Nonradial oscillation of strange stars in d dimensions

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Abstract. The influence of the dimensions on the f and p_1 pulsation modes from strange quark stars, in the Cowling approximation, are investigated. For that purpose, the d-dimensional nonradial pulsation equations ($d \geq 4$) are numerically integrated considering that the Schwarzschild-Tangherlini line element describes the spacetime outside the object. We found that the fluid pulsation modes could become larger than those obtained in four dimensions. In four dimensions, the f pulsation mode is nearly constant, and for high total masses, it increases monotonically and quickly with the total mass. In this mass interval, the f frequencies grow for the spacetime dimensions between 4 and 6 and decay for d larger than 7. Concerning the p_1 pulsation modes, we found that they increase with the spacetime dimension and decline with the increment of the total mass.

1. Introduction

The detection of gravitational waves (GW) coming from a system of two black holes orbiting one another [1], accomplished by the LIGO and Virgo collaborations, is consistent with what general relativity predicted. A few months after this event, GWs originating from the merger of two neutron stars were reported in [2]. This last event, together with its electromagnetic counterpart [3], shows a new way to study observational astrophysics.

Persuaded by theories involving extra dimensions and stimulated by the idea that GWs could be the way to prove their existence, the study of some astrophysical phenomena in extra dimensions has been carried out by different authors. Within the Einstein's theory of gravity, for example, the implications of extra-dimensional spacetimes on the static equilibrium configurations [4, 5], radial stability [6], compactness [7, 8], and gravitational collapse [9, 10] have been theoretically addressed.

Inspired by these articles, within the Cowling approximation approach, we analyze the dependence of the fluid pulsation modes of stable strange stars with extra dimensions (see Ref. [6]). For our aim, the stellar structure equations [11] and the non-radial oscillation equations [12, 13], both modified for the inclusion of the extra dimensions, are integrated numerically. Throughout this work, we use the units $c = 1 = G_4$, where c and G_4 represent the speed of light and the gravitational constant in four dimensions, respectively.

2. Higher-dimensional general relativity equations

2.1. Einstein field equation and energy-momentum tensor

To investigate the fluid pulsation modes f and p_1, in the general relativity context in a d-dimensional spacetime, we consider the field equation ($d \geq 4$) of the form [14]:

$$G_{\mu\nu} = \frac{d-2}{d-3} S_{d-2} G_d T_{\mu\nu}. \tag{1}$$

On the left-hand-side of Eq. (1), $G_{\mu\nu}$ stands the Einstein tensor. The right-hand-side contains the area of unitary hypersphere $S_{d-2} = 2\sqrt{\pi^{(d-1)}}/\Gamma\left(\frac{d-1}{2}\right)$, where Γ represents the gamma function, with $(d-2)\,S_{d-2}/(d-3)$ and G_d being respectively 8π term and the Newton's gravitational constant in $d=4$. In addition, $T_{\mu\nu}$ is the stress-energy tensor where

$$T_{\mu\nu} = \rho_d U_\mu U_\nu + p_d \left(U_\mu U_\nu + g_{\mu\nu}\right), \tag{2}$$

with ρ_d representing the energy density, p_d being the fluid pressure, and U_μ stands for the velocity of the fluid in a d-dimensional spacetime, where $U_\mu U^\mu = -1$. The aforenamed Greek indices μ, ν, etc., run from 0 until $d-1$.

2.2. Stellar structure equations

We regard that the fluid distribution in the hypersphere is depicted by the line element in d dimensions [6]:

$$ds^2 = -e^{2\Phi(r)}\, dt^2 + e^{2\Lambda(r)}\, dr^2 + r^2 \sum_{j=1}^{d-2} \left(\prod_{i=1}^{j-1} \sin^2\theta_i \right) d\theta_j^2, \tag{3}$$

with $\Phi = \Phi(r)$ and $\Lambda = \Lambda(r)$ being functions depending on the radial coordinate r.

To investigate the stellar equilibrium configurations in higher-dimensional spacetime, we integrate the set of equations

$$\frac{dm}{dr} = S_{d-2} r^{d-2} \rho_d, \tag{4}$$

$$\frac{dp_d}{dr} = -(p_d + \rho_d) \left[\frac{\frac{S_{d-2} G_d p_d r}{(d-3)} + \frac{m G_d}{r^{d-2}}}{1 - \frac{2m\, G_d}{(d-3) r^{d-3}}} \right], \tag{5}$$

$$\frac{d\Phi}{dr} = -\frac{1}{(p_d + \rho_d)} \frac{dp_d}{dr}. \tag{6}$$

The parameter m plays the role of mass function and represents the gravitational mass inside the radial coordinate r. Eq. (5) shows the hydrostatic equilibrium equation, also known as the Tolman-Oppenheimer-Volkoff equation [11], modified to insert the effects of the extra dimensions [6].

