q}}\frac{\pi\omega}{(\widetilde{q}^2+m_D^2)^2}, \quad (50)$$

which follows from $\lim_{\omega\to 0}\left[\ln\frac{\omega+\widetilde{q}+i\epsilon}{\omega-\widetilde{q}+i\epsilon}\right] = -i\pi$. By inserting Eqs. (28) and (32) to Eq. (23), the non-extensive correction term of the temporal component of resummed retarded gluon propagator in the order of $(q-1)^1$, denoted as $G^*_{R,(1)}$, within the static limit, is obtained as

$$\lim_{\omega\to 0} G^*_{R,(1)}(Q) = -\frac{m^2_{D,R,(1)}}{(\widetilde{q}^2+m_D^2)^2} - i\frac{m^2_{D,R,(1)}}{2\widetilde{q}}\frac{(\widetilde{q}^2-m_D^2)\omega}{(\widetilde{q}^2+m_D^2)^3}. \quad (51)$$

We explicitly determine the temporal component of the resummed symmetric gluon propagator, G^*_F. To facilitate calculation, we will use the following identity:

$$\lim_{\omega\to 0}(1+2f_{q,B,(0)}(\omega))\mathrm{sgn}(\omega) = \frac{2T}{\omega} + \cdots. \quad (52)$$

Utilizing the set of equations ((28), (32), (42), and (52)) into Eqs. (24) and (25), the expressions of G^*_F up to order $(q-1)^0$ and order $(q-1)^1$, in the static limit, are respectively derived as

$$\lim_{\omega\to 0} G^*_{F,(0)}(Q) = -i\frac{2\pi T m_D^2}{\widetilde{q}(\widetilde{q}^2+m_D^2)^2}, \quad (53)$$

$$\lim_{\omega\to 0} G^*_{F,(1)}(Q) = -i\frac{m^2_{D,R,(1)}}{2\widetilde{q}}\frac{4\pi T(\widetilde{q}^2-m_D^2)}{(\widetilde{q}^2+m_D^2)^3} - i\frac{2\pi T[m^2_{D,F,(1)}-m^2_{D,R,(1)}]}{\widetilde{q}(\widetilde{q}^2+m_D^2)^2}. \quad (54)$$

4 Dielectric permittivity and heavy quark potential in a non-extensive QGP

Based on the modified HTL resummed gluon propagator in the static limit, we further investigate the in-medium heavy quark potential in the presence of non-extensivity. In the QGP, the heavy quark potential can be obtained by modifying the vacuum potential with the dielectric permittivity [65,66,78].

4.1 Dielectric permittivity

As described in Refs. [65,66,78], the dielectric permittivity (ε), which encodes in-medium effects such as temperature, magnetic field, and non-extensive effects, is obtained by using the temporal component of the 11-part of the HTL resummed gluon propagator in the static limit. It is expressed as

$$\begin{aligned}\varepsilon^{-1}(\widetilde{q}) &= \lim_{\omega\to 0}\widetilde{q}^2 G^*_{11}(Q)\\ &= \lim_{\omega\to 0}\widetilde{q}^2\left(G^*_R(Q)+G^*_A(Q)+G^*_F(Q)\right)/2, \quad (55)\end{aligned}$$

where $(G^*_R(Q)+G^*_A(Q))/2 = \mathrm{Re}\,G^*_R(Q)$. By inserting Eqs. (50), (51), (53), and (54) into Eq. (55), the dielectric permittivity in the presence of small non-extensivity is determined as:

$$\begin{aligned}\varepsilon^{-1}(\widetilde{q}) = &\frac{\widetilde{q}^2}{\widetilde{q}^2+m_D^2} - \frac{\widetilde{q}^2 m^2_{D,R,(1)}}{(\widetilde{q}^2+m_D^2)^2}\\ &- i\frac{\pi T\widetilde{q}(m_D)^2}{(\widetilde{q}^2+m_D^2)^2} - i\frac{\pi T\widetilde{q}(\widetilde{q}^2-m_D^2)}{(\widetilde{q}^2+m_D^2)^3}(m_{D,R,(1)})^2\\ &- i\frac{\pi T\widetilde{q}\left[(m_{D,F,(1)})^2-(m_{D,R,(1)})^2\right]}{(\widetilde{q}^2+m_D^2)^2}. \quad (56)\end{aligned}$$

As q approaches 1, both $m_{D,R,(1)}$ and $m_{D,F,(1)}$ vanish, the standard equilibrium form of $\varepsilon^{-1}(\widetilde{q})$ is reproduced.

