Nuclear structure studies in mirror nuclei

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Abstract. The nuclear structure of the A=31 and A=47 mirror couples produced by two fusion evaporation reactions has been elaborated, utilizing the Doppler-shift attenuation method. Excited states in ^{31}P and ^{31}S were populated using the 1p and 1n exit channels, respectively, of the reaction ^{20}Ne + ^{12}C, while in ^{47}Cr and ^{47}V couple excited states were populated based on ^{28}Si + ^{28}Si reaction, as products of $2\alpha n$ and $2\alpha p$ exit channels. The A=31 mirror couple was studied utilizing Piave-Alpi accelerator of the Laboratori Nazionali di Legnaro with GASP multidetector array and for A=47 one - with the EUROBALL array using XTU Tandem also in Legnaro. In both cases the lifetime measurements in mirror couples at the same experiment open possibilities for investigations of isospin symmetry. Determined B(E1) strengths in the mirror nuclei ^{31}P and ^{31}S allow to extract the isoscalar component, which can reach up to 24% of the isovector one. The B(E1) values can be modeled by the Equation of motion method. In the case of A=47 mirror couple, the quadrupole moments can be described by shell-model calculations.

1. Introduction

The study of nuclei with equal number of protons and neutrons opens interesting possibilities for investigation of symmetries in nuclei. It is also a test for the role of the Coulomb effects on the nuclear structure. Mirror nuclei are among these nuclei and a comparison of their structure provides information for the charge symmetry of the nuclear forces and isospin symmetry. According to the Siegert theorem the E1 operator is completely isovector in the long wavelengths limit. If the nuclear force is charge independent the E1 transitions in the N=Z nuclei between levels with the same isospin are forbiden, while the E1 transitions between analogue states in the mirror nuclei have the same transition strengths. Nonetheless, these generally accepted assumptions and theoretically derived rules need to be confirmed by carefully designed experiments.

We report here on the results from lifetime measurements, which aim to test the isospin symmetry conservation in the A=31 region, through a comparison of the E1 strengths of the transitions depopulating the $7/2^-$ analogue states above 4.4 MeV in the mirror nuclei ^{31}P and ^{31}S [1]. The experimental findings are compared with beyond mean field and self-consistent calculations. For the mirror nuclei ^{47}Cr and ^{47}V the behaviour of the Coulomb energy differences (CED) as a function of angular momentum has been presented as an evidence for nuclear structure changes [2]. Our interest to investigate the A=47 couple was to check whether these changes are reflected by the analogue B(E2) transitions in both nuclei. The measured B(E2) values could be used to test the performance of the *fp* shell model in this mass region.

Figures 1 and 2 are an excellent illustration of the excitation energies symmetries in these two couples of mirror nuclei. Here we summarize our findings related to CED and specifically to the origin of the differences in the transition probabilities for the analogues states. Figure 1 shows the partial level schemes of ^{31}P and ^{31}S. The partial level schemes of A=31 mirror couple is reported in Ref. [3] and latter modified in the work [4]. The mirror E1 transitions $7/2^-_1 \rightarrow 5/2^+_{1,2}$ clearly show different strengths. To verify if the different intensities of those transitions observed in both nuclei correspond to different B(E1) values we performed the experiment reported in Ref. [1]. Figure 2 shows the partial level schemes of ^{47}Cr and ^{47}V from Ref [2]. For this mirror couple we investigated the possible differences of B(E2) values for the transitions $23/2^- \rightarrow 19/2^-$, $19/2^- \rightarrow 15/2^-$ and $15/2^- \rightarrow 11/2^-$. In this spin region nuclear structure changes are reported in the works of [2] and [5] and this was our motivation to measure lifetimes of the states.

