Primordial power spectrum from a matter-ekpyrotic scenario in loop quantum cosmology[1]

Bao-Fei Li*, Sahil Saini† and Parampreet Singh*

*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

†Department of Physics, Guru Jambheshwar University of Science & Technology, Hisar 125001, Haryana, India

The general matter bounce scenario, including an Ekpyrotic field to avoid anisotropic instabilities, is studied in a loop quantized isotropic and homogeneous FLRW setting. The matter bounce scenario provides a convenient way to include quantum corrections from the bounce in the perturbations originating in the far past, which also produce a scale invariant power spectrum. LQC provides the right setting for studying quantum corrections in a matter bounce scenario as the bounce in LQC occurs entirely due to quantum geometrical effects without needing any exotic matter fields to avoid the singularity. A detailed exploration of this general matter-Ekpyrotic scenario in spatially flat FLRW spacetime in LQC filled with minimally coupled dust and Ekpyrotic scalar field is studied with the help of numerical simulations. Various features of the background dynamics are shown to be robust under variations in initial conditions and choice of parameters. We use the dressed metric approach for the perturbations and obtain a scale invariant power spectrum for modes exiting the horizon in the dust dominated contracting phase. In contrast to previous studies considering a constant equation of state for the Ekpyrotic field, we found that the magnitude of the power spectrum changes during the evolution. The scale invariant section of the power spectrum also undergoes a rapid increase in its magnitude in the bounce regime, while its scale invariance is unaffected. We argue that apart from increasing the magnitude, the bounce regime may only substantially affect the modes outside the scale invariant regime. However, the spectral index is found to be too close to unity, thus inconsistent with the observational constraints, necessitating further modifications of the model.

Keywords: Loop quantum cosmology; bouncing cosmology; primordial power spectrum.

1. Introduction

Early universe cosmology provides one of the most promising avenues to search for signatures of quantum gravity. Singularity free bouncing cosmologies of LQC provide a convenient setting for this. Inflation being the leading model has been studied widely with LQC backgrounds. Since there is no singularity, it is possible to extend the inflationary scenario further back in time upto the Planck regime of the quantum bounce to include quantum gravity signatures. These studies have led to predictions that are in excellent agreement with standard inflation and CMB observations at small angular scales but are closer to observations at larger scales than standard inflation.[2] The quantum bounce which lies to the past of this inflationary regime is a cataclysmic event which changes the course of the evolution of the universe, thus is expected to contribute significantly to the quantum gravity signatures in the early universe. In this regard, it's natural to consider alternatives such as the matter-bounce scenario where the perturbations start out in the far past in the contracting branch, hence getting imprinted by the highly quantum phenomena as they pass through the quantum bounce. Due to a duality between inflation and matter-bounce, a scale invariant spectrum of perturbations is obtained in both of them.[3] An Ekpyrotic field is generally also included to guard against a potential BKL instability which may occur due to unchecked growth of anisotropies during the contracting phase.[4] An Ekpyrotic field is a scalar field with a negative-definite potential, thus having an equation of state larger than unity. As the bounce is approached in the contracting phase, the Ekpyrotic field having an equation of state larger than unity is expected to dominate the anisotropies which evolve as a^{-6}, where a denotes the scale factor.[5] Thus combining these two ingredients we obtain the the matter-Ekpyrotic scenario,[6] in which a scale invariant spectrum is produced by matter perturbations which exit the horizon during the contracting phase.

An important ingredient of the matter-Ekpyrotic scenario is a non-singular bounce which allows the scale invariant perturbations to pass through without changing them substantially. This is where LQC provides an advantageous framework. In LQC, the quantum bounce arises purely due to quantum geometry effects without requiring any exotic matter content violating the weak energy conditions.[7] Thus the marriage of the matter-Ekpyrotic scenario with an LQC bouncing background provides a compelling model for obtaining quantum gravity signatures in the early universe. Using effective dynamics of LQC, a nonsingular model with an Ekpyrotic potential was obtained[8,9] but it was found that a viable cyclic model cannot be realized unless another matter field or anisotropies are present.[10] Previous studies of the matter-Ekpyrotic model in LQC have either restricted to a special case of a constant equation of state for the Ekpyrotic field,[11] or when the general case has been considered,[12] the analysis has been limited to qualitative predictions due to difficulties associated with the deformed algebra approach used for perturbations in dealing with ultraviolet modes near the bounce. Thus a detailed study of the general matter-Ekpyrotic scenario in LQC analyzing the impact of the Ekpyrotic phase and the bounce on the perturbations has been lacking so far. Our work aims to fill this gap and provide a basis for further exploration.

