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$F(R)$ gravity dark energy model of an interaction between dark radiation and dark matter

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Abstract In this work, we will study the late-time dynamic evolution of the $F(R)$ gravity dark energy model of an interaction between dark radiation and dark matter. We explore the possibility of the apparent extra dark radiation being linked directly to the physics of cold dark matter (CDM). In particular, we consider a generic scenario where dark radiation, as a result of an interaction, is produced directly by a fraction of the dark matter density effectively decaying into dark radiation. The decay process is controlled by its rate $Q = \alpha H \rho_{dm}$, where α is the (constant) dimensionless parameter quantifying the strength of the decay mechanism, for the weak coupling between dark matter and dark radiation $\alpha \ll 1$. We will solve the field equations numerically by using the statefinder function y_H to appropriately represent the field equation. The $F(R)$ gravity model we consider is described by a nearly R^2-model at early epochs and at late-time the model mimic the Λ-Cold-Dark-Matter model (ΛCDM), and we fine tune the parameters to achieve viability and compatibility with the latest Planck constraints at late-time. Furthermore, we consider the behavior of several well-known statefinder quantities, we find that $statefinder$ diagnostic and Om diagnostic can not only break the degeneracy of different parameter values in the $F(R)$ gravity dark energy model, but also effectively distinguish the difference between the $F(R)$ gravity model and the ΛCDM model. At last, we compared the theoretical results with the $SNIa$ data.

1 Introduction

The current cosmic acceleration is supported by various observations such as Supernovae Ia ($SNIa$) [1], large scale structure (LSS) [2] with baryon acoustic oscillations (BAO) [3], cosmic microwave background (CMB) [4] radiation and weak lensing [5]. Dark energy is a mystery in modern theoretical cosmology, and several theoretical proposals can in principle describe successfully this cosmological era [6]. Modified gravity constitutes a very powerful and natural possibility to unify the physics of the early time inflation epoch with that of the late-time acceleration stage, under the frame of a common theory [7] and this without the need to introduce any extra fields (scalar, spinor, or other) as dark components.

The most prominent and simplest of the modified gravity is $F(R)$ gravity [8–18]. At first time the unification of the inflation with dark energy epoch in $F(R)$ gravity was proposed in [10]. The successful operation of WMAP and Planck, as well as the large-scale galaxy survey and Hubble Space Telescope observations have provided us with a large amount of observational data, which represents the era of "precision cosmology" has arrived. However, when fit to measurements of the early universe, the ΛCDM model finds results inconsistent with observations of the late universe. These include the persistent Hubble tension as well as the milder S_8 tension [19]. One possible explanation for this difference is that the dark matter (DM) is unstable in whole or in part, if the decay products are new massless or very light dark states, such that a small fraction of the DM is effectively converted into relativistic "dark" radiation. Some literature suggests that this conversion can ease the possible tension between CMB measurements and large scale structure observations, so this paper studies the $F(R)$ gravity dark energy model in which dark radiation interacts with dark matter [20–23]. We explore the possibility of the apparent extra dark radiation being linked directly to the physics of cold dark matter (CDM). we consider dark radiation as a result of an interaction, is produced by the direct decay of the dark matter density. The decay process is controlled by its rate $Q = \alpha H \rho_{dm}$, where α is the (constant) dimensionless parameter quantifying the strength of the decay mechanism, for the weak coupling between dark matter and dark radia-

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tion $\alpha \ll 1$, H is the Hubble rate in the spatially flat $FLRW$ universe, and ρ_{dm} is the energy density of the decaying cold dark matter [24]. We will solve the field equations numerically by using the statefinder function y_H to appropriately represent the field equation. The $F(R)$ gravity model we consider is described by a nearly R^2-model at early epochs and at late-time the model mimic the Λ-Cold-Dark-Matter model (ΛCDM). By fine-tuning the parameters, we produce a viable late-time phenomenology. Furthermore, we study the behavior of several well-known statefinder functions include $statefinder$ diagnostic and Om diagnostic. We find that the statefinder can not only break the degeneracy of different parameter values in this model, but also can effectively distinguish the $F(R)$ gravitational dark energy model from the ΛCDM model. Finally, we compare the theoretical results with the $SNIa$ data.