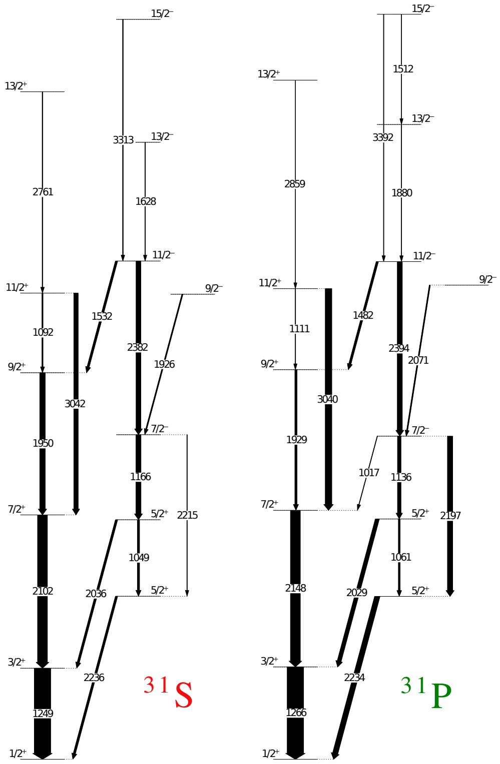

Figure 1. Partial level schemes of the mirror couple A=31, redrawn on the basis of the data in Ref [3] and Ref. [4]. The width of the arrows indicates the relative intensities of the transitions.

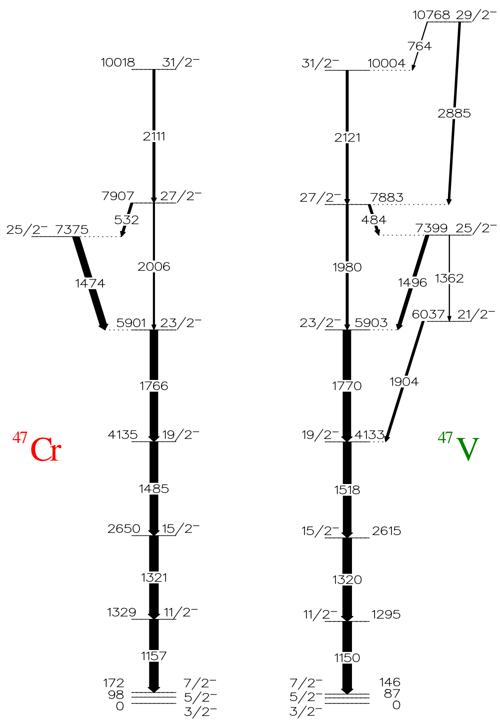

Figure 2. Partial level schemes of the mirror couple A=47, redrawn on the basis of the data in Ref [2]. The width of the arrows indicates the relative intensities of the transitions.

2. Experiments and data analyses

^{31}P and ^{31}S were produced simultaneously by the reactions ^{12}C(^{20}Ne, p) and ^{12}C(^{20}Ne, n) using a carbon target with a 33-MeV ^{20}Ne beam from the Piave-Alpi accelerator at the Laboratori Nazionali di Legnaro (LNL). The carbon target consisted of 0.75 mg/cm^2 of ^{12}C onto a 10 mg/cm^2 gold foil stopper. The target was surrounded by GASP array [6] in its configuration II, and charged particles were registered with EUCLIDES silicon ball [7].

Excited states in ^{47}Cr and ^{47}V were populated using ^{28}Si + ^{28}Si, at 110 MeV. The beam of ^{28}Si was provided by the XTU Tandem accelerator of LNL. The EUROBALL spectrometer in its configuration III [8] was used to detect γ-rays. Neutrons were registered with the Neutron Wall [9] and charged particles with the ISIS silicon ball [10]. In both experiments the ancilary detectors played a very important role, since transition energies in mirror nuclei are very close and in order properly to describe the lineshape in DSAM analysis one needs to select only the nucleus of interest. For the lighter mirror couple the γ-γ coincidence matrices for the nucleus of ^{31}P were sorted whereby the registration of protons was required. For A=47 mirror nuclei, events with at least two Ge segments firing in coincidence with one neutron were accepted by the event trigger [11].

In order to determine precisely the transition probabilities, the branching ratios of the transitions deexciting the level of interest, their mixing ratios and the lifetimes are needed.

