

SHELL MODEL FEATURES FROM BARE POTENTIALS

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Shell model features are investigated for the Deuterium and ^4He with a semirealistic potential, using the Hybrid Multideterminant method applied to the bare Hamiltonian. For a sufficiently large single-particle space we found that few Slater determinants account for most of the binding energy of ^4He . Using only one Slater determinant with all symmetries restored with the appropriate projectors to good quantum numbers, we can account for about 84% of the binding energy of ^4He .

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1. Introduction

The Shell model is the fundamental tool used to study the structure of nuclei. The basic idea is that the nucleons move in an average potential generated by the other nucleons. The single-particle wave functions of the nucleons in such an average potential can be used to construct antisymmetrized product wave functions that in turn are used to diagonalize the many-body Hamiltonian. A major obstacle in this description is that the nucleon–nucleon potential is strongly repulsive at small distance between the particles. Nowadays there are several methods that are used to lessen the problem. In several descriptions, the bare Hamiltonian is replaced with an effective Hamiltonian which preserves the low-energy part of the spectrum and no longer has the repulsive core. The Lee–Suzuki method (Ref. [1,2]), the G -matrix method (see for example Ref. [3]), the unitary correlation operator method (Ref. [4]), are methods that belong to this class. In other approaches, the many-body wave function is assumed to be a product of correlation functions (or, more generally, correlation operators) which suppress the probability of having nucleons close together and thus account for

the effects of the repulsive core, and a Slater determinant (see for example Ref. [5]). It is common notion that shell effects are properties of these effective Hamiltonians or of the residual Slater determinant, in the second class of approach. Also, it is widely acknowledged that the use of single-particle states as a basis, requires an extremely large number of Slater determinants to reach a reasonable description of the system, if bare Hamiltonians are used. This however induces to some considerations.

Let us consider for example the Lee–Suzuki approach. In this approach the many-body Hilbert space is divided into two parts, the so-called P -space or shell model space, and the remaining Q -space, the excluded space. The main goal of the approach is to find a similarity transformation X such that the Hamiltonian H becomes $\mathcal{H} = X^{-1}HX$. The requirement on the operator X is that the transformed Hamiltonian $\mathcal{H}$ should not have matrix elements between the P -space and the Q -space, hence only the P -space can be used to obtain the low-energy part of the spectrum of the Hamiltonian H . The P -space submatrix of the transformed Hamiltonian is the effective Hamiltonian. The P -space and the Q -space are obtained with the projectors P and Q respectively acting on the full Hilbert space. As a consequence, if ϕ is a wave function with components in the P -space only,

$$\mathcal{H}\phi = (P + Q)\mathcal{H}(P + Q)\phi = P\mathcal{H}P\phi + Q\mathcal{H}P\phi.$$

Therefore, if $Q\mathcal{H}P = 0$ (the decoupling condition), then $\mathcal{H}\phi = P\mathcal{H}P\phi$ and thus the effective Hamiltonian does not connect states in the P -space with the Q -space, that is, it does not scatter P -space components into the Q -space. If ϕ is an eigenstate of $P\mathcal{H}P$ then $X\phi$ is an eigenstate of the bare Hamiltonian H with the same eigenvalue. There is not a unique solution to the decoupling equation $Q\mathcal{H}P = 0$ and typically a solution of the form $X = \exp \omega$ with $\omega^2 = 0$ is assumed. Differently stated, ω has only non-zero matrix elements of the type ω_{QP} , moreover the wave function ϕ is the P -space part of the eigenstate of the bare Hamiltonian and $\omega\phi$ is the Q -space part of the eigenstate.

We are free to select the P -space. A convenient choice is the Hilbert space spanned by a set of physically motivated single-particle states. However if we increase the size of the P -space we end up recovering the full Hilbert space as a limit. If shell effects are properties of the effective Hamiltonian in the P -space, then these shell effects should remain as we increase more and more the P -space and hence shell effects should be also seen using bare Hamiltonians. Clearly, this motivates the study of shell effects in simple bare Hamiltonians. Certainly we do not suggest that this is the method to be used for realistic calculations. The method of constructing effective Hamiltonians is practical, reasonable accurate, and unavoidable.