The paper is organized as follows: in Sect. 2, we quantify the generation of dark radiation and solve the background solution. In Sect. 3, we propose and discuss a theoretical model of $F(R)$ gravity. We introduce the function y_H and we rewrite the Friedmann equation with y_H and its derivative. Furthermore, in Sect. 4, we numerically study the late-time behavior of the $F(R)$ gravity dark energy model. In Sect. 5, we explore the evolutionary behavior of the $F(R)$ dark energy model by diagnostics. In Sect. 6, we present the cosmic behavior observably transparently. Finally, the paper summarizes the results of this paper.

2 Interaction between dark radiation and dark matter

If dark radiation and dark matter together belong in the dark zone, then an interaction between the two is possible. General coupling (at the background level) can be described by the energy balance equation [25–29]

$$\dot{\rho_{dm}} + 3H\rho_{dm} = -Q \tag{1}$$
$$\dot{\rho_{dr}} + 3H(1+w)\rho_{dr} = Q, \tag{2}$$

where the ρ_{dm} and ρ_{dr} are the dark matter and dark radiation energy densities, respectively, and $H = \dot{a}/a$ is the Hubble rate, where a is the scale factor and overdots denote derivatives with respect to time. The positive energy transfer rate Q indicates the direction of energy transfer from the dark matter to the dark radiation. This means that dark matter and dark radiation are no longer redshifted as before. And we need a covariant form of the energy–momentum transfer. We adopt the four-vector form of energy–momentum transfer introduced in [30]

$$Q = \Gamma\rho_{dm}, \tag{3}$$

where Γ is the constant interaction rate, in the dark matter framework, $\Gamma > 0$ corresponds to the decay of dark matter towards dark energy. Many forms of Γ have been studied in the literature [31–35]. Here, we explore the simple case of $\Gamma = \alpha H$, where α is the constant and H is the Hubble rate. Implied behind this Γ form, as described in [25], is the assumption that the interaction rate varies over time and not in space, which explains why H replaces the interaction rate in Eq. (3). This form of Γ can be generated from different models of dark matter decay. Later, we show a $F(R)$ gravity model of dark matter decay to dark radiation and show that the Γ of the above form can be easily implemented in nature. For $\Gamma = \alpha H$, the background evolution is easily solved

$$\rho_{dm} = \rho^0_{dm} a^{-(3+\alpha)} \tag{4}$$
$$\rho_{dr} = \rho^0_{dr} a^{-3(1+w)} + \frac{\alpha}{\alpha - 3w}\rho^0_{dm} a^{-3}(a^{-3w} - a^{-\alpha}), \tag{5}$$

superscript 0 indicates values today. For $w = \frac{1}{3}$, Eq. (5) can be further reduced to

$$\rho_{dr} = \beta a^{-4} + \frac{\alpha}{1-\alpha}\rho^0_{dm} a^{-(3+\alpha)}, \tag{6}$$

where β is a constant. The first term in Eq. (6) appears as the standard radiation density, and the second term appears as a fluid with the equation of state of $\alpha/3$. For the weak coupling between dark matter and dark radiation, $\alpha \ll 1$, yielding the $\beta \sim \rho^0_{dr}$.

3 $F(R)$ gravity dark energy model

The four-dimensional action of $F(R)$ is generally given by a general function of the Ricci scalar R

$$S = \int \frac{F(R)}{2\kappa^2}\sqrt{-g}d^4x + S_m, \tag{7}$$

where $S_m = \int d^4x L_m, \kappa^2 = 8\pi G$, L_m stands for the Lagrangian of the perfect matter fluids that are considered present. g denotes the determinant of the metric tensor $g_{\mu\nu}$. If $F(R) = R$, it can return to the action of general relativity. Varying the action by the metric, we obtain the equations for gravitational field

$$F_R(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - [\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]F_R(R) = \kappa^2 T_{\mu\nu}, \tag{8}$$

where $F_R(R) = \frac{dF(R)}{dR}$, ∇ is the covariant derivative with respect to the coordinate, $\Box = g_{\mu\nu}\nabla_\mu\nabla_\nu$, $T_{\mu\nu}$ is the energy–momentum tensor defined by the relation