The static equilibrium equations are integrated from the center to the surface of the object. In the center ($r=0$), we consider the conditions $m(0)=0$, $\Lambda(0)=0$, $\Phi(0)=\Phi_c$, $p_d(0)\,G_d = p_{cd}\,G_d$, and $\rho_d(0)\,G_d = \rho_{cd}\,G_d$; where the parameters $p_{cd}\,G_d$ and $\rho_{cd}\,G_d$ are respectively the central pressure and the central energy density. The compact object's surface ($r=R$) is attained when $p_d(R)\,G_d = 0$. At this point, the interior metric connects smoothly with the Schwarzschild-Tangherlini exterior spacetime [15, 16] where $e^{2\Phi(R)} = e^{-2\Lambda(R)} = 1 - \frac{2G_d\, M}{(d-3) r^{d-3}}$, with $MG_d/(d-3)$ standing the total mass of the object.

2.3. Nonradial oscillation equations

Perturbed stars can present different modes of oscillations depending on the restoring force that acts on them. They form a distinct family of oscillation modes [17]. E.g., we have the f-mode, favored for the gravitational waves emission, and p-modes, where the pressure is the restoring force and with frequencies higher than the f-modes. One of the methods used to investigate the oscillation modes is the relativistic Cowling approximation, where the metric perturbations are neglected during the fluid oscillations. In $d = 4$, this method displays a difference of around 20% and 10% to the results found by a relativistic numerical form for f and p_1-modes [18], respectively. With this motivation, we find that this method is used to study, e. g., how rotational speed [19, 20], crustal elasticity [21], and anisotropy [22] affect fluid pulsation modes coming from compact stars.

The higher-dimensional fluid Lagrangian displacement vector is assumed of the form [23]

$$\varsigma^k = \left[\frac{e^{-\Lambda}}{r^{d-4}}\tilde{Q}, -\tilde{Z}\left[\prod_{i=1}^{j-1}\frac{1}{\sin^2\theta_i}\right]\frac{\partial}{\partial\theta_j}\right]r^{-2}Y_l^m, \tag{7}$$

where k goes from 1 to $d-1$ and j runs from 1 to $d-2$. Eq. (7) contains the parameters $\tilde{Q} = \tilde{Q}(t,r)$ and $\tilde{Z} = \tilde{Z}(t,r)$, which are functions of both the temporal t and radial coordinate r, and the harmonic functions in d dimensions $Y_l^m = Y_l^m(\theta_1, ..., \theta_{d-2})$. Then the perturbations of the velocity, δU^μ, can be placed as

$$\delta U^\mu = \left(0, e^{-\Phi}\frac{d\varsigma^r}{dt}, e^{-\Phi}\frac{d\varsigma^{\theta_i}}{dt}\right). \tag{8}$$

It is worth mentioning that Eq. (8) can be reduced to the form reported in [24] assuming $d = 4$.

With these variables, the fluid pulsation equations can be derived by considering the variation of the conservation of energy-momentum tensor ($\delta(\nabla_\mu T^{\mu\nu}) = 0$). Considering $\delta g_{\mu\nu}$=0, we get $\nabla_\mu(\delta T^{\mu\nu}) = 0$. The explicit forms with $\nu = r, \theta$ are

$$(p_d+\rho_d)\frac{e^{\Lambda-2\Phi}}{r^{d-2}}\frac{\partial^2\tilde{Q}}{\partial t^2} - \frac{\partial}{\partial r}\left[p_d\left[\frac{1}{e^\Lambda r^{d-2}}\frac{\partial\tilde{Q}}{\partial r} + \frac{l(l+d-3)}{r^2}\tilde{Z}\right]\Gamma_1 + \frac{\tilde{Q}}{e^\Lambda r^{d-2}}\frac{dp_d}{dr}\right]$$
$$-\left[\frac{d\rho_d}{dr}+\frac{dp_d}{dr}\right]\frac{d\Phi}{dr}\frac{\tilde{Q}}{e^\Lambda r^{d-2}} + \frac{dp_d}{dr}\left[\frac{dp_d}{d\rho_d}+1\right]\left[\frac{1}{e^\Lambda r^{d-2}}\frac{\partial\tilde{Q}}{\partial r} + \frac{l(l+d-3)}{r^2}\tilde{Z}\right] = 0, \tag{9}$$

