Lepton flavour violating processes in an S_3-invariant extension of the SM

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Abstract. A variety of lepton flavour violating effects related to neutrino oscillations and mixings will be systematically discussed in the framework of a minimal S_3-invariant extension of the Standard Model. After a brief review of some results on neutrino masses and mixings, we will give explicit analytical expressions for the matrices of the Yukawa couplings and the results of a computation of the branching ratios of some selected flavour-changing neutral current (FCNC) processes, as well as, the contribution of the exchange of neutral flavour-changing scalars to the anomaly of the magnetic moment of the muon, in terms of the masses of the charged leptons and the neutral Higgs bosons. It will also be shown that the $S_3 \times Z_2$ flavour symmetry and the strong mass hierarchy of the charged leptons strongly suppress the FCNC processes in the leptonic sector and give a nearly tri-bimaximal neutrino mixing matrix. The contribution of the FCNCs to the anomaly of the magnetic moment of the muon is small but non-negligible.

1. Introduction

The discovery that neutrinos have non-vanishing masses and mix among themselves much like the quarks do [1–19], brought out very forcefully the need of extending the Standard Model to accommodate in the theory the new data on neutrino physics in a coherent way, free of contradictions, and without spoiling the Standard Model's many phenomenological successes. At the same time, the number of free parameters in the model had to be drastically reduced to give predictive power to the theory. These two seemingly contradictory demands are met by a flavour symmetry under which the families transform in a non-trivial fashion. In the minimal S_3-invariant Extension of the Standard Model [20–26], the concept of flavour and generations is extended to the Higgs sector in such a way that all the matter fields - Higgs, quark and lepton fields, including the right handed neutrino fields - have three species and transform under the flavour symmetry group as the three dimensional representation $\mathbf{1} \oplus \mathbf{2}$ of the permutational group S_3. A model with more than one Higgs $SU(2)_L$ doublet has tree level flavour changing neutral currents whose exchange may give rise to lepton flavour violating processes and may also contribute to the anomalous magnetic moment of the muon. The phenomenological success of the model will be tested by verifying that all flavour changing neutral current processes and the magnetic anomaly of the muon, computed in the S_3-invariant extended form of the Standard Model, agree with the experimental values.

2. The Minimal S_3-invariant Extension of the Standard Model

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak and electromagnetic interactions. Prior to the introduction of the Higgs boson and mass terms, the Lagrangian is chiral and invariant with respect to permutations of the left and right fermionic fields.

The six possible permutations of three objects (f_1, f_2, f_3) are elements of the permutational group S_3. This is the discrete, non-Abelian group with the smallest number of elements. The three-dimensional real representation is not an irreducible representation of S_3. It can be decomposed into the direct sum of a doublet f_D and a singlet f_s, where

$$\begin{aligned} f_s &= \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \\ f_D^T &= \left(\frac{1}{\sqrt{2}}(f_1 - f_2), \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3)\right). \end{aligned} \tag{1}$$

The direct product of two doublets $\mathbf{p_D}^T = (p_{D1}, p_{D2})$ and $\mathbf{q_D}^T = (q_{D1}, q_{D2})$ may be decomposed into the direct sum of two singlets $\mathbf{r_s}$ and $\mathbf{r_{s'}}$, and one doublet $\mathbf{r_D}^T$ where

$$\mathbf{r_s} = p_{D1}q_{D1} + p_{D2}q_{D2}, \quad \mathbf{r_{s'}} = p_{D1}q_{D2} - p_{D2}q_{D1}, \tag{2}$$

$$\mathbf{r_D}^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} - p_{D2}q_{D2}). \tag{3}$$

The antisymmetric singlet $\mathbf{r_{s'}}$ is not invariant under S_3.

Since the Standard Model has only one Higgs $SU(2)_L$ doublet, which can only be an S_3 singlet, it can only give mass to the quark or charged lepton in the S_3 singlet representation, one in each family, without breaking the S_3 symmetry.