2. Background dynamics of the matter-Ekpyrotic scenario in LQC

LQC uses the techniques of non-perturbative canonical quantization of LQG on symmetry reduced cosmological models such as the isotropic and homogeneous FLRW model. The canonical quantization of gravity in LQG is carried out in terms of the Ashtekar-Barbero connection and their conjugate triads instead of metric variables. The discrete quantum dynamics obtained in LQC is found to be well approximated by a continuum effective description in terms of differential equations on a smooth manifold, giving modified versions of Friedmann and Raychaudhuri equations that include quantum corrections that lead to a singularity free evolution in which the big bang is replaced by a bounce. The effective Hamiltonian describing the dynamics of loop quantized spatially flat FLRW model in terms of symmetry-reduced Ashtekar-Barbero connection and its conjugate triad, namely c and p, is given by[7]

$$\mathcal{H} = -\frac{3v}{8\pi G\gamma^2\lambda^2}\sin^2(\lambda b) + \mathcal{H}_{\rm m}, \tag{1}$$

where $\lambda = \sqrt{\Delta}$. Here $\Delta = 4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2$ is the minimum area eigenvalue in LQG. The matter Hamiltonian $\mathcal{H}_{\rm m}$ is given by

$$\mathcal{H}_{\rm m} = \frac{{p_\phi}^2}{2v} + vU(\phi) + \mathcal{E}_{\rm dust}, \tag{2}$$

and the Ekpyrotic potential $U(\phi)$ is taken to be,

$$U(\phi) = \frac{-2u_o}{e^{-\sqrt{\frac{16\pi}{p}}\phi} + e^{\beta\sqrt{\frac{16\pi}{p}}\phi}}, \tag{3}$$

which has the shape of an asymmetric exponential well. Here u_o, p and β are parameters taking positive values. The potential (3) is negative definite. The width of the potential increases with an increasing p, and the degree of asymmetry is controlled by β.

We obtain the following Hamilton's equations from the above Hamiltonian constraint,

$$\dot{b} = -\frac{3\sin^2(\lambda b)}{2\gamma\lambda^2} - 4\pi G\gamma P, \tag{4}$$

$$\dot{v} = \frac{3\sin(2\lambda b)}{2\gamma\lambda}v, \tag{5}$$

$$\dot{\phi} = \frac{p_\phi}{v}, \qquad \dot{p}_\phi = v\,U_{,\phi}, \tag{6}$$

where $U_{,\phi}$ stands for the differentiation of the potential with respect to the Ekpyrotic field and P is the isotropic pressure $P = -\frac{\partial\mathcal{H}_{\rm m}}{\partial v}$. It can be shown that the above equations lead to a following modified Friedmann equation of the form

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{7}$$

where $\rho_c = 3/(8\pi G\gamma^2\lambda^2) \approx 0.41\rho_{\rm Pl}$ is called the critical energy density in LQC. The Hubble rate is generically bounded, and vanishes and turns around when the energy density reaches a maximum ρ_c. This is when the bounce occurs. The matter content satisfies the continuity equation $\dot{\rho} + 3H(\rho + P) = 0$, where ρ and P are given respectively by

$$\rho = \frac{\dot{\phi}^2}{2} + U(\phi) + \frac{\mathcal{E}_{\rm dust}}{v}, \quad P = \frac{\dot{\phi}^2}{2} - U(\phi). \tag{8}$$

