Torsion Geometry 5-Fold Symmetry, Anholonomic Phases, Klein Bottle Logophysics, Chaos, Resonance: Applications Towards a Novel Paradigm for the Neurosciences and Consciousness

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Abstract. We discuss the torsion geometries as the universal dynamical setting for the five-fold symmetry and its relation to nonorientable surfaces of selfreference embodying a supradual logophysics, rooted in the Möbius strip and Klein Bottle. We frame the discussion in terms of image-schemas in cognitive semantics and their disruption stemming from supraduality and nonorientability. We present the relation with anholonomic phases, chaos and the brain-mind as an integrated dynamical system. We discuss nonorientability as the characteristic transcendental metapattern of resonant connection, pattern formation and recognition. We present the torsion geometry and nonorientability in psychophysics and the neurosciences. We discuss the homology of the torsion geometry of physical space or spacetime with that of the unconscious modelization by the brain-mind of the kinematics of objects in physical space and in the perception of music, elaborating on the Principle of Complementarity of cognitive psychology due to Shepard. We elaborate on the phenomenological construal of experience, the world as experience. We discuss the supraduality of the liminal states of consciousness and the basis of awareness in memory. We introduce the cognitive psychology foundations of memory on nonorientability and its relations to the short-memory space and its quantumlike nature, and the hyperbolicity of the psychophysics of vision. We discuss the chaotic behaviour of dynamical systems as a manifestation of supraduality as their nonorientability. We propose a basis for the source of consciousness, the Hard Problem, given by the principles of selfreference and hetero-reference which generate the Klein Bottle supradual logophysics. We present the supradual logophysics of neuron cytoskeletal structures, its relation to torsion, resonances, topological and geometrical phases and the microtubule dynamics in terms of nonlinear buckling patterns and nonorientability, and still the torsion geometry of the irreversible thermodynamical processes supporting interactions-at-a-distance. We discuss the primal relation of torsion, nonorientability and memory, particularly arising in the buckling of microtubules. We discuss the primal morphogenesis of the cell as a tensegrity structure, torsion and the indiscernibility of elastodynamics, electromagnetic and gravitational wavefronts as morphogenetic fields. We discuss nonorientability as the metaform pattern of connection and resonance, particularly of interaction-at-a-distance. We apply it to a topological allosteric effect mediated by electromagnetic fields. We discuss the topological chemistry paradigm, particularly of organic chemistry where conformation superposition is crucial, its relation to anaesthetics and its application to the ORC OR theory of Penrose and Hameroff for consciousness as arising from the collapse of the wave function which we relate to the multiconformation in the topological chemistry paradigm.We propose a model of continuous signal processing in digital terms which allows an optimal reconstruction -from digital to continuous- in terms of the Nyquist-Shannon theorem whose constraints on frequencies and bandwidths is naturally interpreted in the terms of the 2:1 harmonics of nonorientability in the Möbius strip or Klein Bottle**.** We discuss the non-dual logic of the tubulin code and the resonator nature of the neuronal cytoskeleton. We propose that the topological phases appearing in the cytoskeleton may correspond to the topological anholonomity, namely, nonorientability, as was elicited empirically by the Bandyopadhyay group, the existence of anholonomity corresponding to the 360° rotation characteristic of the topological anholonomity, Möbius strip. We propose the Matrix Logic representation of the Klein Bottle logic as the basis for microtubule computations, and discuss the relation to quantum computation, topological entanglement and the topological coherence/decoherence Klein Bottle cycle. We also relate it to holography, the brain's integration and Velmans' Reflexive Monism extended by Rapoport. We discuss the multiconformation orientable and nonorientable resonators electronic devices and antennas as classical-quantum realisations of microtubule coherent behaviour. We discuss the relation of life and consciousness and the topological nonorientable embodiment of memory. We introduce the dynamical reduction program for the collapse of the quantum state in terms of the torsion stochastic differential geometry of the quantum geometry of Quantum Mechanics as projective space, and particularly the stochastic extension of the Schroedinger equation to account for the coupling of quantum system and environment, say a measurement apparatus.

Thus we identify a realisation of the dynamical reduction program which is based on the torsion geometry, yet supports an agent-free collapse of the quantum state, as an objective albeit random process due to quantum fluctuations. We shall discuss the present supradual logophysics in several aspects such as 1) microtubule structure and dynamics, 2) the orientable and nonorientable conformations of organic molecules, 3) the relation with anaesthetics and the altered states of consciousness and 4) confront them with the tenets of Penrose & Hameroff ORC OR theory for the origin of consciousness, not only the topological chemistry but their choice of a superposition of null torsion as in General Relativity, which renders a trivial selfreferentiality, with the topological chemistry multiconformations which requires non-null torsion. The nontrivial selfreferentiality is both proper to the mind and the torsion geometry, be that on the physical, chemical, perceptual or cognitive domains. All in all, we propose that consciousness is neither based on panpsychism nor the collapse of quantum states, nor exclusively on the control of superposition of conformations of organic chemistry, but rather on the torsion-nonorientable geometry-topology based on the principles of selfreference and hetero-reference, operating in all domains: that of meaning,the mental-anatomical-physiological domain, structural processes of matter energy and in-formation, be that physical, chemical, biological, cognitive and perception. We present a remarkable connection between them. Our approach rather than multidisciplinary will be transdisciplinary, a possibility supported on supraduality - transcending the dual-logic based logophysics- from which the theory will follow in a rather smooth way.

1. Introduction

Given the universality of the Golden Mean it is quite remarkable that its association with the universal dynamical geometries of nonlinear vortical morphology, namely the torsion geometries, is unacknowledged. This is still quite remarkable given that one of the most basic example of torsion geometries is none other than the Lie groups of continuous symmetries which sustains much of theoretical physics, originally introduced by Sophus Lie to give a structural support for perception (Pribram [196]).These geometries are basic to physics, cosmology, biology, chemistry, perception, cognition, music, anatomy-physiology, phenomenology as the world as created/experienced – rather than conceived as given data. These geometries introduce morphogenesis as well as pattern formation and pattern recognition through their vortical nonlinearity. We remark, pattern formation operates on Nature at large, particularly the brain-mind, while pattern recognition is proper to the latter. The Klein Bottle being the metaform for both (Rapoport,[228,231-234,236], Carlsson [31,32]), it is natural to conceive vortical motions as the dynamical metaform of Nature associated to the Golden Mean and the Fibonacci series, as shown by John Bell Pettigrew in *Design in Nature,* published in 1907 [18]. Actually, D'Arcy Thompson Wentworth, considered the father of mathematical biology was mentored by Bell Pettigrew. Although he curated his collection and succeeded him at St Andrews, he failed to acknowledge the primality of vortices but partially and almost entirely he did so with Bell Pettigrew's contribution to his own work [13]. The fact that torsion geometry is tied to the pentagon allows to elicit another remarkable extensions of the 5-fold symmetry. Indeed, already in the Principle of Complementarity in cognitive psychology (Shepard [250-253]), torsion supports the association of cognition, torsion and nonorientable topology, as crucial to the ongoing mental formation of models of the kinematics of extended objects (Rapoport [236]). Nonorientability as in the surfaces of selfreference, Möbius strip, Klein Bottles, not only support this modeling, but are also basic to embody memorization, mental and physiological modeling of visual and somatosensory experience (Rapoport [222,226,228,229,231-236]). The defining characteristic one-sidedness of nonorientable surfaces supports the integration of Outside and Inside. Thus they transcend the duality inherent to Aristotelian-Boolean logic [230-236]. Another surface of selfreference, yet orientable, the 2d-torus, provides a banal selfreference and a topology for dualism (Rapoport [235]). Thus these nonorientable surfaces embody the disposition for cognitive capabilities by providing the metaforms –mother-form or form of forms- of a supradual logophysics which further supports a transdisciplinary unification of the material world of matter and energy, information, the psyche, as well as the imaginal domain and that of sign processes (Rapoport [236]) associated

to cognition as meaning. Returning to torsion as the topological shape instrumental for embodying memorization, the Golden Mean appears coding brain harmonics as the quantum of thought associated with short-memory span (see Weiss &Weiss [280]) related to the quantization of subjective visual space, to be discussed below.

The first purpose of this article is to briefly present the pentagonal torsion geometries and its association to nonorientability and the anholonomic geometro-topological phases pervasive to physics, to set a more comprehensive bearing for the 5-fold symmetry. Remarkably, this realises Felix Klein's Erlangen program (see Sharpe & Chern [111]) and unseparably the gestaltic modeling construal of the world as experience, to which we shall also refer in this setting. Furthermore, it is a geometry of chance prophesied by Pascal, which considers Brownian motion as a geometry with continuous nondifferentiable paths with a stochastic character, Stochastic Differential Geometry (Elworthy [47], Ikeda & Watanabe, [89], Rapoport [201,206,209]), developed in theoretical physics (Rapoport [200-210, 216-221,227]) and fluid-dynamics (Rapoport [201-217]). In particular this was elaborated by Rapoport for the Schroedinger equation on space or spacetime as torsion geometries associated to Brownian motion, or the Schroedinger equation for open -i.e.non-isolated- systems, say subjected to measurements, evolving in the so-called quantum geometry of complex projective space [218,219]. Both the linear and nonlinear Schroedinger equations reveal the underlying torsion geometry and its relations to Brownian motion. Moreover, this is revealed as well in the linear and nonlinear Dirac-Hestenes for spinor-operator fields of Relativistic Quantum Mechanics, and its equivalence to the Maxwell equation, in the setting of Clifford bundles (Rapoport [207,208,217]). In Stochastic Differential Geometry the torsion field describes the average velocity while the Brownian fluctuations describing the fluctuations are related to a diffusion matrix related to a square-root form of a metric which can be positive definite [201,203,206]or non-positive definite as in Einstein's Relativity theory, or Minkowski space (Oron & Horwitz [180]). Here the Fokker-Planck operator of the diffusion process is the Laplacian operator of a Riemann-Cartan-Weyl covariant derivative, with trace-torsion (Rapoport [201]). The notion of a non-differentiable structure of spacetime at Planck scale as the geometry of quantum fluctuations and its relevance to the measurement problem of Quantum Mechanics has become the physical basis for developing schemes for the so called collapse of the wavefunction and its role as a form of "quantum gravity" related to the alleged emergence of consciousness as in (Penrose & Hameroff [74,75,192]). Remarkably enough, rather than torsion what is considered is the derivative curvature field, which at Planck scale would grow rather than a flat structure being the case (Percival [188]). However, independently of these considerations of a Stochastic Differential Geometry as the geometry of fluctuations, 't Hooft has shown [87] that the singularities of a "quantum foam" have for event horizon a Möbius strip arising from the antipodal identification of projective space, as described by Rapoport, which is the case of visual perception and the arousal of the sense of selfhood, selfconsciousness (Rapoport [236]).

With respect to philosophy in relation to which science has mostly developed as unrelateable forms of cognition and of probing the world and phenomena at large [230] the supradual Klein bottle logophysics and their interweaving by 'gluing' several Klein Bottles, namely HyperKlein Bottles as forms of multi-distinctions proper to complexity ([230,236]), realise Heidegger's claim (see Malpas [155]) that phenomenology is topologically grounded (see Rapoport[230,236]; Rosen [241]). It provides for a transdisciplinarian unification of science [223], knowledge systems at large, music (Mazzola [159], Tymozcko [274], Payeron [185]) and its cognition (Merrick,[320]).

As second -yet the major- topic of this article, we shall make the case of the transdisciplinarian unification of science in terms of this supradual logophysics, extending and

applying the previous discussion to develop a novel paradigm for the neurosciences and consciousness, based on the principles of selfreference and hetero-reference proper to the foundations of torsion geometry and nonorientability, and to cognition.

2. Nonorientability and torsion geometries: the 5-fold symmetry
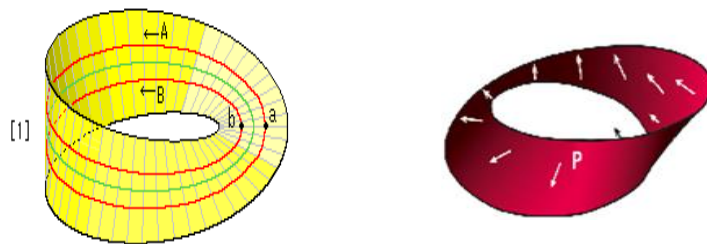

Figure 1 (Möbius strip; left figure from Rapoport, [228] CC):The green line is a centre line. Suppose the red line extends counterclockwise from point to turn back and arrives at point b which is the opposite side of point, so to return to b again another complete 360° turn is required. The same is the case of the red line which extends one more round, thus gets back to the starting point a. The red line never went across the centre line. Nevertheless the red line runs both sides of the centre line. CC

Another way of expressing this is what is *above* the centre line turns to become *below*, and the converse as well: Top and Bottom are continuously intertransformable. Top and Bottom are the elements of one of the most basic universal image-schemas -as conceived in cognitive linguistics after Johnson and Lakoff [96,139]. While dualism in an orientable surface assigns to this image-schema a fixed dual categorization, the nonorientabiity of the surface belies their dual character. Indeed, the red line looks like parallel lines when we see them partially. But it is not true. Route A and route B are actually a single route. This belies the top/down duality. Maurits Cornelis Escher represented this supraduality with his famous *Ascending and Descending* litograph. The green line looks a median or a central reservation on a road, a safe place from collision with whatever transits outside of it. However, it does not divide Möbius strip into two: thus the supraduality just expressed; or non-duality, would we stick to the habit of dualism as primal. Together with this, what we may describe as whatever is placed inside turns to be placed outside, and the converse. So another universal image-schema which is construed as a dual categorical divide is also belied. This is another manifestation of the supradual nature of Möbius strip. Philosophers Wittgenstein and Bachelard contested the dual Inside/Outside categorization., usually phrased as the CONTAIN image-schema (see Wittgenstein, IV 35, 4023 [282], Bachelard [9]). The notion developed in cognitive linguistics is that our thoughts, our conceptualization and theoretizations at large, are anchored in figures of speech denoting a spatial disposition, and these image-schemas -as they are called- are cross-culture universals. CONTAIN -and dualism, at large- is paradoxically both eminent and transparent in our construal of the representation of the world as already manifest in language, actually hegemonic based on the dual onto-epistemology, after Aristotelian-Boolean logic. In system theory, CONTAIN introduces a divide of form and content. In mathematics set theory this image-schema established its formalization which were further applied to attempt the axiomatical formalization of mathematics as a whole, the program faltered, proved unrealizable. It is problematic indeed, to consider any language as if exterior to its context -understood literally, a complex of physical, biological, cognitive, psychological, social, or economic conditions-, producing at times unresolvable paradoxes as Russell's invocation of an infinite set that contains itself. CONTAIN is indeed a very important image-schema, transcendental in establishing the dual onto-epistemology, i.e. based on Aristotelian logic. Its fundamental laws or principles are the principle of identity, the principle of non-contradiction and the principle of no third value, tertium non datur -but true and false, for its two possible values. We shall

this discuss further with respect to Quantum Cognition, the biophysics of action potentials and to chaotic systems.

CONTAIN pertains either to the real space where we suffer on kicking a stone, the abstract space of the mathematician and physicist, and the imaginal space of a prophet, poet, shaman or dreamer. Whereas the latter two may experience as if real that Inside and Outside are connected, this is unlikely to the previous formalizing subjects. This is a privilege which on delivery to this world from the maternal womb every newborn experiences, with no due need of immediate reflection about the transformation. However, the fecunded ovum starting from a cleavage on its surface, turns outside-in, the blastopore invagination as a first step of development towards the maturation of a viviparous organism. As a stage of this inward flexing and folding development of a morphomechanics of tissue surfaces, the exterior surface of the ovum turns outside-in migrating to constitute the nervous system which the later outgoing brain finalises turns outwards to become the eyes, the progeny of a turning Outside-Inside-Outside development. Invertebrate *Hydra* makes of its -in principle – eternal being this perpetual transformation. Most of the matter in the Universe is produced by supernovae remnants of collapsing stars. Cassiopeia A for one such star turned Outside-Inside ([232]). CONTAIN image-schema and the cosmologies constructed on its terms appear banal in terms of such transformations. An ideally elastic 2-sphere might be turned inside-out with no discontinuities nor creases to produce another 2-sphere with the previous Inside and Outside interchanged, through mediating nonorientable surfaces such as the Klein Bottle or Morin's surface (Smale's Paradox [4,158,230]). In distinction with the ideally elastic 2sphere, the ovum requires a discontinuity [230,231]. We are to more surprises:

If we cut the Möbius strip along the green line, its length becomes doubled. It plays a crucial role in the process of duplication and transcription of DNA (Rapoport [233]).There is no opposite lane on Möbius strip. This is the topological origin of the 2:1 harmonic. It is also the origin of a protoform of Newton's Third Law, albeit it does not require, in distinction with Newton's formulation, a dualistic assumption, nor the instantaneous causality that this law implicitly assumes; causality, if any, is embodied by the 2:1 resonance produced by the 180º twist. Indeed, consider a normal (i.e. perpendicular) vector to Möbius strip; if we move it along any curve in a 360° turn as before, we would find it pointing in the opposite turn will return the vector to the samepoint further coinciding with its original configuration. Rather than having an action and a reaction as distinct, in the Möbius strip and Klein Bottle, the opposite and equal modulus of normal vectors is a resultant of the 2:1 harmonic produced by the nonorientable torsion, not an hypothesis for the foundations of physics at large. Thus it appears that this harmonic is more fundamental to physics than Newton's Third Law. Also, this is the case of the origin of the 4π symmetry of relativistic quantum mechanics (Rapoport [228]). The Möbius strip is a topological (Berry)-phase (an)holonomy. We shall return to this below.Would we cut and unfold the Möbius strip while keeping the length of the edge with notches, it turns into a trapezoid and the length and slope of the notches lines are distorted; see figure 4. This distortion is of great physical import. Indeed, while the fundamental rule of the sum of vectors, say forces, on Euclidean space is valid in the case of the parallelogram, in the case of Möbius strip unfolded i.e. the trapezoid, this is no longer the case. In fact, this property is shared by spaces with torsion, in which parallelograms don't close so that the sum of two vectors does not yield the diagonal which is ill-defined. Thus, instead in the non-closure of the parallelogram formed by the vectors u_P and v_P at P and their parallel transport, $u_R^{\parallel}$ at R and $v_Q^{\parallel}$ at Q, a fifth side T(u,v) closes the infinitesimal vectors. Prolonging T(u,v) to the line joining P and Q, we have the trapezoid whose completion is depicted in red, the inferior triangle at the right. Would we start with u_P and v_P of equal length then we can form a regular pentagon by scaling T(u,v) by a factor of the lengths of u_P (or v_P), $|v_P|$, and as $|v_P|T(u,v)/|T(u,v)|$ whose length coincides with theirs. (For the latter normalization a metric compatible with the parallel transport is required (Rapoport [201]).Thus the regular pentagon, i.e. all sides of equal length- is formed by considering torsion and defining parallel translation in terms of it, and further a metric as described. An example of this was introduced by the geometrization of dislocations of crystals by Kondo at the 1940s. In dislocation theory the torsion is called the Burgers vector, and as it will turn out, is a geometrical anholonomy.
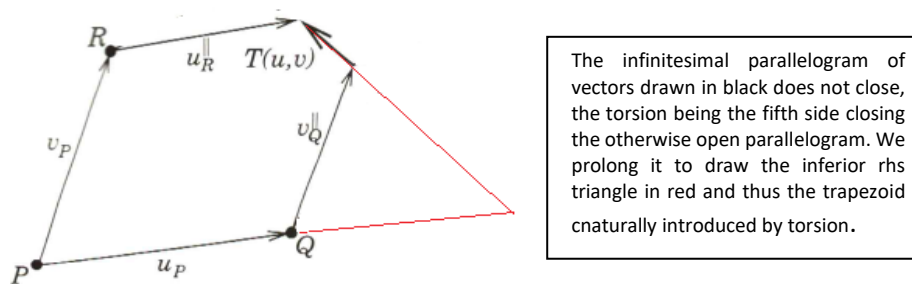
The infinitesimal parallelogram of vectors drawn in black does not close, the torsion being the fifth side closing the otherwise open parallelogram. We prolong it to draw the inferior rhs triangle in red and thus the trapezoid cnaturally introduced by torsion.