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g_{\mu\nu}}. \tag{9}$$

Next, consider the spatially flat universe described by the Friedman–Lemetre–Robertson–Walker (FLRW) metric

$$ds^2 = -dt^2 + a(t)\sum_{i=1}^{3} dx_i^2, \tag{10}$$

where $a(t)$ is the scale factor. By varying the Eq. (8) with respect to the metric, we obtain the gravitational equations of motion

$$3F_R H^2 = \kappa^2 \rho_m^{tot} + \frac{F_R - F}{2} - 3H\dot{F}_R, \tag{11}$$

$$-2F_R\dot{H} = \kappa^2(\rho_m^{tot} + P_m^{tot}) + \ddot{F}_R - H\dot{F}_R, \tag{12}$$

where "dot" denotes derivative with respect to cosmic time. $H = \frac{\dot{a}}{a}$ is the Hubble parameter and ρ_m^{tot},P_m^{tot} stand for the matter fluids energy density and the corresponding pressure respectively. We can rewrite the equation in Eqs. (11) and (12), as follows

$$H^2 - (F_R - 1)(H\frac{dH}{dlna} + H^2) + \frac{1}{6}(F - R) + H^2 F_{RR} \frac{dR}{dlna} = \frac{\rho_m^{tot}}{3}, \tag{13}$$

with R denoting the scalar curvature as usual, which can be rewrite as follows

$$R = 12H^2 + 6H\frac{dH}{dlna}. \tag{14}$$

In order to better quantify the late time behavior of the $F(R)$ gravity model, we shall introduce the statefinder function y_H [36–39]

$$y_H \equiv \frac{\rho_{DE}}{\rho_m^{tot(0)}} = \frac{H^2}{m_s^2} - \frac{1}{1-\alpha}a^{-(3+\alpha)} - \chi a^{-4}, \tag{15}$$

with $\rho_m^{(0)}$ denoting the energy density of the cold dark matter at present time, $m_s^2 = H_0^2\Omega_{dm}^{(0)} = 1.37\times10^{-67}eV^2$ is the mass scale and χ is defined as $\chi = \frac{\rho_{dr}^{(0)}}{\rho_{dm}^{(0)}}$, where $\rho_r^{(0)}$ is the present time radiation energy density. By dividing Eqs. (13) by m_s^2, we get

$$\frac{1}{m_s^2}\frac{dR}{dlna} = \Bigg[-y_H + (F_R - 1) \times\left(\frac{H}{m_s^2}\frac{dH}{dlna} + \frac{H^2}{m_s^2}\right) - \frac{1}{6m_s^2}(F - R)\Bigg]\frac{1}{H^2F_{RR}}, \tag{16}$$

where $F_{RR} = \frac{\partial^2 F}{\partial R^2}$. Differentiating Eq. (15) with respect to the variable lna, we have

$$\frac{dy_H}{dlna} = \frac{2H}{m_s^2}\frac{dH}{dlna} + \frac{3+\alpha}{1-\alpha}a^{-(3+\alpha)} + 4\chi a^{-4}, \tag{17}$$

then we have

$$\frac{H}{m_s^2}\frac{dH}{dlna} + \frac{H^2}{m_s^2} = \frac{1}{2}\frac{dy_H}{dlna} + \frac{1+\alpha}{2(\alpha-1)}a^{-(3+\alpha)} - \chi a^{-4} + y_H. \tag{18}$$

By using Eqs. (14) and (17),we have

$$\frac{dy_H}{dlna} = \frac{R}{3m_s^2} - 4y_H - a^{-(3+\alpha)}, \tag{19}$$

then differentiating Eq. (19) with respect to the variable lna, we get

$$\frac{d^2y_H}{dlna^2} = \frac{dR}{3m_s^2 dlna} - 4\frac{dy_H}{dlna} + (3+\alpha)a^{-(3+\alpha)}, \tag{20}$$