In this work we consider a simple semirealistic bare Hamiltonian. The Modified Afnan–Tang S3 potential (MATS3) (Ref. [6]), which consists of the following terms dependent on the distance r between the nucleons

$$v = v_s(r)I_{01} + v_t(r)I_{10} + v_o(r)(I_{00} + I_{11}), \quad (1)$$

where I_{ST} is the projector to the values (ST) of the spin and isospin of the two nucleons, has been widely used in testing many-body methods and central correlations (see for example Refs. [4,7-9]). The singlet, triplet and odd potentials v_s, v_t, v_o are a sum of Gaussians and contain a moderately strong repulsive core. The largest values of the core (at $r = 0$) are 811.0 MeV, 630.3 MeV and 1000.0 MeV in the singlet, triplet and odd channels, respectively. The Afnan–Tang S3 potential does not include the odd-channel. We consider the case of the Deuterium in a semi-analytical method and the case of ${}^4\text{He}$. The results discussed in this work can be understood in the following way, especially the ones relative to the closed shell case of ${}^4\text{He}$.

A given number of major oscillator shells can be used to define a truncated bare Hamiltonian. We found that few Slater determinants are necessary to obtain the ground-state energy of this truncated bare Hamiltonian. As the number of major oscillator shells is increased, we obtain better approximations to the bare Hamiltonian in the full Hilbert space and again few Slater determinants account for the, now much lower, binding energy. In the largest single-particle space that we were able to use, a single Slater determinant properly projected to good angular momentum and parity $J^\pi = 0^+$ accounts for about 84% of the binding energy of ${}^4\text{He}$. We evaluated the binding energy in the full Hilbert space using the effective Hamiltonian obtained with the Lee–Suzuki method. In constructing the effective Hamiltonian, we have used the variant of the method which gives a hermitian effective Hamiltonian in a way very similar to the no-core shell model approach of Ref. [10].

Therefore, the problem with shell effects, at least for ${}^4\text{He}$ and for the potential used in this work, does not seem to originate from the fact that the bare Hamiltonian requires a very large number of Slater determinants, but rather, that the truncated bare Hamiltonian matrix is a poor approximation to the one evaluated in the full Hilbert space. That is, the number of single-particle states may not be sufficiently large. Differently stated, the exact binding energy of the bare truncated Hamiltonian problem is much smaller than the one in the full space.

The method used in evaluating the binding energy for both the truncated bare and the effective Hamiltonian (for comparison) is the Hybrid Multideterminant method (Ref. [11]), whereby the many-body wave function is written as a linear combination of symmetry unrestricted Slater determinants and exact quantum numbers are restored with the appropriate projectors.

That is

$$|\psi\rangle = \hat{P} \sum_{\alpha=1}^{N_w} g_{\alpha} |\phi, \alpha, n\rangle |\phi, \alpha, p\rangle. \quad (2)$$

The labels n and p refer to neutrons and protons respectively, $\hat{P}$ is the projector that restores the exact quantum numbers and $|\phi, \alpha, \tau = n, p\rangle$ is a Slater determinant built from the generalized creation operators

$$c_n^{\dagger}(\alpha, \tau) = \sum_{i=1}^{N_s} U_{i,n}(\alpha, \tau) a_{i,\tau}^{\dagger}. \quad (3)$$

that is

$$|\psi\rangle = \sum_{\alpha=1}^{N_w} g_{\alpha} \prod_{\tau=n,p} c_1^{\dagger}(\alpha, \tau) c_2^{\dagger}(\alpha, \tau) \dots c_{N_{\tau}}^{\dagger}(\alpha, \tau) |0\rangle. \quad (4)$$