Figure 2 (From Rapoport [228] CC):The five-fold symmetry of torsion geometry. Notice that the torsion can be introduced with either chirality, clockwise or anticlockwise¸ which introduces a minus sign to distinguish them. As a torsion tensor or differential 2-form this makes of them noncommutative, specifically anticommutative: on transposing the covariant indices of the torsion tensor or as vector-valued differential form of degree 2, a minus sign is introduced . Noncommutation, say as in the product of matrices, is the crucial property which supports Quantum Mechanics, and is key to Quantum Cognition, the application of the mathematical formalism to model cognitive processes such as information decision process, concepts and conceptual reasoning, human judgment, and perception, cognition in general [2,29,37,38,100,101]. This arises from the contextuality of Quantum Mechanics as a theory of physics and of cognitive processes. The Klein Bottle is a body of contextuality, to be introduced below.
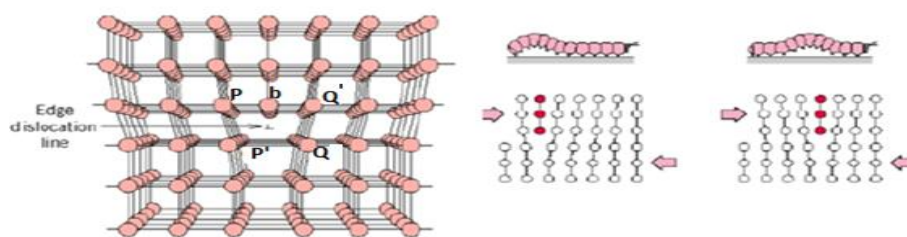

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Figure 3 (From [228] CC): We introduce torsion by shear in a lattice; in the rhs by a caterpillar moving in the surface of a perfect crystal. In the lhs we have a perfect lattice, as is the case of a discrete space-time rendering of the homogeneous spacetime of General Relativity, but in the central area in which a dislocation is the case. We see then the transition from parallelograms that close (null torsion as in General Relativity), to non-closing and the formation of a fifth side, the torsion field. The figure above shows the meaning of torsion. In an otherwise perfect crystal (whose vertices are described in red), i.e. free of inhomogeneities an edge dislocation is produced. This is done either by removal (i.e., introduction of singularities) of atoms of the crystal, as the figure shows, or by introducing extra material; both are inhomogeneities. Torsion can be introduced by shear (i.e., the relative motion of two planes) as figure 3 shows, on the plane itself. (The other class of dislocations are the screw dislocations involving a shear on the direction transversal to the plane, which acting together with a shear dislocation we get the figure of DNA). Think of a caterpillar which moves a lattice a step at a time, and the shear produces the torsion of the crystal; this shear produces a vortical motion on the vertical plane to the shearing plane under the mixing of layers for small that it can be. Another analogy is that of a rug, which moving in the perfect background of the homogeneous crystal; local changes affect the whole structure; the analogy strikingly applies to the crease (the folded rug) formation in the gastropore invagination in Embryology (Rapoport, 2011c, 2014,2016a,b). It can also be produced by a hole in the surface, producing an embryological expansion wave. Thus, it is an action-dependent participative geometry introduced in terms of inhomogeneities by the subject, say the caterpillar, or more basically the photon.

This stands in stark contrast with the *homogeneous* situation of a Cartesian ideal geometry exterior to the subject, which corresponds in the continuum limit in which the atoms of the crystal approach indefinitively, i.e. the continuum hypothesis of Einstein, which due to the lack of a singularity, it corresponds to the zero torsion metric-based geometry of General Relativity (Cartan [33],Rapoport & Sternberg [200,204];Hehl et.al. [81-83], Vargas [276]). In short, to have loci, a geometrical reference at all and in the first place – quite literally! -self-referentially dislocations are needed: These are the inhomogeneities that put up a geometrical locus. The most basic dislocation is produced by a photon, a quantum particle, a singularity of an electromagnetic field, or still, a low dimensional submanifold of the wavefront eikonal propagation of light rays (Rapoport [220,221]); *no duality of wave and particle*. They play an important role not only as topological anholonomies to be discussed below but as a kind of square root of the null state (the "mind Apeiron", or vacuum state of the mind) in Matrix Logic as Penrose twistors [220,221,223] The parallelograms where inhomogeneities are present do not close,while in the perfect crystal they do close indicating a self-referential trivial loop: by default is indeed a loop, i.e. closed without mediations. This is also the case of the metric-derived spacetime geometry of General Relativity. Instead, in the former case a pentagon is produced. The fifth newborn side at the upper right side of the centre, joining Q' with b, is the torsion –the self-referential mediator; it is necessary and sufficient to the effect of completing the self-referential closure of the otherwise closed parallelogram; we shall explain this further below. As the figure 4 shows in the centre, the torsion appears producing a trapezoid with vertices P, Q, P' and Q', since the upper side is now longer than the lower one. Yet trapezoids are themselves Möbius strips cut transversally, say along the line on the Möbius strip below joining P (P') and Q (Q') below, and laid open on a plane, which by identifying the opposite lateral sides PP', QQ', on a previously 180º twist of the trapezoid, re-establishes thus the Möbius strip, as shown below:
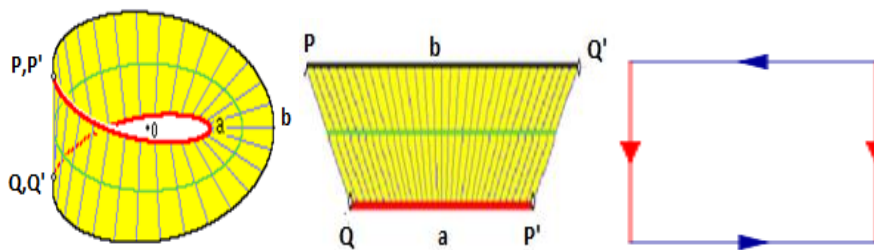

Figure 4 (From [228] CC): The Möbius strip, its unfoldment (as the completion of the dislocated parallelogram in fig. 2 above and the identifications producing the Klein Bottle, respectively.
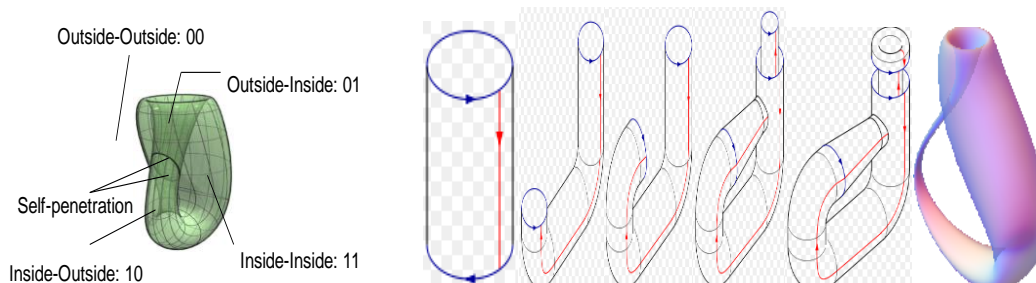

Figure 5. *Left* (From [224] CC) Natural logic of 4 states (the Klein Bottle Logic, KBL) which has two digits representation: Inside-Inside (represented as 11), Inside-Outside (represented by 10), Outside-Inside (represented by 01) and Outside-Outside (represented by 00). The mediation states arise from self-penetration absent in the mechanical-dual-membrane of biological cells and in the Cartesian conception of object-in-space-before-subject. They are paradoxical states associated to time waves

(Rapoport [224,234]), as introduced in terms of non-dual de Morgan logic by Kauffman ([112]). They express the imaginary logic values proposed by Spencer-Brown in his calculus of indications [303] as reentrant states of the primal distinction: torsion. These time waves are crucial to Cantor's diagonal construction for his argument of a continuum, which due to his unacknowledged choice of Boolean logic he was unaware of.They underly the paradoxical unending tail of the dyadic expansion of most real numbers (Hellerstein, [304], [226,231]); for an update on these issues see (Kauffman et.al. [131])

From the KBL appears the genetic code with its 64 elementary codons, and furthermore the genome and its association to the Dirac algebra of Quantum Mechanics. The latter is also based on the KBL reinterpretation of the recursive Universal Rewrite System (NURS) of Rowlands [243] . Since NURS topologically is based on the Klein Bottle sustaining a coupling of a fermion singularity with the rest of the universe as its dual element, this coupling evokes a boson interaction of the fermion with the universe for its dual. Indeed, in NURS bosons of spin 0 and spin 1 can be produced by fermion-antifermion combinations while boson condensates can be formed in terms of fermion-fermion combinations. In other words, the Klein Bottle with its 2:1 harmonics supports fermions and bosons. We shall return to this upon discussing the cytoskeletal neuron architecture and action potentials.

 Right: (sequential construction of the Klein Bottle; rendered with Mathematica 8 using the parameterization provided by Robert Israel; uploaded by Wridgers. Creative Commons SA BY 3.0 File: Klein bottle translucent.png, Wikipedia)

Möbius strip although it surmounts the dualism of CONTAIN it is still embedded in Euclidean 3d space, as contained in it. The Klein Bottle is entirely different: rather than contained in what seems ambient space, it is selfcontained through the selfpenetration. Thus we see that dualism as supported by CONTAIN is inappropriate to theoretization vis-à-vis this selfpenetrating surface. However, in practical terms the Klein Bottle can be construed in ambient 3d space by using liquid crystals further solidified as exposed at the London Museum of Science, in the crystal figures by artist-extraordinaire Alan Bennett, HyperKlein Bottles.The latter introduce image-schemas which transcends hierarchy through heterarchies, functional organisations which disrupt the iterated nesting of CONTAIN as in the Babushkas Russian dolls [230,236].

Klein Bottle can be represented in 4d or 5d removing its selfpenetration, thus serving as the metaform for both pattern recognition and pattern formation (Carlsson), which is basic to the patterns of the brain-mind formed by eigenstates harmonics of the brain connectome (Atasoy et.al. [8], Rapoport & Perez [234], Rapoport [236])

Notably, the most basic example of a manifold which bears a torsion geometry is a Lie group of continuous symmetries! The so-called structural coefficients which describe the operations of infinitesimal symmetries, the Lie Algebra, stand for the components of the torsion tensor ([232]). It seems that conceptual astraying from the geometry of Lie symmetries which lay at the very basis of theoretical physics, in particular since the inception of gauge theories, has produced a cognitive dissonance that made of metrics, i.e., distances, as the basic structure of the geometrization of physics after Einstein, rather than symmetries. A Poincaré group gauge theory of gravitation with torsion and spin was somewhat exceptional (Goenner [67], Hehl et.al. [81-83]). So, given that symmetries provide the basic geometry of physics, as well as chemistry, which also is the case of biology, in addition of vortices as the dynamical form of torsion, we are led to examine what are the inherent symmetries of the Möbius strip and the Klein Bottle. While elementary dual image-schemas CONTAIN and ON - the latter as in the expression "on the table" pointing to above/below categorization- are surmounted by the Möbius strip and Klein Bottle, the left/right categorization in terms of specular symmetry is also compromised by considering chirality as in enantiomeric pairs. Handedness is no longer global. This is of crucial importance to chemistry and biology, as discovered by Pasteur, which we shall refer to (Rapoport [228]) for further graphical information.

3. The induction of selforganisation and nonorientability

The topological entanglement supported by nonorientabiity is crucial to the overall coherence of genomes, in which separate genomic configurations are interlinked in time and in space, establishes a coherence in time and in space as a form of non-Darwinian evolution linked to palindromic sections. This operates through resonant harmonics given by the proportions of codons to mirror-codons, which

is almost perfectly equal to one, and to the proportion of the 32 most frequent codon-mirror codons pairs to the 32 less frequent pairs -discovered by Perez [113], which is 2 to 1; both being the signature of non-orientability (Rapoport [224,233]). The Golden Mean plays a crucial role in genomes and in harmonic principles at large, for the material world, cognition and its experience, the phenomenology of music [320]. Genomes are liquid crystals, omnipresent in biological systems, in which nonorientability is crucial to their coherence (Bouligand [24,25]).

The remarkable fact is that the topology of liquid crystals is amenable to control, and to the induction of topological changes. These can be produced by the expedience of introducing into them colloidal particles; thus topological changes arise due to the incompatibility of constraints, namely the anchoring conditions on the particle surfaces with the alignment imposed by the cell boundaries, or at *large distances* (Machon and Alexander [153,154]; Melle [164]; Musevic [171]). So, forms which are pervasive to organic chemistry such as knotted defects, Möbius strips and molecular motors, can be induced. This amounts to a general principle of nature for the topological generation of *complexity* as intricate structures in which nonorientable surfaces act as a kind of "glue", simply by inducing them through emplacing colloidal particles on cholesteric or in nematic liquid crystals, such as DNA (Chow [36]). The sole factor appears to be the nonorientable topology of the director field of the crystal, which underlies the formation of these intricate structures in terms of topological dislocations (generically, torsion geometries). It operates through the elastical adaptation of the director field to the extraneous colloidal particles, as in the previous references. But rather than the non-orientable topology being erased, more intricate robust structures appear to be the case, still carrying as their progeny, the nonorientable structure which make them possible. Thus, complexity –as intricate structures *progenies* of foldedness, though no genomes are involved- appears to be related to non-orientability and its elastic deformation to ensure its preservation under disturbances!

The addition of particles as inductors of orders can be interpreted in very general terms as the introduction to asymmetry to a system which otherwise already has an ordered structure producing constraints to those which may already be acting. The existence and relevance of constraints in any form, which usually appear as boundary conditions, are crucial to selforganisation and life [231,232].

4. Berry topological phases, (an)holonomies, torsion and nonorientability

The paradigm of theoretical physics as spacetime or state-space geometries (say, the projective Hilbert space of quantum mechanics), which in General Relativity was predicated in terms of homogeneity and geodesic motion, the latter as an extension of linear free fall motion, is clearly invalid vis-à-vis the omnipresence of vortical nonlinear motion in Nature. First of all, vortices are indissociable from singularities which stand as organising centres. Furthermore, the rotational motion establishes cyclicity which is selfreferential and yet hetero-referential since the cyclical motion also refers to the centre. So we have two principles which support vortical inhomogeneities: Namely selfreference and hetero-reference, to be discussed below. They are basic to cognition and awareness, and to perception in the first place as the pinwheel hypercolumn structure of the primary visual cortex. Thus, in the primal stage of processing the visual system has for functional topology the Klein Bottle (Swindale [266] Tanaka [268, 269]). Furthermore the primary visual cortex arises from selforganisation as a π-periodic structure (Miller [128]) which we related to the above hypercolumnar Klein Bottle functional architecture ([234,236]) and was independently interpreted as produced by a Berry quantum phase (Marcer & Rowlands [156]): The Klein Bottle as the topological anholonomity given by gluing two opposite chirality Möbius strips already mentioned [228]. Secondly, vortices cannot be identified as objects occupying space or space-time, say as in classical physics or General Relativity; vortices are geometries, dynamic as such. Furthermore they are eminently relational and processual, and they do not produce a space or spacetime as a container with absolute "Inside" and "Outside" categories. Instead these are connected dynamically, at times the differences, if any, are blurred or undefinable, uncertain: generically, vortices mix Insides and Outsides. Yet the geometry of inhomogeneity which embodies vortices is that of torsion, and ultimately at the Planck scale level, associated to quantum spin (Ross [244]), with spin-density for the macroscopic source of torsion in the extended theory of gravitation (Hehl et.al. [81-83]). Torsion stands as the fundamental geometry of physics, for which cyclical

processes are the most fundamental (Dolce [45]). Particularly torsion geometry is the case of i) quantum physics (Rapoport, as already indicated), ii) the fundamental interactions as spacetime symmetries rather than internal symmetries (Rapoport &Tilli [202]), iii) fluid and magneto-hydrodynamics -the most pervasive form of matter in the cosmos, iv) biology (Rapoport, [224,225,229,230, 232-236,238]), v) the topological chemistry paradigm, vi) in the Klein Bottle form of the Mendeleev Periodic Table of Elements which is the unfoldment of the Golden spiral as a standing wave (Boeyens & Lavendis [22]), vii) perception and cognition [213,220,223], and viii) crucial to music and its experience [159,251,274]. Torsion geometry is the case of fluids as the paramount matter organisations of the cosmos at all scales, particularly liquid crystals. The latter have nonorientable topologies pervasive to biology [225, 231,232], genomes (Rapoport, [233]), cholesterics and nematics (Machon [153,154)], and coherent water domain crucial to cell metabolism (Ling [135], Rapoport [225]). They are also crucial to the topological chemistry paradigm (Mezey [118], Flapan [51], Bonchev 136, Sokolov [258]) , and to the topological entanglement of molecules having multiple conformations (Putz & Ori [197])

We have already introduced geometrical torsion as the natural geometry of nonorientability, yet we wish to examine this in a different guise: As a primal form of "panpsychism", more properly of pansemiosis -the world constituted by signs processes eliciting meaning-, which is proper to the supradual logophysics through the action of referral to. However, where the psyche, if any, though unnecessary in general, is related to selfreference and hetero-reference, as already discussed. Indeed, it appears as the completion of a space or spacetime dislocation which closes a fractured cycle taken around a singularity, thus blending selfreference (to produce the closure of the path around a singularity) and hetero-reference (as the alter of the singularity). Selfreference and hetero-reference are the two fundamental cognitive principles, however present as a vortical process of geometrical comparison which Nature itself produces. Alternatively it appears from cutting open non-transversally the Möbius strip to render either the trapezoid as the crystal relaxed description of the homogenized dislocated parallelogram or the pentagon, described in the figures 2 and 3 above (see Batterman [16], Figs. 3 & 4 where it is identified as an anholonomy). Thus we can identify torsion as the primal geometrical anholonomy to be discussed below.

The name anholonomy is a major source of confusion since some authors call it as holonomy. In theories of gravitation torsion is related to the so called anholonomic nonintegrable systems of reference, and coherently with this in the realm of theoretical physics at large,it has been called anholonomy. The term has been derived from classical mechanics, The terms ''holonomy'' and ''anholonomy'' derive from the classical mechanics of systems evolving under certain constraints. If the constraint is integrable, i.e. the evolution of the system does not depend on the particular path, and leads to a reduction in the number of degrees of freedom, it is called ''holonomic.'' Nonintegrable constraints, i.e. path dependent motions, are called''anholonomic'' or ''nonholonomy'' an in gravitation or quantum physics. Geometers apparently do not respect this distinction calling anholonomies ''holonomies.'' Berry took this reversal of usage to be ''a barbarism'' [288,289]. The name anholonomy as called by the physicists which have mostly remained. They are also called alternatively quantum phases, geometric phases or Berry phases. He discovered that in the cyclic trajectories in the quantum state-geometry given by projective Hilbert space adding to it a circular U(1)-phase symmetry , antipodal quantum states carry a phase. This phase is elicited upon completion of the cylic path in which an initial state is transported by a rule of parallel transport, the torsion appearing as the end result. Due to its its rendering of an understanding of seemingly disparate phenomenon in terms of a unifying concept, the discovery of Berry's phase is considered fundamental to physics (Lyre [152]). More of this below.

A cursory definition of anholonomy is as follows. We transport a system on a selfreferential– i.e., closed –path in the abstract space of states of a system to further discover the fact that the state variables return to their initial values ,and that there is no local rate of change for quantity S. However, it appears there is a global change in S's value at the end of the closed path in the complete system of states. Thus, in manifest physical appearance there is a difference with the initial state on returning to it through a closed loop in state-space. This final state is called the anholonomity. Two observations are in order. Firstly is that the notion of transport is more precisely of parallel transport acting on what is being transported, say a state of a system or a vector in spacetime say a crystal. It is a relational construct.The path upon which the parallel transport can be differentiable or continuous non-differentiable. The latter

is the case of Stochastic Differential Geometry, the torsion geometry of Brownian motion already introduced, elaborating on Edward Nelson's conception of the Schroedinger equation arising from random fluctuations of the vacuum and Newton's second law with acceleration derived from the *mean* velocity of the random motions [175], the torsion drift vectorfield [201,206,208-211]. The notion of parallel transport in the setting of torsion geometries was introduced by Elie Cartan. It is called a Cartan or affine connection [33]. Its centrality is apparent in being the core instrument in theoretical physics in the modern extension of Klein's Erlangen Program (Sharpe & Chern [111]). It further extends to the psyche through the Principle of Complementarity of Shepard ([250-253]): Namely the unconscious modeling of the mind by introjection of the physical world in terms of representations which are further conditioned by embodied constraints [236]. Secondly a cyclic character about which the transport is carried out, as in the case of torsion and nonorientability. Thirdly, the ultimate cognitive operation is of establishing a difference and furthermore, second-order differences arising from the primal one (Johansen [294]). In other words, the primal perception *and* principle for creation, in the first place, is the perception of a distinction, a difference, its metaform the Klein Bottle Second-order differences allow for the eliciting of meaning and understanding, as proposed by cybernetist Gregory Bateson.

Let us identify primal forms of geometric-topological anholonomies in the very structure of the supradual Klein Bottle logophysics. Indeed, the primal topological anholonomy is embodied as the 2:1 harmonics of the Möbius strip and Klein Bottle decurring from their nonorientability. We recall Fig.1: When we take a vector normal to the surface at some point of a Möbius strip points *upwards* (action) and we further transport longitudinally this vector to the same point placed on the "other" side after a 360° percourse completing thus half a closed path, the normal vector now points *downwards* (reaction). Another way of understanding the anholonomy of the Möbius strip is as follows. Given a point on the vortex as centre of coordinates would we consider the radius vector from the centre to the point on the surface, then after a complete 360° rotation the radius returns to the same point but the normal vector does not, thus the anholonomy of 360° degrees of the Möbius strip. Another complete turn of 360° will complete the path to close it and yield the same upper-pointing vector. Möbius strip is the case of a topological anholonomy, and is founded on non-abstract-physical space terms, not a state-space, which nevertheless is the case. Actually, this state-space is given by Möbius strip and the normal vectors to it, and the parameter space is subsumed by two values, +1 and -1. They describe the upper-pointing and downward-pointing direction of normal vectors. Thus the nonorientability allows for a characterization either as a "real"-space or state-space anholonomy, somewhat of a Platonian archetype which very much manifests as real. The latter issue of the abstract or real character of anholonomies is a subject of discussion (Lyre [152]; Bliokh et.al. [21]).

As for the space or spacetime anholonomy we have already identified the torsion, say of a dislocated crystal, a viscous fluid or a quantum system, the latter being the velocity field and the logarithmic differential of the wave function, respectively. It provides for the selfreferential closure of the path around a singularity, which when mapped into a relaxed configuration elicits the torsion (Burgers vector) as the anholonomy (Batterman [16]).

In the quantum geometry of state-spaces, projective Hilbert space, the path transports a wave function which acquires a phase upon completion by closure. Yet this does not depend on the closed path itself in distinction of the configuration "real"-space. However, the quantum geometry does have a singularity which is the excluded null state. Furthermore, antipodal identification of quantum states produces nonorientability (Bliokh et.al. [21], Rapoport [228]). Both singularities are usually unacknowledged, whereas nonorientability has lately been found omnipresent. More of this below.