We use the above set of equations along with suitable initial conditions provided at the bounce to numerically analyze the background dynamics of this model. We also carry out simulations to study the effect of changes in the potential parameters and the relative proportion of the initial dust energy density compared to the Ekpyrotic field energy density. We find that LQC leads to a non-singular bounce in all cases. Although there can be multiple bounces, we restrict ourselves to the cases with a single bounce to study the evolution of curvature perturbations. As an example, we show here the evolution for the choice of parameters $u_o = 0.75, p = 0.10$ and $\beta = 5.0$ with the following initial conditions:

$$\phi_B = 0, \qquad \mathcal{E}_{dust} = 2.00 \times 10^{-5}, \qquad p_{\phi_B} = 1.50. \tag{9}$$

The volume at bounce is taken to be unity, while we choose $b_B = \pi/2\lambda$ in order to have the correct classical limit on either side of the bounce. The dust energy density is chosen such that the energy density at the bounce is dominated by the Ekpyrotic field. The initial conditions are taken to satisfy the Hamiltonian constraint which has to be satisfied throughout the evolution. The evolution of the energy densities and the equation of state for these initial conditions is shown in Fig. 1. We note that the contracting phase starts in a dust dominated phase where the total energy density is very small and the universe is classical. Eventually it transitions to a phase dominated by the Ekpyrotic field which lasts through the bounce. Thus we find the equation of state at the bounce is $w_B \approx 5.76$. Thus a distinct phase where $w > 1$ is obtained near the bounce, which is very important to avoid the BKL instability as envisaged. The $w > 1$ regime is obtained only when the scalar field is traversing the very bottom of the potential, and is obtained only for a short duration during the scalar field dominated phase near the bounce. After the bounce the equation of state quickly drops below unity and decreases monotonically as the Ekpyrotic field moves away from the bottom of the potential. We did extensive simulations to study the robustness of the qualitative features of the background dynamics obtained above by varying the parameters of the potential and taking different proportions of the initial dust energy density while still keeping the bounce dominated by Ekpyrotic field. We found that the effect of reducing the proportion of the dust energy density

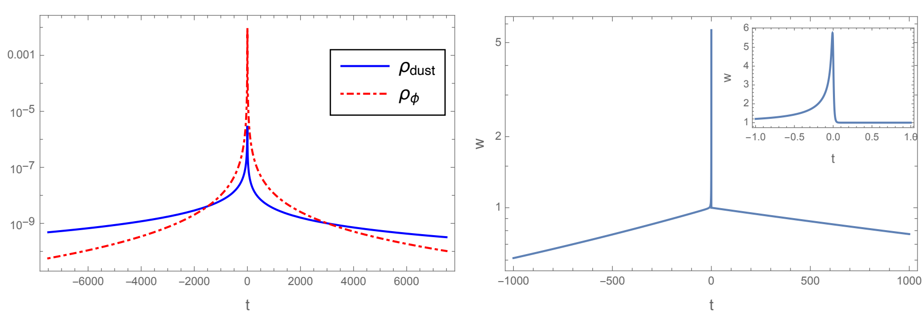

Fig. 1. In the left panel, we see that the energy density of the Ekpyrotic field dominates over the dust energy density in the bounce regime. The right panel shows the behavior of the equation of state when the Ekpyrotic field is dominant and the inset plot depicts the details of the change in the equation of state near the bounce point.

at the bounce has the obvious effect of increasing the duration of Ekpyrotic field dominated regime and the $w > 1$ regime near the bounce. The $w > 1$ regime can also be elongated by widening the potential by increasing p, making it less steep due to which the field takes longer to traverse the bottom of the well. Further, increasing the parameter u_o has effect of increasing both the depth and steepness of the potential. If we increase u_o while keeping the rest of the conditions same, this has the effect of numerically increasing both the kinetic and potential energy of the scalar field at the bounce while keeping the total the same (since the potential is negative-definite, the total energy density is the difference of the numerical values of kinetic and potential energies). Higher kinetic energy means the field climbs out of the well faster, i.e. the duration of the Ekpyrotic field dominated phase is reduced. But a deeper well means that $w > 1$ for most of the duration of this climb. Thus increasing u_o has the effect of decreasing the Ekpyrotic field dominated phase while increasing the $w > 1$ regime. This is illustrated in the following table showing durations of different regimes in the contracting branch (in Planck seconds):

u_o	Ekpyrotic field dominated phase	$w > 1$ regime
1	8.70×10^4	100
0.75	1.50×10^5	65
0.03	3.00×10^5	11
0.008	3.04×10^5	6.5