Hence, in order to impose S_3 as a fundamental symmetry, unbroken at the Fermi scale, we are led to extend the Higgs sector of the theory. The quark, lepton and Higgs fields are

$$\begin{aligned} Q^T &= (u_L, d_L)\ ,\ u_R\ ,\ d_R\ , \\ L^T &= (\nu_L, e_L)\ ,\ e_R\ ,\ \nu_R \ \text{ and }\ H, \end{aligned} \tag{4}$$

in an obvious notation. All of these fields have three species, and we assume that each one forms a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$. The doublets carry capital indices I and J, which run from 1 to 2, and the singlets are denoted by Q_3, u_{3R}, d_{3R}, L_3, e_{3R}, ν_{3R} and H_S. Note that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{5}$$

where

$$\begin{aligned} \mathcal{L}_{Y_E} \ = \ & -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\ & -Y_2^e [\ \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR}\ ] \\ & -Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.}, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{L}_{Y_\nu} \ = \ & -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ & -Y_2^\nu [\ \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR}\ ] \\ & -Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.}, \end{aligned} \tag{7}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ \text{ and } \ \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8}$$

$\mathcal{L}_{Y_D}$ and $\mathcal{L}_{Y_U}$ have similar expressions to $\mathcal{L}_{Y_E}$ and $\mathcal{L}_{Y_\nu}$ respectively.

Table 1. Z_2 assignment in the leptonic sector.

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}. \tag{9}$$

The extended Higgs sector has three $SU(2)$ doublets, in a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$ of the flavour group S_3. The Higgs potential, invariant under S_3, has an additional reflection symmetry $R:\ H_s \rightarrow\ -H_s$. and an accidental permutational symmetry S_2': $H_1 \leftrightarrow H_2$. Hence, $\langle H_1 \rangle = \langle H_2 \rangle$. Then the Yukawa interactions yield mass matrices for all fermions in the theory, of the general form [20]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{10}$$

The Majorana mass for the left handed neutrinos ν_L is generated by the see-saw mechanism. The corresponding mass matrix is given by

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_\mathbf{D}})^T, \tag{11}$$

where $\tilde{\mathbf{M}} = \mathrm{diag}(M_1, M_1, M_3)$.
In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry S_3. The mass matrices are diagonalized by bi-unitary transformations as

$$\begin{aligned} U^\dagger_{d(u,e)L} \mathbf{M}_{d(u,e)} U_{d(u,e)R} &= \mathrm{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \\ U_\nu^T \mathbf{M}_\nu U_\nu &= \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \tag{12}$$

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values.

The mixing matrices are, by definition,

$$V_{CKM} = U^\dagger_{uL} U_{dL}, \quad V_{PMNS} = U^\dagger_{eL} U_\nu K. \tag{13}$$

where K is the diagonal matrix of the Majorana phase factors.

3. The mass matrices in the leptonic sector and Z_2 symmetry

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian Z_2 symmetry. A possible set of charge assignments of Z_2, compatible with the experimental data on masses and mixings in the leptonic sector is given in Table 1. These Z_2 assignments forbid the following Yukawa couplings Y_1^e, Y_3^e, Y_1^ν and Y_5^ν. Therefore, the corresponding entries in the mass matrices vanish, *i.e.*, $\mu_1^e = \mu_3^e = 0$ and $\mu_1^\nu = \mu_5^\nu = 0$.

3.1. The mass matrix of the charged leptons

The remaining three parameters in the mass matrix of the charged leptons $|\tilde{\mu}_2|$, $|\tilde{\mu}_4|$ and $|\tilde{\mu}_5|$ may readily be expressed in terms of the charged lepton masses [22]. The resulting expression for M_e, written to order $(m_\mu m_e/m_\tau^2)^2$ and $x^4 = (m_e/m_\mu)^4$ is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}. \tag{14}$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Notice that this matrix has no free parameters other than the Dirac phase δ_e.