4.2 Real part of in-medium heavy quark potential

Following the approach proposed in [65,66], the heavy quark potential in the non-extensive QGP can be obtained through the convolution of the Cornell potential and non-extensive modified dielectric permittivity,

$$V(\widetilde{q}) = V_{\text{Cornell}}(\widetilde{q})\varepsilon^{-1}(\widetilde{q}). \quad (57)$$

Here, the Cornell potential is given as [27,28]:

$$V_{\text{Cornell}}(r) = -C_F\alpha_s/r + \sigma r, \quad (58)$$

where $r \equiv |\boldsymbol{r}|$ denotes the quark-antiquark separation distance and $C_F = (N_c^2-1)/2N_c$. Here, α_s and σ are phenomenological parameters. α_s represents the strong coupling constant, σ is the string tension which is determined to reproduce the vacuum quarkonium property [79]. The first term corresponds to the Coulombic part, reflecting the asymptotic freedom at small separation distances, whereas the second term represents the string-like part, responsible for color confinement at large separation distances. Accordingly, the Fourier transform of the Cornell potential in the momentum space, $V_{\text{Cornell}}(\widetilde{q})$, is given as [78]

$$V_{\text{Cornell}}(\widetilde{q}) = -\sqrt{(2/\pi)}\frac{C_F\alpha_s}{\widetilde{q}^2} - \frac{4\sigma}{\sqrt{2\pi}\widetilde{q}^4}. \quad (59)$$

Through the Fourier transform, Eq. (57) is transformed into real coordinate space, which is expressed as

$$V(\boldsymbol{r}) = \int\frac{d^3\widetilde{\boldsymbol{q}}}{(2\pi)^{3/2}}(e^{i\widetilde{\boldsymbol{q}}\cdot\boldsymbol{r}}-1)V_{\text{Cornell}}(\widetilde{q})\varepsilon^{-1}(\widetilde{q}). \quad (60)$$

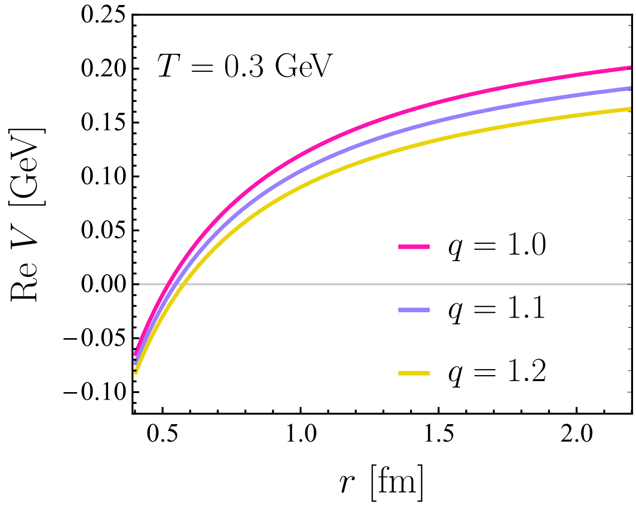

Fig. 1 The real part of heavy quark potential, Re V, as a function of quark-antiquark separation distance r at different values of non-extensive parameter q for $T = 0.3$ GeV and $\mu = 0$ GeV

In the present work, the string tension is chosen as $\sigma = 0.18\ \text{GeV}^2$ [80]. For the coupling constant, we use the one-loop result [81], $\alpha_s(\Lambda^2) = \frac{12\pi}{(11N_c - 2N_f)\ln\left(\Lambda^2/\Lambda^2_{\overline{\text{MS}}}\right)}$, where the renormalization scale Λ is taken to be $2\pi\sqrt{T^2 + \mu^2/\pi^2}$ for quarks and $2\pi T$ for gluons. The parameter $\Lambda_{\overline{\text{MS}}}$ for $N_f = N_c = 3$ is determined as 176 MeV requiring $\alpha_s(1.5\ \text{GeV}) = 0.326$ is satisfied to match the lattice data [82].

By inserting the real part of Eq. (56) into Eq. (60), we compute the real part of the potential, denoted as Re V, which to order $(q-1)^0$ is written as follows:

$$\text{Re}\,V_{(0)}(r,T) = -C_F\alpha_s\, m_D\left(\frac{e^{-m_D\, r}}{m_D\, r} + 1\right) + \frac{2\sigma}{m_D}\left(\frac{e^{-m_D\, r} - 1}{m_D\, r} + 1\right). \quad (61)$$

The non-extensive correction term of Re V to order $(q-1)^1$ is obtained as

$$\text{Re}\,V_{(1)}(r,q,T) = C_F\alpha_s\frac{m^2_{D,R,(1)}}{2m_D}\left(e^{-m_D r} - 1\right) + \frac{\sigma m^2_{D,R,(1)}}{m_D^3}\left(\frac{2-(2+m_D r)e^{-m_D r}}{m_D r} - 1\right). \quad (62)$$

The first and second terms on the right side of Eqs. (61)–(62) are the HTL and string-like parts of the potential, which are related to the short-range perturbative Yukawa and long-range non-perturbative string-like interactions, respectively.

In Fig. 1, we display the real part of the heavy quark potential, Re V, for different values of non-extensive parameter q with varying quark-antiquark separation distances r. As aforementioned, the non-extensive parameter has the typical value in the range of $1 \le q \le 1.2$ in high-energy physics applications of non-extensive statistics. Furthermore, the validity of Eqs. (5) and (6) requires that the value of q not be far from 1. Therefore, we have limited our studies to $1 \le q \le 1.2$. The non-extensive correction alters Re V by modifying the Debye masses, transforming m_D into $\widetilde{m}_{D,R}$. In Fig. 1, we observe that Re V increases rapidly at first and then gradually flattens out as r increases. In the absence of non-extensivity ($q = 1$), Re V becomes flattened (screened). Furthermore, introducing non-extensivity also leads to a shorter Debye screening length (or equivalently, a larger Debye screening mass), which results in Re V flattening with increasing q.