The meaning of the symbols in Eq. (3) and Eq. (4) is the following. The complex coefficients $U_{i,n}(\alpha, \tau)$ and g_{α} are determined variationally minimizing the expectation values of the Hamiltonian $\langle\psi|H|\psi\rangle/\langle\psi|\psi\rangle$, $a_{i,\tau}^{\dagger}$ is the operator that creates a nucleon in the harmonic oscillator single-particle state i and $|0\rangle$ is the particle vacuum. The minimization of the expectation values of the Hamiltonian is carried out using the powerful quasi-Newtonian gradient methods as described in detail in Ref. [11]. We have used two sets of projectors $\hat{P}$. In some cases we have used the projectors to good z -projection of the angular momentum and parity $J_z^{\pi} = J^{\pi}$. In some other cases we have used the full angular momentum projector to good J^{π} . This last type of projector decreases the number of Slater determinants necessary to accurately describe the ground-state. A projected, symmetry unrestricted Slater determinants is equivalent to a very large number of m-scheme Slater determinants as it can be seen by substituting Eq. (3) in Eq. (4). Precisely the inclusion of a very large number of m-scheme Slater determinants in just one symmetry-unrestricted Slater determinant with good quantum numbers restored with the projectors before variation, makes this method the ideal one to study shell effects. The method brings together the advantages of the Quantum Monte Carlo Diagonalization method (QMCD) (Ref. [12]) and of the VAMPIR method (Ref. [13]).

In Section 2 we analyze the case of Deuterium mostly in a semianalytical fashion and in Section 3 we discuss the case of ^4He .

2. Deuterium

For the case of Deuterium we first diagonalize the intrinsic Hamiltonian

$$H = -\frac{\hbar^2}{M}\Delta + v_t(r), \quad (5)$$

since only the triplet channel gives the bound state. M in Eq. (5) is the nucleon mass. We took $\hbar^2/2M = 20.72125 \text{ MeV fm}^2$. The ground-state wave function $\phi_{\text{rel}}(r)$ has an intrinsic energy of -2.222 MeV , r being the relative distance. In order to localize the system we multiply this intrinsic wave function by a normalized Gaussian of the center of mass $\psi_{\text{cm}}(R_{\text{cm}}) = N \exp(-aR_{\text{cm}}^2/2)$ (we selected $a = 1.4478 \text{ fm}^{-2}$)

$$\phi(\vec{r}_n, \vec{r}_p) = \psi_{\text{rel}}(r) \psi_{\text{cm}}(R_{\text{cm}}), \quad (6)$$

r_n and r_p being the position of the neutron and the proton, respectively. The total wave function of Eq. (6) has an angular dependence on $\cos \theta_{np}$, where θ_{np} is the angle between the vectors $\vec{r}_n$ and $\vec{r}_p$. It can be analyzed in terms of the Legendre polynomials P_l , that is

$$\phi(\vec{r}_n, \vec{r}_p) = \sum_{l=0}^{\infty} \xi(r_n, r_p) P_l(\cos \theta_{np}), \quad (7)$$

using the well known identity that relates the Legendre polynomials and the spherical harmonics, one has

$$\phi(\vec{r}_n, \vec{r}_p) = \sum_{l,m} \frac{4\pi}{2l+1} \xi_l(r_n, r_p) Y_{lm}(\hat{r}_n) Y_{lm}^*(\hat{r}_p). \quad (8)$$

In order to disentangle entirely this wave function, we need to expand the radial function $\xi(r_n, r_p)$ in terms of radial single-particle wave functions. Let us discretize the lengths r_n and r_p and consider the eigenvalue problem for the matrix $r_n \xi_l(r_n, r_p) r_p$, that is

$$r_n \xi_l(r_n, r_p) r_p = \sum_k R_{l,k}(r_n) E_{l,k} R_{l,k}(r_p) \quad (9)$$

after inserting into Eq. (8), and redefining the weights $E_{l,k}$ to include the coefficient $4\pi/(2l+1)$, we obtain

$$\phi(\vec{r}_n, \vec{r}_p) = \sum_{l,m,k} E_{l,k} \frac{R_{l,k}(r_n)}{r_n} Y_{lm}(\hat{r}_n) \frac{R_{l,k}(r_p)}{r_p} Y_{lm}^*(\hat{r}_p). \quad (10)$$