For the case of A=31 mirror couple angular correlation analysis was performed using the procedure described in the work [12], and with the code CORLEONE. Comprehensive details about the analysis are presented in the works [13] and [1]. The Ge detectors of the GASP array were grouped into rings according to their polar angles. The results for the transitions $7/2^-_1 \rightarrow 5/2^+_2$ in both mirror nuclei show a dominant E1 character.

Our next step was to determine the lifetimes of the analogue states in the mirror couples utilizing the DSAM approach. In both experiments we used a gate on the Doppler shifted part of the feeding transition [14]. In order to describe the lineshape we need to describe at the same time the process of population of the state of interest and the velocity distribution of the recoil nuclei. For the present analysis we have done Monte Carlo (MC) simulations of the process of slowing-down of the recoils using a modified [14, 15] version of the DESASTOP program [16]. Data from the tables of Northcliffe and Schilling [17] have been used to describe the electronic stopping powers and a correction for the atomic structure of the medium has been made following the procedure from Ref. [18]. We have applied a reduction of the nuclear stopping power [19] as prescribed by the theory of Lindhard, Scharff, and Schiøtt [20]. All of the approaches lead to similar results. The same approach was used for both experiments and in both of them, different methods to determine stopping power were giving good result. One should note, that there is not an universal approach to calculate stopping power, there are some cases when all of the approaches could not fit the lineshape and obviously experimental values for the stopping powers are needed in those cases. When the excited states in the nuclei are populated in the same reaction, like in the present case of mirror nuclei, the errors coming from the determination of the stopping powers are minimized. Please see Figure 3.

For the A=31 mirror couple, according to the MC simulations, the mean velocity of the recoil nuclei when they enter the gold foil (the stopper) is 3.7% of the velocity of light. For the case of A=47 mirror couple, it is – 3.9% of the velocity of light. It is obvious that an evaporation of charged particles from a light compound nucleus will play a significant role in the MC simulations and in the description of the lineshape, see Ref. [11]. In the case of formation of the ^{47}V two α particles and one proton are evaporated from the compound nucleus and we need to underline that without taking into account the process of evaporation of charged particles from the compound nucleus the lineshape could not be fitted.

A comprehensive description of the MC calculations needed for the analysis can be found in Refs. [11, 14, 15, 21]. The good statistics obtained in the experiment for the A=31 mirror couple allows us to use the approach [14] when the gate is set on the shifted part of the direct feeding transition. In this case, the effects of the unknown feeding transition nearly cancel. Within this approach, the timing quality of the gated line shape is improved, compared to the case where the gate includes also fully stopped events that do not provide lifetime information. Moreover, "gating from above" allows the elimination of the uncertainties related to the unobserved feeding of the level of interest which perturbs singles and coincidence measurements when the gate is set on a transition de-exciting a level fed by the level of interest. We have also used the method for lifetime determination with a gate on a transition de-exciting a level fed by the level of interest. Such an approach [21, 22] is used for the study of ^{47}Cr because of its low reaction yields.

3. Results

For the case of A=31 mirror couple we have determined the branching ratios, the M2/E1 multiple mixing ratios and the lifetimes of the $7/2^-_1$ analogue states in both nuclei [1]. The values derived for the M2/E1 multipole mixing ratios are δ = -0.03(7) and δ = -0.07(8) for ^{31}P and ^{31}S, respectively [1].