4.3 Imaginary part of in-medium heavy quark potential

Next, we study the imaginary part of the in-medium heavy quark potential, denoted as Im V, which relates to the inelastic scattering of the light constituents of the medium with heavy quarkonium via exchanged gluons (Landau damping phenomenon) [29].

By inserting the third term in the right-hand side of Eqs. (56) to (60), Im V to order $(q-1)^0$ is computed as

$$\text{Im}\,V_{(0)}(r,T) = -C_F\alpha_s T m_D^2\phi_2(m_D r) - \frac{2\sigma T}{m_D^2}\chi_2(m_D r). \quad (63)$$

Then, inserting the fourth and fifth terms in the right-hand side of Eq. (56) to Eq. (60), the non-extensive correction term of Im V is obtained as

$$\begin{aligned}\text{Im}\,V_{(1)}&(r,q,T)\\ &= -C_F\alpha_s T\Bigg[\frac{(m_{D,R,(1)})^2}{m_D^2}\Big(\psi(m_D r) - \phi_2(m_D r) - \phi_3(m_D r)\Big)\\ &\quad + \frac{(m_{D,F,(1)})^2}{m_D^2}\phi_2(m_D r)\Bigg]\\ &\quad - \frac{2\sigma T}{m_D^2}\Bigg[\frac{(m_{D,F,(1)})^2}{m_D^2}\chi_2(m_D r)\\ &\quad + \frac{(m_{D,R,(1)})^2}{m_D^2}\Big(\phi_3(m_D r) - \chi_2(m_D r) - \chi_3(m_D r)\Big)\Bigg]. \quad (64)\end{aligned}$$

Here, the functions $\phi_n(x)$ is defined by $\phi_n(x) \equiv 2\int_0^\infty dz \frac{z}{(z^2+1)^n}\left[1 - \frac{\sin(xz)}{xz}\right]$, and the function $\chi_n(x)$ is defined by $\chi_n(x) \equiv 2\int_0^\infty \frac{dz}{z(z^2+1)^n}\left[1 - \frac{\sin(xz)}{xz}\right]$, as well as $\psi(x) \equiv 2\int_0^\infty \frac{z^3}{(z+1)^3}\left[1 - \frac{\sin(xz)}{xz}\right]$.

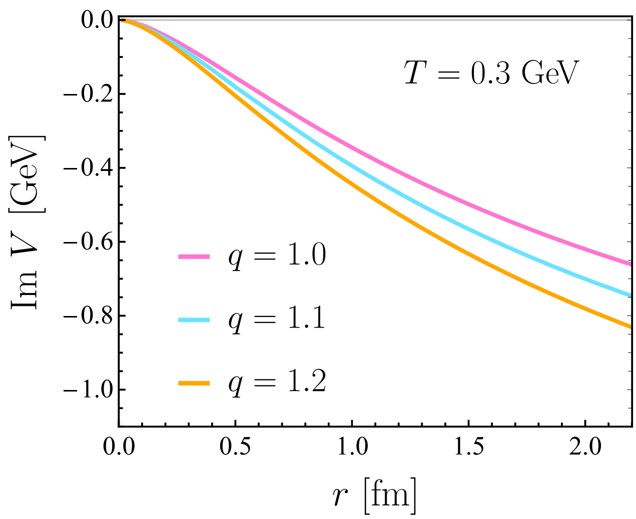

Fig. 2 The imaginary part of heavy quark potential, Im V, as a function of quark-antiquark separation distance r at different values of non-extensive parameter q for $T = 0.3$ GeV and $\mu = 0$ GeV

In Fig. 2, we show the dependence of the imaginary part of in-medium heavy quark potential, ImV, on the quark-antiquark separation distance r at different values of non-extensive parameter q. It is clear that the magnitudes of Im V exhibit an increasing trend with r and q.

5 Non-extensive correction to in-medium properties of heavy quarkonia

Based on the non-extensive modified heavy quark potential, we further investigate the effects of non-extensivity on in-medium static properties of heavy quarkonium states including binding energy ($E_{\rm bin}$), decay width (Γ), as well as melting temperature ($T_{\rm melt}$). To obtain the binding energies of heavy quarkonia, one needs to solve Schrödinger equation for the radial wave function $\psi(r)$ with the real part of in-medium heavy quark potential [38],

$$-\frac{1}{2m_Q}\left(\psi''(r) + \frac{2}{r}\psi'(r) - \frac{l(l+1)}{r^2}\psi(r)\right) + \text{Re}\,V(r)\psi(r) = \epsilon_{ln}\psi(r), \tag{65}$$

where ϵ_{nl} is eigenvalue of heavy quarkonia with principal quantum number n and azimuthal quantum number l, and $m_Q = m_{HQ}/2$ denotes the reduced mass of quarkonium system with m_{HQ} being heavy quark mass. In the limit $r \gg 1/m_D$, the real part of the potential, apart from r-independent terms, can be reduced as

$$\text{Re}\,V \simeq -\frac{2(m_D^2 - m_{D,R,(1)}^2)\sigma}{m_D^4 r}. \tag{66}$$

