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# Overview of the Phenomenology of Lorentz and CPT Violation in Atomic Systems

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Received: 2 November 2019; Accepted: 19 November 2019; Published: 21 November 2019


**Abstract:** This is an overview of recent publications on the prospects of searching for nonminimal Lorentz-violating effects in atomic spectroscopy experiments. The article discusses the differences in the signals for Lorentz violation in the presence of minimal and nonminimal operators and what systems are more sensitive to certain types of Lorentz-violating operators.

**Keywords:** Lorentz violation; standard model extension; CPT violation

## 1. Introduction

Lorentz and CPT symmetry are two of the greatest principles in modern physics. In the last few decades, the exactness of the symmetry has been put into question, and its violation has been pursued as a candidate low energy signal for a quantum theory of gravity. The potential of Lorentz and CPT symmetry as a low energy signal was first proposed following the realization that realistic mechanisms for spontaneous Lorentz- and CPT-breaking in string theory are possible [1,2]. Since then, other studies have suggested that Lorentz- and CPT violation might be low energy signals for several theories beyond the standard model and general relativity such as noncommutative field theory [3–5], loop quantum gravity [6], multiverse scenarios [7], and granular spacetime models [8].

The Standard Model Extension (SME) was introduced as an effective field theory designed to assist in the systematic search for evidence of CPT and Lorentz violation [9,10]. Since the early years of the SME, models for atomic systems in the presence of Lorentz and CPT violation have been proposed [11–15]. Based on these models, experimental bounds on coefficients for Lorentz violation have been reported [16–29]. The original SME, referred to as the minimal SME, only considered the contributions from Lorentz-violating operators of mass dimensions three and four [9,10]. In the last decade, the SME has been extended by considering operators of higher mass dimensions that are called nonminimal operators [30–33]. Relevant to this work is the systematic classification of the Lorentz-violating nonminimal Dirac fermion operators [32]. This classification permitted the study of the prospects of searching for nonminimal Lorentz and CPT violation in atomic spectroscopy experiments. The study of these prospects resulted in three publications [34–36]. The first two publications [34,35] considered light atoms, including exotic atoms such as antihydrogen, positronium, and muonic atoms. The third publication considered heavier atoms that are usually used in high precision spectroscopy experiments or atomic clocks [36]. These publications complement each other, and together, they form a picture of the phenomenology of Lorentz and CPT violation in atomic systems.

This article is intended as a brief overview of the phenomenology of Lorentz and CPT violation in atomic systems based on three recent publications [34–36]. Section 2 is an overview of the perturbative Hamiltonian used in the publications [34–36]. The nonrelativistic coefficients for Lorentz violation are introduced in this section. The next section, Section 3, justifies the use of the perturbation introduced in Section 2. Section 4 discusses the Zeeman-hyperfine transitions of the ground state, which are the most sensitive transitions to Lorentz violation in atomic spectroscopy experiments. Section 5 discusses

the prospects for measuring the electron coefficients that do not contribute to the Zeeman-hyperfine transitions of the ground state. Section 6 addresses the problem of testing CPT symmetry in the presence of Lorentz violation. Section 7 discusses differences in the signals for minimal and nonminimal Lorentz-violating terms. Section 8 gives an overview of what systems are more sensitive to certain kinds of Lorentz-violating operators. Finally, we conclude with a brief outlook in Section 9.

## 2. Classification of the Lorentz- and CPT-Violating Dirac in the Quadratic Lagrange Density for a Dirac Fermion

The first systematic classification of nonminimal Lorentz-violating operators of arbitrary mass dimension was limited to Lorentz-violating photon operators [30]. This work was followed by systematic classifications of nonminimal neutrino operators [31], nonminimal Dirac fermion operators [32], and a more general classification of gauge field theories with nonminimal Lorentz-violating operators [33]. In this article, we will reproduce some of the results presented in [32] as most of the models to be discussed in this review article will be based on the Lorentz violation perturbation terms derived in this reference.

The authors of [32] considered the most general Lorentz-violating Lagrangian density for a free Dirac fermion with flavor $w$, and it has the form:

$$\mathcal{L} = \tfrac{1}{2}\bar{w}_w(\gamma^\mu i\partial_\mu - m_w + \widehat{\mathcal{Q}}_w)w_w + \text{h.c.}, \tag{1}$$

where $w_w$ is the Dirac fermion field operator, $m_w$ the fermion's mass, and $\widehat{\mathcal{Q}}_w$ is a spinor matrix containing the Lorentz-violating terms. The spinor matrix $\widehat{\mathcal{Q}}_w$ can be represented as the linear combination of the spinor matrices $\gamma^I \in \{I, \gamma^\mu, \gamma_5, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}$. The linear expansion of $\widehat{\mathcal{Q}}_w$ is assumed to have the form:

$$\widehat{\mathcal{Q}}_w = \sum_I \widehat{\mathcal{Q}}^I_w\gamma_I = \widehat{\mathcal{S}}_w + i\widehat{\mathcal{P}}_w\gamma_5 + \widehat{\mathcal{V}}^\mu_w\gamma_\mu + \widehat{\mathcal{A}}^\mu_w\gamma_5\gamma_\mu + \tfrac{1}{2}\widehat{\mathcal{T}}^{\mu\nu}_w\sigma_{\mu\nu}, \tag{2}$$

where $\widehat{\mathcal{Q}}^I_w \in \{\widehat{\mathcal{S}}_w, \widehat{\mathcal{P}}_w, \widehat{\mathcal{V}}^\mu_w, \widehat{\mathcal{A}}^\mu_w, \widehat{\mathcal{T}}^{\mu\nu}_w\}$ are the expansion coefficients. The hat on top of the coefficients identifies them as functions of the derivative operator $i\partial_\mu$, and they can be expanded as:

$$\widehat{\mathcal{Q}}^I_w = \sum_{d=3}^{\infty} \mathcal{Q}^{(d)I\alpha_1\alpha_2\ldots\alpha_{d-3}}_w i\partial_{\alpha_1} i\partial_{\alpha_2}\ldots i\partial_{\alpha_{d-3}}, \tag{3}$$

where the coefficients $\mathcal{Q}^{(d)I\alpha_1\alpha_2\ldots\alpha_{d-3}}_w$ are the coefficients for Lorentz violation that are assumed to be constant in an inertial reference frame.

The magnitude of a coefficient for Lorentz violation quantifies the degree of the breaking of the Lorentz symmetry. The indexes in the coefficients refer to the properties of the Lorentz violation operators, and Table 1 contains brief explanations of the indices most relevant to the discussion presented in this work. In this discussion also, we will introduce several types of coefficients for Lorentz violation and the terminology used to identify different subsets of the coefficients. For convenience, the terminology needed for this work is collected in Table 2.

The superscript $d$ of the coefficients is the mass dimension of the Lorentz-violating operator that is multiplied by the coefficient $\mathcal{Q}^{(d)I\alpha_1\alpha_2\ldots\alpha_{d-3}}_w$ in Equation (1) after using the expansions in Equations (2) and (3). The expansions in Equations (2) and (3) consider Lorentz-violating operators of arbitrary mass dimension as there is no upper bound on the mass dimension of the operators.