The lifetime value obtained in our work [1] for the $7/2^-_1$ level in ^{31}P is of $\tau = 597(45)$ fs. The value obtained for the $7/2^-_1$ state of ^{31}S is $\tau = 543(49)$ fs. By measuring the branching ratios, multipole mixing ratios and lifetimes we could then determine the B(E1) values for the two analogue transitions depopulating the $7/2^-_1$ state of the A=31 mirror pair. For the transition $7/2^-_1 \rightarrow 5/2^+_2$ in ^{31}P, the obtained B(E1)= 2.7(2) x 10^{-4} e^2 fm^2, and for the same transition in ^{31}S the B(E1)= 7.2(7) x 10^{-4} $e^2 fm^2$. For the case of A=47 mirror couple, five lifetimes were determined and for the $23/2^-$ of ^{47}Cr we obtained a limit for the lifetime value. For the stronger populated nucleus of ^{47}V the following values for the lifetimes were obtained: for the $15/2^-$, τ = 0.97(10) ps, for the $19/2^-$ - τ = 0.57(4) ps and for the $23/2^-$ - τ = 0.37(5) ps. In the case of ^{47}Cr, for the $15/2^-$, τ = 0.84(12) ps, for the $19/2^-$ - τ = 0.44(6) ps and for the $23/2^-$ - τ less than 0.64 ps. As long as the experimental errors allow to distinguish a difference, in mirror states most of the lifetimes in the nuclei having one proton more are found to be shorter. It is so for the $27/2^-$ state in ^{51}Fe and ^{51}Mn mirror pair [23]. Opposite cases are encountered for the $9/2^-$ isomeric state in A=67 case, ^{67}As and ^{67}Se [24].

4. Discussion

4.1. A=31 mirror couple

Experimentally determined B(E1) values are compared with theoretical predictions of the Equation of Motion Phonon Method (EMPM)[25, 26]. The EMPM utilizes an orthonormal basis of multiphonon states. They are constructed by the Tamm-Dancoff Approximation (TDA). The Pauli principle is taken into account in this theoretical framework. A self-consistent calculation have been performed using a Hamiltonian composed of an intrinsic kinetic operator and the chiral potential NNLO_{sat} [27], and it includes the contribution of the three-body forces. The Hartree-Fock (HF) basis states have been constructed including all harmonic oscillator shells up to $\mathrm{N}_{max} = 6$. There are not any admixtures due to the center of mass motion [28].

Based on the experimentally determined B(E1) values for the analogue $7/2^-_1 \rightarrow 5/2^+_2$ transitions in the mirror A=31 couple one can obtain $\langle J_i|||\mathcal{M}_{\mathrm{IV}}|||J_f\rangle = 0.149(38)$ efm and $\langle J_i|||\mathcal{M}_{\mathrm{IS}}|||J_f\rangle = 0.021(2)$ efm. The ratio $|\langle J_i;T_iT_3||\mathcal{M}_{\mathrm{IS}}||J_f;T_fT_3\rangle/\langle J_i;T_iT_3||\mathcal{M}_{\mathrm{IV}}||J_f;T_fT_3\rangle|$ $= |\langle J_i|||\mathcal{M}_{\mathrm{IS}}|||J_f\rangle\sqrt{6}/\langle J_i|||\mathcal{M}_{\mathrm{IV}}|||J_f\rangle\sqrt{2}|$ for the $7/2^-_1 \rightarrow 5/2^+_2$ transitions is then about 0.24. Using the experimental limit of 1% for the branching ratio of the transition 2213 keV $7/2^-_1 \rightarrow 5/2^+_1$ in ^{31}S we can obtain also a limit for the B(E1) value. This leads to $|\langle J_i|||\mathcal{M}_{\mathrm{IS}}|||J_f\rangle\sqrt{6}$ / $\langle J_i|||\mathcal{M}_{\mathrm{IV}}|||J_f\rangle\sqrt{2}| < 0.6$

From experimental data we derive the B(E1) for ^{31}P for the $7/2^-_1 \rightarrow 5/2^+_2$ transition 2.7(2) x 10^{-4} e^2 fm^2 and theoretical prediction is 2.2 x 10^{-4} e^2 fm^2. For the analogue transition in ^{31}S, the experimental value is 7.2(7) x 10^{-4} e^2 fm^2 and the theoretical prediction is 7.9 x 10^{-4}