The unitary matrix U_{eL} that diagonalizes $M_e M_e^\dagger$ and enters in the definition of the neutrino mixing matrix V_{PMNS} may be written in the polar form as $U_{eL} = P_{eL}\mathbf{O_{eL}}$ [23] where P_{eL} is a diagonal matrix of phases and the orthogonal matrix $\mathbf{O}_{eL}$ can be written as M_e, as follows

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}}x\frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}x\frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x\frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \tag{15}$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$ and $x = m_e/m_\mu$.

3.2. The mass matrix of the neutrinos

According to the Z_2 selection rule, the mass matrix of the Dirac neutrinos takes the form

$$\mathbf{M}_{\nu_\mathbf{D}} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}. \tag{16}$$

Then, the mass matrix for the left-handed Majorana neutrinos, $\mathbf{M}_\nu$, obtained from the see-saw mechanism, $\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}}\tilde{\mathbf{M}}^{-1}(\mathbf{M}_{\nu_\mathbf{D}})^T$, is

$$\mathbf{M}_\nu = \begin{pmatrix} 2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu\rho_4^\nu \\ 0 & 2(\rho_2^\nu)^2 & 0 \\ 2\rho_2^\nu\rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 \end{pmatrix}, \tag{17}$$

where $\rho_2^\nu = (\mu_2^\nu)/M_1^{1/2}$, $\rho_4^\nu = (\mu_4^\nu)/M_1^{1/2}$ and $\rho_3^\nu = (\mu_3^\nu)/M_3^{1/2}$; M_1 and M_3 are the masses of the right handed neutrinos appearing in (9).

The non-Hermitian, complex, symmetric neutrino mass matrix M_ν may be brought to a diagonal form by a unitary transformation, as

$$U_\nu^T M_\nu U_\nu = \text{diag}\left(|m_{\nu_1}|e^{i\phi_1}, |m_{\nu_2}|e^{i\phi_2}, |m_{\nu_3}|e^{i\phi_\nu}\right), \tag{18}$$

where U_ν is the matrix that diagonalizes the matrix $M_\nu^\dagger M_\nu$.

As in the case of the charged leptons, the matrices M_ν and U_ν can be reparametrized in terms of the complex neutrino masses. Then [22, 23]

$$M_\nu = \begin{pmatrix} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3}-m_{\nu_1})(m_{\nu_2}-m_{\nu_3})}e^{-i\delta_\nu} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3}-m_{\nu_1})(m_{\nu_2}-m_{\nu_3})}e^{-i\delta_\nu} & 0 & (m_{\nu_1}+m_{\nu_2}-m_{\nu_3})e^{-2i\delta_\nu} \end{pmatrix} \tag{19}$$

and

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \begin{pmatrix} \cos\eta & \sin\eta & 0 \\ 0 & 0 & 1 \\ -\sin\eta & \cos\eta & 0 \end{pmatrix}, \tag{20}$$

where

$$\sin^2\eta = \frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_1}}, \quad \cos^2\eta = \frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_2}-m_{\nu_1}}. \tag{21}$$

The unitarity of U_ν constrains $\sin\eta$ to be real and thus $|\sin\eta| \leq 1$, this condition fixes the phases ϕ_1 and ϕ_2 as

$$|m_{\nu_1}|\sin\phi_1 = |m_{\nu_2}|\sin\phi_2 = |m_{\nu_3}|\sin\phi_\nu. \tag{22}$$

The only free parameters in the matrices M_ν and U_ν, are the phase ϕ_ν, implicit in m_{ν_1}, m_{ν_2} and m_{ν_3}, and the Dirac phase δ_ν.

3.3. The neutrino mixing matrix

The neutrino mixing matrix V_{PMNS}, is the product $U_{eL}^\dagger U_\nu K$, where K is the diagonal matrix of the Majorana phase factors, defined by

$$diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = K^\dagger diag(|m_{\nu_1}|, |m_{\nu_2}|, |m_{\nu_3}|)K^\dagger. \tag{23}$$

Except for an overall phase factor $e^{i\phi_1}$, which can be ignored, K is

$$K = diag(1, e^{i\alpha}, e^{i\beta}), \tag{24}$$

where $\alpha = 1/2(\phi_1 - \phi_2)$ and $\beta = 1/2(\phi_1 - \phi_\nu)$ are the Majorana phases.