**Table 1.** Definitions of some of the relevant superscripts and subscripts for the coefficients.

| Symbol | Description |
|---|---|
| $d$ | Mass dimension of the Lorentz-violating operator contracted with the coefficient in the Lagrangian density. Used in effective Cartesian and spherical coefficients. |
| $w$ | Specifies the flavor of the Lorentz-violating operator contracted with the coefficient in the Lagrangian density. Used in all coefficients. |
| $j$ | Specifies the rank of the spherical tensor contracted with the coefficient in the one-particle Hamiltonian; $j > 0$. Used in nonrelativistic and spherical coefficients. |
| $m$ | Specifies the component of the spherical tensor contracted with the coefficient in the one-particle Hamiltonian; $m \in \{-j, -j+1, \ldots, j-1, j\}$. Used in nonrelativistic and spherical coefficients. |
| $n$ | Specifies the power of the three-momentum when the the one-particle Hamiltonian is expressed in terms of $E_0$ and $\lvert\boldsymbol{p}\rvert$. Used in spherical coefficients; see Equation (8). |
| $k$ | Specifies the power of the three-momentum when the the one-particle Hamiltonian is expressed in terms of $m_w$ and $\lvert\boldsymbol{p}\rvert$. Used in nonrelativistic coefficients; see Equation (10). |

**Table 2.** Terminology used to refer to certain types of coefficients for Lorentz violation.

| Terminology | Description | Types of Coefficients |
|---|---|---|
| Effective Cartesian | Coefficients for Lorentz-violating operators expressed as Lorentz tensors. | $\mathcal{V}_{w\,\mathrm{eff}}^{(d)\mu\alpha_1\ldots\alpha_{d-3}}$ $\widetilde{\mathcal{T}}_{w\,\mathrm{eff}}^{(d)\mu\nu\alpha_1\ldots\alpha_{d-3}}$ |
| Spherical | Coefficients for Lorentz-violating operators expressed as spherical tensors | $\mathcal{V}_{w\,njm}^{(d)}$ $\mathcal{T}_{w\,njm}^{(d)(0B)}$, $\mathcal{T}_{w\,njm}^{(d)(1B)}$ |
| Nonrelativistic | Linear combinations of spherical coefficients of arbitrary mass dimension $d$. | $\mathcal{V}_{w\,kjm}^{\mathrm{NR}}$ $\mathcal{T}_{w\,kjm}^{\mathrm{NR}(0B)}$, $\mathcal{T}_{w\,kjm}^{\mathrm{NR}(1B)}$ |
| Minimal | Coefficients for minimal operators | Coefficients with $d \leq 4$ |
| Nonminimal | Coefficients for nonminimal operators | Coefficients with $d > 4$ |
| CPT-even | Coefficients for CPT-invariant operators | $\mathcal{V}$-type with even $d$ or $c$-type $\mathcal{T}$-type with odd $d$ or $H$-type |
| CPT-odd | Coefficients for CPT-violating operators | $\mathcal{V}$-type with odd $d$ or $a$-type $\mathcal{T}$-type with even $d$ or $g$-type |
| Spin-dependent | Coefficients proportional to the Pauli matrices in the one-particle Hamiltonian | $\mathcal{T}$-type; or equivalently $g$-type and $H$-type |
| Spin-independent | Coefficients not proportional to the Pauli matrices in the one-particle Hamiltonian | $\mathcal{V}$-type; or equivalently $a$-type and $c$-type |
| Isotropic | Coefficients for rotational scalar Lorentz-violating operators | Spherical or nonrelativistic coefficients with $j = 0$ |
| Anisotropic | Coefficients for Lorentz-violating operators that are not rotational scalars | Spherical or nonrelativistic coefficients with $j > 0$ |

Starting from the Lagrange density (1), a Lorentz-violating perturbation to the one-particle Dirac Hamiltonian was obtained [32]. The form of the perturbation is:

$$\delta h = -\frac{1}{E_0}\left[\widehat{\mathcal{V}}^{\nu}_{\text{eff}} + \widehat{\widetilde{\mathcal{T}}}^{0\nu}_{\text{eff}}\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{m_w i} + \widehat{\widetilde{\mathcal{T}}}^{i\nu}_{\text{eff}}\left(\sigma_i + p_i\frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{(E_0+m_w)m_w}\right)\right]p_\nu, \tag{4}$$

where $E_0$ is the energy of the fermion, $\boldsymbol{p}$ is the three-momentum of the fermion, and $\boldsymbol{\sigma}$ is the Pauli vector. The terms $\widehat{\mathcal{V}}^{\nu}_{\text{eff}}$ and $\widehat{\widetilde{\mathcal{T}}}^{0\nu}_{\text{eff}}$ can be expressed as polynomials of the components of the four-momentum; see Equations (77) and (79) of [32]. This is similar to the expansion in Equation (3) with the reinterpretation of the operator $i\partial_\mu$ as the one-particle four-momentum operator. The coefficients of the expansion, denoted as ${\mathcal{V}_w}^{(d)\mu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}}$ and ${\widetilde{\mathcal{T}}_w}^{(d)\mu\nu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}}$, are called the effective Cartesian coefficients for Lorentz violation.

The operators contributing to the perturbation (4) can be classified into several categories. The operators multiplied by the $\mathcal{V}$-type coefficients are called spin-independent coefficients as they are independent of the spin degree of freedom. In contrast, operators multiplied by the $\mathcal{T}$-type coefficients are called spin-dependent coefficients. The properties of the operators in (1) under CPT transformation are determined by the mass dimension $d$ of the operator. By convention, different letters are used for the coefficients corresponding to CPT-violating operators and for the ones corresponding to CPT-invariant operators. The spin-independent operators with even mass dimensions are CPT-invariant operators, and the coefficients are $c$-type coefficients. In contrast, the spin-independent operators with odd mass dimensions are CPT-violating operators, and the coefficients are $a$-type coefficients. These coefficients are related to the $\mathcal{V}$-type coefficients by:

$${\mathcal{V}_w}^{(d)\mu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} = \begin{cases} -{a_w}^{(d)\mu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} & \text{if } d \text{ is odd} \\ +{c_w}^{(d)\mu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} & \text{if } d \text{ is even} \end{cases}. \tag{5}$$

The spin-dependent terms can also be divided into CPT-invariant and CPT-violating terms. The relation between the $\mathcal{T}$-type coefficients and the other set of coefficients is given by:

$${\widetilde{\mathcal{T}}_w}^{(d)\mu\nu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} = \begin{cases} -{\widetilde{H}_w}^{(d)\mu\nu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} & \text{if } d \text{ is odd} \\ +{\widetilde{g}_w}^{(d)\mu\nu\alpha_1\ldots\alpha_{d-3}}_{\text{eff}} & \text{if } d \text{ is even} \end{cases}, \tag{6}$$

where $H$-type coefficients correspond to CPT-invariant operators and the $g$-type coefficients to CPT-violating operators.