It has been lately acknowledged -and presently taken a widespread momentum- the relevance of the quantum anholonomy and its omnipresence in quantum physics, electromagnetism, acoustics, pressure waves, electronic devices, sound, gravity waves, fluids, crystals, etc. However, this has proceeded as the unacknowledgement of the geometrical anholonomy of ("real") space or spacetime torsion, rather than state-space, say in dislocated crystals as elementary examples of torsion geometry, or quantum fields associated to linear and non-linear Schroedinger and Dirac equations [207,208,217-219]. They reveal the unity of continuity and singularities as discontinuities, much sought for in the search of a theory of quantum gravity. (Yet seemingly impossible to achieve in terms of the metric geometries of General Relativity which assume a continuous spacetime and null torsion; these assumptions imply no

selfreference nor primal singularities, by default, though solutions to the field equations show otherwise). Furthermore they have brought to the fore the fundamental relevance of "non-conventional" (viz. nonorientable) topologies. On the other hand the torsion geometries of fluid and magneto-hydrodynamics, quantum physics and thermodynamics which was revealed in the works by Rapoport have not been acknowledged. Furthermore, torsion has appeared in the qualitative studies of nonlinear systems, particularly in blow-ups, chaos and bifurcations (Yong Wu and Yi Lin [286];Rapoport [228,232,233]). For example, electromagnetic fields with Möbius strip polarizations were first discovered in optics (Freund [54], Bauer et.al. [17], Andrews [7], Bliokh et.al. 21), to later extend this discovery to tidal ocean waves, sound, fluids and particularly turbulence, gases, water-surfaces, gravitational waves, which "show that polarization singularities and Möbius strips are also ubiquitous for them. In contrast to well-studied electromagnetic polarizations associated with the motion of abstract field vectors, acoustic wave polarizations correspond to real-space trajectories of the medium particles" (Bliokh et.al. [21]). The robust quality of these topological singularities is relevant.Thus the abstract state-spaces of quantum mechanics in terms of which quantum anholonomies were first discovered, were anticipated by spacetime ("real-space") effects such as the Aharonov-Bohm effect associated to the primality of electromagnetic potentials, actually torsion potentials as appearing in the trace-torsion differential 1-form of Cartan-Weyl type (Rapoport [202,203-210,212,217-220]) and in the Schroedinger equation for open systems upon measurement of quantum geometry state-space (Rapoport [218]). They were complemented by a plethora of space or spacetime quantum anholonomies associated to such elementary phenomena such as geometrical-optics eikonal propagation of light, sound waves (Deymier & Runge [43]*)*, fluid and pressure waves, phonon vibrations on lattices, metamaterials, etc. The quantum anholonomy has shown its omnipresence (Bliokh et.al. [2]1) to which we can add the universal phenomena of resonance of harmonic systems which has nonorientabilty and quantum and classical anholonomies for their very basis (Dembowski et.al. [122], Deymier & Runge [43], Xu et.al. [287]). It has been empirically verified that resonance arises from tuning harmonic systems through the mediation of nonorientable states (Shelton [249], Xu et.al. [287]).

In *Steps to an ecology of mind,* Bateson's posthumous work on cybernetics, general systems theory and the foundations of biosemiotics, he envisaged the existence of a universal transcendental metaform, a "pattern which connects" (Brier [27]). We identified nonorientability as the characteristic property of the connection pattern (Rapoport,[236]). As already mentioned, the Klein Bottle is the metaform of both pattern formation and pattern recognition in Nature, and in the brain-mind connectome as an harmonic system (Atasoy et.al. [8], Rapoport & Perez [234])

5. Anholonomies, nonorientability and chaos in the brain-mind dynamics

Nonorientability plays a crucial role in the theory of dynamical systems; we already mentioned the blow-ups of nonlinear systems which lead to chaotic attractors. Due to the complexity of nonlinearity which manifests in the asymptotic morphologies given by their attractors, particularly those that arise as periodic orbits that makeup the unstable manifolds, this is perforce a qualitative theory. Topological invariants are basic to characterize the complexity of the invariant manifolds of these attractors. The reason is simple: stretching, folding, twisting, tearing, squeezing are the topological transformations which nonlinear processes undergo in their phase spaces. However, the characterization of the topological properties are much involved. One such topological invariant is an anholonomy, the so called local torsion (Gilmore & Lefranc [61]). Indeed, consider a normal vector to a periodic orbit of the solution flow of a dynamical system. As the flow evolves the base of the vector moves along the orbit, the tip rotates along the direction of the flow. We are already familiar with this, the Möbius strip. However, in this dynamical system flow, when the initial state returns to its starting point, the vector rotates through an angle, Θ,precisely Θ/Π, the local torsion around the periodic orbit. The value of the local torsion is generally not an integer. The latter are relevant to topology, nonorientability and complexity. Say, if the local torsion around a closed period-n orbit is an even integer, then it is possible for two orbits of period n to entwine the original period-n orbit. Otherwise, if the local torsion is an odd integer, the original period-n orbit can be entwined by an orbit that closes after two cycles, i.e. a period-2n orbit. This notion of anholonomy can be extended to nonlinear dissipative systems undergoing Hopf

bifurcations with a continuous symmetry (Ning & Haken [176]), which characteristically produce nonorientable unstable periodic attractors.

Would we consider dynamical systems described by a vectorfield on 3d-Euclidean space, i.e. a field of vectors in 3d space which evolve in time as a parameter, namely describable by a system of ordinary differential equations. Since detailed description is not the issue but rather qualitative models which are useful to capture the essential dynamics in the simplest low-dimensional model. Classical examples are the Van der Pol equation, which models a simple electronic circuit exhibiting oscillations, and the Lorenz system capturing the essence of complexity in weather models. Very often, as is also the case with these examples, the resulting model is three-dimensional due to 3d-vectorfields being the lowest-dimensional continuous time models that can exhibit extremely complicated behaviour, including chaos. To understand the dynamics of such a vectorfield, we search for attractors, their basins of attraction and in particular, the boundaries of these attracting domains. Two-dimensional invariant manifolds often act as basin boundaries, and it is important to know for understanding the qualitative aspects which are the very core of the theory of dynamical systems where they are located and their shapes. They are called separating manifolds, as they separate the state-space into two invariant regions. Indeed, trajectories of the solution flow of the dynamical system falling on one side of the manifold are trapped into to this region since they cannot move through an invariant manifold. However, not every 2d-invariant manifold is a separating manifold. Indeed, a separating manifold must have two sides, namely an inside and an outside and it is the case that the nonorientability of Möbius strip and Klein Bottle unite them.Thus invariant manifolds may be nonorientable and thus nonseparating, and this bears the signature of a supradual logophysics. The particular feature of Euclidean 3d-space is that any nonorientable manifold embedded in it is topologically equivalent to a Möbius strip. Already in 4d this is no longer the case. In (Mindlin &Solari [166]) 2d surfaces contained in the 3d invariant manifolds of periodic orbits were found, clear signatures of both orientable and nonorientable surfaces, invariant tori and Klein bottles in a 4d flow, respectively. Thus the torus and the Möbius strip appear conspicuously while for 4d nonlinear dynamical systems is the twisted torus and the Klein Bottle (Charó et.al. [34,35]). The complex topology of attractors was considered initially as "fragile anomalies", but this was meant in relation to Cantor dust structures, rather than nonorientability, which was much disregarded.

This is of importance to the brain-mind would we consider it as a metastable chaotic system, having electroencephalograms as the wave patterns for the stable functioning, which are organized as a series of 2:1 Möbius strip harmonic series (Rapoport [236]). The chaotic attractors which represent the 2:1 arms of the Hopf bifurcation say of the logistic map, elicit Möbius strips and Klein Bottles which are pervasive to chaotic attractors, the unstable periodic orbits appearing in the models by Lorenz, Rössler and others (Crawford & Omohundro [40], Meiske and Schneider [163]). This transition between chaotic attractors of the nonlinear dynamics of neural circuits are conceived as the functional support for the multistable switching between different perceptions or behaviours, say as in the Necker cube or Gestalt figures, supporting the transition of the brain dynamics between different oscillatory states. This can be conceived as the core of the dynamical systems approach to the neurosciences. Thus the nonlinear metastable dynamics of the brain-mind produced by the cross-influence between the electroencephalogram's octave rhythmic progression, manifests as a sequential transition between non-orientable attractors, providing the basis for the self-organized control of a "communication-through-coherence" (Battaglia et.al. [15]), resonance. Indeed, these different attractors would correspond to different dynamical states of a fixed structural network. However, the metastable behaviour from which the sequential chaotic attractors appear is conceived as a single 'complex' attractor, where the complexity refers to the shape of the manifold drawn by the trajectory, which can be characterized as a set of connected sub-manifolds, each capable of sequestering the trajectory for a limited period of time (Rapoport [236]). They are connected to, or embedded in a larger surface in a way such that the trajectory will find this connection and a new transient will emerge as the trajectory moves off to another submanifold. The previous considerations can be extended with René Thom's Catastrophe theory. Following the Aristotelian revindication of morphology this theory claims the universality of some forms, to account for the sudden abrupt transitions of nonlinear systems between regimes as universal, following small changes in additional parameters. However, examined in a larger parameter space, Catastrophe Theory reveals that such bifurcation points tend to occur as part of well-defined qualitative

geometrical-topological structures. Despite the number of discontinuities in Nature appear infinite in variety, Thom showed that the graphs of these processes could be categorized into a few basic shapes. The jump phenomena occur as the state of the system approaches local phase-space manifolds (attractors) with different (non)orientability properties. In these cases the so-called invariant manifolds describe the asymptotic- in- time geometry of the periodic oscillations, known as Instantaneous Centre Manifolds. The instantaneity is related to the discontinuity in sudden regime change emerging as the change of topology, in terms of orientability to nonorientability and the converse. They show the archetypical form of infinity or 8, the lemniscal projection of the Klein Bottle, of the unstable periodic orbits of chaotic attractors, say of Rössler and Lorenz. Current discoveries indicate the relevance of knots (Kaufmann [126]) to the topologies of chaotic attractors (Charó et.al. [34,35]) suggested in (Gilmore & Letelier [61]), following a theorem by Birman and Williams.

6. On the logophysical topological nature of the brain-mind: emotions, Markov blankets and free energy principle

Nonorientability does not only appear in the chaotic attractors of the dynamical theory mathematical treatment of neurons. Rapoport identified it in the vision of a subject which operates through projectivity, as initiated by the Renaissance Masters which is crucial to the constitution of selfhood: the *topology of consciousness* (Rapoport [236]). The young Sigmund Freud was a neurophysiologist in the making, whose reflections on the psyche have become integral to the current developments of Biological Psychology (Panksepp [182] ,Solms 259-262), and the neurosciences at large. In his *Project for a Scientific Psychology,* Freud associated the brain's resting state as constituting a spatiotemporal structure supported by the brain's neuronal structure for what he conceived as a "psychological structure" of the ego for which he suggested a topological consideration (Northoff [177]). Freud referred to this spatiotemporal anchoring of the psyche as "It must lie on the borderline *between* [our emphasis] inside and outside; it must be turn outwards toward the external world and must envelop the other psychical systems" (quoted by Solms and Panksepp, [126]). It is as if Freud unbeknownst on the Klein Bottle had indicated its role as the integrated agency, psyche. Later on Jacques Lacan temptatively pursued this further, alternatively choosing the 2-torus, which supports a banal form of selfreference [239], and the Klein Bottle, yet with deep intuition indicating the importance of topological knots to the psyche (Miller [165], Hewitson [85], Ragland [199]). This anticipated crucial notions of the psyche's operations which appear to indicate the Klein Bottle as the metaform for the spatialization of the psyche elaborated by Rapoport. Phenomenological theoretizations calling for anchoring phenomenology on topology, such as the works by Heidegger further elaborated by Malpas, with Stern, Rosen and Rapoport's identification of nonorientability for grounding phenomenology, for the latter not restricted to physical space. Striving to find a quantum theory for the brain-mind, particularly in terms of quantum field theory of open systems (Vitiello [278]), Globus referred to observational practices which appear to indicate the spatiotemporal anchoring of the body-mind already displayed by the novelist Marcel Proust. Globus' proposal for the scientific study of consciousness [66] was to extend the third-person account of the subject-observer by the praxis of first-order account. The latter was also proposed in the Radical Neurophenomenology and biogenetic structuralism initiated by Laughlin et.al. [141]. It was later pursued by Varela [276], Petitmengin [189]. Globus [66] pointed out to the ultimate spatial-temporal referral of consciousness somewhat treading on Heidegger's conception for anchoring his phenomenology (Malpas [155]) . The third-person account is the signature of objectivity practiced by science curtailed by the lack of a supradual principle, requiring nevertheless to be completed by a first-order account proper of the selfreference of consciousness. The notion pledged by Globus is the "in-betweeness" of phenomenology -just like Freud-, the world as experience, which he stops short of linking with the Klein Bottle (personal communication to the author) yet linked to the "tilde" and "non-tilde" states of Vitiello [278], Jibu and Yasue [123]; see also [120]. An important comment: as a field anthropologist Laughlin remarked the intervention of rhythmic and resonance elements inducing altered states of consciousness arising in rituals [142], and stressed their crucial character to consciousness rather singularly. Nowadays has resurfaced in the neurosciences the appropriateness of claiming a musical nature to the living brain (Bandyopadhyay [14]), which we suggested to be based on the Klein

Bottle metaform of pattern formation and pattern recognition at the brain connectome in terms of harmonics [234] which may appear as cymatic patterns [8].

The experiences associated with the cognition of music have a signifying richness and ample spectra perhaps unique and nonreproducible by any other sensorial stimulus but sound, besides its abstractness perhaps comparable to the seemingly most abstract mathematics, say topoi and category theory (Mazzola [159]). Returning to the anthropologist Laughlin's interest for music and its embodiment induced by brain entrainment through repetition and mimicry of animal motion through ritualisytic performance, later validated by mirror neuron research, it has been ventured that it is music which presents those consciousness states: They are brought to the mind by music and this is so in all states of consciousness, mostly in states of unawareness (Addis [1]). In other words, Addis claims that music is the rather unique agency which fully activates through the body-brain the spectra of consciousness states and particularly emotions, which are conceived both in the neurosciences and Biological Psychology after Panksepp [182], as the most elementary qualia. The latter conclusion was reached on the studies of the anatomical and physiological evidence that led to conclude that the primary role of consciousness is to scan the proper body states rather than the states of the seemingly independent Exterior world (Solms & Friston [308]). In other words, the primary role of consciousness is interoceptive, directed to the mind's embodiment. The most elementary qualia require no cognitive nor perceptual representations, they are the so-called called affections: hunger, lust, surprise or curiosity, particularly aroused by sound. This is a radical departure of consciousness as primed on abstractions.
The experience of music is both a first-person and third-person issue, in the latter case providing for a universal form of intersubjectivity, yet one which is anchored subjectively, in the sense of interoceptiveness.

The first-person account is very much anchored through emotions, to which we shall return briefly. Remarkably this notion of the embodied mind integrating interoprioception with exteroception is crucial to the body-mind homeostasis-directed cybernetics. Freud's conceptualization of the psyche has influenced the cognitive neuroscience, for which the primal form of consciousness is grounded on emotics operating through the limbic system (Damasio [41,42], Solms [261,262]). More recently Craig has identified the middle insula as crucial to homeostatic sentience, and interoception and exteroception integration, leaning on the motor aspect -rather than feelings- in the motor-perception cycle which Turvey identified as a Möbius strip [273] and whose somatotopographic representation is supported by the Klein Bottle [119,281]. See Craig [316.]

The spatialization structure in the neurosciences are referred to the mainly unconscious modeling activity of the psyche in terms of a Markov blanket, a 4-state logophysics proposed by Friston [55] which was identified as the Klein Bottle by Rapoport ([234,236]), further identifying the somatosensory and visual systems Klein Bottle topographic representations of the body periphery through touch and vision, respectively, as key Markov blankets integrations.

These considerations have arised through the search for the minimal conditions to ascribe feelings to an entity, to be alive. A fundamental property of biological self-organising systems is their tendency to resist the second law of thermodynamics. Friston characterized this functional property as emerging in the conditions of ergodic random dynamical system that naturally arises within any such system that possesses a Markov blanket. They are characterized by systems consisting in two sets: ('sensory' and 'active' states. They further influence each other in a selfreferential cycle fashion: [active] external states [Outside–Outside; Klein Bottle-wise, as in the other states of the blanket] cause sensory states which influence—but are not influenced by—internal states [Outside-Inside], while internal states [Inside–Inside] cause active states which influence—but are not influenced by—external [Inside–Outside] states" (Rapoport [236])

Together with a free energy principle the Markov blankets are the core of the developments in neurosciences conceiving the brain-mind as operating through a Bayesian inferential process Actually Solms identified any living biological system enclosed by a boundary as a Markov blanket ([262]; see also [317]). Such is the case of the living cell organised with its membrane which is not amenable to identification as a dual-logic gate (Rapoport [225], Ling [135]).The cell's cytoplasm appear to operate through the "like likes like" principle, introduced by Richard Feynman, which is crucial to the cell's metabolism (Pollack [193])], Ling [135]). and to the formation of coherent water domains. Of course,

Feynman's principle expresses non-duality. This substantiates Spencer-_Brown's conceptualization of worlds arising from a reentering system defined by a primal distinction/boundary [303], a reentering which cannot be supported by a 2d surface other than the selfpenetrating Klein Bottle (Rapoport[231,232]). Boundaries rather than operating in a dualistic setting as merely defining the system by the CONTAIN image-schema, as that what remains Inside -and thus a somewhat passive object- and what is left Outside, are constitutive to the systems bounded by them rather than mere tokens of separation (Celliers [311]).

The nonorientability decurring from antipodal identification of projectivity in the case of the visual field, is crucial to the topology of consciousness, as we already referred to. In Algebraic Topology the notion of blowup is introduced as a geometric transformation which replaces a subspace of a given space with all the directions pointing out of that subspace. For instance, the replacement of a point by the projectivized tangent space at that point. Remarkably we can generate the Möbius strip as the blow-up of the origin of R^2 replacing the plane with the projectivized tangent space. This is related to the pinhole singularities of the visual cortex (Petitot [190]) and its periodic architecture with its Klein Bottle topology, and to the purported origin of the Universe through a Big Bang [236]. However, the underlying conception proper to this is the so called resolution of singularities in Algebraic Topology, which conceives it as zooming on a point, rather than an explosion. There is nothing that is that has not already been, as another form, perhaps. No ex-nihilo. As already observed the event-horizon of the Planck-scale singularities of the vacuum elicit non-orientability as the topology of the infinitesimal boundary of the experientable. In short, the speculation of a subquantum void elicits a frontier to cognition, which has the signature of nonorientability.

Whatever, this reflects the notion of spacetime as a product of the mind, conceived as a "geometric agent", which Piaget identified at the early stage of the child's cognitive development. Kant: "Space is a necessary a priori intuition, that lies at the root of every awareness. One cannot imagine its negation, although one can imagine it to be empty. It is the necessary condition for the possibility of perception, and not a consequence of perception, it is an intuition a priori, the necessary basis for external perceptions". (p. 38) (quoted from Northoff [177]) While space may be an existential precondition to perception, the denial of the space-construal nature by the embodied mind has been disproved.

7. The Principle of Complementarity in cognitive psychology, torsion and nonorientability in the homology of Inside and Outside representations

Shepard's Principle of Complementarity conceives visual experience as arising from a priori embodied structures, constraints operating on the construal of experience. Constraints arise from perception as framed by the visual system, an ecological psychology. The crucial constraints arise from the perceived locally Euclidean 3-dimensionality of space and of the objects placed as if contained in it. This appears in principle as the usual conception of Cartesian space as a container, yet framed by perception. However, objects appear through two-dimensional surfaces of their boundary, both of them as characteristics of the perceived environment and our actions on them. So rather than staying on the 3-dimensionality of objects, perception arise from their boundaries, and these are 2-dimensional. We can conceive of this in terms of the complementarity of form, related to symmetry, with function, related to asymmetry, first suggested by Pierre Curie. Biologist cum crystallologist Lima de Faria upholded this principle for the very foundation for biology as selforganisation, and he argued that it may carry from the level of elementary particles to higher forms [148].

This complementarity is already a Klein Bottle embodiment, as the right hand side rectangular figure (see fig.3) shows its formation: two equally directed sides -the symmetry aspect- produce the cylindrical shape, and the identification of the other directed sides -the asymmetrical aspect- produces in turn the torsion towards the selfpenetration. Steven Rosen has called this a synsymmetry (Rosen [241]). Without the asymmetrical aspect of synsymmetry Nature would be timeless, inert, functionless, and without the symmetrical aspect the morphological regularity would not stand to shape it, as if inert, yet in pregnant openness to change.

According to this, phenomenological space is 2-dimensional rather than 3-dimensional, alike in the holography principle of physics [313], but embodied , for fundamental difference. Remarkably, the

visual receptive fields of the cortex are mathematically modeled as Gabor functions, of holography theory. The built-in symmetry of these functions, as appear in their phases, is the Klein Bottle, 2-dimensional. In other words: Phenomenology is related to 2-dimensional construal of experience, in which holography is sustained by Gabor functions of receptive fields, yet to be able to perform pattern recognition the selfpenetration of the Klein Bottle needs to be removed, and this is carried out in 4d or 5d (Rapoport [228]; Rapoport & Perez 2018 [234]).

At the basis of this mental construal which is further related to the physiologics of visual perception, we retrieve torsion and nonorientability as the geometry-topology of the unconscious ongoing modelization of the living brain of the kinematics of objects (Shepard [250,252,253], Carlton and Shepard [49]). Actually, Shepard departed from the experience of music: the Tritone Paradox motion inside an octave which moves as in Escher's *Ascending and Descending* pivoting on the tritone halfway [251], a Möbius strip [228])). The former was further pursued by Risset in the paradox of rhythm perception (Risset [237],Vernooij et.al. [277])). Shepard further discovered that this geometro-topological principle extended to the unconscious modelization by the brain-mind of the kinematics of objects offered to the subject.