The above form is the Coulombic potential as encountered in the hydrogen atom problem, but with the fine structure constant $\frac{2(m_D^2 - m_{D,R,(1)}^2)\sigma}{m_D^4}$. We are only interested in the 1S ground state ($n = 1,\ l = 0$) of charmonium (J/Ψ) and bottomonium (Υ), the associated radial wave function is given as $\psi(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$. The medium effects are encoded in the Bohr radius (a) of the heavy quarkonium system.

Following Refs. [38,80], the binding energies of heavy quarkonia are determined by the difference between the asymptotic value of the real part of the potential and the associated eigenvalue,

$$E_{\rm bin} = \text{Re}\,V(r \to \infty) - \epsilon_{nl}. \tag{67}$$

Finally, the binding energies of heavy quarkonium ground states are derived as

$$E_{\rm bin} = (m_D^2 - m_{D,R,(1)}^2)^2 \sigma^2 m_{HQ}/m_D^8. \tag{68}$$

Taking $q \to 1$, Eq. (68) recovers to the standard result presented in Refs. [65,83]. In this work, the charm and bottom masses are taken as $m_c = 1.275$ GeV and $m_b = 4.66$ GeV, respectively.

In a first-order perturbative theory, by using the obtained non-extensive modified imaginary part of the potential and folding with the radial wave functions of heavy quarkonia, the decay widths (Γ) of heavy quarkonia can be obtained [78], which are computed as

$$\Gamma = -\frac{\int d^3\boldsymbol{r}(\psi(r))^2 \text{Im}\,V(r)}{\int d^3\boldsymbol{r}|\psi(r)|^2}. \tag{69}$$

Based on the binding energies and decay widths of heavy quarkonia, we can further estimate the melting temperatures ($T_{\rm melt}$) of heavy quarkonia, which are determined by a common criterion that the binding energy coincides with the decay width for a quarkonium state, that is, $\Gamma(T_{\rm melt}) = E_{\rm bin}(T_{\rm melt})$ [84].

Next, we discuss how the non-extensivity of medium influences the binding energies and decay widths of heavy quarkonia. In Fig. 3, we depict the binding energies (solid lines) and decay widths (dashed lines) for J/Ψ (left panel) and Υ (right panel) as functions of temperature. We observe that as temperature increases, the binding energies of heavy quarkonia rapidly decrease, signifying that the binding between the quark and antiquark becomes increasingly incompact. Concurrently, the decay widths of heavy quarkonia gradually increase with the rising temperature. We note that the non-extensive correction has a dual effect: it further suppresses the binding energies of heavy quarkonia and elevates their decay widths. This results in the intersection point of these quantities (intersection points of solid and dashed lines of the same color in Fig. 3) moving towards lower tempera-

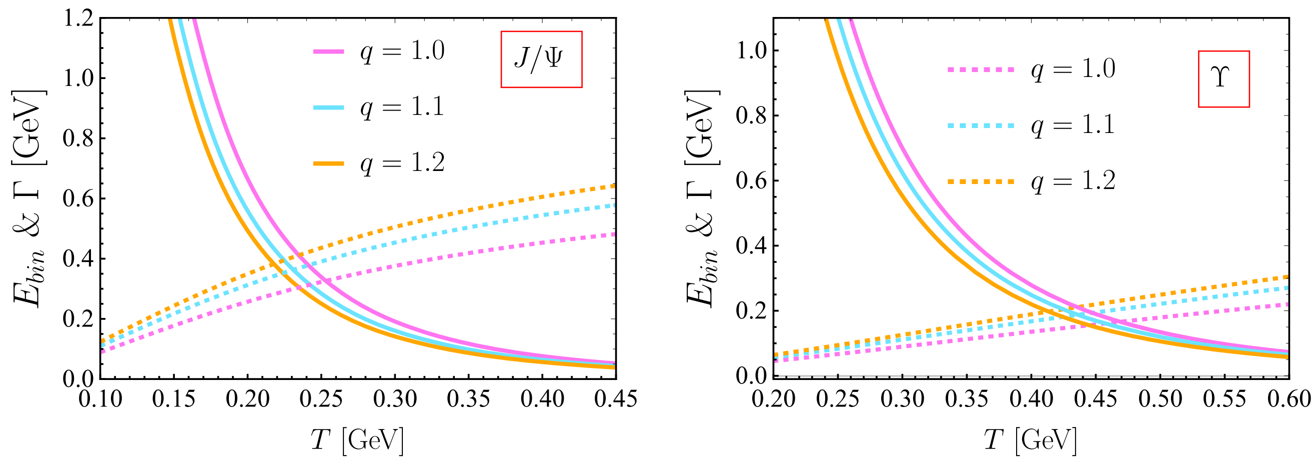

Fig. 3 The binding energies E_{bin} (solid lines) and decay widths Γ (dashed lines) of charmonium J/Ψ (left panel) and bottomonium Υ (right panel) as a function of temperature at different values of the non-extensive parameter q. In all plots, a fixed temperature of $T = 0.3$ GeV and zero chemical potential ($\mu = 0$ GeV) are chosen