The perturbation Hamiltonian (4) can be expressed in momentum-space spherical coordinates instead of Cartesian coordinates. The three-momentum $\boldsymbol{p}$ is the product of its magnitude $|\boldsymbol{p}|$ and direction $\hat{\boldsymbol{p}}$. The unit vector in the direction of the three-momentum can be represented as a function of the polar and azimuthal angles as $\hat{\boldsymbol{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The direction of the Pauli vector can be indicated in terms of the direction of the three-momentum by introducing a helicity basis with unit vectors $\hat{\boldsymbol{\epsilon}}_\pm = (\hat{\boldsymbol{\theta}} \pm i\hat{\boldsymbol{\phi}})/\sqrt{2}$ and $\hat{\boldsymbol{\epsilon}}_r = \hat{\boldsymbol{p}}$. After these changes, the Hamiltonian has the generic form:

$$\delta h = h_{w0} + h_{wr}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\epsilon}}_r + h_{w+}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\epsilon}}_- + h_{w-}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\epsilon}}_+. \tag{7}$$

The explicit expressions for the terms $h_{w0}$, $h_{wr}$, $h_{w+}$, and $h_{w-}$ can be found in Equations (85) and (87) in [32]. As an example, consider the expression for $h_{w0}$,

$$h_{w0} = \sum_{d=3}^{\infty}\sum_{n=0}^{d-2}\sum_{j}\sum_{m=-j}^{j} E_0^{d-3-n}|\boldsymbol{p}|^n\, Y_{jm}(\hat{\boldsymbol{p}})\, {\mathcal{V}_w}^{(d)}_{njm}, \tag{8}$$

where the sum over $j$ is restricted to $j \geq 0$ and $j \in \{n, n-2, n-4, \ldots\}$. The coefficients ${\mathcal{V}_w}^{(d)}_{njm}$ are called the spherical coefficients for Lorentz violation. The spherical coefficients for Lorentz violation are linear combinations of the effective Cartesian coefficients for Lorentz violation. The relation between the two sets of coefficients is explained in detail in Section IV of [32]. In the Equation (8), the symbol

$Y_{jm}(\hat{\boldsymbol{p}})$ represents the spherical harmonics and the subscripts $j$ and $m$ of the spherical coefficients label the indices of the corresponding spherical harmonic. The index $d$ is the mass dimension of the operator, and the index $k$ is the power of the magnitude of the three-momentum. The relation between the indices $j$, $d$, and $n$ for all the different types of coefficients is summarized in Table III of [32].

The Hamiltonian (7) is only valid at linear order on the coefficients for Lorentz violation. At this order, the energy $E_0$ can be assumed to be the energy for a free fermion given by $E_0 = \sqrt{|\boldsymbol{p}|^2 + m_w^2}$. In nonrelativistic systems, the ratio $\boldsymbol{p}/m_w$ is a small number that can be used to expand the energy as a Taylor series. Using the binomial formula, we have that:

$$E_0 = m_w \sqrt{1 + \left(\frac{|\boldsymbol{p}|}{m_w}\right)^2} = m_w \sum_{k=1}^{\infty} \binom{\frac{1}{2}}{k} \left(\frac{|\boldsymbol{p}|}{m_w}\right)^{2k}, \tag{9}$$

where $\binom{j}{k}$ is the binomial coefficient. Using this formula, we can express Equation (8) as:

$$h_{w0} = -\sum_{kjm} |\boldsymbol{p}|^n \, {}_0Y_{jm}(\hat{\boldsymbol{p}}) \mathcal{V}^{\mathrm{NR}}_{wkjm}, \tag{10}$$

where $\mathcal{V}^{\mathrm{NR}}_{wkjm}$ is the linear combination of all the spherical coefficients $\mathcal{V}^{(d)}_{wnjm}$ that are proportional to the same power of $|\boldsymbol{p}|$ after replacing the energy in Equation (8) in terms of $|\boldsymbol{p}|$ by using Equation (9). The index $n$ in Equation (8) is the power of $|\boldsymbol{p}|$ when the Hamiltonian was represented as a function of the energy and three-momentum, and it is different from the index $k$ in Equation (10) that corresponds to the power of $|\boldsymbol{p}|$ after replacing the energy using Equation (9).

The coefficients $\mathcal{V}^{\mathrm{NR}}_{wkjm}$ are called the nonrelativistic coefficients and are the observable coefficients in most nonrelativistic experiments. The term observable effective coefficients means that the Lorentz-violating shift to the observables in nonrelativistic experiments can be expressed as linear combinations of the nonrelativistic coefficients. The nonrelativistic coefficients are defined in Equations (111) and (112) of [32]. For instance, consider the definition of $\mathcal{V}^{\mathrm{NR}}_{wkjm}$,

$$\mathcal{V}^{\mathrm{NR}}_{wkjm} = \sum_d m_w^{d-3-k} \sum_{q \leq k/2} \binom{(d-3-k+2q)/2}{q} \mathcal{V}^{(d)}_{w(k-2q)jm}. \tag{11}$$

The nonrelativistic coefficients are the linear combination of coefficients for Lorentz violation of arbitrary mass dimension multiplied by powers of the fermion's mass $m_w$. The mass dimension of the nonrelativistic coefficients can be determined with some basic dimensional analysis. If the operator multiplied by the coefficient $\mathcal{V}^{(d)}_{wnjm}$ has mass dimension $d$, then the coefficient has mass dimension $4-d$. The mass dimension of the nonrelativistic coefficient $\mathcal{V}^{\mathrm{NR}}_{wkjm}$ is the mass dimension of $\mathcal{V}^{(d)}_{wnjm}$ multiplied by the mass dimension of $m_w^{d-3-k}$. Putting the pieces together, we can conclude that the mass dimension of $\mathcal{V}^{\mathrm{NR}}_{wkjm}$ is equal to $1-k$.

In many nonrelativistic experiments, it is impossible to distinguish between the spherical coefficients that contribute to the same nonrelativistic coefficient [34–36]. For that reason, Lorentz violation effects in atomic systems usually result in bounds on the nonrelativistic coefficients for Lorentz violation. Exceptions to this rule are Lorentz-violating models that consider contributions due to the electromagnetic fields [37] or boost effects [35,36]; see Section 8.

## 3. Hierarchy and the Lorentz-Violating Perturbation

The Lorentz-violating corrections to the free propagation of the electron and the proton in the hydrogen atom are expected to be responsible for the dominant Lorentz- and CPT-violating effects if we consider all the possible Lorentz-violating operators [12,35,38]. The previous statement needs some clarification. In the context of models for Lorentz violation in atomic systems, there are two kinds of small parameters. The first kind is the expansion parameters used to obtain corrections to the atomic

energies using perturbative methods. Examples of these parameters are the ratio $|\boldsymbol{p}|/m_w$ between the magnitude of the three-momentum and the mass of the electron, the ratio $m_w/M$ between the mass of the electron and the mass of the nucleus, and the fine structure constant $\alpha$. These parameters introduce a hierarchy on the atomic corrections.

The second kind of small parameter is the coefficients for Lorentz violation. The coefficients for Lorentz violation are considered small parameters to be measured, but before measuring them, we cannot compare two coefficients for Lorentz violation. For example, we cannot tell which one of the following dimensionless terms $c_{w\,200}^{(4)}$, $c_{w\,210}^{(4)}$ or $m_w\, a_{w\,200}^{(5)}$ is greater. A common practice is not to assume a hierarchy between the coefficients for Lorentz violation in the absence of experimental bounds. We consider all the coefficients to be independent of each other. We also usually consider only linear contributions due to the coefficients for Lorentz violation; therefore, any hierarchy on the perturbative corrections is due to the atomic expansion parameters. For each coefficient for Lorentz violation, we have a perturbative series that has a similar hierarchy as the usual atomic corrections in the absence of Lorentz violation. For example, the Lorentz-violating contributions to the energy shift proportional to the same coefficient can be classified or ranked in terms of the nonrelativistic expansion that is the expansion on the small parameter $|\boldsymbol{p}|/m_w$.