Remarkably, there is an homology of the geometry of dynamics of space or spacetime and the geometry of phenomenology, the world as experienced, particularly in the mathematical modelization of the living brain-mind. Phenomenology points out to symmetries, nonorientability through the Cartan torsion geometry that underlies the Erlangen Program as of XXth century (Sharpe and Chern [111]), and the very organisation of the visual cortex with its π-periodic hypercolumnar structure underlying the Klein Bottle and operating as the visual receptive fields [234]). This is further supported by the human heart which rather than a pump it operates as a Möbius strip torsional motion, as elicited by Francisco Torrent Guasp; see Kocika et.al. [104]). Or still further, the body architecture of fascia -the connective tissue system [129] made of liquid crystals which extends from the cellular cytoskeleton to connect the whole body and covers all tissues, muscle and bones, as lemniscal one-dimensional projections of either the Klein Bottle or Möbius strip. We already mentioned the somatosensory topographic map of touch with its Klein Bottle topology, which models on the primary somatosensory cortex the skin periphery supporting thus the unity of action and perception, an homology which comprises the lemniscal trace of the gait of mammals, flying and liquid inhabiting species. The homology extends to the distortions of the perception of space and time, through subjective Lorentz transformations, which show that "the subjective psychophysical laws of motion perception are relative to the observer's reference frame and that the Lorentz transformations govern the relationships between time, distance, and velocities of perceived events" (Caelli et.al. [114]). The visual system psychophysics does not comply with the Galilean transformations but appears to repose on hyperbolicity. Remarkably, there is a connection between hyperbolicity and nonorientability as shown in (Rapoport [228])).

Furthermore, there is a physiological connection between the subjective space-time and quantum physics structures through what is conceived as the visual component of short-time memory, VSTM. The capacity of VSTM is set both by information load and by number of objects*:* A racing driver or a fighter pilot may reach a limiting velocity of processing of information in which there is a perceived time dilation and space contraction which upon the subject's attention fixation on the phenomenon it may turn to be the final instant before crushing into an obstacle.

The point is that short-time memory is quantized as discovered by Juan Pascual-Leone [184], a former disciple of Jean Piaget. He pioneered cognitive psychology, and in studying child development claimed that topology is a primer on framing the cognition of children. According to Piaget, at the early states of cognitive developments the child puts up elementary spatial representations which *"are none other than those relations which the geometricians tell us are of a primitive character, forming that part of geometry called Topology, foreign to notions of rigid shapes, distances, and angles, or to mensuration and projective relations"* (see Piaget & Inhelder [319]). This does not disappear with the child's development. Indeed, spatial representations that form the elements of perception are not "based on a fixed homogeneous space in the sense of geometry these representations have a dynamic situation-dependent nature involving topological, projective and metrical aspects involving a complex structure of parameters and interactions with other primitives" (Mausfeld, p.86 [115]) which participate with is conceived as an innate system of organisation which support/produce appearances. This system is

attributed to the brain as a "geometrical engine" (Koenderik [105]). The fact that they are mostly transparent to the final outcome operates a dimensional reduction, a process of integration which rather than working through discrimination reduces the information to a small number of parameters referred to as dimensions (Pang et.al. [183]). They early appeared in the pioneering visual research of Hubel and Wiesel who primed object orientation as experienced in the primal visual cortex where visual information is represented: topologically, a Klein Bottle. In the dynamical systems approach to the neurosciences the dimensional reduction is referred to the low dimensional attractors, which we discussed in a previous section. This also suggests that together with the spikes, neuron action potentials attribution of a dual logic, there is a coordination which in that paradigm is related to mitigation of the discrete character attributed to them and the necessity of reducing noise. We shall return to this below.

However, psychologists have discovered that attention has " *a quantitative dimension in terms of the maximum number of items to which a person can attend at one time"* (Weiss & Weiss [280]). The limits were indeed the case, yet rather than fixed and uncontextual,they have a plastic character subjectable to learning and other factors. In distinction with symmetries as the logophysical constraints in the unconscious modeling of object kinematics, in the case of attention it is the range of the number of objects or mental states which an observer can attend to, which also led to the so-called magic rule of 7 plus minus 2. Pascual Leone discovered that this space of objects subjected to attention appears to follow up an Einstein-Bose statistics, it is a bosonic-like space of objects vis-à-vis attention [280]. From this followed the proposal by Weiss & Weiss "… *that memory span has to be understood as the quantum of action of thought. In fact, this quanta of action represent macroscopic ordered states in the sense of quantum mechanics*". VSTM representations are stored by means of sustained firing of action potentials, which corresponds to a supradual logophysics, to be discussed below. So the visual space on which subjective relativity operates through VSTM is quantized: Unlike the theory of spacetime of objects as contained in physical "real" space, where in principle the homogeneity and continuity is posited, the visual space of experience as entertained by attention, does not need to search for a physical quantum basis: The singularity is the viewing subject, very much alike in the very definition of torsion when we supersede the primality of physical space as if exterior to our nature. *The subject as the primal distinction,* as offered by Spencer-Brown [303], and identified as a torsion geometry by Rapoport [231].

8.   The hard problem of consciousness and the torsion of cognition

We have presented the relation of the five-fold structure to dynamical torsion geometries and nonorientable topologies pervasive to Nature. Rather than a purely algorithmic static structure it turned out to be related to a vortical universal dynamics which comprises both the material and cognitive domains. Particularly, so called *chaotic systems appear to indicate a supradual logophysics*, an issue which has been disregarded. These topologies embody harmonics and thus transpired that Nature is fundamentally related to harmonic principles, as already arises in genomes, in the brain connectome, and in the Principle of Complementarity of cognitive psychology, further substantiated in (Rapoport [236]).The mysterious nature of resonance has turned out to be supported on nonorientability,which is also evident in the dynamical system models of neurons, the polyrhythmicity of motion as reflected by the brain-mind, and the topographic maps of the visual and somatosensory modes. Actually the latter somatosensory topography which is crucial to the constitution of the 'body image' does so by further integration of the projection zones of the other sensory modalities, themselves consisting of topological maps of different sensory organs (Solms and Panksepp, [126]; Craig [316]). Thus, rather than a Klein Bottle topology for the somatosensory modality, this further integration indicates a HyperKlein Bottle. The supradual logophysics is keenly related to selfconsciousness, on the one hand by the 'body image' of selfhood, as experienced by the subject contemplating a distant point at the horizon [236]). While the so called Hard Problem of consciousness as the quest to identify the 'source of consciousness' points to emotions, as sustained by Biological Psychology after Panksepp and upholded in the neurosciences (Solms & Turnbull [259]), there is much empirical evidence that resonance is the 'gluing' phenomenon producing the synchronicity of the brain-mind as a whole, and of many-minds (Hunt & Schooler [88]). This biological base for emotions somehow contrasts with the prevalent notion that consciousness stems from complex computation among a network of simple neurons which each receives and integrates

synaptic inputs to a threshold which performs a kind of implicit gate to a dual-logic firing/non-firing logic, which as we shall discuss below, is currently disproved. The other approach to understand consciousness queries on the origin of cognition as how it originates from cognitive processes accompanied or driven by phenomenal conscious experience and subjective feelings, referred to by philosophers as 'qualia'. This is the so qualified "Hard Problem" of consciousness. Whatever the difficulty is, we face the problem of understanding the nature of distinctions between awareness and the major form of consciousness. After all, "qualia" do not disappear in consciousness at all, whatever the form or degree. We see the redness of an apple while awaken or dreaming, not to mention the intensity of qualia in other altered states of consciousness. Much of what is unavailable to awareness and performs in seemingly autonomic form which never is the case as the so-called Autonomous Nervous Systems – a component of the peripheral nervous system. It regulates involuntary physiological processes , say heart rate, blood pressure, etc.; it is not isolated from the environment through its connection to the Central Nervous System. Its coupling to it appears to be embodied as a Klein Bottle topology as the somatosensory and visual systems, cogent to the unity of action and perception (Turvey [273]). The same problem arises in attempting to define Life, from the perspective of dualism [230], since it already denies a continuity from the physical-chemical level to the biological one, as viruses show to be mediating states which remarkable have a remarkable geometric complexity. Distinctions between conscious and unconscious are not amenable to be conceived in dual terms, as we shall meet upon discussing anaesthetics as well as other states of altered states of consciousness, those of enhancement of its functionality. To the query posed by the Hard Problem, Biological Psychology has pointed out to emotions as having the inherent quality from which experience can be anchored to. Solms: "An explanation of experience will never be found in the function of vision—or memory, for that matter—or in any function that is not inherently experiential. The function of experience cannot be inferred from perception and memory, but it *can* be inferred from feeling… affection" (Solms, [261,262]). As for the function of affects " feeling enables complex organisms to register—and thereby to regulate and prioritize through thinking and voluntary action—deviations from homeostatic settling points *in unpredicted contexts"*. So they cannot be conceived as performing in terms of dual logic, rather analogics appears to be the case. As uniquely elaborated by Antonio Damasio, from his clinical work with brain-body damages or amputations, emotions also provide for the basis of the most complex forms of consciousness, as presented in his extraordinary *The Strange Order of Things* ([41,42]). Consciousness is primally interoceptive, it registers the state of the subject, not of the object world. "The sentient subject is first and foremost an affective subject. Only then can it experience perceptual and cognitive representations... The subject of consciousness is primary. The secondary (perceptual and cognitive) form of consciousness is achieved only when the subject of consciousness feels its way into its perceptions and cognitions, which are unconscious in themselves" (Solms , ibid). This selfreferential nature of elementary consciousness does not breed in isolation but rather in relation to the environment, in mammals in the maternal uterus and interacting with it through the umbilical chord nurturing matter and through sound, as the most fundamental form of sensing in direction to alterity. The complex mammal-to-be-born with relation to the maternal being is a HyperKlein Bottle. Resonance as the most fundamental primarily exteroceptive form of communication of the feeling of what happens as referred to the Outside Other.

Emotions are supported by selfreference and hetero-reference, self and alter-directed and/or experienced, supradually. Empathy in its wide spectra appears to be related to resonance, as the experience of music shows to be the case. While a form of panpsychism which evades identification might be attributed to Nature (Skrbina [257],) the supradual logophysics of the Klein Bottle is based on the principles of selfreference and hetero-reference, both torsion and nonorientability, *as if independently of the existence of a subjective agency*. It is known in neurophysiology that selfreference is fundamental to the brain-mind, and as the mirror-neuron phenomenology shows it is undissociable from hetero-reference: There is no Self without Other, as the so called mirror neurons phenomenology has exposed. Interoception is unseparable from our perspectival alter-related mutual construal be that of reality, the imaginal or ideational domains, mimetics, having resonance and empathy for its vehicles (Baiano et.al. [10], Rizzollati et.al. [238], Ferrari et.al. [50]): Its chameleonic nature a disproof of the hegemonic attribution of the primality of the dual logophysics.

As for the origin of emotions has been proposed that the interoception scanning primary intention of consciousness establish a sense of value related to the preservation of existence. Spinoza gave it a name, conatus [308]. It is a selfreferential process which elicits intentionality, selfreferentially: preservation of being. It is being sensed as the different states of homeostasic regulation. The dynamics of this process is embodied as a Markov blanket [55,307], which we already identified as a Klein Bottle architecture.

9. The supradual phenomenology of neuronal action potentials and topological phases

An important role of the receptive field -whose Gabor function representation first introduced in holography has a phase with a builtin Klein Bottle symmetry (Tanaka [268,269])- is the enabling of cell-to-cell communication through the action potentials. All of information of the outside world arrives through patterns of action potentials created at one of five sensory organs. Action potentials occurs when the membrane potential of a specific cell location rapidly rises and falls; the current paradigm attribute this to ion current transport. This depolarization in turn causes adjacent locations to similarly depolarize. Action potentials occur in several types of animal cells, called excitable cells, which include neurons, muscle cells, endocrine cells and in some plant cells. Action potentials support the propagation of signals along the neuron's axon toward synaptic boutons, the axon terminals, small swellings situated at the ends of an axon. Synaptic boutons are typically the sites where synapses with other neurons are found, and neurotransmitters are stored here to communicate with other neurons via these synapses. These signals can then connect with other neurons at synapses, or to motor cells or glands.

Let us start by noting that the neural network operation is not a matter of separate elements. This already dispels the notion that a dual gate phenomenology could support this connectivity, it rather elicits a contextuality, which already the Klein Bottle embodies. Indeed, a typical neuron receives inputs from thousands other neurons through the contacts on its dendritic tree called synapses. The inputs produce electrical transmembrane currents that change the membrane potential of the neuron. Synaptic currents produce changes, the so called postsynaptic potentials (PSPs). Small currents produce small PSPs; larger currents produce significant PSPs that can be amplified by the voltage-sensitive channels embedded in the neuronal membrane and lead to the generation of an action potential or spike – an abrupt and transient change of membrane voltage that propagates to other neurons via a long protrusion called an axon. Such spikes are the main means of communication between neurons. Spikes tune the time for cognition and manage the times of firing, so they perform as time-operators.

We continue by dispelling the notion that complex behaviour may be necessarily related to a complex network of neurons, a connectome dynamics (Atasoy et.al. [8]), but rather is based from an ecological integration of the living organism with its environment. Such an integration is the Klein Bottle embodiment. A paramount example is *Paramecium,* a unicellular organism, which swims in its habitat. *Paramecium* is a single spiking "neuron"; there is no connectome to this organism, and yet it can accomplish a variety of ecologically relevant tasks. It performs and controls complex navigation in crowded multisensory environments, practices accordingly social behaviour, sexual mating, and spatial memory might also be the case. Its motions with multiple cilia -mainly constituted by microtubules- is vortical. However, this does not arise from the complexity of the cell but "rather from the interaction between this spiking cell and the environment, together with the exploratory properties conferred by the pseudo-random nature of the effect of a spike [referring to the trial-and-error pragmatics of *Paramecium* in search for nurture]. This highlights the importance of embodiment and coupling with the environment, which are increasingly appreciated in cognitive science and philosophy of mind" (Brette [26])

In general neurons do not fire on their own; they fire as a result of incoming spikes from other neurons. If wished their identity as action is not entirely proper but a resultant of the endogenous *and* exogenous influence. As already said, there is no Self without Alter. In other words the CONTAIN image-schema appears to be inapplicable to the individual neuron in a collective of neurons . Hence, neurons are said to fire and yet they can be characterized *as being fired*. This, we claim, is the signature of the Klein Bottle supradual logophysics. Thus, there is an inherent "circularity" to the behaviour of neurons, which being associated to a joint selfreference and hetero-reference, it is associated to a

HyperKlein Bottle cybernetics, rather than a dual logic associated to the membrane and its electrochemical potential. Yet, not only a collective of neurons are involved in the spike formation and propagation, neurotransmitters are required for this propagation. The usual paradigm claims that action potentials are caused by ion currents which diffuse inside the neuron and when reaching and surpassing a threshold the spike is produced.

The mathematical model for action potentials as arising from ionic currents is the Hogdkin-Huxley model (HHM), which describes neuronal dynamics in terms of activation and inhibition of voltage-gated conductances. An important hypothesis of the HHM is that neurons are dynamical systems, and thus described by a system of ordinary nonlinear differential equations. HHM is used together with the claim that the continuous action potentials can be reduced to sudden appearance of peaks called "spikes" to posit the dual logic; this is done in terms of the individual neuron presumably upon surmounting a threshold voltage which thus fires producing the spike or otherwise stays quiescent not producing it at all. In HHM biological neurons are dual-logic gates working through an "integrate-and-fire" scheme based on comparison, a difference-producing-differences process, after Bateson: Firstly, |a cell body (soma) and several branched dendrites receive and integrate synaptic inputs as action potentials. Then, a comparison between the action potential and the integrated threshold potential at the axons initiation segments takes place, when the latter reaches the former value then a dual all-or-nothing "spike", or action potential as output and transmitted across the axon. The issue left unresolved by this dual logophysics is what makes the comparison, in the first place. In closer examination it further produces an infinite regress proper to its first-order cybernetics based on dual logic onto-epistemology [234,236].

However, further examination of this model does not support its in dual logic preconception, which is rather hegemonic in the sciences (Rapoport [230]). Izhikevich: "The irony is that the Hodgkin-Huxley model does not have a well-defined threshold; it does not fire all-or-none spikes; and it is not an integrator, but a resonator, it prefers inputs having certain frequencies that resonate with the frequency of subthreshold oscillations of the neuron" (p.2 [91]). As for establishing an adequate methodology, Fitzhugh concluded his mathematical modeling of threshold phenomena by saying that "…many neuronal properties are invariant under continuous, one-to-one transformations of the coordinates of phase space and fall within the domain of topology…the most logical procedure in the description of a complex biological might be to characterize the topology of its phase space, then to establish a set of physically identifiable coordinates in the space, and finally to fit differential equations to the trajectories" [315]. Upon examining the HHM attractors, nonorientable twists were discovered in the phase space chaotic attractors (Guckenheimer & Labouriau [70]). Rather than subthreshold, a conceptual hindrance of the assumption of dual logophysics, the more appropriate term would be subcellular or cytoskeletal.

However, it has been empirically verified that neurons operate as resonators (see Izhikevich [91]); hence, dual logic is not the case for the basis of neuronal phenomenology. Yet this resonator character turned out to be the core of what is conceived as a novel paradigm for the neurosciences elaborated by Anirban Bandyopadhyay and collaborators, the Hindu group [14,64,246,254-256]. Already in 2014 Hameroff and Penrose [75] appealed to the empirical findings of this group as support for their ORC OR theory, and very recently further promoted them for a novel paradigm for the neurosciences. Their elaboration is quite distinct with the present Klein Bottle logophysics, disregarding the torsion geometry and topological issues, not mentioning the topological phases in the cytoskeleton and the shape of action potentials, nor the deep connections to cognition and perception, topological chemistry and nonorientability of liquid crystals, which are crucial to the supradual Klein Bottle logophysics.

Already visual receptive fields do not operate in terms of dual logic: the consideration of context such as central locus and periphery elicits a non-dual logic by which neurons are in a state of superposition of firing and non-firing, inhabitation and activation superposed, yielding a 4-state Klein Bottle logophysics (Rapoport [228]).

Redirecting our attention to the so called cytoskeletal structures and processes we first remark their association to the filamentary structures of the neuron. Focusing the attention on the spikes in disregard of the phenomena being produced on the subthreshold and before actual spikes firing is very much in ignorance of the complexity and information associated to it. Indeed, the current spike dual-logic based paradigm ignores all components beneath the neuron membrane - fundamentally the

cytoskeleton, tagging them as inconsequential: the prejudice of the neural membrane as a dual gate is hegemonic. We turn to this now.

The cytoskeleton is basically made of proteins, the most remarkable proteins are the molecular motors, the so called motor molecules which play a crucial role in the cytoskeletal dynamics. Indeed, motor proteins operate transporting membrane-enclosed organelles or changing the shape of structures built from cytoskeletal filaments (Alberts et.al. [5]). In other words, in the latter case motor proteins operate as topological agencies. They bind to a polarized cytoskeletal filament and use the energy derived from repeated cycles of ATP hydrolysis to move steadily along it. Dozens of different motor proteins coexist in every eukaryotic cell. They differ in the type of filament they bind to (either microtubules or actin), the direction in which they move along the filament, and what they carry.

The cytoskeleton is a protein lattice which has three kinds of filaments, microtubule made of tubulin, neurofilaments, and microfilaments (actin). Microtubules are hollow cylindrical tubes with outer diameter of 25 nm and inner diameter of 14 nm with thirteen linear tubulin chains ('protofilaments') aligning side-to-side to form the hollow microtubule Given the large surface area inside and outside the tube, ordered water domains are considered to be bound to microtubules. These ordered water domains are crucial to metabolism and to the overall coherence of organisms [79,80,135,168-170,193]. Due to their uniaxial symmetry rather than having the topology of the abstract projective space (Machon and Alexander [153,154]), given that organisms are bounded rather than extending to infinity, they are characterized by Möbius strips due to the antipodal identifications of points at the boundaries supporting nonorientability (Rapoport [228]). Remarkably, in the case of vision this supports the establishment of a sense of selfhood ([236]). In other words, already coherent water domains have the selfreferential nonorientable shaping as a form of protoconsciousness.

Using the quantum field theory for condensed matter systems it has been argued that microtubule cavities with water in their vicinity could support superradiance, i.e. collective infrared photon emission by water molecules known as superradiance. Pollack showed that coherent water domains are produced by electromagnetic absorption by regular water, and upon electromagnetic emission a cycle of coherence/decoherence of water is produced. Also a coupling between the water molecules inside the cavity of the microtubule and the tubulins that build up the microtubule walls may produce superradiance (Pollack [193]). In the case of coherent water domain, the supradual like likes like principle operates towards their formation, hence we may venture whether this also applies to the allosterics of water in the coupling of tubulin aggregating into microtubules, as a shaping typical of aromatic molecules behaviour.

It has been argued that the ordered water domains environment of microtubules could produce a dissipationless transfer of energy on neuronal microtubules due to the collective coherent behaviour. This transport would be supported by electromagnetic sine-Gordon solitons (Georgiev [59]). They are solutions to a non-linear Schroedinger equation, and as such associated to a torsion geometry of vacuum Brownian fluctuations (Rapoport [219]). Remarkably sine-Gordon solitons support Möbius strips as the edge of knots in the 2-torus phase-space of coupled oscillators [116]. They appear as the dynamical system attractors of the "mental" rotations discovered in the cognition of music and in the nonlinear dynamical systems of coupled oscillators say of two tonically spiked neurons , whose behaviours is represented by the strange attractors of their dynamics (Izhikevich, [91]). We shall return to this below.