We can properly normalize the radial wave functions by integration and redefine again the weights $E_{l,k}$. The normalization of the wavefunctions implies the sum rule

$$\sum_{k,l} (2l+1) E_{l,k}^2 = 1. \quad (11)$$

Equation (10) shows that the wave function has been recast in a shell model expansion. For this interpretation to be valid some conditions must be met. Even if the intrinsic wave function is suppressed at short distances, only few weights must contribute appreciably to the sum rule of Eq. (11). Moreover, the single-particle radial wave functions must be localized around the origin and, since they do not satisfy an oscillation theorem in any obvious way, should have approximately the right nodal structure. It is not simple to determine accurately these radial wave functions, since for large l, k values their probabilities in the total deuterium wave function are very small. The nodal structure of these radial wave functions can be different from the one obtained by diagonalizing a single-particle Schroendinger equation. Here we are diagonalizing a two-particle wave function. In Table I we show the largest values of the probabilities $E_{l,k}^2$ for some l values.

TABLE I

Probabilities E_{lk}^2 of having single-particle states l, k in the Deuterium wave function. The sum rule of Eq. (11) obtained with these states is 0.9928.

l	k	$E_{l,k}^2$	l	k	$E_{l,k}^2$
0	1	0.8072767	1	1	0.0332132
0	2	0.0194795	1	2	0.0018147
0	3	0.0014325	1	3	0.0003395
0	4	0.0004355	1	4	0.0001639
0	5	0.0001581	1	5	0.0000192
0	6	0.0000213	1	6	0.0000105
2	1	0.0067855	3	1	0.0014879
2	2	0.0006597	3	2	0.0001814
2	3	0.0001923	3	3	0.0000863
2	4	0.0000875	3	4	0.0000259
2	5	0.0000134	3	5	0.0000057
4	1	0.0004020	5	1	0.0001221
4	2	0.0000570	5	2	0.0000197
4	3	0.0000297	5	3	0.0000095
4	4	0.0000096	5	4	0.0000034

As it can be seen from Table I, the probabilities of having a given l, k single-particle state in the deuterium wave function decrease very rapidly with increasing l or k . However, neglecting small components in the expansion of the wave function in Eq. (10), can give sizable errors to the energy.

In Figs. 1–3 we plot the dominant radial wave functions up to $l=2$. As it can be seen, the nodal structure is not similar to the one of the harmonic oscillator, or to the one obtained diagonalizing a single-particle Schroedinger equation. These nodal structures can be different depending on the two-particle wave-function, sometimes nodes are missing and sometimes there are more nodes than one would expect. Moreover, sometimes they appear in intervals where the radial wave function is rather small and thus their absence or presence is irrelevant. Structures like the one displayed in Table I and in Figs. 1–3 appear also by choosing a different nucleon–nucleon

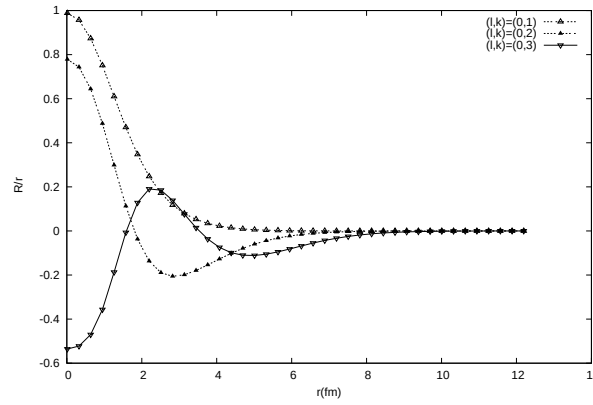


Fig. 1. $I = 0$ Radial wave function R/r .

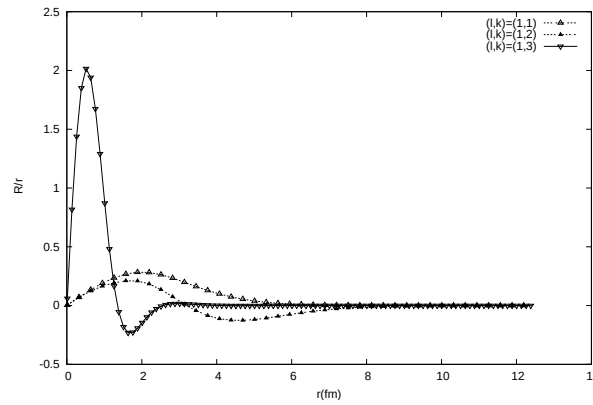


Fig. 2. $I = 1$ Radial wave function R/r .