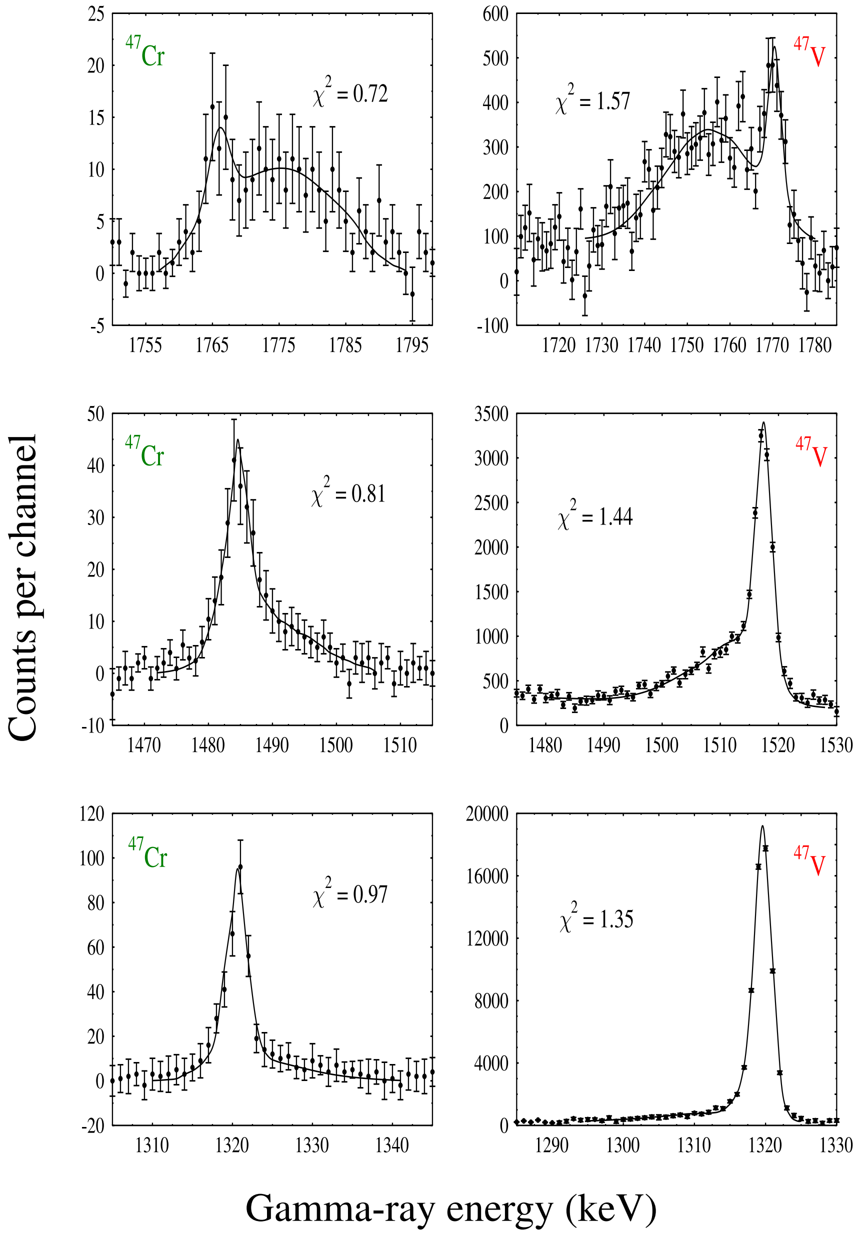

Figure 3. Illustration of the lifetime analysis in ^{47}Cr and ^{47}V. Fits of the data for different angles. On the left-hand side spectra for forward angles in ^{47}Cr are shown, while on the right-hand side backward spectra in ^{47}V are presented. The accuracy of the fits are indicated by χ^2 values. More information is presented in Ref. [11].

e^2 fm^2. There is a very good agreement between the experimentally determined B(E1) values and the EMPM model values. With the same theoretical approach for the analog transitions in the A=31 couple we obtain $\langle J_i|||\mathcal{M}_{\rm IV}|||J_f\rangle = 0.147$ efm and $\langle J_i|||\mathcal{M}_{\rm IS}|||J_f\rangle = 0.027$ efm, also in an excellent agreement with the experimental ones. Due to the Pauli principle the proton (neutron) hole couple to the neutron (proton) TDA phonons and hence to the phonon excitation of the core. The model allow us to describe the B(M2) values as well.