By using the following relations,

$$\frac{d}{dlna} = -(z+1)\frac{d}{dz}, \tag{21}$$

$$\frac{d^2}{dlna^2} = (z+1)\frac{d}{dz} + (z+1)^2\frac{d^2}{dz^2}, \tag{22}$$

we can express the Friedmann equation in terms of the statefinder y_H and it reads

$$\frac{d^2y_H}{dz^2} + J_1\frac{dy_H}{dz} + J_2 y_H + J_3 = 0, \tag{23}$$

where the dimensionless functions J_1, J_2, J_3 are defined as follows

$$J_1 = \frac{1}{(z+1)} \times \left(-3 - \frac{1}{y_H + \frac{1}{1-\alpha}(z+1)^{3+\alpha} + \chi(z+1)^4} \frac{1-F_R}{6m_s^2 F_{RR}} \right), \tag{24}$$

$$J_2 = \frac{1}{(z+1)^2} \times \left(\frac{1}{y_H + \frac{1}{1-\alpha}(z+1)^{3+\alpha} + \chi(z+1)^4} \frac{2-F_R}{3m_s^2 F_{RR}} \right), \tag{25}$$

$$J_3 = -(3+\alpha)(z+1)^{\alpha+1} - \frac{(1-F_R)\left[\frac{1+\alpha}{1-\alpha}(z+1)^{3+\alpha} + 2\chi(z+1)^4\right] + (R-F)/(3m_s^2)}{(z+1)^2\left(y_H + \frac{1}{1-\alpha}(z+1)^{3+\alpha} + \chi(z+1)^4\right)} \frac{1}{6m_s^2 F_{RR}}. \tag{26}$$

Also, the $Ricci$ scalar as a function of the Hubble rate and of the redshift is equal to

$$R = 12H^2 - 6HH_z(1+z). \tag{27}$$

So the $Ricci$ scalar is an implicit function of the statefinder parameter y_H and can be expressed in terms of it as follows

$$R(z) = 3m_s^2\left[-(z+1)\frac{dy_H(z)}{dz} + 4y_H(z) + (1+z)^{3+\alpha}\right]. \tag{28}$$

In order to study the late-time evolution of the universe using an $F(R)$ gravity approach, we need to solve Eq. (8) numerically which demands determining the initial conditions. We consider the following physically motivated choice of initial conditions at $z_f = 10$ [6,39–43]

$$y_H(z_f) = \frac{\Lambda}{3m_s^2}\left(1 + \frac{1+z_f}{1000}\right), \tag{29}$$

$$\left.\frac{dy_H(z)}{dz}\right|_{z=z_f} = \frac{1}{1000}\frac{\Lambda}{3m_s^2}, \tag{30}$$

where $z_f = 10$ and $\Lambda \simeq 11.89 \times 10^{-67} eV^2$. From the obtained numerical solution $y_H(z)$, we can evaluate all the relevant physical quantities, such as the energy density parameter $\Omega_{de}(z)$, the dark energy EoS parameter. The energy density parameter $\Omega_{de}(z)$ is given by

$$\Omega_{de}(z) = \frac{y_H(z)}{y_H(z) + \frac{1}{1-\alpha}(z+1)^{3+\alpha} + \chi(z+1)^4}. \tag{31}$$

The dark energy EoS parameter is

$$w_{de}(z) = -1 + \frac{1}{3}(z+1)\frac{1}{y_H(z)}\frac{dy_H(z)}{dz}. \tag{32}$$

4 Late-time dynamic evolution

We next introduce the results of numerical analysis of the $F(R)$ gravity dark energy model, and it is worth noting that we add the R^2/M^2 terms in each function, where $M = 3.04375 \times 10^{22}$ eV [6,43]. The R^2 term has a key role in the unification of the early-time and the late-time eras, since it is the dominant term in the evolution of the early universe, $R \sim H_I^2$ where H_I is the inflationary scale, but becomes insignificant compared to the other terms at late times where R becomes comparable to the cosmological constant. We mainly consider the $F(R)$ gravity in the logarithmic form.