Therefore, the theoretical mixing matrix V_{PMNS}^{th}, is given by

$$V_{PMNS}^{th} = \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta e^{i\delta} & O_{11}\sin\eta - O_{31}\cos\eta e^{i\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta e^{i\delta} & -O_{12}\sin\eta - O_{32}\cos\eta e^{i\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta e^{i\delta} & O_{13}\sin\eta + O_{33}\cos\eta e^{i\delta} & O_{23} \end{pmatrix} \times K, \tag{25}$$

where $\cos\eta$ and $\sin\eta$ are given eq. (21) O_{ij} are given in (15), and $\delta = \delta_\nu - \delta_e$.

To find how our results are related to the neutrino mixing angles we make use of the equality of the absolute values of the elements of V_{PMNS}^{th} and V_{PMNS}^{PDG} [28], that is

$$|V_{PMNS}^{th}| = |V_{PMNS}^{PDG}|. \tag{26}$$

This relation allows us to derive expressions for the mixing angles in terms of the charged lepton and neutrino masses.

The magnitudes of the reactor and atmospheric mixing angles, θ_{13} and θ_{23}, are determined by the masses of the charged leptons only. Keeping only terms of order (m_e^2/m_μ^2) and $(m_\mu/m_\tau)^4$, we get

$$\sin\theta_{13} \approx \frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4}}, \quad \sin\theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1+\frac{1}{4}x^2-2\tilde{m}_\mu^2+\tilde{m}_\mu^4}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4}}. \tag{27}$$

The magnitude of the solar angle depends on charged lepton and neutrino masses, as well as, on the Dirac and Majorana phases,

$$|\tan\theta_{12}|^2 = \frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}} \left(\frac{1 - 2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_3}}} + \left(\frac{O_{11}}{O_{31}}\right)^2 \frac{m_{\nu_3}-m_{\nu_1}}{m_{\nu_2}-m_{\nu_3}}}{1 + 2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}}} + \left(\frac{O_{11}}{O_{31}}\right)^2 \frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}}} \right). \tag{28}$$

The dependence of $\tan\theta_{12}$ on the Dirac phase δ, see (28), is very weak, since $O_{31} \sim 1$ but $O_{11} \sim 1/\sqrt{2}(m_e/m_\mu)$. Hence, we may neglect it when comparing (28) with the data on neutrino mixings.

The dependence of $\tan\theta_{12}$ on the phase ϕ_ν and the physical masses of the neutrinos enters through the ratio of the neutrino mass differences, it can be made explicit with the help of the unitarity constraint on U_ν, eq. (22),

$$\frac{m_{\nu_2}-m_{\nu_3}}{m_{\nu_3}-m_{\nu_1}} = \frac{(|m_{\nu_2}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{29}$$

4. Neutrino mass spectrum

In the present model, the numerical values of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are determined by the masses of the charged leptons only, in very good agreement with the experimental values [11, 12, 29],

$$(\sin^2\theta_{13})^{th} = 1.1 \times 10^{-5}, \quad (\sin^2\theta_{13})^{exp} \leq 0.046, \tag{30}$$

and

$$(\sin^2\theta_{23})^{th} = 0.5, \quad (\sin^2\theta_{23})^{exp} = 0.5^{+0.06}_{-0.05}. \tag{31}$$

In this model, the experimental restriction $|\Delta m_{12}^2| < |\Delta m_{13}^2|$ implies an inverted neutrino mass spectrum, $|m_{\nu_3}| < |m_{\nu_1}| < |m_{\nu_2}|$ [20].