To illustrate the idea, we need to study the form of the nonrelativistic expansion for a free Dirac fermion. Using Equation (9), we obtain:

$$E_0 = \sqrt{|\boldsymbol{p}|^2 + m_f^2} \simeq m_f \Big(1 + \frac{1}{2}\Big(\frac{|\boldsymbol{p}|}{m_w}\Big)^2 - \frac{1}{8}\Big(\frac{|\boldsymbol{p}|}{m_w}\Big)^4 + \dots\Big). \tag{12}$$

The contributions at different orders in the expansion have the generic form $m_w(|\boldsymbol{p}|/m_w)^n$. Even in the case of a Dirac fermion in the presence of an external electromagnetic field, we can expand the Hamiltonian in terms of the small parameter $|\boldsymbol{p}|/m_w$ using the Foldy–Wouthuysen transformation [39]. In the particular case of the hydrogen atom, the Coulomb-potential term appears at the first-order in the nonrelativistic expansion, but it is suppressed by a factor of the fine structure constant $\alpha$ that makes the Coulomb term of the same size as a second-order term such as the nonrelativistic kinetic energy $|\boldsymbol{p}|^2/2m_w$.

Consider the term $E_0|\boldsymbol{p}|^2 c_{w\,200}^{(6)}$ that contributes to the one-particle Hamiltonian (8). If we want to determine the dominant contribution from the coefficient $c_{w\,200}^{(6)}$ to the energy shift, we can use the nonrelativistic expansion of the energy and get:

$$E_0|\boldsymbol{p}|^2 c_{w\,200}^{(6)} \simeq m_f \Big(1 + \frac{1}{2}\Big(\frac{|\boldsymbol{p}|}{m_w}\Big)^2 - \frac{1}{8}\Big(\frac{|\boldsymbol{p}|}{m_w}\Big)^4 + \dots\Big)|\boldsymbol{p}|^2 c_{w\,200}^{(6)}. \tag{13}$$

Using this result, we can recognize that a term of the form:

$$m_f|\boldsymbol{p}|^2\, c_{w\,100}^{(6)} = m_w \left(\frac{|\boldsymbol{p}|}{m_w}\right) (\,|\boldsymbol{p}|\, a_{w\,200}^{(6)}\,), \tag{14}$$

is a zero-order term in the nonrelativistic expansion, and it is the dominant contribution from the coefficient $c_{w\,200}^{(6)}$. In the context of the nonrelativistic expansion of the Lorentz-violating perturbation, this term is a large term of the order of the rest energy of the particle, and it is greater than any Lorentz-violating term proportional to $c_{w\,100}^{(6)}$ that is produced by the electromagnetic interaction in the atom. However, its contribution to the atomic energy is really small because it is proportional to a coefficient for Lorentz violation. The crucial point is that we know that any term that is proportional to both the coefficient and an interaction term such as the Coulomb potential will be smaller than this term.

We can also consider the term of the form:

$$\frac{|\boldsymbol{p}|^4}{m_w}\, c_{w\,100}^{(6)} = m_w \left(\frac{|\boldsymbol{p}|}{m_w}\right)^2 \left( |\boldsymbol{p}|^2 c_{w\,200}^{(6)} \right) \tag{15}$$

and recognize that it is a second-order term in the nonrelativistic expansion; it is not the dominant contribution from the coefficient $c_{w\,200}^{(6)}$, and it can contribute at the same order as a Coulomb potential term that is proportional to the same coefficient; for that reason, in order to study this term, we must consider the Lorentz-violating electromagnetic interaction terms [12,35,38]. Fortunately, in practice, we can ignore this term and only consider the term $m_f |\boldsymbol{p}|^2\, c_{w\,100}^{(6)}$ that dominates over the Lorentz-violating terms that contain electromagnetic interactions.

Going back to the statement at the beginning of this section, the dominant contribution to the atomic spectrum is obtained by considering only the dominant free-propagation corrections to the proton and the electron for each coefficient. The implication is that it is enough to consider the perturbative Hamiltonian (7) in order to study the dominant Lorentz-violating effects in the spectrum of an atom [35,36].

## 4. Hyperfine Transitions and Anisotropic Terms

The best limits on Lorentz-violating operators obtained from atomic spectroscopy experiments are from hyperfine transitions of the ground state [34–36]. In the standard atomic theory, effects that depend on the total angular momentum of the atom, such as the hyperfine structure, are suppressed. For this reason, in general, hyperfine structure transitions have lower frequencies than gross structure transitions. On the other hand, many of the dominant Lorentz-violating terms are anisotropic, and their expectation values depend on the atomic total-angular-momentum quantum number $F$. For example, consider the term $m_w g_{w\,010}^{(4)(0B)}\, \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\, Y_{10}(\hat{\boldsymbol{p}})$. This is the dominant contribution of the coefficient $g_{w\,010}^{(4)(0B)}$ to the perturbation Hamiltonian as the other contributions are suppressed by powers of $|\boldsymbol{p}|/m_w$. Because this term depends on the spin expectation value, it does contribute to hyperfine structure transitions [34–36]. What makes this kind of term special is that its contribution has the same size as the gross structure or hyperfine structure transitions. However, because the hyperfine transitions are usually more sensitive to smaller frequency shifts than gross structure transitions, then the hyperfine structure transitions are more sensitive to the coefficient $g_{w\,010}^{(4)(0B)}$ than other types of transitions [34–36].

At first-order in perturbation theory, the anisotropic terms in the Lorentz-violating Hamiltonian affect the spectrum in a fashion that is analogous to the presence of small external electric and magnetic fields. For example, some of the leading-order Lorentz-violating energy shifts have a structure that resembles the Zeeman and Stark effects [12–15,34–36]. This is a challenge because transitions that are insensitive to Zeeman or Stark effects may also be insensitive to these Lorentz-violating effects. To understand this statement, we need to understand what are the common tests for Lorentz violation in atomic systems.

The most common tests for Lorentz violation are sidereal and annual variations of the transition frequency [16–23]. The idea behind these tests is to compare the transition frequency of the atom at different velocities and orientations relative to a fixed inertial reference frame. For convenience, the Sun-centered frame is used as the fixed reference frame [40]. The best approach to control the orientation of the atom is to introduce an external magnetic field in the z-direction in the instantaneous laboratory frame. Because of the presence of the magnetic field, the stationary states of the system are quantum states of the z-component of the total-angular-momentum $F_z$ relative to the laboratory frame. As the applied magnetic field rotates with the Earth, the stationary states are rotated adiabatically around the Sun-centered frame, and we can test the rotational symmetry of the atomic spectrum. Similarly, the velocity of the atoms changes as the atoms are accelerated in the Sun-centered frame due to the rotation of the Earth around its axis and the motion of the Earth around the Sun. In this

scenario, the Lorentz-violating terms appear as small corrections to the Zeeman levels that depend on the annual and sidereal time [12–15,34–36].

If we are forced to use applied magnetic fields, then we want to reduce the uncertainty due to the magnetic fields. A common method is to use transitions insensitive to the linear Zeeman effect. Examples of this kind of transition are the clock transitions in hydrogen masers and cesium atomic fountain clocks. This is a bad idea in the context of Lorentz violation. Whatever makes these transitions insensitive to the linear Zeeman effect also makes them insensitive to linear effects due to other uniform anisotropic external fields such as the anisotropic Lorentz-violating background fields [35,36]. In other words, these transitions are insensitive to the dominant CPT- and Lorentz-violating effects. Still, there is one advantage of having transitions that are insensitive to the dominant Lorentz-violating terms. We measure a frequency by comparing it to another frequency. We need to know if the Lorentz-violating model predicts any variation in the reference frequency in order to search for time variations of a transition frequency [13]. Using the perturbation (7), we know that the hydrogen maser and the cesium standards are insensitive to the dominant Lorentz-violating effects and are good reference frequencies for time variation studies of transition frequencies [35,36].