We shall begin with the following function,

$$F(R) = R + \frac{R^2}{M^2} + \frac{b}{1 + log(R/R_0)}, \tag{33}$$

Where the log represents the logarithm with bottom 10, b and R_0 are free parameters with dimension eV^2 (the dimension of natural units is $[m]^2$). We selected the value $b = 0.5m_s^2$ and $R_0 = 220m_s^2$.

Figure 1 depicts the image of the evolution trajectory of the logarithmic form $F(R)$ gravity dark energy model of the energy density of the dark energy and the state parameter w_{de} as the function of the redshift z. It can be seen that the evolution of the energy density parameter of the $F(R)$ model satisfies the law of the evolution of the universe, and when the values of α are 0, 0.05 and 0.1 respectively, the effect on the dark energy energy density Ω_{de} is small. The evolution trajectory of the state parameter w_{de} with redshift z describes the accelerating expansion universe and tends to -1, indicating that the model is biased towards the ΛCDM model and the Big Rip will not appear in the future, and the value of α has little effect on w_{de}.

The predicted values of dark energy EoS parameters and dark energy density parameters are compared with the constraints of Planck's cosmological parameters in 2018 [44]. It is found that when the coupling parameter $\alpha = 0$, the today's values of dark energy EoS parameters and dark energy den-

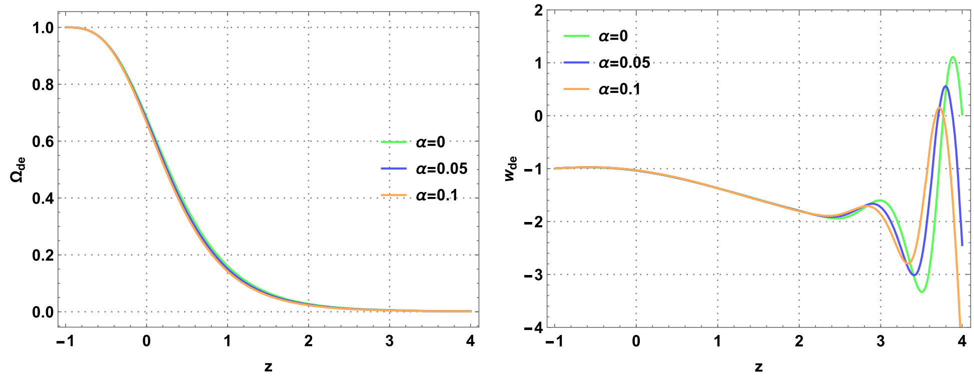

Fig. 1 Plots of the energy density parameter $\Omega_{DE}(z)$(left plot), the dark energy EoS parameter $w_{de}(z)$(right plot) as functions of the redshift for the logarithmic form $F(R)$ model of Eq. (33) for different α

Table 1 The today's values of parameters of the $F(R)$ dark energy model (the present values of EOS parameter w_{de0}, the present values of dark energy energy density Ω_{de0})

Parameters α	$F(R)$ 0	0.05	0.1
w_{de0}	− 1.038	− 1.034	− 1.031
Ω_{de0}	0.683	0.675	0.665

sity parameters of the $F(R)$ gravitational dark energy model are closer to the observed data. When the coupling parameter is 0.1, it deviates greatly from the observed results (Table 1).

5 Diagnostics

5.1 *Statefinder* diagnostic

Since many dark energy models explain the accelerated expansion of the universe well, to distinguish them, we need to use some discriminatory tools. So various tools have been developed to distinguish between dark energy models. The Hubble parameter $H \equiv \frac{\dot{a}(t)}{a(t)}$ is the first derivative of the scale factor, the deceleration parameter $q \equiv -\frac{\ddot{a}}{aH^2}$ is the second derivative of the scale factor, while the $Statefinder$ diagnostic is related to the third derivative of the scale factor, and the $Statefinder$ diagnostic parameter [45–48] is defined as

$$q = -1 - \frac{\dot{H}}{H^2}, j = \frac{\ddot{H}}{H^3} - 3q - 2, s = \frac{j-1}{3\left(q - \frac{1}{2}\right)}. \quad (34)$$

Figure 2a shows the evolution trajectory of q where α takes different values. It can be found that the evolution trajectory of the deceleration parameter q with redshift z satisfies the transition from the decelerated expansion period to the accelerated expansion period, and the smaller the value of α is, the shorter the time required for the universe to enter the accelerated expansion.