As can be seen from eqs. (28) and (29), the solar mixing angle is sensitive to the neutrino mass differences and the phase ϕ_ν, but is only very weakly sensitive to the charged lepton masses. If we neglect the small terms proportional to O_{11} and O_{11}^2 in (28), we get

$$\tan^2\theta_{12} = \frac{(\Delta m_{12}^2+\Delta m_{13}^2+|m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(\Delta m_{13}^2+|m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{32}$$

From this expression, we may readily derive expressions for the neutrino masses in terms of $\tan\theta_{12}$ and ϕ_ν and the differences of the squared masses of the neutrinos masses

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu\tan\theta_{12}} \frac{1-\tan^4\theta_{12}+r^2}{\sqrt{1+\tan^2\theta_{12}}\sqrt{1+\tan^2\theta_{12}+r^2}}, \tag{33}$$

where $r^2 = \Delta m_{12}^2/\Delta m_{13}^2 \approx 3 \times 10^{-2}$.

The other two masses, $|m_{\nu_1}|$ and $|m_{\nu_2}|$ are immediately obtained from the knowledge of $|m_{\nu_3}|$ and Δm_{12}^2 and Δm_{13}^2.

The cosmological upper bound on the sum of neutrino masses sets a lower bound for $\cos\phi_\nu$ [17]

$$\sum |m_\nu| \leq 0.17\ eV \quad \longrightarrow \quad \cos\phi_\nu \geq 0.55 \tag{34}$$

Since, for small values of ϕ_ν, the neutrino masses change very slowly with $\cos\phi_\nu$, in the absence of any other experimental information, we set $\phi_\nu = 0$ in our formulas. Hence, we find

$$|m_{\nu_2}| \approx 0.056eV \quad |m_{\nu_1}| \approx 0.055eV \quad |m_{\nu_3}| \approx 0.022eV, \tag{35}$$

where we used the values $\Delta m_{13}^2 = 2.6 \times 10^{-3} eV^2$, $\Delta m_{21}^2 = 7.9 \times 10^{-5} eV^2$ and $\tan\theta_{12} = 0.667$, taken from [13].

5. V_{PMNS}^{th} and the tri-bimaximal form

Once the numerical values of the neutrino masses are determined, we may readily verify that the theoretical mixing matrix, V_{PMNS}^{th}, is very close to the tri-bimaximal form of the mixing matrix [30],

$$V_{PMNS}^{th} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} + \delta V_{PMNS}^{tri}, \tag{36}$$

where $\delta V_{PMNS}^{tri} = V_{PMNS}^{th} - V_{PMNS}^{tri}$. From eq. (25), the correction term to the tri-bimaximal form of the mixing matrix comes out as

$$\delta V_{PMNS}^{tri} \approx \begin{pmatrix} 1.94 \times 10^{-2} & -2.84 \times 10^{-2} & -3.4 \times 10^{-3} \\ 2.21 \times 10^{-2} & 1.5 \times 10^{-2} & -8.2 \times 10^{-6} \\ 1.8 \times 10^{-2} & 1.24 \times 10^{-2} & 3.1 \times 10^{-10} \end{pmatrix}. \tag{37}$$

A more complete discussion of the deviation from the tri-bimaximal form in the framework of the minimal S_3-invariant extension of the SM can be found in [25].

6. Flavour Changing Neutral Currents (FCNC)

Models with more than one Higgs $SU(2)$ doublet have tree level flavour changing neutral currents. In the Minimal S_3-invariant Extension of the Standard Model considered here, there is one Higgs $SU(2)$ doublet per generation coupling to all fermions. The flavour changing Yukawa couplings may be written in a flavour labelled, symmetry adapted weak basis as

