Effectiveness of Empirical Mode Decomposition in Search for Gravitational Wave Signals

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Abstract

The Hilbert-Huang transform is a novel, adaptive approach to time series analysis that does not make assumptions about the data form. This algorithm is adaptive and does not impose a basis set on the data, and thus the time-frequency decomposition is not limited by time-frequency uncertainty spreading. Because of its high time-frequency resolution, it will have important applications to the detection of gravitational wave signal. As a first step, we demonstrate a possibility of the application of a Hilbert-Huang transform to the search for gravitational waves.

1 Introduction

The Hilbert-Huang transform (HHT) is the combination of the well-known Hilbert spectral analysis and the empirical mode decompositon developed recently by Huang et al.[1]. It presents a fundamentally new approach to the analysis of time series data. Its essential feature is the use of an adaptive time-frequency decomposition that does not impose a fixed basis set on the data, and therefore, unlike Fourier or Wavelet analysis, its application is not limited by the time-frequency uncertainty relation. This leads to a highly efficient tool for the investigation of transient and nonlinear features. The HHT is applied to various fields including materials damage detection [2], biomedical monitoring [3] [4], etc. Because gravitational wave detectors, such as LIGO, Virgo and LCGT, have a great variety of nonlinear and transient signals, the HHT has the promise of being a powerful new tool in the search for gravitational waves. We will, therefore, demonstrate a possibility of the application of the HHT to data analysis of gravitational waves.

2 Brief Description of Hilbert-Huang Transform

The HHT consists of two components; empirical mode decomposition (EMD) and Hilbert spectral analysis. In this section, we introduce briefly both components of HHT. It will be shown that the Hilbert transform (HT) can lead to an apparent time-frequency-energy description of a time series. However, this description may not be consistent with physically meaningful definitions of instantaneous frequency and instantaneous amplitude, since the HT is based on Cauchy's integral formula of holomorphic functions tending to zero fast enough at infinity. The EMD can generate components of the time series whose HT can lead to physically meaningful definitions of these two instantaneous quantities, and hence the combination of EMD and HT provides a more physically meaningful time-frequency-energy description of a time series.

Hereafter, we assume that the input $x(t)$ is given by sampling a continuous signal at the discrete time, $t = t_i$ for $i = 0, 1, \cdots, N$.

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2.1 Empirical Mode Decomposition

The EMD has implicitly a simple assumption that, at any given time, the data may have many coexisting oscillatory modes of significantly different frequencies, one superimposed on the other. The each component is defined as an intrinsic mode function (IMF) that satisfies the following conditions: (i) In the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one. (ii) At any data point, the mean value of the upper and the lower envelopes defined using the local maxima and the local minima, respectively, is zero.

With the above definition of an IMF, we can then decompose any function through a sifting process. The sifting starts with identifying all the local extrema and then connecting all the local maxima (minima) by a cubic spline to form the upper (lower) envelope. The upper and lower envelopes usually encompass all the data between them. Their mean is designated as $m_1(t)$. The difference between the input $x(t)$ and $m_1(t)$ is the first protomode, $h_1(t)$, namely, $h_1(t) = x(t) - m_1(t)$. By construction, h_1 is expected to satisfy the definition of an IMF. However, that is usually not the case since changing a local zero from a rectangular to a curvilinear coordinate system may introduce new extrema, and further adjustments are needed. Therefore, a repeat of the above procedure is necessary. This sifting process serves two purposes; (a) to eliminate background waves on which the IMF is riding and (b) to make the wave profiles more symmetric. The sifting process has to be repeated as many times as is required to make the extracted signal satisfy the definition of an IMF. In the iterating processes, h_1 can only be treated as a proto-IMF, which is treated as the data in the next iteration: $h_1(t) - m_{11}(t) = h_{11}(t)$. After k times of iterations, $h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t)$; the approximate local envelope symmetry condition is satisfied, and h_{1k} becomes the IMF c_1, that is, $c_1(t) = h_{1k}(t)$.

The approximate local envelope symmetry condition in the sifting process is called the stoppage criterion. The several different types of stoppage criterion were adopted. In this article, we use the Cauchy types of stoppage criterion [1]:

$$\sum_{i=0}^{N} \left|m_{1k}(t_i)\right|^2 \bigg/ \sum_{i=0}^{N} \left|h_{1k}(t_i)\right|^2 < \epsilon \tag{1}$$

with a predetermined value ϵ.

The first IMF should contain the finest scale or the shortest-period oscillation in the signal, which can be extracted from the data by $x(t) - c_1(t) = r_1(t)$. The residue, r_1, still contains longer-period variations. This residual is then treated as the new data and subjected to the same sifting process as described above to obtain an IMF of lower frequency. The procedure can be repeatedly applied to all subsequent r_n, and the result is $r_{n-1}(t) - c_n(t) = r_n(t)$. The decomposition process finally stops when the residue, r_n, becomes a monotonic function or a function with only one extremum from which no more IMF can be extracted. Thus, the original data are decomposed into n IMFs and a residue obtained, r_n, which can be either the adaptive trend or a constant: $x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t)$.

2.2 Hilbert Spectral Analysis

The purpose of the development of HHT is to provide an alternative view of the time-frequency-energy paradigm of data. In this approach, the nonlinearity and nonstationarity can be dealt with better than by using the traditional paradigm of constant frequency and amplitude. One way to express the nonstationarity is to find instantaneous frequency (IF) and instantaneous amplitude (IA). This was the reason why Hilbert spectral analysis was included as a part of HHT.