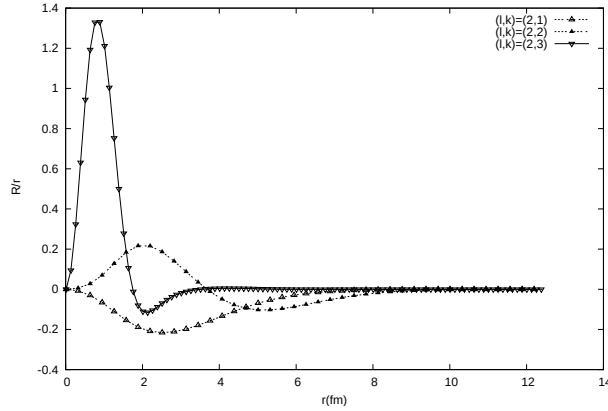


Fig. 3. $I = 2$ Radial wave function R/r .

potential (for example the Malfliet–Tjon V potential of Ref. [14]). These structures are qualitatively present even if we replace the repulsive core with a much stronger one, although we need more l, k radial wave functions to have a good approximation to the ground state. Moreover, if we consider mathematical models without the repulsive core, the nodal structure becomes similar to the familiar one where $k - 1$ is the number of nodes.

Since the set of single-particle wave functions appearing in Eq. (10) is complete, it can in principle be used as the set of spherical single-particle wave functions, instead of the harmonic oscillator wave functions, in order to use the Hybrid Multideterminant method for many-body calculations. We preferred not to do so since the Lee–Suzuki prescription is very simple to implement in an harmonic oscillator basis and computationally very expensive in any other basis. For Deuterium we performed this analysis to show in a simple case how typical shell structure emerges. The real interesting case is ^4He which will be discussed in the next section. Before leaving this section it is worth mention that this analysis is particularly simple in the case tensor forces are not included.

3. ^4He

We considered an harmonic oscillator single-particle basis to define a truncated bare Hamiltonian. The harmonic oscillator frequency enters the calculation only via the matrix elements of the Hamiltonian. Explicitly the MATS3 potential is given as follows. Strengths are in MeV and distances in fm. The singlet, triplet and odd potentials are of the form $\sum_{i=1}^3 w_i \exp -\gamma_i r^2$. For the singlet $w_1 = 1000.0$, $w_2 = -166.0$, $w_3 = -23.0$ and $\gamma_1 = 3.0$, $\gamma_2 = 0.8$ and $\gamma_3 = 0.4$. For the triplet $w_1 = 1000.0$, $w_2 = -326.7$, $w_3 = -43.0$ and $\gamma_1 = 3.0$, $\gamma_2 = 1.05$ and $\gamma_3 = 0.6$. For the odd potential $w_1 = 1000.0$, $w_2 = w_3 = 0.0$ and $\gamma_1 = 3.0$.

We considered several levels of truncation. We selected $\hbar\Omega = 47$ MeV, as discussed below, and evaluated the intrinsic energy (*i.e.* the center of mass energy has been removed). As a first level we considered only the $0s, 1p$ single particle states ($N_{\text{ho}} \leq 1$). In this case the ground state energy evaluated with a single Slater determinant (with a $J^\pi = 0^+$ projector) is positive and the addition of several Slater determinants does not produce to a bound state.

Considering instead 3 major oscillator shells (that is $N_{\text{ho}} \leq 2$) and again using the full angular momentum projector with one Slater determinant, ($N_w = 1$) the energy is -4.446 MeV, for $N_w = 2$ the energy is lowered to -5.081 MeV, with $N_w = 3$, it becomes -5.194 MeV and with $N_w = 5$, -5.212 MeV. Comparing the results for increasing number of Slater determinants we can see that 5 Slater determinants are sufficient to describe the ground state (there is still some room for a further small decrease in the energy). Between 1 and 5 Slater determinants the decrease of the energy is 0.776 MeV indicating that just one Slater determinant accounts for most of the binding energy of this severely truncated Hamiltonian problem.

