ReneSANCe event generator for high-precision e^+e^- physics

R Sadykov[1], A Arbuzov[2], S Bondarenko[2], Y Dydyshka[1,3], L Kalinovskaya[1], L Rumyantsev[1], and V Yermolchyk[1,3]

[1] Dzhelepov Laboratory for Nuclear Problems, JINR, Joliot-Curie 6, RU-141980 Dubna, Russia
[2] Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie 6, RU-141980 Dubna, Russia
[3] Institute for Nuclear Problems, Belarusian State University, Bobruiskaya 11, 220006 Minsk, Belarus

E-mail: `sadykov@cern.ch`

Abstract. We present a new version of the Monte Carlo event generator `ReneSANCe`. The generator takes into account complete one-loop electroweak (EW) corrections, higher order QED corrections in the leading logarithm (LL) approximation and some higher order EW corrections to processes at e^+e^- colliders with finite particle masses and arbitrary polarizations of initial and final particles. `ReneSANCe` can produce events in the full phase space and this option is available for a wide range of center-of-mass energies.

1. Introduction

`Renewed SANC` Monte Carlo `event generator` (`ReneSANCe`) [1] simulates processes at e^+e^- colliders. The following processes are fully implemented:

- Bhabha scattering ($e^+e^- \to e^-e^+$) [2],
- Higgs-strahlung ($e^+e^- \to ZH$) [3, 4],
- s-channel annihilation ($e^+e^- \to \mu^-\mu^+,\ \tau^-\tau^+$) [5].

The generator is based on the `SANC` modules [6] which contain complete one-loop and some higher-order electroweak radiative corrections. Polarization and masses of all particle can be taken into account. The generator efficiently operates in collinear regions and in a wide range of center-of-mass energies. Other processes can be easily added.

2. The SANC framework and products

The `SANC` framework is organized in the way shown in 1. The calculation of the virtual diagrams has been performed in the unitary gauge and R_ξ gauge using dimensional regularization. The diagrams were evaluated by means of `FORM` [7]. Tensor integrals have been reduced to linear combinations of a fundamental set of scalar integrals in terms of the standard scalar Passarino-Veltman functions. The obligatory checks were of the type the gauge invariance by eliminating the dependence on the gauge parameters, cancellation of ultraviolet poles, checking various symmetry properties and the Ward identities. Covariant amplitudes and form factors of $ffbb \to 0$, $4b \to 0$, and $4f \to 0$, where f is a fermion and b is a boson, type processes are calculated analytically for the case of anihilation into vacuum. Helicity amplitudes obtained using internal procedures of the `SANC` system. The latter are translated to `FORTRAN` modules for specific processes with NLO EW and QCD corrections. The modules are utilizing `LoopTools` [8] and `SANClib` packages for loop integral evaluations.

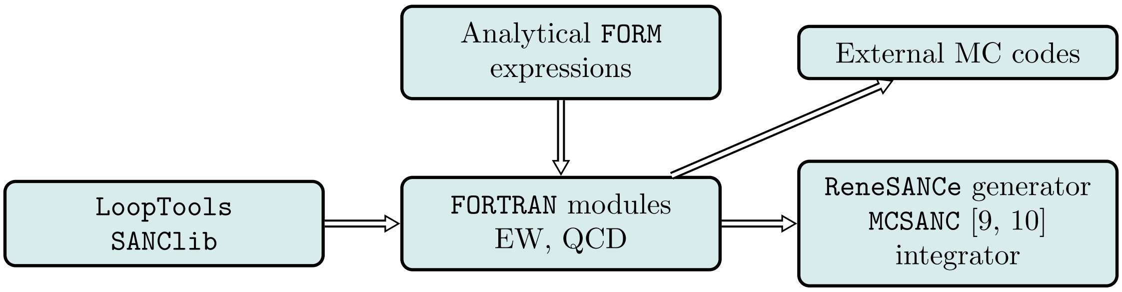


Figure 1. The `SANC` framework.