3. Scalar power spectrum using the dressed metric approach

In this section, we study perturbations around the background spacetime studied in the previous section. As mentioned above, due to the duality with the inflationary epoch,[3] the perturbations in dust matter that exit the horizon during the contracting phase yield a scale invariant spectrum. The dust dominated phase in our background dynamics occurs in the far past, away from the bounce when the universe is classical. Thus we must start perturbations in the far past and then evolve them through the Ekpyrotic phase and the bounce upto the expanding branch.

Since the Ekpyrotic field contributes negligibly to the total energy density in the far past and the equation of state $w \approx 0$, thus we ignore it while considering perturbations in this phase. In this regime the total energy density is far below the Planck density and quantum gravity modifications are negligible. Thus the perturbation equation in terms of the Mukhanov-Sasaki variable also takes its classical form in this phase,

$$\nu_k'' + \left(k^2 - \frac{z''}{z}\right)\nu_k = 0, \tag{10}$$

where the prime represents the differentiation with respect to the conformal time. And we choose initial conditions such that, the above state corresponds to the Bunch-Davies vacuum in the asymptotic past:

$$\lim_{\eta \to -\infty} \nu_k = \frac{e^{-ik\eta}}{\sqrt{2k}}. \tag{11}$$

The above equations are only valid in the matter dominated phase. As perturbations evolve towards the bounce, the Ekpyrotic field starts dominating and the energy density starts increasing towards the Planck scale. Since the Ekpyrotic field dominated phase overlaps with the bounce regime, we cannot use classical equations to describe the perturbations in the Ekpyrotic field dominated phase. Of the various approaches in LQC that consider perturbations in the quantum spacetime, we work with the dressed metric approach.[13] This avoids the Jeans instability encountered near the bounce in the previous study based on deformed algebra approach.[12] In the dressed metric approach, the quantum spacetime is approximated by a differential manifold with a dressed metric for sharply peaked semi-classical states. The effective description of the quantum perturbations is then provided by a Mukhanov-Sasaki variable with an evolution equation of the form

$$\nu_k'' + \left(k^2 + \Omega^2 - \frac{a''}{a}\right)\nu_k = 0, \tag{12}$$

where Ω^2 can be written in terms of the background variables as

$$\Omega^2 = 3\kappa\frac{p_\phi^2}{a^4} - 18\frac{p_\phi^4}{a^6\pi_a^2} - 12a\frac{p_\phi}{\pi_a}U_{,\phi} + a^2U_{,\phi\phi}. \tag{13}$$

Here $\kappa = 8\pi G$ and π_a is the conjugate momentum of the scale factor. The required background quantities in the above equations are to be calculated using the effective background dynamics described in section 2. The power spectrum is given by

$$\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2}\frac{|\nu_k|^2}{z^2}, \tag{14}$$

with $z = a\dot{\phi}/H$. The modes that exit the horizon in the matter-dominated phase of contraction are scale invariant, and it is these modes that will be observable today. However, numerical simulations are important to study the impact of the Ekpyrotic field dominated phase and the bounce on the spectrum of these perturbations. In particular, the scale invariance of spectrum is to be preserved in order to be consistent with observations. Fig. 2 shows the evolution of scalar power spectrum at different stages.