Other methods used to reduce the uncertainty due to the magnetic field, such as averaging over Zeeman pairs, can also eliminate contributions due to the anisotropic Lorentz-violating terms [36]. For example, consider the measurement of the hyperfine transition of the ground state of antihydrogen [41]. In the experiment, two frequencies were averaged to suppress the contribution due to the magnetic field. This process also eliminated the contributions from the dominant CPT-violating terms. In other words, the sensitivity of the measurement to CPT violation is suppressed compared to other kinds of tests that could be done using the same system. A method that can be used to eliminate the magnetic field without eliminating the contribution of the Lorentz-violating terms is to extrapolate the frequency to the zero-magnetic-field value [36]. The dominant CPT-violating terms are independent of the magnitude of the magnetic field, and they will contribute to the extrapolated zero-field frequency. Another method that has been proposed is to compare the $\sigma$ and $\pi_1$ antihydrogen transitions [42]. The $\sigma$ transition is insensitive to the dominant CPT-violating terms, and it can be used as a reference frequency for searching for a sidereal variation of the Lorentz violation-sensitive $\pi_1$ transition [35].

Averaging over Zeeman pairs does not always cancel all the contributions due to Lorentz violation. For example, a Lorentz symmetry test with cesium fountain clocks cannot use the standard clock transition as it is insensitive to the dominant Lorentz-violating terms. Time variation frequency studies with cesium fountain clocks were done using an averaged pair of hyperfine-Zeeman transitions [22]. The process used to eliminate the linear Zeeman shift also canceled the contributions from the $g$-type and $H$-type spin-dependent coefficients, but it allowed contributions from $a$-type and $c$-type spin-independent ones [22,36]. The most successful method for eliminating the linear Zeeman effect without eliminating the Lorentz-violating terms has been the use of comagnetometers [18,19,36]. The nonrelativistic $g$-type and $H$-type coefficients for Lorentz violation with $j = 1$ produce small corrections to the Zeeman levels; however, the corrections are not proportional to the gyromagnetic ratios and are by the method used to eliminate the linear Zeeman shift in the comagnetometer.

The only spatially isotropic terms that can contribute to the Lorentz-violating shift to the atomic spectrum are spin-independent operators that depend only on the magnitude of the three-momentum [34–36]. These isotropic terms do not contribute to Zeeman-hyperfine transitions, and for that reason, all the terms that contribute to these transitions are anisotropic. The Lorentz-violating frequency shift for the Zeeman-hyperfine transitions depends on the orientation of the magnetic field and the boost velocity of the instantaneous laboratory frame relative to the Sun-centered frame. Only considering the rotation of the instantaneous laboratory frame due to the rotation of the Earth is not enough to impose bounds on all the coefficients for Lorentz violation that contribute to the transition frequencies. Models for space based experiments such as for the Atomic Clock Ensemble in Space (ACES) [43] and for experiments on turntables have been considered to impose bounds on a greater set of coefficients for Lorentz violation [35,36].

## 5. Isotropic Terms and Optical Transitions

The isotropic term in the laboratory frame has the form $\mathcal{V}_{f\,k00}^{\mathrm{NR,lab}}|\boldsymbol{p}|^k$, where the superscript "lab" is a reminder that these coefficients are not constant and uniform because the laboratory frame is not an inertial reference frame. The isotropic term does not contribute to the frequency shift for the hyperfine-Zeeman transitions of the ground state, and for that reason, it cannot be measured using the experiments mentioned in Section 4. This term does contribute to any gross structure transition such as optical transitions. The best candidates to study the isotropic term are optical transitions such as the 1 s–2 s transition in hydrogen [35] or clock transitions used in optical clocks [36].

The isotropic term in the laboratory frame is independent of the orientation of the magnetic field, and it is not canceled by any process that cancels the contributions from anisotropic external fields [36]. The implication is that the isotropic term can be studied using optical clocks without requiring the optical clock to operate in a different way than usual. The drawback is that the isotropic term is insensitive to changes in the orientation of the laboratory frame that is the dominant signal for Lorentz violation. However, it is sensitive to boost effects, which are suppressed by a factor of $10^{-4}$ compared to the dominant rotation effects.

An isotropic coefficient for Lorentz violation in the laboratory frame can be expressed in the Sun-centered frame as:

$$\mathcal{V}_{f\,k00}^{\mathrm{NR,lab}} = \mathcal{V}_{f\,k00}^{\mathrm{NR,Sun}} + \beta_{\oplus} f_{ann}(T) + \beta_L f_{sid}(T), \tag{16}$$

where $\mathcal{V}_{f\,k00}^{\mathrm{NR,Sun}}$ is the isotropic coefficient in the Sun-centered frame, $T$ is the time in the Sun-centered frame, $\beta_{\oplus} = 10^{-4}$ is the orbital speed of the Earth, and $\beta_L = 10^{-6}$ is the rotational speed of the Earth at the Equator. The function $f_{ann}(T)$ is a linear combination of coefficients for Lorentz violation with terms that vary with the first harmonic of the annual frequency and $f_{sid}(T)$ the same, but the terms vary with the first harmonics of the sidereal frequency. The explicit expression for Equation (16) can be found in Equation (63) in [35]. The best way to impose constraints on the coefficients that contribute to $f_{ann}(T)$ and $f_{sid}(T)$ is by searching for annual and sidereal variations of the optical transitions in the first harmonic of the sidereal and annual frequency. The first term in Equation (16) produces a constant shift that will be the same independent of wherever on the surface of the Earth the experiment was done. This constant shift cannot be constrained by studying the time variation of the transition frequency under consideration. However, the isotropic term in the Sun-centered frame has been constrained by comparing the 1 s–2 s transitions frequency of hydrogen and antihydrogen [36] or by comparing the experimental and theoretical values for the 1 s–2 s transitions in positronium [35] and muonium [34].

## 6. The Problem of Testing CPT Symmetry Using Different Frames

A breaking of CPT symmetry implies Lorentz violation in interacting local field theories [44]. This result is also observed in the non-gravitational sector of the SME, where all the local CPT-violating terms that can be added to the Lagrangian density also break Lorentz symmetry [9,10]. If we expect CPT violation to emerge as small corrections to the standard model of particle physics, then we expect CPT violation to be accompanied by Lorentz violation. This observation implies that CPT tests that compare the properties of a system and its CPT counterpart must be conducted in the same laboratory frame. Otherwise, Lorentz-violating effects that are not CPT-violating effects might be responsible or might cancel any discrepancy between the two systems [37].