3. Polarization

A state with arbitrary polarizations of the initial e^+ and e^- can be described by polarization vectors

$$\vec{P}_{e^\pm} = (f_\pm \sin\theta_\pm \cos\phi_\pm, f_\pm \sin\theta_\pm \sin\phi_\pm, \pm f_\pm \cos\theta_\pm),$$

where $f_\pm \in [0;1]$ are polarization degrees, $\theta_\pm \in [0;\pi]$ are polar angles, and $\phi_\pm \in [0;2\pi]$ are azimuthal angles (figure 2).

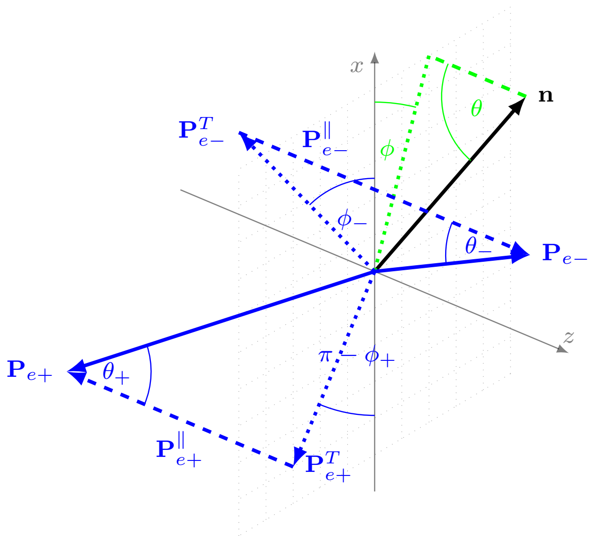


Figure 2. Polarization vectors.

The matrix element squared for a process with initial e^+ and e^- can be written as

$$|\mathcal{M}|^2 = \sum_{\lambda_{e^+}\lambda_{e^-}\lambda'_{e^+}\lambda'_{e^-}} \rho^{e^+}_{\lambda_{e^+}\lambda'_{e^+}} \rho^{e^-}_{\lambda_{e^-}\lambda'_{e^-}} \mathcal{H}_{\lambda_{e^+}\lambda_{e^-}} \mathcal{H}^*_{\lambda'_{e^+}\lambda'_{e^-}}$$

with helicity density matrices

$$\rho^{e^+} = 1/2 \begin{pmatrix} 1 + P^{\shortparallel}_{e^+} & P^{\perp}_{e^+} e^{-i(\phi_+ - \phi)} \\ P^{\perp}_{e^+} e^{i(\phi_+ - \phi)} & 1 - P^{\shortparallel}_{e^+} \end{pmatrix}, \ \rho^{e^-} = 1/2 \begin{pmatrix} 1 + P^{\shortparallel}_{e^-} & -P^{\perp}_{e^-} e^{i(\phi_- - \phi)} \\ -P^{\perp}_{e^-} e^{-i(\phi_- - \phi)} & 1 - P^{\shortparallel}_{e^-} \end{pmatrix}.$$

Here $P^{\shortparallel}_{e^\pm} = \pm f_\pm \cos\theta_\pm$, $P^{\perp}_{e^\pm} = f_\pm \sin\theta_\pm$, and $\mathcal{H}_{\lambda_{e^+}\lambda_{e^-}}$ are helicity amplitudes for a process with initial e^+ and e^-.

4. Cross section at one-loop level

The cross section of processes at the one-loop level can be divided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where σ^{Born} is the Born level cross section, σ^{virt} is the virtual (loop) contribution, σ^{soft} is soft photon bremsstrahlung one, and σ^{hard} is hard photon bremsstrahlung one (with energy $E_\gamma > \omega$). The auxiliary parameters λ ("photon mass") and ω (an arbitrary soft cut-off) are disposed of after summation.

5. Higher order improvements

Higher order improvements get added through the $\Delta\rho$ parameter: $s_W^2 \to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2$. At the two-loop level, the quantity $\Delta\rho$ contains two contributions:

$$\Delta\rho = N_c X_t \left[1 + \rho^{(2)} \left(M_H^2/m_t^2\right) X_t\right] \left[1 - \frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2 + 3)\right],$$

where $X_t = \sqrt{2} G_{\text{F}} m_t^2/(16\pi^2)$.