It is clear from the plots that a scale invariant spectrum is already generated at the end of the dust dominated phase during contraction. However, in contrast to previous studies relying on constant equation of state for the Ekpyrotic field,[11] the

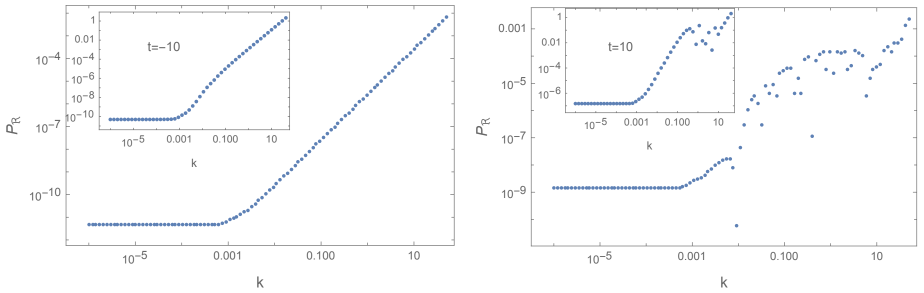

Fig. 2. The power spectra at different times are depicted: in the left panel, the power spectra are evaluated at the transition time from dust dominated to Ekpyrotic field dominated regime in the contracting phase ($t \approx -1.49 \times 10^3$) and at $t = -10$ (in the inset plot); while in the right panel, the power spectra are evaluated at the transition time in the expanding phase ($t \approx 3.02 \times 10^3$) from Ekpyrotic field domination to dust domination and at $t = 10$ (in the inset plot).

magnitude of the scale invariant power spectrum changes over time. Although the scale invariance is preserved through the bounce, a rapid increase in the magnitude of the power spectrum is observed from before the bounce to after the bounce seen by comparing the inset plots. Further, an oscillatory regime reminiscent of inflationary scenarios is seen for $k \geq 10^{-2}$. A comparison of the oscillatory regime at different times as seen in the right panel of Fig. 2 shows that different modes become oscillatory as they exit the horizon in the expanding phase. In contrast to earlier works on the matter-Ekpyrotic scenario in LQC dealing with special cases as well as in contrast to inflationary scenarios, the magnitude of the spectrum in the scale invariant regime gradually decreases as the universe expands. The qualitative behavior of the spectrum remains the same when we vary the proportion of the initial dust energy density relative to the Ekpyrotic field density. However, the range of modes that show scale invariance may get shifted. When the dust energy proportion is decreased, that leads to a shorter duration of dust dominated phase during contraction. Since only those modes that exit during the dust dominated contraction show a scale invariant spectrum, range of modes that show scale invariance is decreased. This may be used to put a constraint on the duration of Ekpyrotic field allowed by observed range of scale invariant modes.

Lastly, in order to compare with observations, we compute the spectral index of the spectrum given by:

$$n_s = 1 + \frac{\mathrm{d}\ln P_{\mathcal{R}}}{\mathrm{d}\ln k}. \tag{15}$$

The spectral index is plotted in Fig. 3. We find that the spectral index is larger than unity in the scale invariant regime and is unfavored by recent CMB observations that put tight constraints on the spectral index: $n_s = 0.9649 \pm 0.0042$ (68%CL).[14] We find that this cannot be ameliorated by either changing the parameters of the scalar field potential or changing the proportion of initial dust density to Ekpyrotic field density. Thus making further modifications of this model necessary.

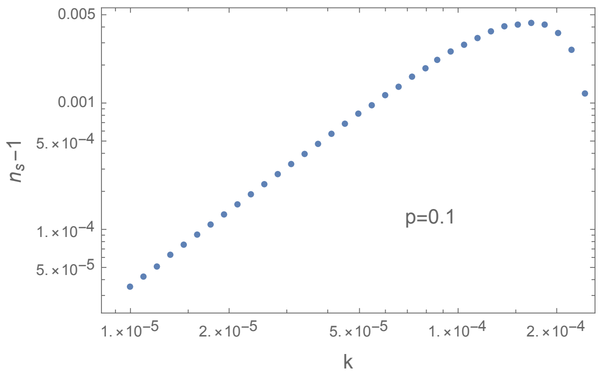

Fig. 3. For the power spectrum displayed in Fig. 2, we evaluate the spectral index n_s at the transition point $t \approx 3.02 \times 10^3$ in the expanding branch and find n_s is larger than unity in the scale invariant regime of the power spectrum.