$$\begin{aligned} \mathcal{L}_Y^{\rm FCNC} = & \left(\overline{E}_{aL} Y_{ab}^{ES} E_{bR} + \overline{U}_{aL} Y_{ab}^{US} U_{bR} + \overline{D}_{aL} Y_{ab}^{DS} D_{bR}\right) H_S^0 \\ & + \left(\overline{E}_{aL} Y_{ab}^{E1} E_{bR} + \overline{U}_{aL} Y_{ab}^{U1} U_{bR} + \overline{D}_{aL} Y_{ab}^{D1} D_{bR}\right) H_1^0 + \\ & \left(\overline{E}_{aL} Y_{ab}^{E2} E_{bR} + \overline{U}_{aL} Y_{ab}^{U2} U_{bR} + \overline{D}_{aL} Y_{ab}^{D2} D_{bR}\right) H_2^0 + \text{h.c.} \end{aligned} \tag{38}$$

The Yukawa couplings of immediate physical interest in the computation of the flavour changing neutral currents are those defined in the mass basis, according to $\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$, where U_{eL} and U_{eR} are the matrices that diagonalize the charged lepton mass matrix defined in eqs. (12). We obtain [23]

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{39}$$

Table 2. Leptonic FCNC processes, calculated with $M_{H_{1,2}} \sim 120\ GeV$.

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \to 3\mu$	8.43×10^{-14}	2×10^{-7}	B Aubert *et al* [33]
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7}	B Aubert *et al* [33]
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B Aubert *et al* [34]
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B Aubert *et al* [35]
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}	U Bellgardt *et al* [36]
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M L Brooks *et al* [37]

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{40}$$

where $\tilde{m}_\mu = 5.94 \times 10^{-2}$, $\tilde{m}_e = 2.876 \times 10^{-4}$ and $x = m_e/m_\mu = 4.84 \times 10^{-3}$. All the non-diagonal elements are responsible for tree-level FCNC processes. If the S_2' symmetry in the Higgs sector is preserved [31], $\langle H_1^0 \rangle = \langle H_2^0 \rangle = v$.

The amplitude of the flavour violating process $\mu \to 3e$, is proportional to $\tilde{Y}^E_{\mu e}\tilde{Y}^E_{ee}$ [32]. Then, the leptonic branching ratio,

$$Br(\mu \to 3e) = \frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\nu\bar{\nu})} \tag{41}$$

and

$$\Gamma(\mu \to 3e) \approx \frac{m_\mu^5}{3 \times 2^{10}\pi^3} \frac{\left(Y^{1,2}_{\mu e} Y^{1,2}_{ee}\right)^2}{M^4_{H_{1,2}}}, \tag{42}$$

which is the dominant term, and the well known expression for $\Gamma(\mu \to e\nu\bar{\nu})$ [28], give

$$Br(\mu \to 3e) \approx 2(2 + \tan^2\beta)^2 \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4, \tag{43}$$

taking for $M_H \approx 120\ GeV$ and $\tan\beta = 1$ we obtain $Br(\mu \to 3e) = 2.53 \times 10^{-16}$, well below the experimental upper bound for this process, which is 1×10^{-12} [36].

Similar computations give the numerical estimates of the branching ratios for some others flavour violating processes in the leptonic sector. These results, and the corresponding experimental upper bounds are shown in Table 2. In all cases considered, the theoretical estimations made in the framework of the minimal S_3-invariant extension of the SM are well below the experimental upper bounds [23].

7. Muon anomalous magnetic moment

In the minimal S_3-invariant extension of the Standard Model we are considering here, we have three Higgs $SU(2)$ doublets, one in the singlet and the other two in the doublet representations of the S_3 flavour group. The Z_2 symmetry decouples the charged leptons from the Higgs boson in the S_3 singlet representation. Therefore, in the leading order of perturbation theory there are two neutral scalars and two neutral pseudoscalars whose exchange will contribute to the anomalous magnetic moment of the muon. Since the heavier generations have larger flavour-changing couplings, the largest contribution comes from the heaviest charged leptons coupled to the lightest of the neutral Higgs bosons.