The master formula for ISR corrections in the LL approximation:

$$\sigma^{\text{LLA}} = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \mathcal{D}_{ee}(x_1)\mathcal{D}_{ee}(x_2)\sigma_0(x_1, x_2, s)\Theta(\text{cuts}),$$

where $\sigma_0(x_1, x_2, s)$ is the Born level cross section of the annihilation process with reduced energies of the incoming particles. $\mathcal{D}_{ee}(x)$ is the probability density for an electron with an energy fraction x to be found in the initial particle.

$$\begin{aligned}
\mathcal{D}_{ee}(x) &= \mathcal{D}^{\gamma}_{ee}(x) + \mathcal{D}^{\text{pair}}_{ee}(x), \\
\mathcal{D}^{\gamma}_{ee}(x) &= \delta(1-x) + \frac{\alpha}{2\pi}(L-1)P^{(1)}(x) + \left(\frac{\alpha}{2\pi}(L-1)\right)^2 \frac{1}{2!}P^{(2)}(x) \\
&\quad + \left(\frac{\alpha}{2\pi}(L-1)\right)^3 \frac{1}{3!}P^{(3)}(x) + \left(\frac{\alpha}{2\pi}(L-1)\right)^4 \frac{1}{4!}P^{(4)}(x) + \mathcal{O}(\alpha^5 L^5), \\
\mathcal{D}^{\text{pair}}_{ee}(x) &= \left(\frac{\alpha}{2\pi}L\right)^2 \left[\frac{1}{3}P^{(1)}(x) + \frac{1}{2}R_s(x)\right] \\
&\quad + \left(\frac{\alpha}{2\pi}L\right)^3 \left[\frac{1}{3}P^{(2)}(x) + \frac{4}{27}P^{(1)}(x) + \frac{1}{3}R_p(x) - \frac{1}{9}R_s(x)\right] + \mathcal{O}(\alpha^4 L^4).
\end{aligned}$$

We can separate pair corrections into singlet ($\sim R_{s,p}$) and non-singlet ones ($\sim P^{(n)}$), taking into account both by default. The large logarithm is $L = \ln(s/m_e^2)$, where $\sqrt{s}$ is chosen as factorization scale.

6. Code structure

The `ReneSANCe` is written as a set of modules (figure 3). We use a version of `libucl` for program settings management and store actual settings as a 'key':'value' map. The work with command line is done using header only library `CLI11`. After all the settings are merged together, they are checked to be valid according to the YAML Schema files, which describe data format and provide clear human- and machine-readable documentation. If any error is found, the program is terminated with the corresponding error message. Then the Info class calculates all the derived parameters and initializes `SANC` modules.

For generating unweighted events, according to the differential cross section, and calculating the total integral we use an adaptive algorithm `mFOAM` [11] that is a part of `ROOT` [12] program.