Table 1 Melting temperatures of charmonium (J/Ψ) and bottomonium (Υ) states at different values of q. The results at $q = 1$ are compared with lattice QCD result [85]

States	$T_{\text{melt}}^{q=1}$	$T_{\text{melt}}^{q=1.1}$	$T_{\text{melt}}^{q=1.2}$	Lattice QCD
J/Ψ	0.254	0.232	0.219	0.267
Υ	0.468	0.431	0.411	0.440

tures, implying that the non-extensive effects lower the melting temperatures of heavy quarkonia, leading to an earlier quarkonium dissociation.

The melting temperatures of both J/Ψ and Υ at different values of q are summarized in Table 1. In the absence of non-extensivity, our computed melting temperatures align reasonably well with lattice data.

6 Summary

Heavy quarkonia are useful in probing the nature of the medium around them by modifying their properties. In this work, we studied how the non-extensivity of the medium is imprinted on the properties of heavy quarkonia by calculating the in-medium heavy quark potential. We first revisited the gluon self-energy and resummed gluon propagator in a non-extensive medium using the HTL resummation in the real-time formalism. In the massless limit, we found that the non-extensivity of the medium leads to distinct shifts in the retarded/advanced and symmetric Debye masses. Specifically, in the leading order of $(q-1)$, the retarded/advanced Debye mass at zero chemical potential is modified from $\frac{g^2T^2}{6}(2N_c + N_f)$ to:

$$\frac{g^2T^2}{6}2N_c\left[1+(q-1)\left(\frac{18\zeta(3)}{\pi^2}-2\right)\right] + \frac{g^2T^2}{6}N_f\left[1+(q-1)\left(\frac{27\zeta(3)}{\pi^2}-2\right)\right],$$

whereas the symmetric Debye mass at zero chemical potential is modified from $\frac{g^2T^2}{6}(2N_c + N_f)$ to:

$$\frac{g^2T^2}{6}2N_c\left[1+(q-1)\left(\frac{36\zeta(3)}{\pi^2}-3\right)\right] + \frac{g^2T^2}{6}N_f\left[1+(q-1)\left(\frac{54\zeta(3)}{\pi^2}-3\right)\right].$$

Based on the non-extensive modified resummed gluon propagators, we derived the dielectric permittivity of the medium, which is then used to compute the heavy quark potential. We discussed how the potential is deformed in the presence of non-extensivity. We found that as the non-extensive parameter increases, the real part of the potential becomes flatter due to the enhanced color screening effect. On the other hand, the magnitude of the imaginary part of the potential increases as the quark-antiquark separation distance and the non-extensive parameter increase.

Using the obtained real part of the potential, we solved Schrödinger equation for radial wave function of the quarkonium state, which allows us to determine the binding energies of heavy quarkonia. The imaginary part of the potential was used to calculate the thermal decay widths of heavy quarkonia by folding with the probability density. Inheriting traits from in-medium heavy quark potential, the binding energies of J/Ψ and Υ decrease, whereas thermal decay widths of J/Ψ and Υ increase with rising temperature and the non-extensive parameter. Subsequently, we estimated the melting temperatures of J/Ψ and Υ. Our findings indicated that the increase in the non-extensivity of the medium lowers the melting temperature, leading to earlier heavy quarkonium dissociation.

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Data Availability Statement This manuscript has no associated data. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.]

Code Availability Statement This manuscript has no associated code/software. [Authors' comment: Code/Software sharing not applicable to this article as no code/software was generated or analysed during the current study.]

Appendix A: Gluon self-energy in the presence of non-extensivity using real-time formalism

We derive the gluon self-energy, mainly focusing on the one-loop contribution from quarks, within the HTL perturbative theory and non-extensive statistics, using real-time formalism. Utilizing Eqs. (7) and (13), as well as applying the Feynman rule, the one-loop contribution from quarks to the retarded gluon polarization tensor depicted in Fig. 4 can read as

$$\begin{aligned}\Pi_R^{\mu\nu,\mathrm{quark}}(Q)&\\ &= -iN_f g^2\int\frac{d^4K}{(2\pi)^4}\{\mathrm{Tr}[t_b\gamma^\mu S_{11}(K)t_a\gamma^\nu S_{11}(P)]\\ &\quad-\mathrm{Tr}[t_b\gamma^\mu S_{21}(K)t_a\gamma^\nu S_{12}(P)]\} \qquad (A.1)\\ &= -iN_f\frac{g^2}{4}\int\frac{d^4K}{(2\pi)^4}4(K^\mu P^\nu+K^\nu P^\mu-g^{\mu\nu}K\cdot P)\\ &\quad\times[\Delta_F(K)\Delta_R(P)+\Delta_A(K)\Delta_F(P)\\ &\quad+\Delta_R(K)\Delta_R(P)+\Delta_A(K)\Delta_A(P)], \qquad (A.2)\end{aligned}$$