As the next case we considered $N_{\text{ho}} \leq 3$. In this case for $N_w = 1$ the energy is $E = -9.202$ MeV, for $N_w = 2$ $E = -10.268$ MeV, for $N_w = 3$, $E = -10.446$ MeV and with $N_w = 5$ we obtain $E = -10.52$ MeV. Again, as before, 5 Slater determinants describe sufficiently well the ground state and just one Slater determinant accounts for about 87% of the binding energy of the truncated bare Hamiltonian problem.

Next we considered $N_{\text{ho}} \leq 4$. The results for energies in MeVs, evaluated again with the $J^\pi = 0^+$ projector before variation, are -15.913 , -17.102 , -17.230 and -17.429 for $N_w = 1$, $N_w = 2$, $N_w = 3$, $N_w = 5$, respectively. Again we see the previous pattern, that is, one Slater determinant accounts for most of the binding energy of the bare truncated problem and few Slater determinants accurately describe the ground state since increasing the number of Slater determinants, the energy converges.

We kept increasing the single-particle basis in order to approach more and more the full space. Since the calculations become time consuming using the $J^\pi = 0^+$ projector, we performed the calculations using just one Slater determinant. For $N_{\text{ho}} \leq 5$ the energy is $E = -19.229$ MeV. For $N_{\text{ho}} \leq 6$ we obtained $E = -22.388$ MeV. For $N_{\text{ho}} \leq 8$, we included in the single-particle basis all states with $l \leq 3$ and for $l > 3$ we could include only the single-particle states $(n, l) = (0, 4)$, $(n, l) = (1, 4)$ and $(n, l) = (0, 5)$ since the calculation becomes too time consuming (all these calculations were performed on personal computers). In this last case the energy with $N_w = 1$ is -24.077 MeV.

The largest set of single-particle states we were able to use, consists of the following states. For $l = 0$, $n = 0, 1, \dots, 8$; for $l = 1$ and $l = 2$, $n = 0, 1, 2, 3, 4$; for $l = 3$, $n = 0, 1, 2, 3$; for $l = 4$, $n = 0, 1, 2$ and $n = 0$, $l = 5$ the total number of single-nucleon states is 460 and with one Slater determinant, we obtained $E = -25.202$ MeV.

We determined the most convenient harmonic oscillator frequency using $N_{\text{ho}} \leq 8$ and just one J_z^π projected Slater determinant. We changed the frequency and selected the one that gives the lowest energy. The partial J_z^π was used to keep down the computational cost. In order to have an idea of how the energy changes with the $\hbar\Omega$, we considered (in MeV) $\hbar\Omega = 30, 35, 40, 45, 47, 50$ and obtained for the energy the values (in MeV) $-13.55, -14.62, -15.18, -15.40, -15.42, -15.40$, respectively. As it can be seen, despite the very large change in the values of $\hbar\Omega$, the energy does not change much. The value of the frequency determined in this way, was also used for the other truncated problems with the full J^π projector.

The harmonic oscillator frequency is just a parameter that defines the variational wave-function. Any value is acceptable and of course the most convenient is the one that gives the lowest energy for the bare truncated problem. We opted for this method rather than repeat the full set of calculations with several frequencies, since the computational cost would be several times larger.

For comparison we also have evaluated the ground state energy using a renormalized Hamiltonian obtained using the Lee–Suzuki prescription along the same lines of the no-core shell model approach (Ref. [10]). The P -space consists of all the Slater determinants that can be formed using 5 major oscillator shells (we considered $\hbar\Omega = 33$ MeV in this case) and the $P + Q$ -space, of all Slater determinants that can be built with $N_{\text{ho}} \leq 200$. The value for the ground state energy is as follows. Using again the $J^\pi = 0^+$ projector, for $N_w = 1, 2, 3, 5, 10$ we obtained $E = -29.318$, $E = -29.685$, $E = -29.932$, $E = -29.994$ and $E = -30.030$, respectively. The calculation performed with the truncated bare Hamiltonian in the largest single-particle space that we were able to use ($E = -25.202$) with just one Slater determinant, accounts for 84% of the binding energy.