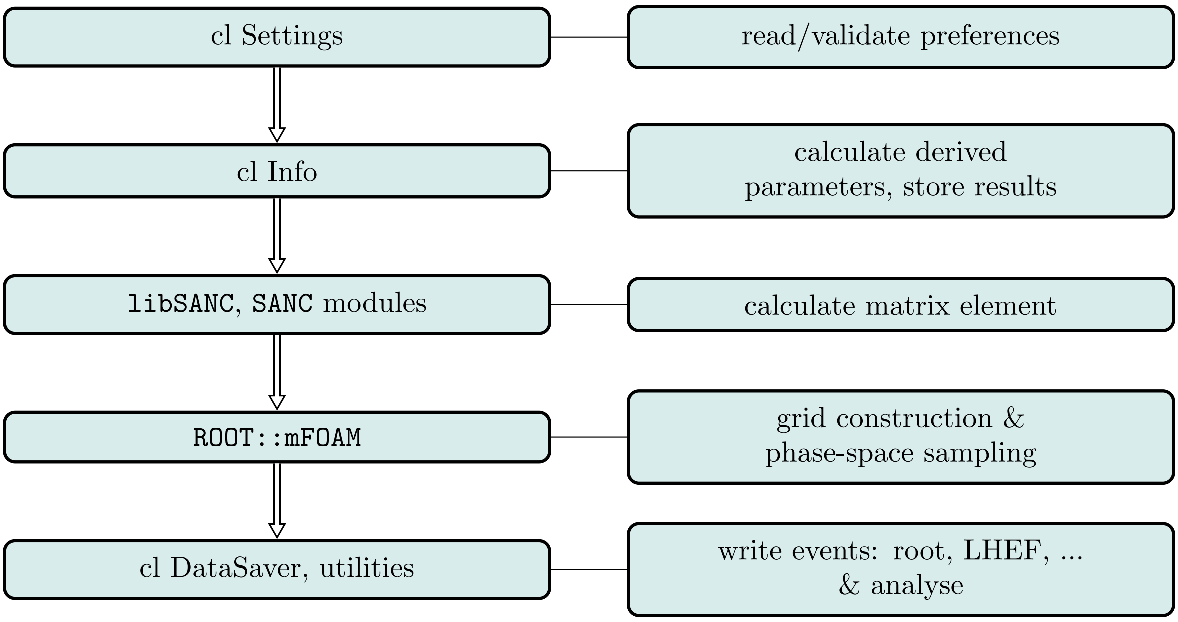


Figure 3. Code structure.

7. Some results

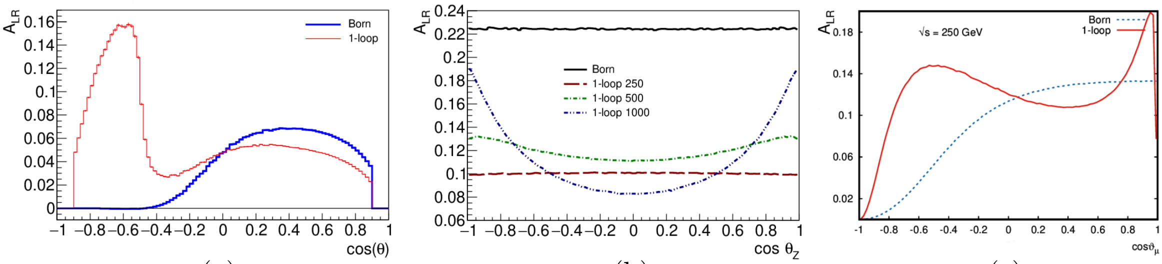
In table 1 and table 2 we present the Born and one-loop cross sections and relative corrections $\delta = \sigma^{\text{1-loop}}/\sigma^{\text{Born}} - 1$ for different polarizations. In figure 4 we show distributions in $\cos\theta$ for left-right asymmetry $A_{\text{LR}} = (\sigma_{\text{RL}} - \sigma_{\text{LR}})/(\sigma_{\text{RL}} + \sigma_{\text{LR}})$.

Table 1. Cross sections and relative corrections at $\sqrt{s} = 250$ GeV for $e^+e^- \to e^-e^+$[2] and $e^+e^- \to ZH$[3].

P_{e^+}	P_{e^-}	$\sigma^{\text{Born}}_{e^-e^+}$, pb	$\sigma^{\text{1-loop}}_{e^-e^+}$	$\delta_{e^-e^+}$,%	$\sigma^{\text{Born}}_{ZH}$, pb	$\sigma^{\text{1-loop}}_{ZH}$	δ_{ZH},%
0	0	56.676(1)	61.73(1)	8.9(1)	225.59(1)	206.77(1)	−8.3(1)
0	−0.8	57.774(1)	62.59(1)	8.3(1)	266.05(1)	223.33(2)	−16.1(1)
−0.6	−0.8	56.273(1)	61.88(1)	10.0(1)	127.42(1)	111.67(2)	−12.4(1)
0.6	−0.8	59.275(1)	63.29(1)	6.8(1)	404.69(1)	334.99(1)	−17.2(1)

Table 2. Cross sections and relative corrections at $\sqrt{s} = 250$ GeV for $e^+e^- \to \mu^-\mu^+$[5].