Since superradiance is not exclusive to microtubule interaction with the coherent water domains- indeed it is the case of genomes as real-space nonorientable configurations [233]- Georgiev argues that it further requires to examine the interaction of microtubules terminals with MAPs (microtubule applied proteins) or motor proteins. We shall see later that this produces nonlinear elasticity buckling patterns as a solitary wave on a Möbius strip surface.

Cytoskeletons conform a tensegrity structure, a scaffold-like structure made of proteins intermittently folding and unfolding, assembling and disassembling (Ingber [117]). In neurons microtubules prevail and are uniquely stable; their collapse is related to the evanescence of consciousness, say under anaesthesia. In the general case microtubules are highly dynamic as they alternate in a cycle of alternative depolymerization and polymerization, known as dynamic instability. In this sense they resemble the superradiance cycle of formation and disgregation of coherent water

domains by alternative absorption and emission of electromagnetic radiation already introduced. Notably Voeikov proposed this cycle as the very signature of the origin of life [279]. Some specific MAPs bind to the terminal sector of microtubules thus controlling their dynamics and attachments to other cellular structures. Other specific MAPs attach to microtubules along their entire length such as α-tubulin and β-tubulin thus modulating microtubule functionality. The molecular patterns generated by them conform the so called tubulin code which control microtubule behaviour. Remarkably, the tubulin code does *not* operate as a dual switch, but rather analogically as a fine regulator (Janke & Magiera [93]). In other words, the dual logophysics of microtubule functionality *is not* the case.

The notion of the cell as a tensegrity architecture provides a model to understand the processes subjacent to cell shape, movement and cytoskeletal mechanics, cells sensing and response to mechanical forces as well as cell differentiation and tissue development. Thus they perform as a cognitive-like structure which has adaptative behaviour. This is particularly notable in the behaviour of ciliate colonies -cilia are microtubules- for which the vortical metaform of motion as a shape of in-formation acting as the logophysical agency at some level being reproduced at another level of organisation: They manifest a natural intelligence as a mimetic reproduction of vortices (Ben Jacob [19]).This isomorphism surges from the common nonorientability of genome sections [233], the lemniscal 1d-projections of Möbius strip inscribed as the gait patterns in different media -swimming in the former case [18], nonorientability of sound and electromagnetic waves [7 ,43], and still in the topological multiconformation of organic molecules, to be discussed below. The latter is basic to ORC OR theory in the superposition of protein conformations.

Filaments look alike nano-wires which are densely packed in neurons, and as a tensegrity structure they are somewhat reductively considered as providing a tensile resisting skeleton. Thus, they resemble electronic devices such as resonators, which upon assembling on Möbius strip shape, the signals transmitted by them are somewhat liminal to the classical-quantum usual divide, to be later discussed. Tensegrities model the mechanical behaviour of the cell as it emerges from physical interactions among the different molecular filament systems that form the cytoskeleton and its role orienting much of the cell's metabolic and signal transduction machinery. Furthermore it elicits that a single physical control parameter, cell shape distortion through gradual variations can switch cells between distinct gene programs (e.g. growth, differentiation and apoptosis). This process can be viewed as a biological phase transition, in which mechanical distortion of cells and of their cytoskeleton through cell surface integrin receptors profoundly affect cell behaviour.

That the cytoplasm is such a tensegrity structure having tensile and compressive elements is well established and particularly microtubules are under compression due to tension generated by actin filaments and the so called cytomusculature (Hameroff, [73]). Microtubules form a scaffoldlike selfsupportive structure without contact among them. Furthermore, there is a complex torsion architecture organized in terms of microtubules with contractile actin filaments spirally winding around them, actually with the two opposite directions alike a spinning top which may cause rotational oscillations. The figure that appears is of a cytoskeletal tensegrity dynamical structure "twisting back and forth, even "rockin' and rollin'!" (Hameroff, p.170 [73]). Currently, this complex structure associated to resonances has been empirically probed by Bandyopadhyay and collaborators, using novel scanning techniques.

10. Introducing the torsion dynamics of microtubules

Astrophysicist N.A.Kozyrev following his discovery of the existence of a nonlocal field of stars along their past, present and future percourse in time as an holographic-like anticipative memory (Kozyrev, [108-110]; Lavrentiev [143-145], Eganova [46], Korotaev [106]) which he attributed to a spinning torsion field, he studied the behaviour of spinning tops. Seemingly, Kozyrev was not aware of the joint work of Einstein and Cartan introducing torsion to spacetime geometry (Debever [298]) which failed to identify the significance of torsion as a gravitational field element due to the omission of macroscopic spin (Goenner [67]). Kozyrev's findings of anomalous behaviour of weight variations of spinning tops were reproduced by Hayasaka and Takeuchi [78]. Remarkably, Kozyrev's initial hypothesis was that the existence of an irreversible thermodynamical process as the local manifestation of the action-at-a-

distance. His conception of time was radically different to the usual time-parameter descriptor of change. According to Kozyrev in these irreversible processes what manifests is that time has a stufflike character: On the one hand being heterogeneous its manifestation depends on what the system it acts upon is like; secondly it establishes a primality of cause-effect, so time is more about *timing* (i.e. a time-operator) and its form is vortical, and thus the effect of this timing is the functionality and form of the system itself. Alike a vortex it selforganizes viz-a-viz the environment and the morphing effect is the system itself. A similar conception in relation to torsion and blow-ups of nonlinear systems was developed by Soucheng OuYang and Jeffrey Yi Lin Forest (see [286]).

For a proof of Kozyrev's findings of a torsion field which operates as an anticipative holographic memory it only takes to look at the centre of our galaxy, the Milky Way, to find a megaparsec size Möbius strip of light (Molinari et.al. [291]). In Bohm's conception of the quantum world as structured in terms of an "explicit" and "implicit" orders in terms of the so-called quantum potential, which is related to the electromagnetic torsion potential under the eikonal light-ray constraint and also arising in the Schroedinger equation as a torsion field, Bohm and Hiley- invoked a "prespace" [292,294]. This prespace is a byproduct of the nonlinear interference wave patterns from which in-formation surges by beaming ligthwaves upon them. They described this prespace as topological, and algebraically by Clifford algebras [28].The shapes of the latter whenever their phases possess an inhomogeneous distribution- which is as well the case of holograms, are non-orientable Möbius strips (Freund [54], Bauer et.al. [17], Bliokh et.al. [21]).

Would we consider the living cell, say a neuron, its cytoskeleton, water and interactions with vacuum fluctuations and still producing and absorbing electromagnetic waves and vibrations, we find a very complex fractal tensegrity architecture in which different periodicities operate in a ample range of frequencies and resonances elicit the overall coherence. Actually the investigation of this architecture has led to conceive the cytoskeleton as a time-crystal by which the cytoskeleton is under perpetual change in a pattern which is periodically repeated [14]. In any given cycle the system may be in a phase in a non-equilibrium state, in which the system appears to be in a thermodynamical irreversible state.

So returning to Kozyrev's conception of time as an operator causing an irreversible process as it morphs in different vortical structures such as proteins, tubulin, microtubules, we can further examine the torsion geometry of this.

In the study of the torsion geometry of thermodynamical irreversible processes and of Brownian motions (Rapoport [210,227]), or still in the equivalence of the Maxwell equation of electromagnetism and the Dirac-Hestenes equations of relativistic quantum mechanics upon examining the Helmholtz decomposition (extended to differential forms on manifolds, the deRham-Kodaira-Hodge theorem) of the trace-torsion differential one-form (or its metric- conjugate vector field) we identified the terms related to irreversibility. It is the non-exact gradient trace-torsion terms: a divergenceless co-closed term and an harmonic term. The harmonic term is both divergenceless and closed, its differential is null. These can be also thought as electromagnetic one-forms, the harmonic term has a zero-field, alike an Aharonov-Bohm potential term [206,208,217]. These terms may support topological anholonomies and are further related to the phenomenon of resonance as interaction-at-a-distance. Furthermore, what this elicits is a dynamical torsion given by differential one-forms, like the electromagnetic potentials which in fact is a trace-torsion Cartan-Weyl 1-form, say Q, yet whose electromagnetic field dQ -d being the differential operator- is a differential 2-form, say F as in electromagnetism, which is null [84]: In other words a dynamical torsion producing null "curvature". This is in stark contrast with null torsion metric geometry with non vanishing curvature which is postulated by Penrose and Hameroff in their ORC OR theory. We claim that this choice of geometry -as in General Relativity- defined by metrics, to be entirely unrelated to what Hameroff described as the rock-and-roll microtubule having tubulin helices morphing inhomogeneously as an invertible gyroscope either supported on any of its extremes, producing an inhomogeneous vortex at the same time.

It is this behaviour which we suggest to be the underlying process for the collapse of microtubules subjected to tubulin loads, while in the present setting the coherence is no longer an action-at-a-distance; more of this below. In other words, the collapse has to do with a thermodynamical irreversible process in which rotations indeed intervene as Hameroff early observed to be the case of microtubules dynamics. Alternatively can be conceived as the vorticity turbulence producing term of the velocity of a viscous

fluid satisfying the Navier-Stokes equations of fluid-dynamics, or the equations of magneto-fluid-dynamics [211-217]. This is in stark contrast with null torsion metric geometry with non vanishing curvature which is postulated by Penrose and Hameroff in their ORC OR theory, to be discussed below. We early suggested that the continuity of the cytoskeleton with the extracellular matrix together with the confinement of the nonorientable coherent water domains basic to cellular metabolism, rather than making of a cell's membrane a semipermeable dual gate it performs an integration supported by local nonorientable membrane topology. According to Ingber, integrin receptors, critical mechanosensitive molecules and cellular components integrate the cell's Inside and Outside. So we can identify the *nonorientability* of Möbius strip elasticity as a supradual protoform of agency, a topological "protoconsciousness", though not necessarily ascribed to a living agent, but a function arising from the interaction of the microtubule with the environment, particularly by additional load producing a change of shape and order. This is still more patent upon considering that crystalline ribbons made of a compound of niobium and selenium, NbSe3 - non ascribable to the constituents of a living structure- in view of its inherent rigidity, which would be expected to prevent it from either bending or twisting, may conform as Möbius strips (Tanda et.al. [270]).

For a proof of concept, we note that selfassembled viral membranes were shown to have scallop-rodlike units selforganising into Möbius conformations at the membrane edge (see Gibaud et.al. [60]). We must recall that viruses are crystal structures *intermediary* to geometrical lifeless structure and living organisms. Indeed they carry genetic material, reproduce and evolve ascribed to natural selection. Alternatively, in the self-organising theory, free of Darwinian natural selection attribution, which is conspicuous to crystal structures, the natural selection process can be left aside, as argued by Lima de Faria. However viruses lack the key characteristics, such as cell structure, that are generally considered necessary criteria for defining life.

Given the importance of proteins to the cytoskeleton, we proceed to the shapes and regularities which are elicited by them. Palindromic sequences are abundant in proteins. Palindromes, say "neuquen", "ana", "somanydynamos", are topologically supported by collapsed 2-tori and Möbius strips, are crucial to genomes and their evolution (Rapoport [233,234]). Studies have shown that reversing the sequence leads to the production of the same 3d protein fold or still its enantiomer (Sheari et.al. [117]). This is most remarkable since it indicates that the 3d structure of proteins may be related to a lower dimensional -actually a 2d Möbius strip- metaconfiguration, the untwisted orientable case corresponding to the symmetric former case. Thus, the principle of "multiple identity" or, if wished "split personality" appears also to be the case of proteins, contradicting thus the principle of identity basic to dual logic.

This is still more notable following the topological studies of protein domains in terms of relational representation of protein domains using the locations of the backbone atoms and the hydrogen bonds, which allows a construal of a combinatorial object – a so-called *fatgraph.* Despite that the space of possible protein structures appears vast and continuous, and the complexity of the relationship between primary, secondary and tertiary structure levels, fatgraphs allow for a *discrete* representation from which a 2d-*surface* appears as the ultimate representation. This surface can be topologically represented as gluing 2-tori (viz bagel-shaped) and Klein Bottles. The former represents a banal selfreferential figure where CONTAIN is structured surrounding an excluded central singularity in a rigid relation -a hole which is part of the whole yet as if excluded (Rapoport [235]). The latter embodies the dynamical contextualization of protein domains vis-à-vis the environment relatable to the instability of protein conformations. In other terms, a juxtaposition of duality and supraduality, of permanence and mutability. The gluing of orientable 2-torus and Klein Bottles that lie as the topological structure of proteins elicits the multiconformation of proteins as possible modifications between the two forms of organisation vis-à-vis the environment: the closed organisation given by the tori, the open-closed given by the Klein Bottle elements, and the possibility intertransformations due to electron delocalization. Given that the Klein Bottle through its Hadamard matrix representation is the single operator which allows to transcend classical computations it is these elements which support this transcendence as quantum computations (Aharonov [3]). Furthermore, since the Klein Bottle operates on its own superposition states to produce Boolean states as a topological decoherence, as much as reiterating its action now on the Boolean states recovers its own coherence as topological superposition, then these nonorientable elements of protein

domains control not only the quantum computations but also the transformations to dual logic elements and futher the reconstitution of quantum computation in the second phase of the cycle. Actually, as observed in (Hameroff et.al., [74]) "protein conformational dynamics suggest that protein conformational states—fundamental information units ('bits') in biological systems—are governed by quantum events, and are thus perhaps akin to quantum bits ('qubits') as utilized in quantum computation … signaling, communication and conductivity in MTs, and theoretical models have predicted both classical and quantum information processing in MTs". As related to their relevance "Quantum processes such as electron tunneling, delocalization, and superposition could, therefore, couple to nuclear locations and influence conformational changes, particularly if the quantum processes are collective and self-organized. Proteins may be designed to amplify these quantum processes. Within proteins ideal sites for electron delocalization are hydrophobic pockets (sites of anaesthetic effect), in particular those containing aromatic rings like that of tryptophan" (ibid.). We shall return to this upon discussing ORC OR theory.

Tubulin dimers are arranged not only in vortical shaped protofilament chains forming thus the microtubules. They further form a particular skewed hexagonal lattice whose crystal structure gives rise to helical winding patterns with regular repeat intervals. Increasingly steep winding patterns complete one cycle around the cylinder at a certain number of tubulins above where the cycle started. These numbers, 3, 5, 8, 13, 21 etc. follow the well known Fibonacci series and whose sequential ratios F_N / F_{N+1} tend to approximate the Golden Mean, reproducing thus the five-fold symmetry which we identified already as a torsion geometry anholonomy. From the association between torsion and nonorientability presented in §2, it is *coherent to claim that topological anholonomies, i.e. nonorientability would appear in microtubular conformations*. We shall discuss below a Möbius strip buckling pattern as a solitary wave arising from the nonlinear elasticity response of (nonorientable) microtubules which would ensue from the coupling of tubulin to the anaesthetic factor.

The association of microtubules with a torsion geometry and nonorientability points to the supradual Klein Bottle logic, in contrast to Hameroff and Watt' s take that microtubules operate as switching Boolean matrices processing binary bits states as they move through microtubules and proteins associated to them [72]. Rather than Boolean matrices and further recalling the analogical nature of the tubulin code, this points out to Matrix Logic, which has fuzzy logic, Boolean logic and quantum logic for subcases. Matrix Logic is the matrix form of the Klein Bottle logic introduced by August Stern, it is homologous to Quantum Mechanics for 2-state systems [265]. Rather than Matrix Logic being circumscribed to logical operators these are jointly logical-quantum operators. Quantum statements are interchangeably expressed as logical connections. It is noteworthy to recall that Orlov showed that Quantum Mechanics can be derived from Boolean logic expressed as matrices ([179]) . In Matrix Logic invertible operators are also considered; their eigenvalues are not restricted to the real numbers proper to measurements processes of quantum mechanical systems but may be complex numbers. Indeed, these logical-quantum operators are not necessarily self-adjoint, the latter restriction as in Quantum Mechanics for closed systems which is the case of Orlov's rendering of Quantum Mechanics from Boolean logic in matrix form. Non selfadjoint operators appear in the case of quantum open system, say the cytoskeleton. First we discuss some crucial relations of this to Quantum Information Theory.

11. The fundamental role of the Klein Bottle in Quantum Information theory and a theory of consciousness

The Klein Bottle plays a core role in Quantum Information theory central to quantum computation, through its 2x2 matrix representation (up to a scaling factor): the Hadamard matrix. This theory is constructed from elementary quantum building blocks, namely qubits, rather than bits as the unit of classical information, and logical gates. In quantum computation the gates are drawn from a universal set of gates, namely, a set which can be used to perform general quantum computations (Nielsen & Chang [174]). It is the Hadamard gate which supports transcending Boolean logic. In fact, in quantum computation it is the single logical gate that transcends the computation of all Boolean functions through the Toffoli reversible gate,which together with the Hadamard gate constitute a universal set of logical

gates (Aharonov [3]).Thus, they constitute the logical gates for which any possible operation on a quantum computer can be expressed as a finite sequence of them. The notion that neuron microtubules would support quantum computations has been raised by Hameroff and Penrose, yet they were mainly concerned with microtubules as a Boolean gate architecture and the Hadamard gate was not considered in their models [72-76,192]. This is quite remarkable since it is the Hadamard gate which may bring a single qubit into a superposition state, the latter being the qubits which their theory of consciousness deals with. Furthermore as already noted, the tubulin code is not dual logic.

12. Topological folding, nonlinear elasticity in the buckling of microtubules, memory and the origin of life

Remarkably there is a close relation between the Möbius strip as a physical space topology and nonlinear elasticity, which has been unnoticed by Penrose & Hameroff, which we turn to discuss. The phenomenon of buckling of structures, say tubulin or microtubules, as in the Euler instability, does not correspond to linear load induction but to a sudden instability due to a load of tubulin that may lead to the collapse of the structure (de Pablo et.al. [297]). The response of an elastic Möbius strip to shear stresses is intrinsically nonreciprocal and nonlinear, although the local stress-strain relation is linear. A direct consequence of non-orientability is that no stress distribution can yield homogeneous shear deformations over a Möbius strip. The topological anholonomy constrains the ribbon to remain undeformed at one point whatever the magnitude of the applied stress. The associated buckling patterns were found to propagate as solitary waves on a Möbius strip; this is a zero-energy cost process. Furthermore the Möbius strip's elasticity may support several conformations and posses a form of mechanical memory as the static deformations encode their stress history. According to Bartolo & Carpentier ([12]): " Nonorientable surfaces offer a paradigmatic example of static mechanical memory. Information is coded and stored by the temporal variations of the stress. Information is read measuring the shear angle and deleted releasing the applied stresses". Thus, for the elastic Möbius strip, the coding of *memory as stuff-stress* reduces to a *single* variable, the shear angle: As in Occam's Razor, the principle of parsimony is the case. At this point we characterize that while memory might be stufflike as it is embodied as nonorientable shape, most basically it is a function, very much alike consciousness is (Solms [260]). Microtubules have a notorious involvement in learning and memory (Woolf [285]). In this multistability of buckling patterns of elastic Möbius strips there is a common behaviour with regards to the hybrid organic aromatic molecules which are present in the neuron, which may also assume different shapes according to π-electron delocalization, leading to both orientable and nonorientable conformations, to be discussed below.

In the mathematical formalization of cognition in the wake of Shepard's Principle of Complementarity, Leighton noted that memory cannot be formalized only in terms of symmetries and the Erlangen Program. Indeed, in the investigation of the psychological relationship between shape and time, Leyton argued compellingly that shape is used by the mind to recover the past and as such it forms a basis for memory, for which Rapoport argued that torsion is the basis for any memorization (Leyton [147]). Leyton identified a twist on a plane as the primal shape embodiment of memory. However, what was left unmentioned by Leighton is that this primeval coding can further produce nonorientability, establishing thus a permanence of the memory produced by the twisted folding of a plane, as noticed in (Rapoport, [236]). In other words, the function of memory in producing shape is introduced by nonorientability, embodied as shape, though it might not be notorious, just like consciousness is not necessarily so. As for the role of memory in biological evolution and development, any development is governed by the tension between the persistence of the past, and an anticipation of the future as proposed in the Klein Bottle logophysics [225,229]. Similarly Stuart Kauffman proposed that in biology evolution in the present is somewhat conditioned by the future future as possibility [295]. So any development is governed by the tension between the persistence of the past, and an anticipation of the future. This points out to Heidegger's notion of time-space, time as a nonlinear shape operating as mediating past and future through the present [298]. Where this shape of time-space is most notorious is in the experience of music yet it also transpires in literature, the so called chronotopes of Bakhtin. One such chronotope is Dante's description of the trip to Hell in search of Beatrice, a Möbius strip journey (Bethea [314].)

To make further the case of topological folding and biology, they are the basis for the *symbolic meaning* establishment of living organisms. On the one hand they are crucial to the morphomechanics of the establishment of their bodyplans during development [232,233]. Furthermore they are crucial to the symbol-matter complementarity of biological systems [257,312]. Pattee identified the relevance of folding to the timing process crucial to metabolism [290]. Indeed, he described folding as operating to transform the passive symbolic gene expressions into the dynamic rate control of enzymes. Pattee: " Folding transforms what are essentially rate-independent syntactically coded sequences into rate-dependent functional controls. Protein folding is a highly parallel process with so many degrees of freedom that is difficult to model even on supercomputers". Consequently, Rapoport followed by identifying topological folding as the operation that turned the material world to the symbolic world of organisms, the very origin of life [235]. *As in the origin of life, the basis of consciousness is, in essence, what we claim to be the case*. Meaning would then have an origin, in the interplay of matter and symbols, both for life in the first place and for consciousness, later for evolution, conceived in terms of folding and palindromic sectors of genomes [233].