A detailed description of the "isoscalar" component is reported in Ref. [29]. Two different origins of the "isoscalar" component are suggested. The first one is connected to higher-order terms in the transition operator while the second one is based on level mixing, caused by the "isovector" term of the Coulomb interaction. Following Ref. [29] the input of the higher-order terms in the transition operator is not significant. Concerning the contributions of higher-lying levels [30] to the wavefunction, they are individually minor, but if every mixing term is in phase with the E1 amplitude of the level of interest [31, 32] they could together become large. This assumption is confirmed by the agreement of the model predictions with the experimentally determined data. The origin of non-preserving of the isospin symmetry in this case is caused by breaking of the charge symmetry of the two- and three-body parts of the chiral potential, utilized in the theoretical approach, which uses the Coulomb interaction. Detailed information for the isospin breaking terms can be found in Refs. [33, 34, 35].

A detailed study of the components of the total wavefunctions in case ot A=31 mirror nuclei is presented in Ref. [1].

4.2. A=47 mirror couple
Very interesting phenomena can be investigated in the structure of nuclei belonging to the middle of the 1 $f_{7/2}$ shell. Up to intermediate spins these nuclei exhibit rotational behavior. The reason for this collective behavior is the existence of enough valence particles. Shell model predictions [36] for the mass region of A=47 show that B(E2) values in the spin region described above show a behavior like for the rotational model. Values of the quadrupole moments reported in Ref. [11] systematically decrease when the spin is increasing. In the case of ^{47}Cr the Qt's show that their increase is excluded. The lifetimes measured by us [11] in the spin region ($15/2^-$ - $23/2^-$) of the mirror nuclei ^{47}Cr and ^{47}V are influenced by contributions to the wavefunction of excited states where significant changes of the Coulomb energy differences are observed. In Ref. [2] these changes are explained as a proof of changes of the structure of the nuclei as well as changes of their shape. We have measured the lifetimes of the I=15/2 $\hbar$ and I=19/2 $\hbar$ states and we have obtained a limit for the I=23/2 $\hbar$ state. Obviously we need to measure lifetimes for the states above I=19/2 $\hbar$ level in order to elucidate the structure of ^{47}Cr at intermediate spins. Upon going to higher spins the opportunity for collectivity is limited and for the band termination state we need to account only for single nucleon excitations. Approaching the band termination state all valence protons in the A=47 mirror couple are aligned and the Coulomb Energy Difference values are similar to values at low spins [37]. The shell model calculations are describing very well the CED with a small difference for the level $I^{\pi}=19/2^-$, which points out to the fact the model cannot describe completely the shape changes.

The nuclear structure in mirror nuclei should be very similar due to the fact that nuclear forces are symmetric with respect to the charge. An excellent example of the symmetry of excitation energy of both A=47 mirror nuclei is shown in Figure 2. Based on the lifetime measurement in ^{47}Cr and ^{47}V we can conclude that the symmetry is also present for the transition quadrupole moments of the analogue states. The one additional proton in the ^{47}Cr nucleus results in slightly shorter lifetimes in this nucleus.

5. Conclusions
Our DSAM measurements of the lifetimes in the A=31 mirror couple allow us to determine the B(E1) values of the first $7/2^-$ analogue states in both nuclei. Based on the B(E1) values obtained we can conclude that the isospin symmetry is broken and as a consequence there is a big induced isoscalar component to the Hamiltonian. The predictions of Equation of Motion Phonon Method show an excellent agreement with the experimentally determined values and confirm the violation of isospin symmetry.

In addition we report on an EUROBALL femtosecond measurement of lifetimes in ^{47}Cr and ^{47}V utilizing a procedure with a gate on the shifted part of the feeding transition. Next to the symmetry of the excitation energies, we observe also symmetry in the transition probabilities with increasing the spin in the yrast band. The full pf shell model calculations describe well the quadrupole transition moments in both mirror nuclei – ^{47}Cr and ^{47}V.

5.1. Acknowledgments
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