Using the truncated bare Hamiltonian, we also evaluated the fractional number of neutrons and protons in the n, l, j orbits. The results are shown in Table II. From Table II we can see clearly the shell closure of ^4He . We can also infer that higher l values should have been included, since the value of $\hbar\Omega = 47$ MeV is large.

Using this large single-particle space, we also performed a calculation using the J_z^π projector. In this case the number of Slater determinants necessary to describe the ground state is much larger. The results for the energy are the following. For $N_w = 1, 2, 3, 5, 10, 15, 25$ the energies are (in MeVs)

TABLE II

Fractional number of neutrons (protons) f_n (f_p) for the projected Hartree–Fock in the largest space discussed in the text.

n	l	j	f_n	f_p	n	l	j	f_n	f_p
0	0	1/2	1.637818	1.638007	1	0	1/2	0.265322	0.265207
2	0	1/2	0.034468	0.034500	3	0	1/2	0.010066	0.010059
4	0	1/2	0.002290	0.002288	5	0	1/2	0.000906	0.000900
6	0	1/2	0.000203	0.000202	7	0	1/2	0.000095	0.000092
8	0	1/2	0.000005	0.000005	0	1	3/2	0.009915	0.009884
0	1	1/2	0.004204	0.004238	1	1	3/2	0.004461	0.004440
1	1	1/2	0.002094	0.002093	2	1	3/2	0.001926	0.001931
2	1	1/2	0.000941	0.000932	3	1	3/2	0.000399	0.000393
3	1	1/2	0.000190	0.000195	4	1	3/2	0.000090	0.000091
4	1	1/2	0.000042	0.000041	0	2	5/2	0.003157	0.003157
0	2	3/2	0.002003	0.001928	1	2	5/2	0.002947	0.002945
1	2	3/2	0.001957	0.001944	2	2	5/2	0.000939	0.000938
2	2	3/2	0.000617	0.000621	3	2	5/2	0.000289	0.000286
3	2	3/2	0.000185	0.000184	4	2	5/2	0.000118	0.000119
4	2	3/2	0.000073	0.000073	0	3	7/2	0.002229	0.002223
0	3	5/2	0.001644	0.001664	1	3	7/2	0.001499	0.001507
1	3	5/2	0.001120	0.001115	2	3	7/2	0.000464	0.000465
2	3	5/2	0.000348	0.000346	3	3	7/2	0.000196	0.000195
3	3	5/2	0.000145	0.000146	0	4	9/2	0.000994	0.000996
0	4	7/2	0.000783	0.000778	1	4	9/2	0.000742	0.000745
1	4	7/2	0.000591	0.000590	2	4	9/2	0.000334	0.000335
2	4	7/2	0.000262	0.000262	0	5	11/2	0.000492	0.000498
0	5	9/2	0.000413	0.000417					

−16.097, −21.054, −23.316, −25.156, −26.494, −26.955, and −27.476, respectively. Comparing the results obtained with 15 and 25 Slater determinants, the energy decrease is 531 KeV, indicating, that convergence has not been completely reached. Presumably the exact ground state energy for this truncated bare problem is several hundred KeV lower.

In conclusion, we have performed ground state energy calculations using bare Hamiltonians, in a truncated shell model space. The results indicate that one or few Slater determinants (with all symmetries restored by the appropriate projectors before variation) account for most of the binding energies of the exact, truncated bare problem, and that, by increasing the

shell model space, this feature remains and it reasonably approximates the binding energies obtained with microscopically constructed effective Hamiltonians. We considered of course the best case of a closed shell nucleus. It should be stressed that the size of the shell model space necessary for reasonable convergence strongly depends on the potential employed in the calculations. The softer the potential, the smaller the shell model space necessary for reasonable convergence. The “harder” the potential, the larger the shell model space required for convergence.

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