Even if the measurement of a system and its CPT counterpart is done in different reference frames, we could use these results to test CPT symmetry by using a model for Lorentz violation. We can use the model to transform the results from one frame to the other keeping track of all the Lorentz-violating effects. In this case, the validity of the CPT test will be limited as it depends on the particular model for CPT and Lorentz violation used. For example, consider the recent comparison between the value of the 1 s–2 s transition in hydrogen [45] and antihydrogen [46]. As these two values were measured in different reference frames, in principle, we should not compare the values without considering how

Lorentz violation could impact these results. Using the Lorentz- and CPT-violating corrections for the 1 s–2 s transition in hydrogen presented in [35], a model for comparing the two measurements of the 1 s–2 s transition was developed [36]. The model considered only the isotropic contribution in the Sun-centered frame to the frequency difference between the 1 s–2 s transition in hydrogen and antihydrogen; see Section 5. In other words, even if the frequencies were measured in distinct reference frames, there are corrections to the frequency difference that are independent of the frames used in the measurements, and these terms correspond to the constant term in Equation (16). To justify this model partially, some of the anisotropic or frame-dependent contributions to the frequency difference can be disregard using results from time variation studies of transition frequencies in hydrogen [16,20]. More time variation studies in hydrogen and antihydrogen are needed to justify experimentally the absence of many of the anisotropic terms that were not considered in the model. Fortunately, some of these time variation studies are expected to happen soon [42]. The approach used to create the model for the frequency difference between the 1 s–2 s transition in hydrogen and antihydrogen cannot be replicated for the Zeeman-hyperfine transitions of the ground state as all the terms that contribute to these transitions are anisotropic in the Sun-centered frame. Measurements of hyperfine transition frequencies of the ground state for hydrogen and antihydrogen in the same location are pursued to avoid any contributions from CPT-invariant Lorentz-violating operators [42].

## 7. Difference in the Signals for Minimal and Nonminimal Lorentz-Violating Terms

For experiments in laboratories on the surface of the Earth, the minimal Lorentz-violating operators could produce sidereal variations in the first and second harmonic of the sidereal frequency [11–14]. In the context of atomic spectroscopy experiments, we can understand this result from the following observations. The minimal Lorentz violation $a$-type and $c$-type coefficients are contained in the nonrelativistic coefficients $\mathcal{V}^{\mathrm{NR}}_{w_{kjm}}$ with $j \leq 2$, and similar relations hold for the spin-dependent terms [32]. We can break the time-varying transition frequency shift in the Sun-centered frame in terms of harmonics of the sidereal frequency [35,36]. If we ignore boost effects, we can break the Sun-centered-frame transition frequency shift $\delta\nu$ in the following way,

$$\delta\nu = \sum_{m=0}^{\infty} \left( A_m \cos m\omega_{\oplus} T + B_m \sin m\omega_{\oplus} T \right), \tag{17}$$

where $\omega_{\oplus} \simeq 2\pi/(23\ \mathrm{h}\ 56\ \mathrm{m})$ is the sidereal frequency and $T$ is the time in the Sun-centered frame. The amplitudes $A_m$ and $B_m$ of the $m^{\mathrm{th}}$-harmonics are linear combinations of the coefficient $\mathcal{V}^{\mathrm{NR}}_{w_{kjm}}$ and $\mathcal{V}^{\mathrm{NR}}_{w_{kj(-m)}}$ [35,36]. In other words, the absolute value $|m|$ of the index $m$ of the coefficient for Lorentz violation in the Sun-centered frame indicates the harmonic of the sidereal frequency that contributes together with the coefficient $\mathcal{V}^{\mathrm{NR}}_{w_{kjm}}$ in the frequency shift. The absolute value of the index $m$ is related to the index $j$ by $0 \leq |m| \leq j$ and for the minimal operators $0 \leq |m| \leq 2$. As expected, the minimal operators can only produce sidereal variations in the first and second harmonic of the sidereal frequency.

In principle, the nonminimal Lorentz-violating operators can produce variations with all harmonics of the sidereal frequency. However, nonminimal coefficients with an index $j$ cannot contribute to the energy shift of every atomic energy level. The maximum value of the index $j$ that can contribute to the energy shift depends on the angular momentum quantum numbers of the energy level [35,36]. For instance, for the ground state of hydrogen, the angular moment quantum numbers are $L = 0$ for the orbital angular momentum, $J = 1/2$ for the total electron angular momentum, and $F = 0$ or $F = 1$ for the total atomic angular momentum. Based on the angular momentum quantum numbers, we can conclude that only spin-independent terms with $j = 0$ and spin-dependent ones with $j \leq 1$ can contribute to the energy shift. A consequence of this observation is that even in the presence of nonminimal terms, we should expect only first-harmonic sidereal variations of Zeeman-hyperfine transitions of the ground state of hydrogen or the 1 s–2 s transition in hydrogen. These are the same

signals predicted by the minimal SME, and for that reason, experimental constraints on these signals already existed and were used to impose bounds on nonminimal coefficients [34–36].

In the minimal case, there was no advantage in considering transitions involving energy levels with high angular momentum quantum numbers. However, the only way to use an atomic spectroscopy experiment to search for Lorentz-violating operators with a high value of $j$ is by using transitions that involve high angular momentum states. In general, a transition could be millions of times more sensitive to Lorentz violation than another transition, but because that more sensitive transition only involves low angular momentum states, it will be sensitive to a small set of Lorentz-violating operators, and if the less sensitive transition involves high angular momentum states, it can provide the best bounds on coefficients for Lorentz violation on nonminimal operators that cannot be studied with the more sensitive transition; see Section 8.

Another difference in the phenomenology of atomic spectroscopy in the presence of minimal and nonminimal Lorentz violation is that the nonminimal terms depend on a higher power of the three-momentum, and this means that the number of transitions that can be affected by Lorentz violation increased significantly compared to the minimal case [35]. Furthermore, this means that the sensitivity of the experiment to the nonminimal coefficients will be dependent on the expectation values of the momentum, and that will make some systems more sensitive to some nonminimal operators than others as is the case with muonic hydrogen and muonium, as discussed in [34].

## 8. Best Bounds on and Prospects for Coefficients for Lorentz Violation from Spectroscopy Experiments

Table 3 contains the best bounds on the nonrelativistic spin-dependent coefficients for Lorentz violation. The first column in the table specifies the type of nonrelativistic coefficient, and the other columns specify the best bounds on the electron, neutron, proton, and muon coefficients. A time variation study of hyperfine-Zeeman transition frequencies of the ground state of hydrogen is responsible for the best bounds on nonminimal electron coefficients obtained in atomic spectroscopy experiments [16,35]. The bounds obtained on the coefficients $g_{e011}^{\mathrm{NR}(0B)}$, $H_{e011}^{\mathrm{NR}(0B)}$, $g_{e011}^{\mathrm{NR}(1B)}$, and $H_{e011}^{\mathrm{NR}(1B)}$ are in the order of $10^{-27}$ GeV [35]. The superscript $e$ in the coefficients means that these coefficients correspond to electron operators. Technically, this experiment also has the best bounds on nonrelativistic proton coefficients, but better bounds on proton coefficients might be obtained by just replacing the nuclear model used in [36].

An experiment using a $^3$He-$^{129}$Xe comagnetometer imposed limits of the order of $10^{-33}$ GeV on Lorentz-violating operators in the nucleon sectors of the SME [18,36]. To assign these bounds to proton or neutron Lorentz-violating operators, we need to use a nuclear model. A simple nuclear model assumes that only the neutron operators contribute to the Lorentz-violating frequency shift, and using this simplistic model, bounds of the order of $10^{-33}$ GeV on the neutron coefficients $g_{n011}^{\mathrm{NR}(0B)}$, $H_{n011}^{\mathrm{NR}(0B)}$, $g_{n011}^{\mathrm{NR}(1B)}$, and $H_{n011}^{\mathrm{NR}(1B)}$ were obtained [36]. From a more realistic nuclear model, we expect to get contributions from both nucleons with smaller contributions from the proton than from neutron operators. For instance, more realistic nuclear models showed that in the context of the minimal SME, the corrections due to the proton operators were only suppressed by a factor of five compared to the neutron operators [29]. Because the comagnetometer experiment is $10^6$-times more sensitive than the hydrogen experiment, we expect that by using a more realistic nuclear model, we will get better bounds on the proton coefficients from the comagnetometer experiment than from the hydrogen experiment. The best bounds on the proton or neutron coefficients depend on the nuclear models used in the derivation of the Lorentz violation shift. However, in general, the best bounds on the nucleon coefficients will be from hyperfine-Zeeman transitions of the ground states [16,22,35,36], as expected from the discussion in Section 4.