where $P=K+Q$, $t_{a,b}$ are the generators of color group, and $\not{K}\Delta_{R/A/F}(K)=S_{R/A/F}(K)$. The minus sign in front of the second square bracket of Eq. (A.1) appears due to the vertex of type-2 field [86]. In deriving Eq. (A.2) from Eq. (A.1), we utilized the Eq. (9) and Eq. (10), and performed the trace over the gamma matrices as well as suppressed the color indices. It should be noted that $\Delta_R(K)\Delta_R(P)$ and $\Delta_A(K)\Delta_A(P)$ in the integrand of Eq. (A.2) are zero upon integration over k_0. Given our focus on the temporal component of the gluon self-energy, we set $\mu=\nu=0$ directly to obtain $(K^\mu P^\nu + K^\nu P^\mu - g^{\mu\nu}K\cdot P)=(k_0p_0+\boldsymbol{k}\cdot\boldsymbol{p})$. Consequently, the temporal component of $\Pi_R^{\mu\nu,\mathrm{quark}}(Q)$, denoted as $\Pi_R^{\mathrm{quark}}(Q)$, is expressed as

$$\begin{aligned}\Pi_R^{\mathrm{quark}}(Q)&\\ &= -iN_f g^2\int\frac{d^4K}{(2\pi)^4}(k_0p_0+\boldsymbol{k}\cdot\boldsymbol{p})\\ &\quad\times[\Delta_F(K)\Delta_R(P)+\Delta_A(K)\Delta_F(P)] \qquad (A.3)\\ &= -N_f g^2\int\frac{d^4K}{(2\pi)^3}(k_0p_0+\boldsymbol{k}\cdot\boldsymbol{p})\\ &\quad\times\Bigg[\frac{1-2\Theta(k_0)f^+_{q,F}(k_0)-2\Theta(-k_0)f^-_{q,F}(-k_0)}{P^2+i\,\mathrm{sgn}(p_0)\epsilon}\delta(K^2)\\ &\quad+\frac{1-2\Theta(p_0)f^+_{q,F}(p_0)-2\Theta(-p_0)f^-_{q,F}(-p_0)}{K^2-i\,\mathrm{sgn}(k_0)\epsilon}\delta(P^2)\Bigg] \qquad (A.4)\\ &= 2N_f g^2\int\frac{d^4K}{(2\pi)^3}\Bigg\{\Big[(\Theta(k_0)\left(f^+_{q,F}(k_0)+f^-_{q,F}(k_0)\right)\\ &\quad+\Theta(-k_0)\left(f^+_{q,F}(-k_0)+f^-_{q,F}(-k_0)\right)\Big]\delta(k_0^2-\boldsymbol{k}^2)\\ &\quad\times\frac{k_0(k_0+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}})}{(k_0+\omega)^2-(\boldsymbol{k}+\widetilde{\boldsymbol{q}})^2+i\mathrm{sgn}(k_0+\omega)\epsilon}\Bigg\} \qquad (A.5)\\ &= N_f g^2\int\frac{dkk^2d\Omega_k}{(2\pi)^3}\frac{1}{k}\left(f^+_{q,F}(k)+f^-_{q,F}(k)\right)\\ &\quad\times\Bigg[\frac{k(k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}})}{(k+\omega)^2-(\boldsymbol{k}+\widetilde{\boldsymbol{q}})^2+i\epsilon}\\ &\quad+\frac{-k(-k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}})}{(-k+\omega)^2-(\boldsymbol{k}+\widetilde{\boldsymbol{q}})^2-i\epsilon}\Bigg]. \qquad (A.6)\end{aligned}$$

From Eqs. (A.4) to (A.6), we used the momentum substitution $K\to -P$ and $d\Omega_k=d\phi\sin\theta d\theta$. Here, the vacuum part has been neglected because it is suppressed compared to the medium part in the HTL approximation. Considering that ω/k and $\widetilde{q}/k$ in the HTL approximation are small terms, the square bracket in Eq. (A.6) can be further expanded as follows:

$$\begin{aligned}\frac{k(k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}})}{(k+\omega)^2-(\boldsymbol{k}+\widetilde{\boldsymbol{q}})^2+i\epsilon}&=\frac{2k^2+k\omega+kq\cos\theta}{2k\omega+Q^2-2k\widetilde{q}\cos\theta+i\epsilon}\\ &\simeq\frac{k}{\omega-\widetilde{q}\cos\theta+i\epsilon}-\frac{1}{2}\frac{Q^2}{(\omega-q\cos\theta+i\epsilon)^2}\\ &\quad+\frac{1}{2}\frac{\omega+\widetilde{q}\cos\theta}{\omega-\widetilde{q}\cos\theta+i\epsilon}, \qquad (A.7)\end{aligned}$$