That this may apply to the elasticity of microtubules, actually to the transition of the awaken state to the (partially) unconscious state as under anaesthetics, would be another proof of the homology of the mathematical geometric-topological formalization of the physical world and that of the unconscious modeling by the mind be that of the kinematic of objects or the "motions" arising in music perception (Rapoport, [236]). This memory supports the mental modelization of the kinematics of regular and irregular objects, by which an homology of the modelization of dynamics of point systems on spacetime are interiorized as the mental modelization both in terms of torsion geometry. Already Shepard not only discovered that the unconscious kinematical modelization required torsion geometries but also that nonorientability came with it by terminating the twist with a topological identification, a memory stabilization as already discussed. Furthermore Shepard discovered that "mental" rotations were inherent to the cognition of music, as in the Tritone Paradox [251]. However he suggested that "mental" rotations had no real substrate in the brain, say the neuronal dynamics. However, the nonlinear dynamical systems modeling of motor activity, say of coupled oscillators, where behaviour is represented as the strange attractors of this dynamics, disproved Shepard's claim. Indeed, in the case of polyrhythmic motor performance (Jagacinski et.al. [92]; Turvey [273]) and the rotational dynamics of population of motor cells first discovered by Churchland (Lebedev et.al. [146]) their attractors are nonorientable. Topological phases, retrieved.

13. Anaesthetics, the persistence of consciousness, the analogics of the tubulin code and nonorientability in the topological chemistry paradigm and the multiconformation of aromatic molecules

Hameroff came to collaborate with Penrose from his work in anesthesiology. He based his theoretization on the evidence that upon administering anaesthetics to patients undergoing surgery, the evanescence of the consciousness present at the awaken state holds as anaesthesia blocks consciousness and memory while sparing non-conscious brain activities. Certainly most of the brain activity is beyond awareness (Solms [259-262], Panksepp [182], Solms and Panksepp [126]). However, the previous description of the anaesthetic state as a duality of awareness/unconscious-state is not the case. Particularly emotions are not entirely suppressed in the anaesthetic state, the most common being anxiety, panic, helplessness, etc (Gidron et.al. [65]). Dreams do not subside neither, being much alike dreams while sleeping, and their experience appears to be unrelated to the depth of the anaesthetic state (Scheinen et.al. [247]). Neither memory is inactive, uttered words and conversations can be registered and expressed after recovery (Moerman et.al. [167]). The feeling of what happens may also be the case, in particular in near-death-experiences (Lopez et.al. [150]). Since emotions are considered the primal form if not the very source for consciousness as selfawareness further associated to hetero-reference -as the mimicry elicited by the mirror neuron phenomenology (Rizzolatti et.al.[238]), the attribution of consciousness as emerging from the physical process of the reduction of a quantum state under experience, say a measurement, is left wanting. Indeed, it is hard to conceive this physicalist reductionism: It attributes to this reduction to a single eigenstate the extraordinary physical source for consciousness of an agent-independent measurement what in fact is the emergence of an ampler selfawareness which already

emotions elicit, fundamentally as interoceptive, *rather than exteroceptive as in the measurement process* discussed in physics. (Interoceptive homeostasic scanning regulation may involve measurements operating in a Klein Bottle cybernetics [234]). Furthermore, under certain conditions this interoceptive attention can be literally turned inside-òut, becoming a real world if not hyper-real, the focusing and control of attention playing a crucial role (Laughlin et.al. [140-142]). Hameroff acknowledges this problem and offers for possible explanation: "… abrupt transition from unconscious, or subconscious possibilities to consciousness, and causal selection of particular actions and perceptions, may be seen as "quantum state reduction," or "collapse of the wavefunction" (Hameroff & Penrose [76]). In other words, anaesthesia which elicits the "dark side of consciousness" -as Hameroff calls it- actually comes with the need of showing that its "darkness" is not to be confused with "blackout", at all. This is treating by "normalizing" the situation: both the dark side of consciousness and its non complete evanescence are all related to collapse of the wave function. We shall return to this below.

Remarkably all living molecules are susceptible to anaesthesia and plants are known to counter stress by producing anaesthetics. Actually plants are capable of memory and display different forms of intelligence (Trevawas [271]).This may be prompted by the alterations introduced by anaesthetics to the existing oscillatory modes in proteins which may tend to alter microtubule polymerization and function. The organising behaviour of macromolecular entities, such as tubulin, indicates the existence of a complex and very fast information processing activity. High -frequency neural oscillations have been associated with conscious states, while low-frequency activity has been associated with unconscious states (Cariani [30]). Coherence theories of anaesthesia suggest that general anaesthesia act by disrupting coherent neuronal activity in critical brain structures. They do by preventing the protein conformational changes which participate in the normal functioning of neurons [75,76] . In other words, anaesthesia blocks the multiconformational states of molecules, only the one in action is preserved. Thus it supresses at the level of the awaken state of awareness the supraduality operating these changes.

However, anaesthetics binding to tubulin in neurons disrupt the microtubules at their extreme or sidewise, which otherwise they are stable for a lifetime, while for the other protein structures they are susceptible to their intermittent assembling and disassembling (Karp [97]). The latter is naturally conceived as the breakdown of microtubules by buckling, a core instability to nonlinear elasticity theory.

Given the importance of organic chemistry to the cytoskeletal dynamics which is upholded as basic to the novel paradigm for the neurosciences proposed by Hameroff [76] in terms of aromatic arrangements crucial to anaesthetics, it is important to observe that not only the usual Hückel 2-torus-like aromatics can be the case but rather there is an alternative Möbius strip-like aromatics, also called Hückel- Möbius aromatics. In terms of molecular orbital theory these compounds have in common a monocyclic array of molecular orbitals in which there is an odd number of out-of-phase overlaps, the opposite pattern compared to the aromatic character of Hückel systems. The nodal plane of the orbitals, viewed as a ribbon, is a Möbius strip, rather than a 2-torus, hence the name (Rainer [198)]. While originally it was considered that Hückel- Möbius aromatics was unknown, in the last forty years starting with the synthesis of Möbius strip shaped molecules, a host of such molecules have appeared and furthermore the existence of multistable conformations transiting from nonorientability to orientability and the converse. The nonorientability provides extra stabilization and with this molecules are less reactive than those without this shape (see Janowicz [94]). The existence of multiple conformations as stereoisomers lays at the very basis of Topological Chemistry (Mezey [118]), and the synthesis of catenanes, rotaxanes, knotted molecules such as the Borromean ring, etc., of molecular motors in short, for which Möbius strips may be used to that effect (Breault, [305]). This paradigm led to the bestowal of the 2016 Nobel Prize in Chemistry to J.P. Sauvage, J. Fraser Stoddart and B. Feringa.This coincided with the 2016 bestowal of the Nobel Prize in Physics for the development of topological approach to condensed matter physics and exotic matter to D. Thouless, F. Duncan, M. Haldane and J. Michael Kosterlitz ([107])

We recall that hybridization is the concept used for explaining molecular shapes in the usual approach, of mixing atomic orbitals into new *hybrid orbitals* (with different energies, shapes, etc. than the component atomic orbitals) suitable for the pairing of electrons to form chemical bonds in valence bond theory. As already mentioned, molecules may appear as having multiple conformations, or a "split personality" in a colloquial dualistic description: on the one hand orientable conformations

may turn to be non-orientable, and the converse can also be the case (Rzepa [245]; Sokolov [258]; Stapien et.al. [263]). It is the hybrid orbitals which are considered to support the "split personality" whereby, say graphene, coexists as in the normal *and* the Möbius strip configurations . Graphene is considered to be a peculiar form of aromatic molecule (Popov et.al. [130]).This double configuration is more generally encountered in aromatics which started with Kekule's model of benzene, which is a case of Hückel-Möbius aromatics as well as Hückel aromatics, i.e. the cyclic ring 2-torus shaped planar molecules.

14. The topological chemistry paradigm and superposition in ORC OR theory

With respect to ORC OR the most basic example of multiple conformations and of the switching between them, the "split personality" is that of tubulin itself: it leads to the quantum superposition from which this theory departs from. Hameroff: "… since London forces are quantum mechanical, the electron pair may occupy both states, and the protein exist in a quantum superposition of both open and closed conformations. By going from bottom (quantum superposition) to top (one particular conformation) the protein functions as a qubit" [76].

So, tubulins may operate as two different conformations upon coupling with a pair of quantum coupled electrons in an internal hydrophobic pocket. Each tubulin may occupy two classical conformations (a "black"-open state and a "white"-closed state"), two classical bit states) or exist in quantum superposition of both conformational states (a qubit). The former two states correspond to Boolean states, the latter to a quantum superposition, typical of the Klein Bottle superposition as nonorientable topology, lies at the very basis of ORC OR. We shall discuss this below.

Early in 1973 Sokolov reviewing the nascent paradigm of topological chemistry showed that a modified form of the allosteric effect could be realised in molecules by an electron delocalization mediated by Van der Waals-London forces which could be rendered as a change of electron density shape with a migration of an electron as if a Klein Bottle outside/inside or inside/outside motion. Sokolov proposed that the allosteric coupling of molecules rather than a lock-and-key coupling it operates in terms of topological changes as discussed above. It is to be superseded by a dynamical shape accommodation (Tripathi and Bankaitis [272]. Cosic and Cosic further identified resonant electromagnetic energy transfer at specific frequency for the specific activation and interaction for macromolecules such as proteins, DNA and RNA as the crucial driving force for macromolecules (protein, DNA and RNA) activation and interaction (Cosic & Cosic [39]). We already discussed nonorientability supporting resonance, Hameroff recognized the importance of resonance as the main interaction with the zero-point fluctuations (Penrose & Hameroff [75,76,192]) and electron delocalization in the anaesthetic state and the phenomenon of consciousness (Hameroff et.al. [74]): He observed that anaesthetic gas molecules act in hydrophobic (lipid-like, water-excluding) regions within critical target proteins…by weak van der Waals London forces between the anaesthetic and nonpolar amino acid groups (the same type of endogenous interactions occurring between non-polar amino acid groups in the absence of anaesthetics)". Furthermore "anaesthetics retard electron mobility—the movement of free electrons in a corona… [B]y forming their own London force attractions in hydrophobic pockets, anaesthetics may inhibit electron mobility required for protein dynamics, quantum superposition and consciousness". In other words, anaesthetics control by retarding or impeding the electron delocalization that operate on aromatic molecules quintaessential to topological shaping and the possible orientable to nonorientable reconformations. Nonanaesthetics may be understood as occupying hydrophobic pockets without altering electron mobility, and convulsants as forming cooperative van der Waals interactions, which promote excessive electron mobility and protein dynamics in excitatory proteins.

We observed that biophotons emission could be produced due to change of conformation by re-organisation of chemical bonding, which is known in biology as chemiluminescence [225,229]. Furthermore, chemiluminescence is recognized to be the basic process for syntropic – rather than entropic-organisation of organisms using stored energy of chemical machines, and that is the resonant energy communication of quantum entanglement in organisms (Ho [79,80]). McClare argued that this can produce using a chemical reaction a form for producing useful work. Namely a "molecular energy" mechanism, which depends upon the rapidity of resonant energy-transfer (Mc Clare [160])

In ORC OR it is this quantum superposition which while in superposition, tubulins communicate/ compute with entangled tubulins in the same microtubule, and in other MTs in the same neuron. Quantum states of tubulin/MTs are isolated/ protected from environmental decoherence by biological mechanisms which are related to liquid crystals which may have nonorientable configurations, such as phases of actin gelation, coherent water domains where the polar ordering produce a Möbius strip, and alike tubulin they assemble and disassemble by absorption and emission of light, respectively, coherent pumping and topological quantum error correction.

15. Consciousness states, warps, multiconformation molecules and anaesthetics, Klein Bottle supraduality and Eastern Philosophy

The lingering question is: if after all, whatever the kind of states of consciousness are, under deep anaesthesia or awaken, if in all cases the reduction to a single state is ascribed to the "source of consciousness" what is the source for the differences between them? Would we consider anaesthesia as a special case of induction of a blockage of the possibility of recognition of sensorial experience as if arising from a given exterior world, together with the blockage of transforming this into action due to the induced immovilization?

We are taken to contrast anaesthesia with the experience of sensory deprivation. Say, by immersion into a lightless soundless ambience (incubation, as were known the long periods of sojourns in dark caves (Kingsley [102]) where motion was allowed, the ingestion of hallucinogens, or still the submersion in a water tank with no further restrictions. What is reported in these situations is that rather consciousness receding, fading or becoming impoverished, exactly the opposite is eventuated. Their importance to the transformations of civilizations and human consciousness has been highlighted by several scholars [102,103].

These transitional situations are called warps, and what warps elicit is a supradual logophysics. For instance the hypnagogic warp from the waking phase to the dream phase of consciousness and the inverse *hypnopompic* warp from the dream phase to the waking phase. Paraphrasing Benny Shanon, a cognition psychologist who travelled across South America to experience entheogens for understanding the psyche, what is elicited in these experiences is the "antipodes of the mind" being unified [248]. They involve the autonomic nervous system (ANS) as it channels energy to the sensorium, and does so through two systems considered complimentary: the excitation system and the relaxation system (Gellhorn [57] Gellhorn and Kiely [58]). It is the combination of these systems and the evocation of symbols what elicits the complexity of awareness as a cycle of meaning. This latter cycle also operates in supradual terms (Rapoport [236]). With respect to anaesthetics (consciousness reduced) and hallucinogens (consciousness enhanced), they act in opposite ways (Hameroff et.al., [74]), which we shall discuss below.

 Warps are conceived as "discrete phases, a phase cognized strip of unfolding experience dominating for some time the sensorium, has a causal influence on the organisation of the phase succeeding it, and a causal influence on the end of the phase preceding it" (Laughlin et.al. [142]) . Thus, they are keenly related to memory, and they have an anticipative character, to the effect that perception has an apperceptive nature: Prediction is involved in the selfreferential interoceptive homeostasic regulation and related to a Bayesian inference process and the Markov blankets. We already discussed their Klein Bottle topology.

Laughlin et.al.: "The *awareness* of experience involves cognition about self and world. The root meaning of "awareness" is the same as "wary" and connotes careful attention to the world and detection of danger. Because the definitive characteristic of awareness is recollection, remembering, or recognition of patterns in experience, awareness obviously refers in part to a role played by knowledge in experience. Further, since the *recursive* [my emphasis] quality of segments of experience may be cognized as recurrent, awareness itself is organized into different "states" of consciousness" (p.109; ibid.). So awareness is related to memory, and the selfreferential re-enactment of its "contents". Thus if nonorientability would arise associated to the neuron cytoskeleton, then memory as an inherent behaviour of these material structures would be embodied as nonorientability of the cytoskeleton dynamical tensegrity structure. Upon collapse of such microtubules the Möbius strip shaped soliton arising as the pattern produced by a nonorientable microtubule buckling would be the

memory in a zero-energy process [12]. Thus, through another warp it may in turn be evoked and in this conception "a minimal reentrainment across warps is all that is required for integration of phases into some semblance of a cognizable continuity" (Laughlin et.al., p.121 [142]). The experiences reified by warps do not "lie" in another branch of the universe as Penrose & Hameroff put it [75,76,192]. McGilchrist qualifies the latter invocation as the dominance of the either/or dual logic action proper to the analytical function of the brain left hemisphere over the intuitive direct cognition holistic integration by the right hemisphere [312].

From the topological chemistry paradigm. taking in account the already discussed Van der Waals-London forces operating the multistable topological shaping the research on both the reduced-consciousness anaesthetic states and the enhanced-consciousness-psychedelic states, say Ayahuasca, LSD, DMT (Dimethyltryptamine), we can phrase it like Hameroff et.al. which spell it out in terms of dependences rather than associations [74]. The different states of consciousness as elicited both by the anaesthetic and psychedelic states appear to be related "to quantum processes in hydrophobic pockets. Furthermore these quantum processes are inhibited by anaesthetics which impair electron mobility in van der Waals London forces", i.e. related to the metamorphical condition of aromatic multiconformations. Further: "The same processes are enhanced (hallucinations, but also enlightenment) in the presence of psychedelic drugs, those with indole rings donating electron resonance energy to indole rings in tryptophan within the hydrophobic pocket, forming a collective quantum state. Consciousness depends on quantum states of electrons within hydrophobic pockets in a class of brain proteins". In other words, as already stated anaesthetics block the quantum processes and computations -which ultimately depend on the Klein Bottle-Hadamard operator- by constraining the electron delocalization and thus multiconformation metamorphical topological transformation. In distinction with the latter, the altered-enhanced-consciousness states induced by psychedelic conformations or meditative techniques appear to enhance electron delocalization towards the formation of coherent collective states. Thus they induce a metamorphical supradual logophysics of multiconformation topological transformations proper to the two versions of aromatic molecules. One of these enhanced yet seemingly rare state is called in the Eastern traditions by different names: Nirvana, Satori, Turiya, perhaps others we are not informed of.

Notably, the shamanistic and Shinto roots of Japanese culture, and the later inception of Zen Buddhism have known and continue to practice meditational states. They allow for the first-person account of consciousness which is mostly decried by science as falling into the figments of our imagination, a practice which anyway has been and continues to be the hallmark of the curiosity and enthusiasm of the human condition. They procure to achieve the state of "supreme unification", which has the supradual logophysics at its core. Hagiwara: "the work of the Japanese Zen master Hakuin [Hakuin Ekaku (白隠 慧鶴 1685-1768)] utilized the idea of the Möbius strip (and the similarly paradoxical Klein bottle) in his expression of satori (nirvana). …a crucial visual expression of both Hakuin's satori and the Buddhist Dharma (Law)" (Hagiwara [71]). In the Indian *Advaita Vedânta* philosophy there is a fourth state of consciousness other than the three states of awakeness, dream-filled sleep and deep sleep: beyond deep sleep, Turiya. It is discussed in Verse 7 of the Mandukya Upanishad [178]:

"Not inwardly cognitive, nor outwardly cognitive, not both-wise cognitive, not a cognition-mass, not cognitive, not non-cognitive, unseen, with which there can be no dealing, ungraspable, having no distinctive mark, non-thinkable, that cannot be designated, the essence of assurance, of which is the state of being one with the Self the cessation of development, tranquil, benign, without a second, such they think is the fourth. He is the Self (Atman). He should be discerned" (Olivelle).

Hence, Turiya is transcendent and immanent, the metaform for cognition and perception, the metaform for both pattern formation and recognition, yet neither Outside-Outside nor Inside-Inside, neither Inside-Outside and Outside-Inside ("nor even both together"); beyond perception, invisible, ungraspable and without distinctions, and still supports all the other states of consciousness, the Klein Bottle.

To resume this section: prompting consciousness as related to microtubule dynamics primarily elicited through anaesthetics has brought us to examine states of consciousness other than the awaken

state. They appear to indicate a supradual logophysics where memory is highlighted as a primary function. This will be discussed in terms of microtubule dynamics and buckling patterns below.

17. The topological chemistry paradigm and the origin of life and meaning.

Returning to the supraduality that the multiple "split personality"conformations embody, this is relevant in regards to biosemiotics and the characterization of life as related to artifact-making, for which molecular motors -particularly the cytoskeletal protein motors- and DNA may play the role of paradigmatic examples. According to biosemiotician Barbieri this is setup in terms of the Inside/Outside categorical divide and still in the alledgedly linear character of the geometry of the biological digital codes, and its digital character. Barbieri: "…molecular biology has discovered that the production of genes and proteins requires not only catalysts but also templates. The catalysts join the subunits together by chemical bonds, and the templates provide the order in which the subunits are assembled. It is precisely that order that determines biological specificity, the most important characteristic of life, and that order comes from a molecule that is outside the assembled molecule. This is precisely the characteristic that divides spontaneous objects from artifacts. In spontaneous and in catalyzed processes, the order of the components comes from within the molecules, i.e., is determined by internal factors, whereas in genes and proteins it comes from without, from an external template" ([318], p.11) :

Actually it was biosemiotician Hoffmeyer who first suggested that life appeared as a lipid membrane surface produced an Inside and Outside in an asymmetric relation vis-à-vis an autocatalytic system of compounds, namely the elements of the cytoskeleton and water (see Rapoport [232]). Its protoconsciouness described as "the never-ending *interest* of the insides into their outsides or, in other words, cellular *aboutness* [the term introduced by philosopher Brentano, considered by phenomenological philosophy as a protoform of consciousness]...this "interest" should be understood as a property that ultimately was derived from the primordial membrane itself".

However, as we have just seen and it turned out to be the case of genomes, rather than linearity we have that Nature introduces memory through the torsion encoded as nonorientability and develops its genomic codification in palindromic sequences related to nonlinear non-orientable surfaces, and the digital character of the codifications is derived from the Klein Bottle logic [234]; we shall later discuss the possibility of reconstructing the continuous signal by adequate data sampling of a discrete series. Furthermore, what chemical topology has shown is that these molecular motors can be synthetized from the application of Möbius-like twists that *anticipate* life, so that Barbieri's account appears to be amenable to be understood in terms of the non-orientable surfaces of self-reference, rather than requiring an elaborate distinction between life and non-life as discussed in (Rapoport, [230]): The dualistic ontology cannot actually integrate what it fractured, ab initio. We have already discussed the semiotic sign-processes origin of life and memory, as well as the interpretation of codes in terms of topological nonorientable folding. If life is identified as artifact-making, clearly chemical topology shows that this is unprecise –or still, ontologically mistaken- with regards to the actual non-dual ontology and logophysics that operates as an *ontopoiesis*, *the supradual logos as the source of creativity (*Rapoport, [230]). The semiotic agency that develops as complex structures required for life exist in the chemical domain in terms of *non-orientability* while in biology the multisystem integration of an organism requires an overall control which penetrates in the particular subsystems and selforganises Klein Bottle wise (Neumann [173] ; Rapoport [234]). Chemical topology dispels the claim of a divide between the "analogue world of chemistry and the digital world of life." Still, instead of envisaging the Interior/Exterior duality, the interplay of templates, genes, proteins and catalysts is to be understood in terms of a HyperKlein Bottle cybernetics [230 -234,236].