Figure 2b shows the evolution trajectory corresponding to j for different values of α. It can be seen from the figure that, firstly, with the expansion of the universe, the value of j gradually approaches $j = -1$ of the ΛCDM; Secondly, the j diagnosis in the high redshift region ($z > 0$) can distinguish the evolution curves corresponding to different α better, while in the low redshift region ($z < 0$) the a value has little effect on the evolution of j.

The Fig. 2c depicts the evolution trajectory corresponding to s for 0, 0.05 and 0.1, respectively. It is clear that the evolution curve is well distinguished in the high redshift region ($z > 0$), but is less affected in the low redshift region. The ΛCDM model is equally well distinguished from the $F(R)$ model by s diagnosis.

5.2 Om diagnostic

The $Statefinder$ diagnostic studied above is related to the third derivative of the scale factor, while the Om diagnosis is only related to the first derivative of the scale factor, which is a simpler geometric diagnostic method [49,50]. In the FRW universe, the Om diagnosis is defined by

$$Om(z) = \frac{\frac{H(z)^2}{H_0^2} - 1}{(1+z)^3 - 1}. \quad (35)$$

The Fig. 3 shows the Om evolution trajectory of the $F(R)$ gravity dark energy model with different values of α. It can be seen that Om diagnosis can distinguish the model both high and low redshift areas. We found that the Om diagnosis performed better than the $statefinder$ diagnosis.

6 Cosmological implication from supernovae observations

Since the absolute luminosity of Type Ia is almost constant at the peak of brightness, the distance to $SNIa$ can be determined by measuring its observed (apparent) luminosity. Thus the $SNIa$ is a kind of standard 'candle' by which luminosity distance can be measured observationally.The luminosity

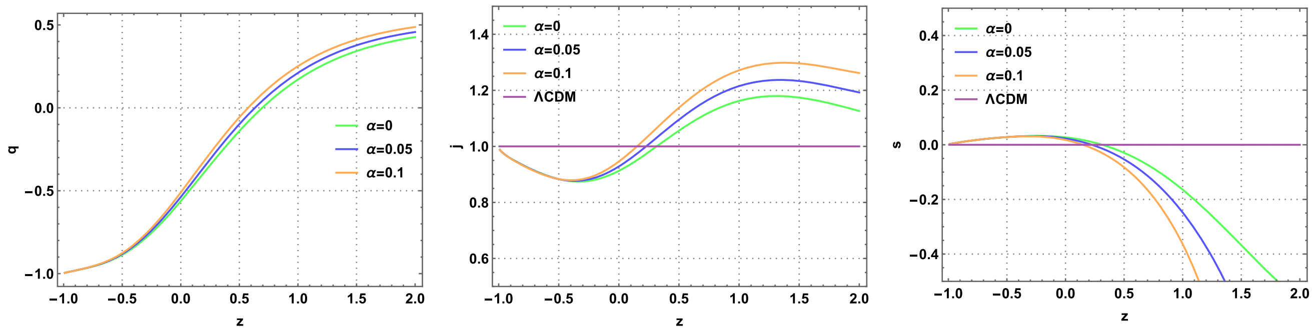

Fig. 2 The effect of dimensionless parameter α on the **a** q, **b** j, **c** s for the $F(R)$ gravity model of the logarithmic form