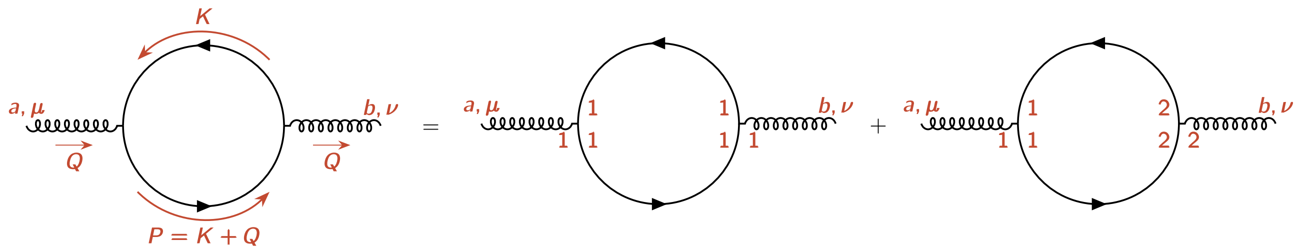

Fig. 4 One-loop diagram for quark contributions to retarded gluon polarization tensor

$$\frac{-k(-k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}})}{(-k+\omega)^2-(\boldsymbol{k}+\widetilde{\boldsymbol{q}})^2-i\epsilon}=\frac{2k^2-k\omega+k\widetilde{q}\cos\theta}{-2k\omega+Q^2-2k\widetilde{q}\cos\theta-i\epsilon}$$
$$\simeq-\frac{k}{\omega+\widetilde{q}\cos\theta+i\epsilon}-\frac{1}{2}\frac{Q^2}{(\omega+\widetilde{q}\cos\theta+i\epsilon)^2}+\frac{1}{2}\frac{\omega-\widetilde{q}\cos\theta}{\omega+\widetilde{q}\cos\theta+i\epsilon},\tag{A.8}$$

where $x=\boldsymbol{k}\cdot\widetilde{\boldsymbol{q}}/(k\widetilde{q})=\cos\theta$. Since the first terms in both Eqs. (A.7) and (A.8) are odd functions of x, they integrate to zero in the range from -1 to 1. Finally, Eq. (A.6) is simplified to the following result:

$$\Pi_R^{\text{quark}}(Q)=N_f g^2\int\frac{kdkd\Omega_k}{2(2\pi)^3}\left(f_{q,F}^+(k)+f_{q,F}^-(k)\right)\times\left[\frac{1-x^2}{[x+(\omega+i\epsilon)/\widetilde{q}]^2}+\frac{1-x^2}{[-x+(\omega+i\epsilon)/\widetilde{q}]^2}\right].\tag{A.9}$$

By utilizing Eqs. (7) and (15), the one-loop contribution from quarks to the symmetric gluon self-energy tensor is expressed as

$$\Pi_F^{\mu\nu,\text{quark}}(Q)=-iN_f g^2\int\frac{d^4K}{(2\pi)^4}\{\text{Tr}[t_b\gamma^\mu S_{11}(K)t_a\gamma^\nu S_{11}(P)]+\text{Tr}[t_b\gamma^\mu S_{22}(K)t_a\gamma^\nu S_{22}(P)]\}\tag{A.10}$$
$$=-iN_f g^2\int\frac{d^4K}{(2\pi)^4}(K^\mu P^\nu+K^\nu P^\mu-g^{\mu\nu}K\cdot P)\times[\Delta_F(K)\Delta_F(P)-(\Delta_R(K)-\Delta_A(K))(\Delta_R(P)-\Delta_A(P))].\tag{A.11}$$

Applying the relation $\Delta_R(K)-\Delta_A(K)=-i2\pi\delta(K^2)\,\text{sgn}(k_0)$, and focusing specifically on the temporal component of $\Pi_F^{\mu\nu,\text{quark}}(Q)$, we obtain

$$\Pi_F^{\text{quark}}(Q)=iN_f g^2\int\frac{d^4K}{(2\pi)^2}(k_0p_0+\boldsymbol{k}\cdot\boldsymbol{p})\Big\{\left[1-2\Theta(k_0)f_{q,F}^+(k_0)-2\Theta(-k_0)f_{q,F}^-(-k_0)\right]\times\left[1-2\Theta(p_0)f_{q,F}^+(p_0)-2\Theta(-p_0)f_{q,F}^-(-p_0)\right]-\text{sgn}(k_0)\text{sgn}(p_0)\Big\}\delta(K^2)\delta(P^2)\tag{A.12}$$
$$=iN_f g^2\int\frac{k^2dkd\Omega_k}{(2\pi)^2}(k(k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}}))\frac{1}{2k}\delta(Q^2+2k\omega-2\boldsymbol{k}\cdot\widetilde{\boldsymbol{q}})\times\left[-2f_{q,F}^+(k)-2f_{q,F}^+(k+\omega)+4f_{q,F}^+(k)f_{q,F}^+(k+\omega)\right]$$
$$+iN_f g^2\int\frac{dkk^2d\Omega_k}{(2\pi)^2}(-k(-k+\omega)+\boldsymbol{k}\cdot(\boldsymbol{k}+\widetilde{\boldsymbol{q}}))\frac{1}{2k}\delta(Q^2-2k\omega-2\boldsymbol{k}\cdot\widetilde{\boldsymbol{q}})\times\left[-2f_{q,F}^-(k)-2f_{q,F}^-(k-\omega)+4f_{q,F}^-(k)f_{q,F}^-(k-\omega)\right].\tag{A.13}$$