Examining the development of structure and tissue differentiation in cells, it was found to be mediated by an active cytoskeleton propagating alike light ways in terms of the Fermat Principle (Gordon [13]). We proposed that the cytoskeleton as a tensegrity has a physical support on optical vortical electromagnetic or sound waves network operating as emmiting and absorbing antennas, with nonorientable polarizations interacting with the coherent liquid crystal nonorientable topology of the intracellular coherent water domains (Rapoport [225,229]). Remarkably, Hameroff has proposed that acupuncture practiced in Traditional Chinese Medicine may be related to holography having microtubules operating as tuned resonators carrying electromagnetic pulses which by either stimulating

or relaxation with a needle applied to the upper epidermis induce membrane ion currents to the effect of reaching a state of homeostasis . (He further suggested that Chi, the Chinese energy field would be coherent photic energy from the Sun and stars which is refracted by the upper epidermis (stratum corneum) further resonating with the microtubules network). We shall further discuss the notion of microtubules as electronic resonator devices and antennas.

18. Cytoskeletal dynamics, morpho-logic, topological phases and nonorientabiity

A coherent network is not exclusive to the brain-mind, genomes with their dynamical nonorientable dynamics resonating with electromagnetic and sound vortices with nonorientable phases or amplitudes, respectively, as if a coordinated orchestra of resonant topological phases. This tensegrity is the basis for a supradual logophysics rather than a dual-logophysics (Rapoport [225,229]). In cellular cytoplasm tensegrity structures are basically produced in terms of geodesic paths which continuously absorb-produce tension, so they are considered in terms of elastodynamics (Gordon [132]), though electromagnetic fields with eikonal light rays propagation can also be a natural interpretation; they develop topological anholonomies (Bliokh et.al. [21]). These differentiation waves support tissue differentiation and meiosis [225,229]. Since the wavefront propagation of gravitational and electromagnetic waves (light rays under the eikonal condition) in matter coincide as proved by Fock in his theory of gravitation [53] , and the latter coincide with the wavefront sets of elastodynamics (Bóna and Slawinski [23]), as wavefronts define the main morphological features, the actual physical content is, in general principles, secondary. *In principle, either the gravitational, electromagnetic and elastodynamical interpretations are undiscernible morphological-wise.* Actually, whatever the interpretation of the physical nature of the waves might be, as long as is consistent with the stuff of the object , environmental interrelations and the morphologies, they are methodologically valid. Certainly, it is hard to conceive the empirical possibility of discerning which is the case.

However, recent experimental settings have allowed to show the filaments deep inside the neural branches check the potential difference across the junctions by sending and receiving electromagnetic signals (Ghosh et.al. [64]; Agrawal et.al. [133], Singh et.al. [254-256]). Filaments and four ordered structures inside a neuron fire a thousand times faster than the ionic current. Filamentary communication is less known to the ionic signal transmission whereas the two are intimately linked through time domains. Novel scanning methods were introduced by the Hindu group, dielectric resonance microscopy (DRM), to operate in two-time domains, milliseconds and microseconds simultaneously, for the first time. It was found that four ordered structures in the cytoskeletal filaments exchange energy ~250 μs before a neuron fires, editing spike-time-gap—key to the brain's cognition. Also the control of the neuron process was achieved, either by deterring from firing above a threshold or initiating a fire by wirelessly pumping electromagnetic signals. The experimental discoveries "support a new dielectric *resonator* model of neuron functioning in various time domains, thus suggesting the dynamic anatomy of electrical activity as information-rich!". Furthermore, a fundamental aspect that has been elicited by the resonator frequencies is that the cytoskeleton operates as a fractal architecture of triplets conformed by triplets supporting quantum topological phases. While tubulin and microtubules are effective insulators with low conductance upon applying to them an alternating electric current at specific "resonant" frequencies, their conductance become extremely high and microtubules excellent conductors. Excitation at these resonant frequencies causes microtubules to assemble extremely rapidly; Fröhlich-type condensation is invoked to describe them [56,151]. Remarkably the same superconductive-like behaviour is the case of Möbius strip electronic resonator devices and antennas to be discussed below.These resonant frequencies show selfsimilar conductance patterns repeating every 3 or so orders of magnitude, over 15 orders of magnitude in the brain, from the quantum world to the electro-encephalogram ("EEG"). From terahertz through gigahertz, megahertz, kilohertz and hertz, conductance at each of these frequencies showed self-similar "triplet-of-triplets" resonance patterns

To resume, there is more to the neurons that ion transport: Indeed, that there is a subcellular cytoskeletal phenomenology to neurons which is subjacent to the process of polarization of neurons and their ensuing depolarization. This cytoskeletal behaviour is continuous, wavelike and thus neurons are conceived as continuous dynamical systems, described by nonlinear differential equations, for which

their stability and the asymptotic state known as attractors are crucial. However, rather than perception being continuous or discrete, exclusive or which is the case of dual Boolean logic, the Klein Bottle logophysics integrates both. We have discussed this already in terms of the warps transitions of awareness states. Notably in morphological terms the abrupt nondifferentiable form of a spike as a wave is a point singularity, i.e. formless, which according to the current paradigm further propagates on the neuron membrane. In other words, since this signal is formless, it cannot generate in-formation as morphogenesis. It stands for the "it out of bit" digital dual-logophysics proposed by Wheeler (see Rapoport [234,236], Floridi [52]). In the terms of semiotics for which signs are architectonical organisation of packets of energy (Taborsky [267]) the singular dotlike packet of the spike as a morphless architecture cannot elicit any meaning at all, it bears no in-formation, no induction of morphology! Actually the identification of topological folding together with a membrane as precursor of life was in terms of folding being a semiotical meaning-conveyor sign as expounded by Hoffmeyer. As the neuronal action potentials are the primal signs of the brain-mind, be that in its preponderantly unconscious operation and its selfawareness mode, the process of meaning assignment cannot be generated nor sustained by a pointlike formless signal. The attribution of a non-differentiable Dirac-delta like ion-based spike has been brought into empirical examination, departing from the notion that there is much more to the neuronal behaviour the ion currents which is based in the continuous so-called subthreshold phenomenology.

19. Nonorientability and the discrete optimal representation of continuous signals: The topographic maps of the sensorium as a discrete sampling reconstruction of a continuous signal

The dual-logic spike paradigm conceives the number of spikes as the discrete variable for a "neural coding" such that objective properties of the world are communicated to the brain in the form of spikes. But the pointlike nature of spikes in this paradigm cannot support any meaning at all, as already discussed. Moreover, the details of the cytoskeletal phenomenology have been obliterated although they support the behaviour of the neuron. In contradistinction perceptual systems construe models introjecting from relations between sensory signals and actions, which are subjected to constraints which determine the outcome of the underlying physical and chemical laws, which have a nonorientable topology for signature. In the visual and somatosensory systems their topographic representation of periphery-to-cortex-to-periphery relations conform a Klein Bottle logophysics as elicited by Werner &Whitsel for the somatosensory system [119,281]. Neurons in the hippocampus represent the rat's position mapping (Mac Naughton et.al. [161,162]) producing a Möbius strip (Rapoport, [236]).

Brette: "Neural codes are inadequate for this purpose because they are unstructured and therefore unable to represent relations…The coding metaphor [following Shannon's statistical theory of communication wrongly deemed information-wise] tries to fit the dynamic, circular [selfreferential Klein Bottle-wise], and distributed causal structure of the brain into a linear chain of transformations between observables, but the two causal structures are incongruent" [26].

Indeed, nonlinearity is incongruent with discreteness, unless we represent the former as a discrete representation of a continuous signal whose maximal frequency, f_{max} is incorporated into a sample rate that permits a discrete sequence of *samples* to capture all the information from a continuous-time signal of finite bandwidth. This is guaranteed by the Nyquist–Shannon sampling theorem in the field of signal processing [157]. It serves as a fundamental bridge between continuous time signals and discrete-time signal processing. Indeed, if the continuous signal has a maximal frequency, say f_{max} a sufficient sample-rate processing is therefore anything larger than $2f_{max}$. Equivalently, for a given sample rate f_s perfect reconstruction is guaranteed possible for a bandlimit less than half the sample rate.

We can consider this duplication, the 2:1 harmonics, as a representation of a continuous signal performed on a non-orientable Möbius strip or a Klein Bottle with length $2f_{max}$, on which we take samples distributed on it that exceed its length. They need not be distributed homogeneously.

Whether the nature of perception is discrete or continuous is unsettled (van Rullen & Koch [137]). Brain function involves oscillations at various frequencies. This could imply that perception and cognition operate periodically, as a succession of cycles mirroring the underlying oscillations. In the case of EEGs we already saw that they operate as a system of 2:1 harmonics conceivable as overlayed

Möbius strips with cross-interaction. Also, the cytoskeleton appears to be organized as a fractal organisation with specific frequencies which appear to organize as chronomes, time structures (Rapoport [226]).

Contrary to earlier views of discrete perception as a unitary sampling rhythm, contemporary evidence points not to one but several rhythms of perception. Would we consider the discrete case, then it is in principle possible to construe a representation of it as if a continuous signal, optimally, by the previous prescription and the metaform of such construal is again, a Möbius surface, as if the continuous model would be the original signal.

This reconstitution by considering the sampled discrete signal as if disposed on a Möbius strip or a Klein Bottle -the 2:1 harmonic being also the case- leads us to consider the Klein Bottle topologies of the visual and somatosensory topographies as such general reconstruction process. In this process the Fourier transform plays the crucial role (see Pribram [125,196]; de Valois [306]) that already the Klein Bottle invariance of the phase of the cortical receptive fields shows that the Klein Bottle as metaform of pattern formation and pattern recognition would be preserved along the reconstruction process. In the contrary perception and cognition would be impossible.

20. Morphology of spikes as revealed by resonance and topological anholonomies

Returning to morphology as the natural analogical codification as in-formation, shape supporting meaning, spikes could not be pointlike. Such a singular signal to be transmitted along an axon, would not survive, it would dissipate instantaneously (Ghosh et.al. [64]). An actual form is required to this effect, "the spike should have the appearance of a ring, encompassing the diameter of a cylindrical axon or dendron…" (Ghosh et.al.., [64])). In other words, the form of the sign should somewhat be accommodated by the architecture which supports the sign as its transmission, in this case a cylinder, the form of the axon or dendrite, as an electrical circuit the connecting "wires" of the neuronal architecture. The suggested form actually observed is a ring, moving pairwise: two confronting rings; see fig.1 in [64]. Furthermore, they are associated to a 3D electric field, which in homogeneous media appear as Möbius strip polarizations transversal to the propagation of the field.

Bandyopadhyay and collaborators used nanotechnology at ambient temperatures to measure quantum resonance conduction in microtubules as a function of applied alternating current frequency. They found self-similar conductance patterns at specific frequencies every few orders of magnitude, i.e., in terahertz, gigahertz, megahertz, and kilohertz, equivalent to the 10^{-4} seconds. These results also were claimed by Hameroff & Penrose to confirm the ORC OR theory.

Action potentials and the cytoskeleton, protein and microtubule morphogenesis of the living brain appear to selforganise as topologies associated to Berry phases related to resonances of vortical dynamics (Bandyopadhyay [14]). They pertain to an intricate cellular tensegrity architecture which responds to alternate currents resonating in ranges which reach terahertz values. This confirmed the hypothesis that neurons are resonators in which the complex architecture of the subcellular level resonate to a now identified spectra of frequencies. Each resonant frequency produces a phase shift: "while passing through a microtubule nanowire, the ac signal shifts its phase by $n\pi=4$, (n = 0,1,2 . . .). The sum of phase shifts for a set of resonance frequencies forming a group is 360°, so completing a phase cycle is not incidental" (Ghosh et.al., [64]). Indeed, 360° is the topological (an)holonomy of the Möbius strip, as already shown. Rather than action potentials being morphless, points, the Hindu group was able to identify them as pairs of luminous annular shapes, two electromagnetic optical vortices sitting above the terminating dendrites. The nonorientability suggested by these empirical findings of luminous structures point to characterize each of them as a "solid" Möbius strip, the pinched-torus Klein Bottle. Remarkably it appears in the modelization of visualization by the moving subject while watching a distant moving point, the so called parallax vision, to be presented elsewhere by the present author (see **https://commons.wikimedia.org/wiki/File:Pinched_Torus_Klein_bottle.jpg)**.

21. Classical-quantum resonator devices as metaphors of the neuron cytoskeletal architecture

We return to the discussion of nonorientability supporting boson-fermion systems. This is generic as can be seen in 1d and 2d electronic and microwave devices, a special class of transmission lines of finite

extent with well defined boundary conditions such as coupled resonators systems embedded on a Möbius strip. As a proof of concept, on producing such a nonorientable embedding of a linear system turns to behave like an infinite yet compact transmission line with remarkable superconductive-like behaviour. These systems are low-pass filters with remarkable superconductive properties. They have been used to design antennas, magnetic resonance and nanodevices, since they allow for efficient compact packing producing an infinite transmission line with half the resonance frequency of the original untorsioned line (Pond [194,195], Rodhe et.al. [239]). They enable, in principle, a Tesla Coil for global wireless transmission of electricity and high-temperature superconductors.They give rise to boson-fermion symmetries, where the non-orientable 4π symmetry are formal patterns of annular resonators with the 2π symmetries (Ballon [11], Hamilton et.al. [77]): Spinor fields arising with 2d devices.

Together with the findings of the Hindu group, this suggests that the cytoskeletal neuron architecture, conceptually at least, can be conceived as a recursive architecture of resonators which may functionally operate alternating in the nonorientable and annular ring forms: This is an alternative to Hameroff & Penrose ORC OR theory to be discussed below. A resonator fractal architecture turned indeed to be the case though nonorientability was not mentioned at all [14], nor by them.

This differs with the quantum reductionism of Hameroff & Penrose who try to anchor consciousness in a rather remarkable superposition, on the one hand the microtubule structures of the cytoskeletal neuronal structures -which we extensively showed to be anchored in a supradual logophysics- and on the other an unknown "quantum gravitation", which these authors invoke as a metaphor for a model which we describe below. They further invoke the neuron dual fire/non-fire paradigm while they focus their attention on the subthreshold cytoskeleton continuous phenomenology of integrate-and-fire neurons where the resonator character of neurons is central and related to a supradual principle. They invoke the findings by the Hindu group on the multifrequency resonant behaviour of the elements of the neuron cytoskeleton as supporting evidence of their ORC OR theory. Remarkably they refrain from mentioning the vortical structures and the topological phases arising in physics and revealed by this group in their investigations of the action potentials arising from the subthreshold cytoskeleton phenomenology. They chose to posit the appearance of consciousness as arising from the decoherence of the purported quantum mechanical phenomenology which they refer to massive particles which somehow alter the coherence of the quantum fluctuations to produce the actual world of experience, one of the many branched world of possible states, consciousness the emergent agency. For them the collapse of the wave function is an agency-independent physical phenomenon, thus the "objective reduction" (OR) qualification, which further "provides the bridge between quantum and classical physics". They depart from extending the Schroedinger equation modified by random terms associated to quantum fluctuations, which somehow are resolved into decoherence by the microtubules.

That quantum physics is indeed the case is evidently associated to the electromagnetic character of the action potentials. We recall that Dirac's equation of relativistic quantum mechanics, both the linear and nonlinear forms, for a Dirac-Hestenes spinor field, and Maxwell equation -as a single equation- for the electromagnetic field, as written in a Clifford bundle, are equivalent, under the proviso of a rotation on the 2d spin-plane [207,208,217]. The latter is not necessarily a 2d-plane, but in principle can be a nonorientable Möbius strip as arises in optical vortices with nonorientable polarizations (Bauer et.al., [17], Bliokh et.al. [21], Freund [54]).

22. Supradual Klein Bottle logophysics, quantum cognition and Matrix Logic

We already discussed the nondual analogic of the tubulin code.The topological phases emerging from the resonant subthreshold phenomenology appears as a supradual logophysics related to the elementary brain phenomenon, which also is the case of cognition and the unconscious modeling by the brain-mind of object-kinematics and the perception of music. Action potentials rather than morphless nondifferentiable spikes are pairs of electromagnetic annular-like optical vortices related to topological phases.

We propose otherwise, incorporating the supradual logophysics of the subthreshold phenomenology, which is supported by topological phases *and* the quantum-mechanical-likeness of the mind, in the sense considered by Quantum Cognition, to be discussed below.

Neuropsychologist cum anthropologist Jacobo Grinberg-Zylberbaum proved that macroscopic entanglement of minds is the case particularly when strong emotions are involved, as elicited in pair of lovers undergoing meditation [68,69]. So the quantum-likeness of the brain is not to be looked upon as exclusively brainwise.

This leads us to topological entanglement, the Klein Bottle. The non-Boolean logic of the mind, operating inferentially, deductively and memorywise -the inversion of operators for backtracking cognitive state- is indeed noncommutative: the Matrix Logic based on the Klein Bottle, after August Stern [265]. In the former, the mind operators as 2x2 matrices operating on cognitive bra-ket states as vectors on a plane. These operators may be invertible and in distinction with the standard physical quantum operators they may be non-hermitean, so real eigenvalues proper of measurement of physical processes are superseded by complex values. They can be written as quantum field creation and annihilation operators, and some are nilpotent, non trivial square roots of the null operator. Particularly they arise as quaternionic light rays giving rise to torsion fields with 3d polarizations represented as Penrose's twistors (Rapoport, [223,220,226]).

The most fundamental noncommutativity in this logic is that TRUE and FALSE operators are noncommutative. This introduces a torsion given by the commutator, which we shall denote as M,

$$[\text{TRUE},\text{FALSE}] = \text{TRUE} \cdot \text{FALSE} - \text{FALSE} \cdot \text{TRUE} = \text{TRUE} - \text{FALSE} = M.$$

The dots in the first identity stand for matrix multiplication. M is the fundamental operator of Matrix Logic. Nonlinear multiplication is turned to additive structure in the third expression. M is nilpotent, $M^2 = 0$. This commutator, i.e. $M = \text{TRUE} - \text{FALSE}$, is the fifth side of the pentagon given by TRUE and FALSE for the other sides of a dislocated parallelogram, alike u and v in Fig.2. Here TRUE $= \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, FALSE $= \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$. Both TRUE and FALSE are idempotent, i.e. $\text{TRUE}^2 = \text{TRUE}$ and $\text{FALSE}^2 = \text{FALSE}$, and so they stand for projection operators.

In addition to the previous antisymmetric form, we have a symmetric form, which together with M becom the two fundamental cognitive operators of Matrix Logic. It is

$$\{\text{TRUE, FALSE}\} = \text{TRUE} \cdot \text{FALSE} + \text{FALSE} \cdot \text{TRUE} = \text{FALSE} + \text{TRUE}.$$

In M we find the two possible orientations, plus 1 and minus 1, *superposed* as either the Möbius strip or Klein Bottle. Matrix Logic is isomorphic to 2-state Quantum Mechanics. Alike Lie groups as torsion manifolds where the commutator of the elements of the Lie algebra yields the structure coefficients, the torsion tensor, the torsion of cognitive space appears as a non-null commutator. The five-fold symmetry of cognitive space. The importance of this lack of commutativity cannot be overstressed: it introduces the notion of a cognitive operator associated to this topological superposition -vis. nonorientability, a logical derivative and topological charge associated to cognitive loops. This is the domain of the mind, for which knots and their tying and untying are fundamental. Rather than this being separated from the geometry of the mind, which is torsion, they are united. We recall that the brain is qualified as a geometric "engine", referring to the visual system and its ecological integration. However as visual cognition working through projectivity, as introduced by the Renaissance Masters, is integrated to the mind by antipodal *topological* identification which produces a Möbius strip, from which selfhood arises unconsciously [236]. This is also the case of the hippocampal spatial mapping of the environment by rats, as elicited by Moser & Moser with the Norwegian University of Science and Technology neuroscience group (Mc Naughton et.al. [161,162]). It also is the case of the unconscious modeling of the kinematics of objects, or still the cognition of music, as elicited by the Principle of Complementarity of cognitive psychology. Remarkably, in the quantum reductionism that aims to anchor consciousness as emerging from Quantum Mechanics, the topological nature of the mind's operations and its relation to torsion are unconsidered altogether. Yet as introduced in this article, it is

in the nonorientable and the torsion primal anholonomies where this logophysical fusion of the mind and the physical -in the ample sense- domains operate in the construal of experience. This further extends to the seemingly abstract topological phases, which are no less "real" than the case of physical space or spacetime, say the Aharonov-Bohm phenomenon, which actually is a trace-torsion differential 1-form. Topological charges is the case of the hypercolumnar architecture of the primal visual cortex and its Klein Bottle topology. It is also the case of condensed matter physics and particularly liquid crystals, the matter of which the brain is made of, and genomes too where nonorientability is eminent in its palindromic sectors (Rapoport [233]).