P_{e^+}	P_{e^-}	$\sigma^{\rm Born}_{\mu^-\mu^+}$, fb	$\sigma^{\rm 1\text{-}loop}_{\mu^-\mu^+}$	$\delta_{\mu^-\mu^+}$,%
0	0	1653.7(1)	4534(1)	174.2(1)
0	−0.8	1804.0(1)	4923(1)	172.9(1)
0.3	−0.8	2257.2(1)	6115(1)	170.9(1)
−0.3	−0.8	1844.0(1)	5047(1)	173.7(1)



(a) (b) (c)

Figure 4. $A_{\rm LR}$ at $\sqrt{s} = 250$ GeV for (a) $e^+e^- \to e^-e^+$[2], (b) $e^+e^- \to ZH$[3] and (c) $e^+e^- \to \mu^-\mu^+$[5].

8. Conclusions

New version `v1.2.1` of Monte Carlo event generator `ReneSANCe` is released. It can produce events with unit weights in ROOT and LHE formats taking into account initial and final state polarization, complete one-loop EW and LL QED corrections. Currently the processes $e^+e^- \to e^-e^+, \mu^-\mu^+, \tau^-\tau^+, ZH$ are fully implemented. The code is available at http://sanc.jinr.ru/download.php and at https://renesance.hepforge.org.

Acknowledgments

This work has been supported by the RFBR grant 20-02-00441.

References

[1] Sadykov R and Yermolchyk V 2020 *Comput. Phys. Commun.* **256** 107445 (*Preprint* `2001.10755`)
[2] Bardin D, Dydyshka Y, Kalinovskaya L, Rumyantsev L, Arbuzov A, Sadykov R and Bondarenko S 2018 *Phys. Rev. D* **98** 013001 (*Preprint* `1801.00125`)
[3] Bondarenko S, Dydyshka Y, Kalinovskaya L, Rumyantsev L, Sadykov R and Yermolchyk V 2019 *Phys. Rev. D* **100** 073002 (*Preprint* `1812.10965`)
[4] Arbuzov A, Bondarenko S, Kalinovskaya L, Sadykov R and Yermolchyk V 2021 *Symmetry* **13** 1256 ISSN 2073-8994 URL `https://www.mdpi.com/2073-8994/13/7/1256`
[5] Bondarenko S, Dydyshka Y, Kalinovskaya L, Sadykov R and Yermolchyk V 2020 *Phys. Rev. D* **102** 033004 (*Preprint* `2005.04748`)
[6] Andonov A, Arbuzov A, Bardin D, Bondarenko S, Christova P, Kalinovskaya L, Nanava G and von Schlippe W 2006 *Comput. Phys. Commun.* **174** 481–517 [Erratum: Ibid. 177, 623–624 (2007)] (*Preprint* `hep-ph/0411186`)
[7] Vermaseren J A M 2000 (*Preprint* `math-ph/0010025`)
[8] Hahn T and Perez-Victoria M 1999 *Comput. Phys. Commun.* **118** 153–165 (*Preprint* `hep-ph/9807565`)
[9] Bondarenko S G and Sapronov A A 2013 *Comput. Phys. Commun.* **184** 2343–2350 (*Preprint* `1301.3687`)
[10] Arbuzov A, Bardin D, Bondarenko S, Christova P, Kalinovskaya L, Klein U, Kolesnikov V, Rumyantsev L, Sadykov R and Sapronov A 2016 *JETP Lett.* **103** 131–136 (*Preprint* `1509.03052`)
[11] Jadach S and Sawicki P 2007 *Comput. Phys. Commun.* **177** 441–458 (*Preprint* `physics/0506084`)
[12] Brun R and Rademakers F 1997 *Nucl. Instrum. Meth. A* **389** 81–86