In the HTL approximation within soft momentum limit $\omega/T\ll 1$, $f_{q,F}^\pm(k\pm\omega)$ can be expanded to the leading order of ω/T,

$$f_{q,F}^\pm(k\pm\omega)\approx f_{q,F}^\pm(k)\mp\frac{\omega q}{T}\exp_q\left(\frac{k\mp\mu}{T}\right)\left[f_{q,F}^\pm(k)\right]^2.\tag{A.14}$$

In Eq. (A.13), the delta function $\delta(Q^2\pm 2k\omega-2\boldsymbol{k}\cdot\widetilde{\boldsymbol{q}})$ can be rewritten as $\frac{1}{2k\widetilde{q}}\delta(Q^2/(2k\widetilde{q})\pm\omega/\widetilde{q}-x)$, subsequently, the term $Q^2/(2k\widetilde{q})$ in the HTL approximation with $k\gg\omega$ becomes irrelevant and can be reasonably removed. Then, Eq. (A.13) is rewritten as

$$\Pi_F^{\text{quark}}(Q)=-iN_f g^2\int\frac{dkk^2}{(2\pi)}\left[f_{q,F}^+(k)(1-f_{q,F}^+(k))+f_{q,F}^-(k)(1-f_{q,F}^-(k))\right]\frac{2}{\widetilde{q}}\Theta(\widetilde{q}^2-\omega^2),\tag{A.15}$$

where we have used the integral over the angle, yielding

$$\int d\Omega_k \delta(\omega/\widetilde{q} \pm x) = 2\pi\Theta(\widetilde{q}^2 - \omega^2). \tag{A.16}$$

Since we are interested in the space-like region within the static limit ($\omega \to 0$), which results in $\Theta(\widetilde{q}^2 - \omega^2) = 1$.

The detailed derivation of the one-loop contribution from gluons (gauge fields) to the gluon self-energy can be found in Refs. [10,87,88]. By repeating the computation procedure in Ref. [10] and applying the non-extensive bare gluon propagators listed in Eqs. (11) and (12), the gluonic contributions (including the gluon-loop, ghost-loop, and tadpole contributions) to the retarded gluon self-energy tensor are given as

$$\begin{aligned}\Pi_R^{\mu\nu,\text{gluon}}(Q) = &-i2N_c g^2 \frac{d^4K}{(2\pi)^4}(K^\mu P^\nu + K^\nu P^\mu \\ &- g^{\mu\nu} K\cdot P)\big[\tilde{\Delta}_F(K)\tilde{\Delta}_R(P) \\ &+ \tilde{\Delta}_A(K)\tilde{\Delta}_F(P)\big].\end{aligned} \tag{A.17}$$

Here, $\tilde{\Delta}_{R/A/F}(K) \equiv G_{R/A/F}(K)$. Using the HTL approximation and only considering the temporal component, a straightforward calculation leads to

$$\begin{aligned}\Pi_R^{\text{gluon}}(Q) = &2N_c g^2 \int \frac{k^2 dk d\Omega_k}{(2\pi)^3}\frac{f_{q,B}(k)}{2k} \\ &\times \left[\frac{1-x^2}{[x+(\omega+i\epsilon)/\widetilde{q}]^2} + \frac{1-x^2}{[-x+(\omega+i\epsilon)/\widetilde{q}]^2}\right].\end{aligned} \tag{A.18}$$

For the one-loop contribution from gluons to the symmetric gluon self-energy tensor, it can be expressed as

$$\begin{aligned}\Pi_F^{\mu\nu,\text{gluon}}(Q) = &-i2N_c g^2 \int \frac{d^4K}{(2\pi)^4}(K^\mu P^\nu + K^\nu P^\mu \\ &- g^{\mu\nu} K\cdot P)\Big[\tilde{\Delta}_F(K)\tilde{\Delta}_F(P) \\ &- \left(\tilde{\Delta}_R(K) - \tilde{\Delta}_A(K)\right)\left(\tilde{\Delta}_R(P) - \tilde{\Delta}_A(P)\right)\Big].\end{aligned} \tag{A.19}$$

In the HTL approximation within the small ω limit, the temporal component of the above equation yields,

$$\begin{aligned}\Pi_F^{\text{gluon}}(Q) = &-i2N_c g^2 \int \frac{dk k^2}{2\pi} f_{q,B}(k)(1+f_{q,B}(k)) \\ &\times \frac{2}{\widetilde{q}}\Theta(\widetilde{q}^2 - \omega^2),\end{aligned} \tag{A.20}$$

which has the same structure as Eq. (A.15) but with $f^{\pm}_{q,F}(k)$ replaced by a non-extensive deformed Bose–Einstein distribution function $f_{q,B}(k)$.

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