A straightforward computation gives

$$\delta a_{\mu}^{(H)} = \frac{Y_{\mu\tau}Y_{\tau\mu}}{16\pi^2}\frac{m_{\mu}m_{\tau}}{M_H^2}\left(log\left(\frac{M_H^2}{m_{\tau}^2}\right)-\frac{3}{2}\right). \quad (44)$$

With the help of (39) and (40) we may write $\delta a_{\mu}^{(H)}$ as

$$\delta a_{\mu}^{(H)} = \frac{m_{\tau}^2}{(246\ GeV)^2}\frac{(2+\tan^2\beta)}{32\pi^2}\frac{m_{\mu}^2}{M_H^2}\left(log\left(\frac{M_H^2}{m_{\tau}^2}\right)-\frac{3}{2}\right), \quad (45)$$

Taking again $M_H = 120\ GeV$ and the upper bound for $\tan\beta = 14$ gives an estimate of the largest possible contribution of the FCNC to the anomaly of the muon's magnetic moment $\delta a_{\mu}^{(H)} \approx 1.7 \times 10^{-10}$. This number has to be compared with the difference between the experimental value and the Standard Model prediction for the anomaly of the muon's magnetic moment [38]

$$\Delta a_{\mu} = a_{\mu}^{exp} - a_{\mu}^{SM} = (28.7 \pm 9.1) \times 10^{-10}, \quad (46)$$

which means

$$\frac{\delta a_{\mu}^{(H)}}{\Delta a_{\mu}} \approx 0.06. \quad (47)$$

Hence, the contribution of the flavour changing neutral currents to the anomaly of the magnetic moment of the muon is smaller than or of the order of 6% of the discrepancy between the experimental value and the Standard Model prediction.

8. Conclusions

In the minimal S_3-invariant extension of the SM the flavour symmetry group $Z_2 \times S_3$ relates the mass spectrum and mixings. This allowed us to compute the neutrino mixing matrix explicitly in terms of the masses of the charged leptons and neutrinos [22]. In this model, the magnitudes of the three mixing angles are determined by the interplay of the flavour $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy. We also found that V_{PMNS} has three CP violating phases, one Dirac phase $\delta = \delta_{\nu} - \delta_e$ and two Majorana phases, α and β, that are functions of the neutrino masses, and another phase ϕ_{ν} which is independent of the Dirac phase. The numerical values of the reactor, θ_{13}, and the atmospheric, θ_{23}, mixing angles are determined by the masses of the charged leptons only, in very good agreement with the experiment. The solar mixing angle θ_{12} is almost insensitive to the values of the masses of the charged leptons, but its experimental value allowed us to fix the scale and origin of the neutrino mass spectrum, which has an inverted hierarchy, with the values $|m_{\nu_2}| = 0.056eV$, $|m_{\nu_1}| = 0.055eV$ and $|m_{\nu_3}| = 0.022eV$. We also obtained explicit expressions for the matrices of the Yukawa couplings of the lepton sector parametrized in terms of the charged lepton masses and the VEV's of the neutral Higgs bosons in the S_3-doublet representation. These Yukawa matrices are closely related to the fermion mass matrices and have a structure of small and very small entries reflecting the observed charged lepton mass hierarchy. With the help of the Yukawa matrices, we computed the branching ratios of a number of FCNC processes and found that the branching ratios of all FCNC processes considered here are strongly suppressed by powers of the small mass ratios m_e/m_{τ} and m_{μ}/m_{τ}, and by the ratio $\left(m_{\tau}/M_{H_{1,2}}\right)^4$, where $M_{H_{1,2}}$ is the mass of the neutral Higgs bosons in the S_3-doublet. Taking for $M_{H_{1,2}}$ a very conservative value ($M_{H_{1,2}} \approx 120\ GeV$), we found that the numerical values of the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds by many orders of magnitude. It has already been argued that small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the gravitational core collapse and shock generation in the explosion stage of a supernova [39–41]. Finally, the contribution of the flavour changing neutral currents to the anomalous magnetic moment of the muon is small but non-negligible and it is compatible with the best, state of the art measurements and theoretical computations.

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