**Table 3.** Best bounds on the imaginary and real part of the spin-dependent anisotropic nonrelativistic coefficients in the Sun-centered frame for electron, proton, neutron, and muon operators.

| Coefficients | Neutron [36] from Xe-He Comagnetometer | Proton and Electron [35] from Hydrogen 1S Splitting | Muon [34] from Muonium 1S Splitting |
|---|---|---|---|
| $H_{w_{011}}^{\mathrm{NR}(0B)}$, $g_{w_{011}}^{\mathrm{NR}(0B)}$ | $4 \times 10^{-33}$ GeV | $9 \times 10^{-27}$ GeV | $2 \times 10^{-22}$ GeV |
| $H_{w_{011}}^{\mathrm{NR}(1B)}$, $g_{w_{011}}^{\mathrm{NR}(1B)}$ | $2 \times 10^{-33}$ GeV | $5 \times 10^{-27}$ GeV | $7 \times 10^{-23}$ GeV |
| $H_{w_{211}}^{\mathrm{NR}(0B)}$, $g_{w_{211}}^{\mathrm{NR}(0B)}$ | $4 \times 10^{-31}$ GeV$^{-1}$ | $7 \times 10^{-16}$ GeV$^{-1}$ | $1 \times 10^{-11}$ GeV$^{-1}$ |
| $H_{w_{211}}^{\mathrm{NR}(1B)}$, $g_{w_{211}}^{\mathrm{NR}(1B)}$ | $2 \times 10^{-31}$ GeV$^{-1}$ | $4 \times 10^{-16}$ GeV$^{-1}$ | $6 \times 10^{-12}$ GeV$^{-1}$ |
| $H_{w_{411}}^{\mathrm{NR}(0B)}$, $g_{w_{411}}^{\mathrm{NR}(0B)}$ | $4 \times 10^{-29}$ GeV$^{-3}$ | $9 \times 10^{-6}$ GeV$^{-3}$ | $2 \times 10^{-1}$ GeV$^{-3}$ |
| $H_{w_{411}}^{\mathrm{NR}(1B)}$, $g_{w_{411}}^{\mathrm{NR}(1B)}$ | $2 \times 10^{-29}$ GeV$^{-3}$ | $5 \times 10^{-6}$ GeV$^{-3}$ | $8 \times 10^{-2}$ GeV$^{-3}$ |

The energy states involved in the Zeeman transitions used in the He-Xe comagnetometer experiment have total angular momentum quantum number $F = 1/2$, and for that reason, the transitions are only sensitive to nonrelativistic coefficients with $j = 1$. The best bounds on nucleon coefficients with $j > 1$ are from the study of hyperfine-Zeeman transitions of the ground state of cesium using a cesium fountain clock and front the sidereal variations studies with a Ne-Rb-K comagnetometer [36]. In the case of the fountain clock, the energy levels involved in the transition have quantum numbers $F = 3$ and $F = 4$, and these high angular momentum states permit contributions from nonrelativistic coefficients with $j \leq 4$ [36]. The experimental constraints obtained with the atomic fountain clock on sidereal variations are only sensitive to proton coefficients if we assume the nuclear model used in [36]. However, we expect that by using a more realistic nuclear model, we can translate the experimental constraints as bounds on neutron and proton coefficients. Overall, the comagnetometer is more sensitive to smaller frequencies than the cesium atomic clock, and the bounds obtained from the comagnetometer are tighter than the bounds obtained from the atomic fountain clock; however, the atomic fountain clock is sensitive to a greater number of coefficients for Lorentz violation than the comagnetometer.

Hyperfine transitions in large atoms involve nuclear-spin flips, and they are not sensitive to electron Lorentz-violating operators. To study the electron operators, we need to consider hyperfine transitions in light atoms or electron transitions such as optical transitions. The hyperfine-Zeeman transitions of the ground state of hydrogen or the 1 s–2 s transition in hydrogen are only sensitive to electron coefficients with $j \leq 1$. The best bounds on electron coefficients with $j = 2$ are obtained from optical transitions in heavy ions such as $^{40}Ca^{+}$ [24,25,36] and $^{171}Yb^{+}$ [23]. These transitions involve energy levels with high angular momentum. For example, the final energy state for the optical transition in $^{171}Yb^{+}$ has quantum number $F = 3$ [23], and it is sensitive to nonrelativistic coefficients with $j \leq 6$. The final energy state for the optical transition in $^{40}Ca^{+}$ has $F = 5/2$ [24,25], and it is sensitive to coefficients with $j \leq 4$. Overall, the hyperfine-Zeeman transitions of the ground state remain slightly more sensitive to Lorentz violation than the optical transition in $^{171}Yb^{+}$; however, the optical transition is sensitive to a greater number of coefficients for Lorentz violation. Unfortunately, to translate the constraints obtained from the optical transition in $^{171}Yb^{+}$ into bounds on nonminimal coefficients for Lorentz violation, a many-body calculation is needed, and at the moment, this type of calculation has only been done for minimal Lorentz-violating operators [23].

Lorentz violation operators that are isotropic in the laboratory frame cannot be studied using Zeeman-hyperfine transitions of the ground state; see Section 5. The best limits on the coefficients that contribute to $f_{sid}$ in Equation (16) are from an annual variation study of the 1 s–2 s transition in hydrogen [20,35]; see the second and third column of Table 4. As mentioned in Section 5, optical clocks are good candidates to improve these bounds. The best bounds on the isotropic CPT-violating electron and proton terms in the Sun-centered frame are obtained from a comparison of antihydrogen

and hydrogen 1 s–2 s transition [36] and for the CPT-even electron term from a comparison between the theoretical and experimental value for the 1 s–2 s transition in positronium [35]. The bounds are shown in Table 5 with the second, fourth, and sixth columns showing the constraints on the electron, proton, and muon isotropic coefficients for Lorentz violation.

**Table 4.** Best bounds from atomic experiments on effective Cartesian coefficients of mass dimensions $d = 5$ and $d = 6$ in the Sun-centered frame for electron [35], proton [35], and neutron [36] Lorentz-violating operators.