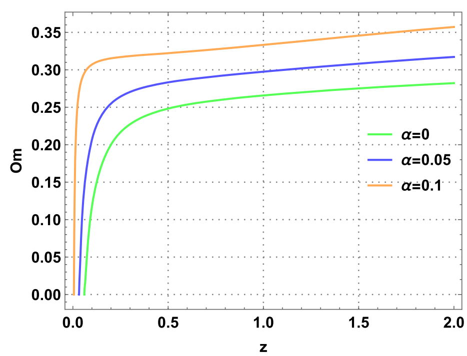

Fig. 3 The effect of dimensionless parameter α on the Om for the $F(R)$ gravity model of the logarithmic form

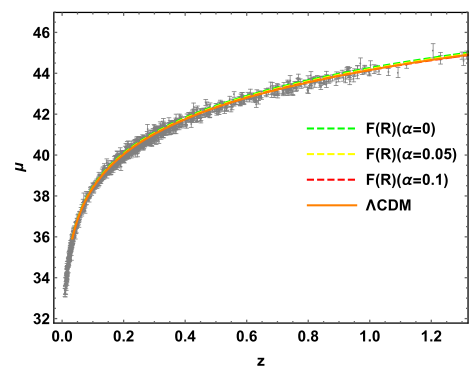

Fig. 4 The theoretically predicted SN distance modulus as a function of the redshift,for logarithmic $F(R)$ gravity model, comparing with the $SNIa$ observational data points(gray bars)from [52], and for comparison we depict the prediction of ΛCDM cosmology with the orange curve

distance redshift relation is a useful tool to explore the evolution of the universe. As the universe expands, the light originating a distant luminous body becomes red-shifted. The flux of a source is determined using the luminosity distance [51]. It is described as

$$d_L = a_0(1+z)r, \tag{36}$$

where r is the radial coordinate of the source. The following shows how the luminosity distance relates to the Hubble parameter

$$d_L = \frac{c}{H_0}(1+z)\int_0^z \frac{dz}{E(z)}. \tag{37}$$

There is another useful observable parameter the distance modulus μ which is related to the luminosity distance by the following formula

$$\mu = m_b - M = 5log_{10}\frac{d_L}{Mpc} + 25. \tag{38}$$

In Fig. 4, we describe the theoretically predicted $SNIa$ distance modulus as a function of z, they are cases with different a values, and ΛCDM cosmological predictions for 1048 SNIa observed data points. As we have seen, the theoretical date agree well with $SNIa$ data. And the larger the value of α, the better the image fits with the ΛCDM. A detailed comparison with observations, namely the joint analysis using data from $SNIa$, baryon acoustic oscillation (BAO), cosmic microwave background (CMB) and direct Hubble observation parameters, is beyond the scope of this work and will be left for future work.

7 Conclusions

This paper studies the $F(R)$ gravity model, which may produce a viable epoch of dark energy and an R^2-like epoch of inflation. We consider a logarithmic model of $F(R)$ gravity for the interaction between dark matter and dark radiation. We formulate the field equation as a function of redshift and quantify our analysis by introducing a state-finding function $y_H(z)$ that essentially measures the deformation of Einstein–Hilbert gravity caused by the $F(R)$ gravitational term. By properly selecting the initial conditions for the final phase of the matter dominance epoch such that the state finder function $y_H(z)$ and its derivatives are relative to redshift, we numerically solve the field equations and derive several physical quantities and the behavior of the statefinder. Specifically, we are interested in the dark energy state parameter, the dark energy density parameter. In addition, we also consider *statefinder* diagnostic and *Om* diagnostic, and the study shows that *Om* diagnostic is relatively effective. Furthermore, we compare the theoretical data with the *SNIa* data and find that the image of distance modulus of the model with an interaction between dark matter and dark energy fits better with the image of distance modulus of ΛCDM model. A detailed comparison with observations, namely, a joint analysis using data from *SNIa*, Baryon Acoustic Oscillation (BAO), cosmic microwave background (CMB) and Hubble direct observation parameters, is beyond the scope of this work and will be left to future projects.

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Data Availability Statement This manuscript has associated data in a repository. [Authors' comment: The associated data of this manuscript is from the publicly available observational data from Ref. [44] of this manuscript.]

Code availability statement The manuscript has no associated code/software. [Author's comment: The Figures are from the publicly available Python language.]

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