For any function $x(t)$, its Hilbert transform (HT) $y(t)$ is

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau, \tag{2}$$

where P is the Cauchy principal value of the singular integral. Although it is not trivial to calculate the Cauchy principal value numerically, the HT can be obtained using the Fourier transform or the FFT of $x(t)$ and the convolution theorem, since the HT is the convolution of $x(t)$ and $1/(\pi t)$. Assuming the function $x(t)$ is the real part of a holomorphic function $F(z)$ on the real axis $z = t$, the HT $y(t)$ will be its imaginary part, that is,

$$F(t) = x(t) + iy(t) = a(t)e^{i\theta(t)}, \tag{3}$$

where

$$a(t) = \sqrt{x(t)^2 + y(t)^2} \qquad \text{and} \qquad \theta(t) = \tan^{-1}\left\{\frac{y(t)}{x(t)}\right\}. \tag{4}$$

Here $a(t)$ is the instantaneous amplitude and $\theta(t)$ is the instantaneous phase function. The instantaneous frequency is given by

$$f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = \frac{1}{2\pi a^2}\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right). \tag{5}$$

With both amplitude and frequency, we can express the amplitude (or energy, the square of amplitude) in terms of a function of time and frequency, $H(\omega, t)$. The marginal spectrum can then be defined as

$$h(\omega) = \int_0^T H(\omega, t)dt, \tag{6}$$

where $[0, T]$ is the temporal domain within which the data is defined. The marginal spectrum represents the accumulated amplitude (energy) over the entire data span in a probabilistic sense and offers a measure of the total amplitude (or energy) contribution from each frequency value, serving as an alternative spectrum expression of the data to the traditional Fourier spectrum.

3 Demonstration of HHT as Gravitational Wave Data Analysis

To illustrate the application of the HHT to data analysis of gravitational waves (GW), we look at the identification of a signal in white Gaussian noise. Our principal motivation is in analyzing data from GW detectors such as LIGO, Virgo and LCGT. GW signals at the sensitivity of the current detector are not expected to show rates exceeding one per year at SNR > 8. The adaptive and high time-frequency resolution features of the HHT are well suited to GW analysis.

We focus in this article on simulations with time series data composed of stationary white Gaussian noise and GW signals well separated in time. As an example, we inject a 20 solar mass black hole binary merger and ringdown signal [5] with SNR = 7 into white Gaussian noise at 16 kHz sampling rate. The merger signal is shown in the top panel of Fig.1 and the time series of signal in white Gaussian noise is shown in the lower panel of Fig.1.

Although, in this article, we do not discuss in detail, we have made the modifications, which is called Ensemble Empirical Mode Decomposition (EEMD) [6], to HHT application to the GW data analysis. The purpose of introducing EEMD is to average over errors in the EMD process. EEMD is also an algorithm, which contains the following steps: (1) Add a white noise series to the targeted data; (2) decompose the data with added white noise into IMFs; (3) repeat steps (1) and (2) again and again but with different white noise series each time; and (4) the ensemble means of corresponding IMFs of the decompositions are obtained as the final result. In this article, we set a EMD ensemble number and EMD stoppage criterion 200 and $\epsilon = 0.01$ respectively.

The top panel of Fig.2 shows the results of EEMD. The green, blue and red lines show 2nd, 3rd and 4th IMFs. The black line shows injected signal. In the 2nd and 3rd IMFs, the signal can be seen, largely separate from the noise. The middle and lower panels of Fig.2 show the instantaneous amplitude and frequency derived from the Hilbert transform of 2nd, 3rd and 4th IMFs. The black line shows the instantaneous amplitude and frequency of injected signal without noise. The instantaneous amplitude and frequency of 2nd plus 3rd IMFs display similar behavior of the instantaneous frequency and power of injected signal. Thus, there is a possibility that we can identify a targeted signal and extract information about the signal frequency and power evolution in time.

4 Summary

In this article, we briefly reviewed the analysis algorithm of the HHT. As a fist step, we demonstrated the application of the HHT to GW data analysis. To illustrate the application of the HHT to GW analysis,

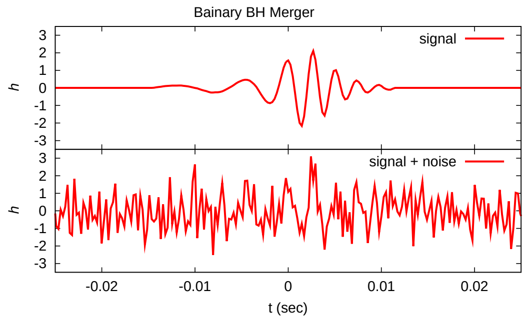

Figure 1: 20 solar mass black hole binary merger and ringdown signal [5]. The upper and lower panels show the pure signal and the signal in white Gaussian noise with SNR=7, respectively.

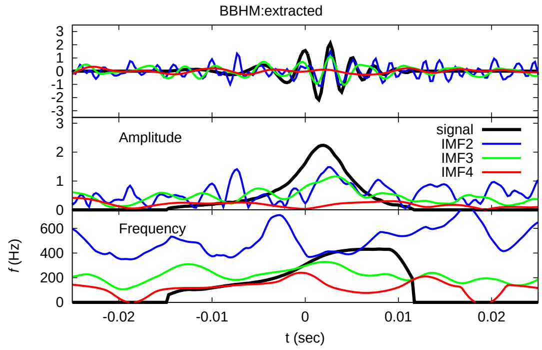

Figure 2: IMFs 2, 3, 4 and original signal. The middle and lower panels display instantaneous amplitude and frequency, respectively.

we looked at the identification of a signal in white Gaussian noise. As the result, we found that there was a possibility that we could identify a targeted signal and extract information about the signal frequency and power evolution in time.

In future, because HHT is empirical method, we need more systematic simulations to investigate the property of the HHT. Many more details of the results of systematic simulations will be discussed elsewhere.

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