$$\frac{p_d+\rho_d}{e^{2\Phi}}\frac{\partial^2\tilde{Z}}{\partial t^2} + p_d\left[\frac{1}{e^\Lambda r^{d-2}}\frac{\partial\tilde{Q}}{\partial r} + \frac{l(l+d-3)}{r^2}\tilde{Z}\right]\Gamma_1 + \frac{\tilde{Q}}{e^\Lambda r^{d-2}}\frac{dp_d}{dr} = 0, \tag{10}$$

where $\Gamma_1 = \left[1+\frac{\rho_d}{p_d}\right]\left[\frac{dp_d}{d\rho_d}\right]$.

Following [23], we consider the perturbative parameters as $\tilde{Q}(t,r) = e^{i\omega t}Q(r)$ and $\tilde{Z}(t,r) = e^{i\omega t}Z(r)$, where ω depicts the eigenfrequency of oscillation. Furthermore, from these two second-order differential equations last equations, Eq. (9) and Eq. (10), we can be derived in two first-order differential equations what are more appropriate for numerical integration. In such a way, we replace the difference $\frac{d}{dr}$ [Eq. (10)]-[Eq. (9)] inside Eq. (10). Thus, we obtain:

$$\frac{dZ}{dr} = 2\frac{d\Phi}{dr}Z - \frac{e^\Lambda}{r^{d-2}}Q. \tag{11}$$

In turn, from Eqs.(10) and (11), we have:

$$\frac{dQ}{dr} = -l(l+d-3)e^\Lambda r^{d-4}Z + \left[\frac{d\rho_d}{dp_d}\right]\left[\frac{\omega^2 e^\Lambda r^{d-2}}{e^{2\Phi}}Z + \frac{d\Phi}{dr}Q\right]. \tag{12}$$

Eqs. (11) and (12) are reduced to those reported in [24] for $d = 4$. We integrate Eqs. (11) and (12) from the center $r = 0$ toward the hypersphere's surface $r = R$. Following [22, 24], with the aim to determine regular solutions at the center, we assume $Q/r^{l+d-3} = -Zl/r^l = C$, with C being a dimensionless constant. Moreover, on the surface of the object is found:

$$\left[\frac{\omega^2 e^{\Lambda} r^{d-2}}{e^{2\Phi}} Z + \frac{d\Phi}{dr} Q\right]_{r=R} = 0. \tag{13}$$

2.4. The profile of the equation of state

The fluid that makes up the compact object is defined by

$$p_d = \frac{\rho_d}{(d-1)} - \frac{d\,\mathcal{B}_d}{(d-1)}, \tag{14}$$

with p_d and ρ_d representing respectively the pressure and energy density, and where $\mathcal{B}_d$ stands a constant. Following [6], we regard $G_d\, d\,\mathcal{B}_d = 240\,[\mathrm{MeV/fm^3}]$.

To determine the oscillations spectrum of compact star, Eqs. (11) and (12) are solved together with the equation of state and the boundary condition (13) for $l = 2$ and different values of d and $\rho_{cd} G_d$ through the Runge–Kutta fourth-order method implemented with the shooting method.