| Coefficient | Electron [35] | Proton [35] | Coefficient | Neutron [36] | Coefficient | Neutron [36] |
|---|---|---|---|---|---|---|
| | $\mathbf{GeV^{4-d}}$ | $\mathbf{GeV^{4-d}}$ | | $\mathbf{GeV^{4-d}}$ | | $\mathbf{GeV^{4-d}}$ |
| $a_{w_{\rm eff}}^{(5)TTX}$ | $<3.4\times10^{-8}$ | $<3.4\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(TXT)}$ | $<1\times10^{-27}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(TXTT)}$ | $<9\times10^{-28}$ |
| $a_{w_{\rm eff}}^{(5)TTY}$ | $<5.6\times10^{-8}$ | $<5.6\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(TYT)}$ | $<8\times10^{-28}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(TYTT)}$ | $<7\times10^{-28}$ |
| $a_{w_{\rm eff}}^{(5)TTY}$ | $<1.3\times10^{-7}$ | $<1.3\times10^{-7}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(TZT)}$ | $<2\times10^{-27}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(TZTT)}$ | $<2\times10^{-27}$ |
| $a_{w_{\rm eff}}^{(5)KKX}$ | $<6.7\times10^{-8}$ | $<6.7\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(TXT)}$ | $<8\times10^{-28}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(TXTT)}$ | $<6\times10^{-28}$ |
| $a_{w_{\rm eff}}^{(5)KKY}$ | $<1.1\times10^{-7}$ | $<1.1\times10^{-7}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(TYT)}$ | $<8\times10^{-28}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(TYTT)}$ | $<7\times10^{-28}$ |
| $a_{w_{\rm eff}}^{(5)KKZ}$ | $<2.5\times10^{-7}$ | $<2.5\times10^{-7}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(TZT)}$ | $<2\times10^{-27}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(TZTT)}$ | $<2\times10^{-27}$ |
| $c_{w_{\rm eff}}^{(6)TTTX}$ | $<3.3\times10^{-5}$ | $<1.8\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(JXJ)}$ | $<4\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(JXJT)}$ | $<9\times10^{-26}$ |
| $c_{w_{\rm eff}}^{(6)TTTY}$ | $<5.5\times10^{-5}$ | $<3.0\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(JYJ)}$ | $<3\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(JYJT)}$ | $<7\times10^{-26}$ |
| $c_{w_{\rm eff}}^{(6)TTTZ}$ | $<1.3\times10^{-4}$ | $<6.9\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)X(JZJ)}$ | $<6\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)X(JZJT)}$ | $<2\times10^{-25}$ |
| $c_{w_{\rm eff}}^{(6)TKKX}$ | $<3.3\times10^{-5}$ | $<1.8\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(JXJ)}$ | $<2\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(JXJT)}$ | $<2\times10^{-25}$ |
| $c_{w_{\rm eff}}^{(6)TKKY}$ | $<5.5\times10^{-5}$ | $<3.0\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(JYJ)}$ | $<3\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(JYJT)}$ | $<7\times10^{-26}$ |
| $c_{w_{\rm eff}}^{(6)TKKZ}$ | $<1.3\times10^{-4}$ | $<6.9\times10^{-8}$ | $\tilde{H}_{w_{\rm eff}}^{(5)Y(JZJ)}$ | $<6\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)Y(JZJT)}$ | $<2\times10^{-25}$ |
| | | | $\tilde{H}_{w_{\rm eff}}^{(5)TJTJ}$ | $<6\times10^{-25}$ | $\tilde{g}_{w_{\rm eff}}^{(6)TJTJT}$ | $<5\times10^{-25}$ |

**Table 5.** Best bounds on the spin-independent isotropic nonrelativistic coefficients in the Sun-centered frame for electron, proton, and muon operators.

| Constraint; Electron | | Constraint; Proton | | Constraint; Muon | |
|---|---|---|---|---|---|
| $\lvert a_{e_{200}}^{\rm NR}\rvert$ | $\sim4\times10^{-9}$ GeV$^{-1}$ [36] | $\lvert a_{p_{200}}^{\rm NR}\rvert$ | $\sim4\times10^{-9}$ GeV$^{-1}$ [36] | $\lvert a_{\mu_{200}}^{\rm NR}\rvert$ | $\sim3\times10^{-5}$ GeV$^{-1}$ [34] |
| $\lvert c_{e_{200}}^{\rm NR}\rvert$ | $\sim2\times10^{-5}$ GeV$^{-1}$ [35] | | | $\lvert c_{\mu_{200}}^{\rm NR}\rvert$ | $\sim3\times10^{-5}$ GeV$^{-1}$ [34] |
| $\lvert a_{e_{400}}^{\rm NR}\rvert$ | $\sim50$ GeV$^{-3}$ GeV$^{-3}$ [36] | $\lvert a_{p_{400}}^{\rm NR}\rvert$ | $\sim50$ GeV$^{-3}$ [36] | $\lvert a_{\mu_{400}}^{\rm NR}\rvert$ | $\sim4\times10^{5}$ GeV$^{-3}$ [34] |
| $\lvert c_{e_{400}}^{\rm NR}\rvert$ | $\sim3\times10^{5}$ GeV$^{-3}$ [35] | | | $\lvert c_{\mu_{400}}^{\rm NR}\rvert$ | $\sim4\times10^{5}$ GeV$^{-3}$[34] |

Table 4 contains bounds on effective Cartesian coefficients obtained from studying boost effects of the 1 s–2 s transition in hydrogen and the Xe-He comagnetometer. The Lorentz-violating frequency shift in the laboratory frame can be expressed in terms of the nonrelativistic coefficients; however, to consider the annual or sidereal variations due to boost effects, we need to boost the frequency shift from the local laboratory frame to the Sun-centered frame. The nonrelativistic coefficients have simple transformation rules under rotation; however, their transformation under boost transformations is quite complicated, and it is easier to expand the nonrelativistic coefficients in terms of the Cartesian effective coefficients before studying boost effects [35,36]. For that reason, bounds due to boost effects are usually on effective Cartesian coefficients.

Finally, the best bounds on the muon nonrelativistic coefficients were obtained from hyperfine transitions of the ground state of muonium and the 1 s–2 s transition in muonium [17,34,47]; see

Tables 3 and 5. The reader should be aware that many of the bounds reported in [34–36] have not been reproduced in this section. Furthermore, the best bounds on minimal coefficients based on models that do not consider nonminimal terms have also been omitted from the discussion.

## 9. Outlook

The current bounds on nonminimal Lorentz-violating operators from atomic spectroscopy experiments are based on experimental studies that were designed to impose bounds on the minimal operators [34–36]. Signals associated only with the nonminimal operators such as sidereal variations in higher harmonics of the sidereal frequency have not been constrained experimentally, and they need to be studied to impose bounds on the nonminimal Lorentz-violating operators. For example, time variation studies of the Zeeman-hyperfine transitions of the ground state of cesium only considered the possibility of time variations in the first and second harmonic of the sidereal frequency [22]. The nonminimal Lorentz-violating model predicts that Zeeman-hyperfine transitions are sensitive to time variations up to the fourth harmonic of the sidereal frequency. Experimental constraints on sidereal variations in the third and fourth harmonic of the sidereal frequency will produce limits on coefficients for Lorentz violation that have not been bounded before. The same situation holds for the time variation studies [23–25] of the optical transitions in $^{171}Yb^{+}$ and $^{40}Ca^{+}$. Sidereal variation studies are also needed in the new field of high precision antihydrogen spectroscopy. The antihydrogen collaborations must consider how to implement time variation studies in their experiments if they want to test CPT symmetry systematically [35,36].

The recent publication [36] on the prospects of testing nonminimal Lorentz violation operators in clock comparison experiments used simplistic models for the electron and nuclear configurations. The advantage of using simple models is that they can be easily applied to a large range of systems and the publication intended to recognize the signals for Lorentz violation in a broad range of systems. Using more realistic models will not change the general form of these signals [36]; however, better models are needed to translate experimental constraints on the signals for Lorentz violation into bounds on coefficients. More realistic models have been used in the context of the minimal SME [22–24,29], and similar calculations are needed in the context of the nonminimal SME.

**Funding:** This work was supported in part by the Department of Energy under grant number DE-SC0010120 and by the Indiana University Center for Spacetime Symmetries.

**Acknowledgments:** The author would like to thank Jay Tasson for the invitation

**Conflicts of Interest:** The author declares no conflict of interest.

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