4. Conclusion

In this paper, we study a general matter-Ekpyrotic scenario in an isotropic and homogeneous FLRW universe in LQC. Using extensive numerical simulations, we establish some general features of the background dynamics and the scalar power spectrum, some of which were not seen in earlier works that either dealt with special cases or were limited to qualitative analysis. A bouncing background cosmology, characteristic of LQC, is obtained where a contracting and an expanding branch connected through a bounce. We find that the background dynamics in each branch is characterized in general by two distinct phases. It starts out in the far past with dust domination and transitions to Ekpyrotic field dominated phase which overlaps with the high energy quantum regime at the bounce. After the bounce there is another transition back to dust dominated universe in the expanding branch. Further we find that the Ekpyrotic field dominated phase generally also contains a phase of ultrastiff equation of state ($w > 1$) leading up to the bounce, which is essential in avoiding the BKL instability. We varied the parameters of the potential and also varied the relative proportion of initial dust density (while keeping it sub-dominant at the bounce), and found the above mentioned features of the background dynamics to hold in general.

We found that the duration of the Ekpyrotic field dominated phase and the $w > 1$ phase can be increased by increasing the width of the Ekpyrotic field potential or by decreasing the relative proportion of initial dust density. In contrast, increasing the depth of the potential has the effect of shortening the Ekpyrotic field dominated phase while increasing the $w > 1$ phase at the same time.

For the study of comiving curvature perturbations, we give our initial conditions in the far past in the dust-dominated classical phase of the contracting branch in order to include the full effect of the quantum bounce on the power spectrum. We employ the classical Mukhanov-Sasaki equation for the dust-dominated contraction phase as the universe is large and classical in this phase. Since the Ekpyrotic field dominated phase overlaps with the quantum bounce regime, we have employed the dressed metric approach to obtain the effective evolution equation for quantum perturbations on a quantum spacetime in this regime. This is a deliberate choice as previous studies employing the deformed algebra approach have faced difficulty in analyzing ultraviolet modes near the bounce. Using the dressed metric approach for the bounce regime has allowed us to fully explore the scalar power spectrum in the bounce regime.

We found that the perturbation modes exiting the horizon during dust dominated phase have a scale invariant spectrum. These modes then maintain their scale invariance as they pass through the Ekpyrotic field dominated phase, the $w > 1$ phase and the bounce, until they start re-entering the horizon in the expanding branch after which they become oscillatory. However, unlike previous studies considering a constant equation of state for the Ekpyrotic field, the magnitude of the scale invariant modes gradually increases during the Ekpyrotic-field dominated contraction, then rapidly increases during the bounce regime and then gradually decreases as the universe expands. This is also in contrast to inflationary scenarios where the magnitude is frozen after the modes exit the horizon during inflation. Further, the duration of the Ekpyrotic field dominated not only impacts the magnitude of the scale invariant power spectrum, but also impacts the range of comoving modes that lie in the scale invariant part of the spectrum.

We also studied the spectral index of the scale invariant part of the spectrum and found that it is inconsistent with current observational bounds. This is due to the fact that perturbation originating in a purely dust-dominated phase of contraction have an exactly scale invariant spectrum, while the observed spectrum has a slight red tilt. Thus changing potential parameters of the Ekpyrotic field potential has no effect on this feature.

Based on our results, further studies are needed to fully explore the possibilities presented by the matter-Ekpyrotic scenario in LQC. In particular, a lower bound on the minimum number of e-foldings required of the $w > 1$ phase needs to be rigorously established by studying the Ekpyrotic scenario in more general anisotropic spacetimes. Further, a suitable extension of the matter-Ekpyrotic scenario needs to be developed where the perturbations finally exit to a radiation-dominated universe. Bounds on the duration of the Ekpyrotic field dominated phase also needs to be established by employing its impact on the range of scale invariant modes and their magnitude. The issue of the inconsistent spectral index can addressed if instead of sourcing perturbations from a purely dust-dominated phase, we have a matter field with slightly negative equation of state. Although, preliminary studies following these lines have been carried out in the literature in various bouncing cosmologies including those of LQC, but rigorous and extensive studies taking the most general matter-Ekpyrotic scenario need to be done.

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