3. Numerical results

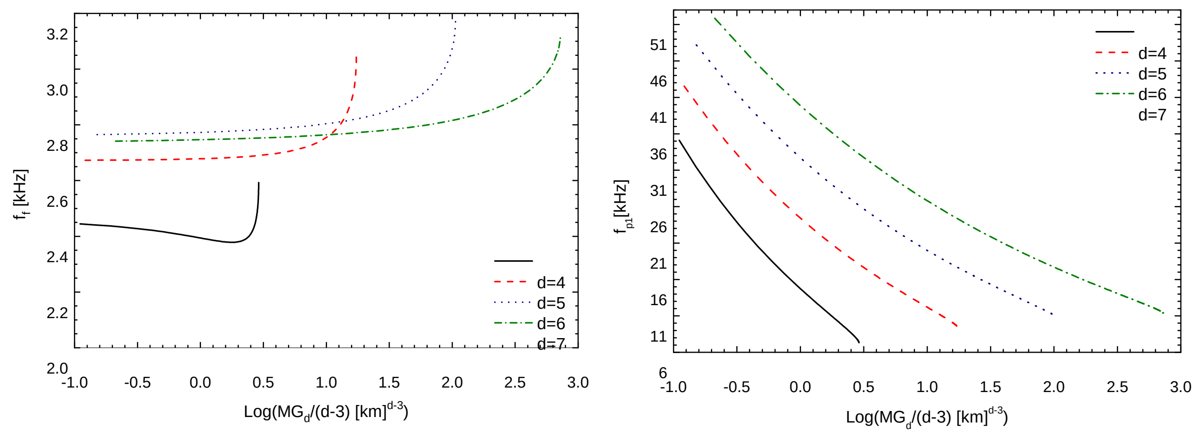

Figure 1. The change of f- and p_1-mode frequencies with the total mass $MG_d/(d-3)$ are shown on the left and right panels, respectively. In both figures, four different dimensions are considered.

The behavior of the f- and p_1-mode oscillation with the total mass $MG_d/(d-3)$ is plotted in the left and right panels of Fig. 1, respectively, for some spacetime dimensions. In figures, stable compact objects against small radial perturbations are considered, review [6].

From the figure, we see that all f-mode frequency curves stay nearly constant [25, 26] and, only in large mass values, it grows quick with $MG_d/(d-3)$. In the four-dimensional case, the curve f-mode frequency decays with the total mass until reaches a minimum value. Hereafter, the curve turns counterclockwise to start increasing its value with the total mass. For higher dimensions, the f-mode is almost constant, and for large total mass values, it increases with $MG_d/(d-3)$. In the same figure, it can be observed the influence of the dimension influence on the f-mode. For total mass interval, we see that the f-mode grows in the dimensions $4 \leq d \leq 6$

and decays for $d \geq 7$. In all dimensions taken into account, all f-mode frequencies are within the interval $2.38 - 3.18$ [kHz] range, which is determined in the respective dimensions $d = 4$ and $d = 6$.

In turn, such as happen in the four-dimensional spacetime, for $d > 4$, we observe that the p_1-are higher than the f-modes frequencies (see [27]). Moreover, it can be seen that the p_1-mode has a monotonic decrease with the increment total mass, thus reaching the lowest p_1-modes at the maximum mass points. Besides, for a total mass range, the p_1-mode frequencies are also affected by the growth of d. For larger dimensions higher p_1-modes are found.

4. Conclusions

In the scope of general relativity, the change of the oscillation spectrum with the spacetime dimension is investigated. For this purpose, the static equilibrium configuration and the nonradial equation within the Cowling approximation in d spacetime dimensions are obtained. For the fluid, it is considered the MIT bag model equation of state extended for d dimensions. It is also assumed that the interior solutions are matched to the exterior Schwarzschild-Tangherlini line element. The f- and p_1-mode frequencies of stable compact objects against radial perturbations are analyzed for some mass $MG_d/(d-3)$ and dimensions d.

By observing the oscillation frequencies of strange quark stars, for a range of masses and dimensions, the f-mode frequencies are essentially constant and exhibit rapid growth for larger masses, in contrast to the p_1 modes which change notably with $MG_d/(d-3)$ and d. Additionally, we found that the minimum and maximum mode frequency f are obtained respectively at $d = 4$ and $d = 6$. On the other hand, the growth of the p_1-mode frequency with the dimension is found. Such as is determined in four dimensions, in higher-dimensional spacetimes the f-modes are smaller than p_1-mode frequencies.

The possibility of detecting f pulsation modes from compact stars, for different masses, and getting nearly constant frequency values in the range $f \sim 2 - 3$ [kHz] with $M/M_\odot \leq 1.7$, in $d = 4$, it would be a good sign of the existence of the of strange quark stars. If the f-modes frequencies are still constant and higher than those obtained in four dimensions for an interval of $MG_d/(d-3)$, it would indicate that quarks can propagate in extra-dimensional spacetimes, and d-dimensional strange quark stars could exist.

Acknowledgments

We thank Fundação de Amparo à Pesquisa do São Paulo, project No. $2013/26258 - 4$. MM is grateful to Conselho Nacional de Desenvolvimento Científico e Tecnológico and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior for a grant.

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