23. The unbroken information phase as nonorientable topology, Matrix Logic and Quantum Mechanics, the nonorientable topological form of coherence/decoherence cycle and holography

In Matrix Logic information is considered as something physically real, no different that space and time, with thinking as its manifestation [265,220,230-236]. In classical physics we resort to classical probability and real numbers. In quantum physics complex numbers are required to describe probability amplitudes and phases, real and imaginary components respectively. But the mind-brain is neither quantum nor classical, but a distinct phase, its "matter" is information, as form-inducing (which we called in-formation) and as data which anyway have a form [31,32,231-234]: *the information phase*. This requires for its description the so called denktors, thinking states, which use both complex probabilities and amplitudes. However, as is the case of the topological paradigm of physics as appears in condensed and soft matter physics, Matrix Logic deals with topological charges, as already the pinwheel architecture of the primal visual system shows to be the case, i.e. the very periodic architecture of the visual cortex, whose topology is the Klein Bottle. The information phase appears as a transition from orientability to nonorientability, as already the unconscious modelling of the kinematics of objects or music cognition requires: a torsion on a plane further glued as the very basis for the appearance of memory as discussed before relative to the Complementary Principle of cognitive psychology. Or still, the appearance of memory of buckling patterns of nonlinear elasticity in microtubules. Upon performing the twist and further stabilizing by the topological identification of the sides of a priorly orientable surface the information phase is constituted. It is free of constraints as imposed by relativistic consideration. All points or events in the priorly bilateral surface become united independently of distance be that of space or time: a topological form of holism and holography as well. We already described though non-formally the relation between the Klein Bottle, i.e, the information phase on the one hand and classical logic as arises in classical physics, say with the validity of Newton's Third Law. We also indicated the primal role of the in-formation phase-Klein Bottle- in producing the classical dual logophysics, i.e. the phase of classical physics in terms of the principles of dual logic, and the further reconstitution of the former from the latter in a selfreferential cycle. This begs for the reformulation of the discussions on the classical world as the topological decoherence of the information phase.

In Matrix Logic logophysical operators, L, act on bra states $<q| = (\bar{q} \quad q)$ and kets $|q> = (\bar{q} \; q)^{tr}$, $L|q>$ with $\bar{q} + q = 1$. Thus, TRUE$|q> = |1>$, FALSE $|q> = |0>$, with $|0> = (1 \; 0)^{tr}$ and $|1> = (0 \; 1)^{tr}$, the true and false states. The scalar true and false values of dual Boolean logic correspond to the eigenvalues of these operators. They are orthogonal. Further consider the orthogonal quantum basis: $|S_+> = (1 \; 1)^{tr}$ and $|S_-> = (1 - 1)^{tr}$, so that $<0|1> = 0$, $<S_+|S_-> = 0$.

These two states can be thought of arising from a surface given by a closed oriented band projecting on the xy plane. Thus to each side of the surface we can associate its normal unit vectors, $(1 \; 0)$ and $(0 \; 1)$. Suppose that we now cut this surface and introduce a twist on the band and we glue it to get thus a Möbius surface. We recall that this is what in cognitive psychology led to identify the genesis of memory, up to the topological identification, as discussed before. Now the surface has lost its orientability and we can identify one side with the other, so that we can generate the superpositions $|S_+>$ and $S_->$. Remarkably, these states are related by a change of phase, namely a rotation of 90 degrees. What the twisting and loss of orientability produced, can be equivalently understood by the fact that TRUE and FALSE are no longer as in Boolean logic and the Aristotelian propositional frame. Indeed, they are no longer dual and what matters is their difference, which in the case of scalar truth values does

not exist. The other state also can be interpreted as a state that represents the fact that the states as represented by vectors, have components standing for truth and falsity values which are independent, so that the Aristotelian link that makes one the trivial reflexive value of the other one is no longer present: they each have a value of their own. The situation is radically different to Aristotelian-Boolean logic in that now (0 0) is another state, 'false and true'. This is the situation of 'Schroedinger's cat' and the Liar Paradox; they are topologically embodied. This state together with (1 1) , 'nor false nor true' state together with (0 1), true, and (1 0) false states we have a 4-state logic in which the logical connectives have been promoted to operators. We have thus returned to Nagarjuna's four-loci logic, the Hindu philosopher who for some reason -seemingly unclarified- called Aristotelian logic the 'student's logic' [299].

We have the expressions

$|0> = (|S_+> + |S_->)/2$ and $|1>= (|S_+> - |S_->)/2$, and

$|S_+> = |0> + |1>$, $|S_-> = |0> - |1>$.

In other words, the classical states are the equally weighted signed superposition states, which themselves are the equal weighted signed Boolean states. While the classical states are ½ weighted the superposition states are not, though we can introduce the normalization factor $1/\sqrt{2}$. Describing the unbroken topological phase by the Hadamard 2x2 matrix H= $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ we obtain:

$H|0> = |S_+>$ and $H|1> = ||S_->$,

thus transforming the classical states into superposed states. In the state of true and false coordinates, the action of H on $|0>$ rotated it by 45° , while the action of H on $|1>$ flipped it. Reversing the previous, we still operate with H but halved, so that

½ $H|S_+> = |0>$ and ½ $H|S_-> = |1>$,

thus transforming the superposed states into the classical states. It is possible still to make the system unitary by normalizing by dividing by $\sqrt{2}$. To resume, through topological transformation tantamount to introduce nonorientability we can transform the classical eigenstates basis to the quantum states basis thus completing a selfreferential cycle upon doing this. Both complete basis of eigenstates are functionally complete and mutually spanning. We also have the decomposition of $M = |S_+> < S_-|$.

We recall that the Hadamard operator is the gate that allows transcending classical Boolean computations. The fact that it is also the interface between the quantum and classical states indicates the supradual primality on the brain-mind. In Matrix Logic quantum (for spin-up and spin-down states) and cognitive identities can be related interchangeably, through a spin logophysical operator,σ, which is a symmetric term of M. The decomposition is M = TIME + σ, TIME being symmetric and unitary, and σ = σ_z the Pauli matrix. This link is a logophysical interface between the quantum and informational realms, in which topology, torsion, logic and the quantum world operate jointly. TIME is tantamount to the operator of phase conjugation, crucial to holography. Yet, we are no longer constrained to measurements appearing through Hermitean operators as their eigenvalues: Indeed, M is non-Hermitean. An important eigenvalue of the logophysical OR operator of the decision process, is the Golden Mean.

The exchange between the two domains can go in both ways, given that the Klein bottle has no independent Inside nor Outside, the quantum realm can be incorporated into the classical cognitive dynamics, while the logical elements can take part in the quantum evolution. Negative probabilities can be understood too in terms of non-orientability. This connection is what we may venture to be the support for the Principle of Complementarity in cognitive psychology, the ecological conception of perception as a system (J J Gibson [300], and to a nonorientable topological monism which we have elaborated in our works and related to Max Velmans' Reflexive Monism [301].

Pribram conceived that memory was 'imprinted' and processed holographically on the brain operating as a parallel computer [125], which we suggested operating in terms of Matrix Logic for which the matricial product is a form of relational connection with several error correcting operations [265]. In holography the brain operates as a phase conjugation logical gate (Mitchell [124]), which points to TIME for its realization. We have suggested that brain interhemispheric coordination and particularly visual binocularity operates as such TIME phase conjugacy (Rapoport, [222]). Brain holographics would operate producing an imaginary image of the quantum emissions, say photons emitted by the perceived object, which rather than being seen are participants of the process. The brain would use adaptive resonance of the input signals with the electromagnetic triplet of triplet already discussed, in its input path to generate the output image representation of the object and further project it as if Outside where we locate and experience the real object -even in hallucinatory states-, as proposed by Velman's Reflexive Monism [323].

As a final comment, the implicit and explicit orders introduced by Bohm are, in principle, incorporated into this linkage, but making this manifest requires elaboration. Also, the formulation of topological chemistry as related to cognition through the informational phase is a natural field to be pursued. With relations to knots as chemical configurations [118] it was examined by Stern [265]

24. On the superposition and interference of as quantum "macroscopic" systems. Penrose & Hameroff's theory revisited

Supraduality rather than duality is also the case of the "interference of minds" as superposition of states elicited in Quantum Cognition, which shows the "quantum-like" nature of the mind, as in decision processes, conceptualization, perceptual multistability, etc. (Aerts [2], Busemeyer [29], Khrennikov [100, 101]). Departing with a criticism of the physicalist reductionism for which Penrose & Hameroff have been mostly disapproved by neuroscientists, Khrennikov modeled the mind as if quantum-mechanical applying the so called Växjö interpretation of quantum mechanics, proposing a Hilbert space formalization of the mind in terms of "contextual probabilities". The Klein Bottle is contextual by nature. Khrennikov did not consider the neurological aspects in particular the supradual continuous subthreshold phenomenology of neurons which subtends the action potentials, nor the tubulin analogical code: The subthreshold phenomenology is indeed proper to quantum topological phases yet as the physical correlate of Quantum Cognition and the Klein Bottle logophysics of perception and cognition. That Boolean logic is not the case of the mind states already transpires in the ambiguity and uncertainty in our daily ponderings. It is the states of superposition which calls our attention, so that besides true and false states and operators, commutative in Boolean logic, a combination of both is also a mind-state as already discussed (Conte et.al., [37,38]). This is their take on the non-dual (viz. supradual) logic quantum-like structure of the mind understood that there is no quantum physical element committed to the mind but rather the interference and superposition of cognitive states. This runs counter the allegation that "Quantum behaviors are difficult to reconcile with our observed "classical" world. For example, we don't encounter superpositions in our perceived world, as the very act of measurement, or of conscious observation seems to cause superpositions to reduce to definite states - "collapse of the wave function." (Hameroff, p.7, [76]). These comments while appropriate to physical quantum behaviour in the atomic and particle scale, fails to acknowledge that in the mental world and its behavioural manifestations, say decision making, superposition, entanglement and disentanglement, or still gestaltic perception, quantum-like behaviour are the normal rather than unobservable or unencounterable events. They exist per se as if independent of actual microtubule behaviour though as claimed by Penrose & Hameroff they can be explained in those terms. In other terms, Penrose and Hameroff implicitly *assume the Cartesian Cut, ab initio*. Then naturally, in such a dualistic setting, it is hard, actually impossible to achieve surmounting the initial dualistic onto-epistemology. For the dual-logic logophysics, the mental domain is projected as an object to the presumably physical world of measurement free of selfreference [230], positivism as its implicit epistemology. According to McGilchrist this is the signature of the breaking of unity produced by the dominance of the left hemisphere over the right hemisphere of the brain [312], and very much responsible for the predicaments of Western culture after Descartes. For the latter see Rapoport [220,222,226,230,236]).

In other words, the 2-value logic is projected to a single value, all things other excluded., as logician and cybernetist Gotthard Günther claimed to be the inexorable case [302]. Günther criticized the nature of logic being related to propositional value, and considered it terms of relations displayed locationally, as appears to be the case of genomes in terms of the Klein Bottle logic [233]. Furthermore, in our previous discussion of the shape-based topological chemistry, or still in the plethora of physical phenomena where topological phases are omnipresent, say liquid crystals, acoustics, phonon modes propagating in lattices, shallow water waves and in scales of experience going down to nanomaterials, optics, they all manifest a supradual logophysics which is not beyond our pragmatics.

Remarkably, nonorientability and topological anholonomy as well as their bearing in the topological chemistry paradigm crucial to their unmentioned at all, as it arises in the superposition of molecular open and closed conformations of proteins, basic to their theory. Penrose & Hameroff do mention the relevance of π- electron delocalization in aromatics towards the formation of dipoles basic to ORC OR and further discussed them with regards to both consciousness blocking and enhancement. As already discussed, Hameroff in his study of computation in microtubules points to the central important of electron delocalizations at the very basis for hybrid conformations of molecules and the relevance to consciousness without mentioning their role in topological changes from nonorientability to orientability and the converse (Hameroff et.al., [74]), and this was carried out to the ORC OR theory entirely.

Most remarkably, Hameroff considered the 6-layered structure of cortices to suggest the so called giant pyramidal neurons which constitute layer V as the probable support for consciousness for being the location apex of the integration of Outside and Inside, the unity of action and perception which for the visual and somatosensory systems is embodied as a Klein Bottle, the latter as elicited by Werner & Whitsel [119,281] and proposed by Turvey as a Möbius strip unity [273] somewhat following the Pragmatist school of the 19th century after William James.

25. The integrated coherence of the brain-mind and the proposal of a non-orientable architecture associated to its synchronicity

Remarkably, in ORC OR theory, it is claimed that microtubules practice quantum computation in intervals in which they are isolated from the environment which they alternate with phases of classical interaction in which coherence is due to synchrony, the 40 Hz oscillation. It is this oscillation which is identified as providing the integration of the brain-mind, which has been identified as the neuronal function proper to consciousness (Buszáki [123])

This synchronization has been suggested to be based on an hexagonal lattice of somatosensory and visual primal cortical areas in which the hypercolumnar structure is unmanifested. In this model Möbius strip like synaptic patterns support the coherence across the cortex maximising synchronous oscillation (Wright et.al. [283, 284]). This lattice would arise by selforganisation during embryogenesis and development of neurons, consistently with the Free Energy Principle (Friston [55],[317]) and operating as a heterarchy of Markov Blankets -Klein Bottles (Rapoport, [234,236] optimizing adaptive learning in an analogical transition from vagueness to crispness of percepts. The Möbius strip would arise as the antenatal organisation of synaptic connection while the receptive field lattice architecture would be the later post-natal development of learned functional connections overlayed on the antenatal connection system. The authors suggest that this organisation could be general across the cortex and the possibility that the underlying principles could extend to the mutual self-organisation of synapses and interactions between cortical areas. Furthermore, it suggests a modular functional synaptic system, which, only in some parts of the cortex, and in some species, is manifest as anatomical ordering into columns. In other words, it would be more basic to the observed periodic pinwheel architecture supporting topological anholonomy, which would appear only locally across the cortex and for some species.

26. Further analysis of Penrose & Hameroff ORC OR theory

Upon Penrose & Hameroff presenting their ORC OR theory following the early findings of the Hindu group they claimed that they provide confirmation of their take that fundamental resonances that arise

in microtubules such as anticipated by them [75,76,192]. This may be the case with regards to the microtubule dynamics of neurons, but to include this with regards to the theory of "quantum gravity" in completion of Quantum Mechanics due to the superposition of protein states in microtubules, might turn to be a step too far, which the authors seem to refrain from stating it explicitly. However, they also refrain from mentioning the vortical structures and the topological phases revealed by this group in their investigations of the action potentials arising from the subthreshold cytoskeleton phenomenology. They chose to posit the appearance of consciousness as arising from the decoherence of the purported quantum mechanical phenomenology which they refer to massive particles which somehow alter the coherence of the quantum fluctuations to produce the actual world of experience, consciousness the mediating agency. For them the collapse of the wave function is an agency-independent physical phenomenon, thus the "objective reduction" (OR) qualification, which further "provides the bridge between quantum and classical physics". They depart from extending the Schroedinger equation modified by random terms associated to quantum fluctuations, which somehow are resolved into decoherence by the microtubules.

Rather than consciousness producing the collapse of the quantum coherence of the superposition of protein multiconformations – which they attribute to a misinterpretation of their theory- they chose to posit the appearance of consciousness as arising from the said collapse. The latter, in their theory, is an agency independent physical process which produces the world of experience, thus the "objective reduction" (OR), a materialism of sorts. It is further produced by a superposition of curvature-gravitational fields which each lump is one of the conformations, which we shall discuss below.

As already discussed in the Klein Bottle logophysics topological superposition is the case of physics, chemistry, cognition, perception, semiosis, etc. Penrose conceives superposition differently. In his theory the superposition is a spacetime geometry property, entirely unrelated to torsion and nonorientability, nor the topological phases of the neuron's cytoskeleton but rather to the curved space of General Relativity. Hameroff: "…superpositions as tiny separated curvatures in spacetime geometry. Were such separations to continue… each spacetime curvature would evolve its own universe…" (p.2,[76]), Penrose deduced that spacetime separations would be unstable, and undergo objective reductions (OR) independent of a subject and based on the quantum uncertainty principle. In their theory, OR would stand for an extension of standard quantum mechanics, as the bridge between quantum- and classical-level physics as a 'quantum-gravitational' phenomenon. Whatever this bridge may originate from, be that environmental decoherence, the participation of an observer through a measurement of observation of any kind, multibranched universe from which a particular one arises from a "choice" or any interpretation of how the classical world of one actual alternative may be taken to arise out of fundamentally quantum-superposed ingredients". So what "quantum gravity" means for this theory, is in principle, as recognized by the authors, any theory in which the standard theory, viz. Schroedinger unitary evolution of quantum states in isolation, is superseded by a theory which considers the environment interacting with the system as a single system. This has been formalized in several proposals as the Dynamic Reduction Program or Collapse Theories (Diósi [44], Ghirardi, [62,63] , Pearle [186], Percival [188], Hughston [309], Adler and Horwitz [310], Rapoport [218]).

The metric-geometric proposal of ORC OR somewhat neglects the possibility of topological superposition embodied as nonorientability, and their omnipresent manifestation, say in liquid crystals pervasive to organisms and the brain itself. which does not presume neither the quantum nor the classical domains as primal. The Klein Bottle embodies and controls the cycle of transition from coherence to decoherence and its reconstitution, as a transition from supraduality to duality and the converse.

Furthermore, rather than consciousness producing the OR, Penrose conceives consciousness ("or an equivalent of it") as emerging from the OR. The OR process is considered to occur when quantum superpositions between slightly differing space-times take place, differing from one another by an integrated space-time measure which compares with the fundamental and extremely tiny Planck (4-volume) scale of space-time geometry. Since this is a 4-volume Planck measure, involving both time and space, we find that the time measure would be particularly tiny when the space-difference measure is relatively large (as with Schrödinger's cat)

To this effect Penrose conceived the selfenergy of the superposition acting as a threshold which upon trespassing it, specific curvature values are selected and the collapse to a single world is

eventuated. This threshold would be connected to the gravitational constant, and though smaller than other constants arising from chemical or other processes, would be of crucial importance for the OR reduction to proceed, in a time range of 20 orders or magnitude smaller than usual quantum processes. Neuronal material would be involved in this reduction in order to produce the infinitesimal displacement and modification of curvature to allow the time threshold to operate in a process in which microtubules play a crucial role. However, rather than the environment playing a role for the reduction, this would be spontaneous and operate independently of the environment and in the cases of major masses, still a quantum entanglement of the environment and the quantum system would operate. Any reduction of either the environment or the system would be accompanied mutually by the other. This mutuality would introduce an uncontrollable stochastic element into the result of the reduction. Consequently any non-random element (which according to Orch OR is non-computable) which may influence the particular choice of state produced from the superposition would be completely masked by this randomness.

Further, according to Penrose a quantum-like unit of phenomenal experience, a "quale" occurs, as a "protoconscious" as a first step for the assessment or constitution of causality. To explain the reduction which Penrose & Hameroff to be alike a musical morphology of several chords and instruments -an "orchestra"- proposed that "microtubules inside brain neurons *orchestrate* quantum vibrational superpositions through resonance, entanglement and memory, guiding wavefunction evolution to threshold for "orchestrated OR" ("Orch OR"), events proposed to result in moments of full, rich conscious experience (more like music than random notes and tones). Sequences of such moments would give rise to our "stream of consciousness"" (Hameroff, p.3 [76])

Let us analyse the above. In their take consciousness rather than primal is somewhat derivative and emergent from a spacetime geometry related to the theory of general relativity (GR). Furthermore, on priming curvature rather than torsion from which in the theory of Cartan connections, i.e. nonnull torsion, they ignored that the covariant differential of torsion yields the curvature: the so-called Bianchi identities of the theory of connections, in differential geometry (Cartan [33] , Hehl [83], Sharpe [111], Sternberg [264] Rapoport and Sternberg [200]). This neglect is quite remarkable since these identities stand for the constitutive relational aspects of the geometries. Their theory does not consider that Minkowski or Euclidean space, with their flat null-curvature metric, can be the case with non-null torsion and still the curvature can be alternatively null or nontrivial, so their curvature is entirely a field which is derivative of a metric. There are several conceptual issues too. As much as our theoretizations are linked to the image-schemas we apply and the logic we chose, concepts and their consistence is a must. Given a metric, say in GR, the geometry is nonrelational -one object distant to the other but no field proper to the interaction. The metric geometry of GR is nonparticipative, and the principles of selfreference and hetero-reference- the two core principles of consciousness - are not its basis, torsion is null and thus nonorientability is not the case. Finally, this notion of spacetime is that of a container, for physical masses, which is the dual-logophysics purported by Descartes and Newton. But consciousness which for ORC OR is to emerge from this passive spacetime is not subjected to CONTAIN, as if a classical though relativistic mass would be the case. Consciousness is not characterized by containment, as the classical notion of matter is, and at its most basic level is relational and undissociable from the principles of selfreference and hetero-reference. Since the geometry of GR is based on metrics, it measures distances, separations between objects, a measure of objectivity which rules out interpenetration and thus setups a logophysics in which the principle of identity is dominant. Metamorphosis is outruled as well as setting up for derivatives of this distantiation pragmatics the hegemony of tertium-non-datur is emplaced: there is not third state but true and false ascriptions. Thus the mind is deleted. Already at its departure ORC OR vindicates a dual logophysics, Aristotelian-Boolean logic for its implicit onto-epistemology. Further, the metaphor of music ascribed to the flow of consciousness begs us to recall that the cognition of music reveals nonorientability (Mazzola [159], Tymoczko, [274] Shepard [250-253],Risset [237]). The stream of music and that of consciousness are strongly related [1].

We can state our case invoking the considerations that led to reveal the Klein Bottle as the metaform of pattern recognition as unappropiate to that effect, using instead the so-called persistent homology ([31,32,127]), which has come to be applied to study neural networks through their topological invariants. The underlying notion is that data has shape (Carlsson [31,32], Rapoport [232-234], be that

the notion that the fundamental parameter to ponder is the discrete number of firings of a neuron. Accordingly the choice of the metric geometry in ORC OR appears to be inorganic, an abstraction from a world of unconnected entities signed by their masses, small for more, and entirely unrelatable, in principle and in concept, to the process they set themselves to explain, consciousness. With regards to the spacetime to emerge from the collapse of the quantum states in their theory, shape is not relevant at all, though it is implicitly the case of the morphology of tubulin in different conformations, particularly superposition states which they indicate their crucial role to their theory.

Their choice of geometry contravenes the very observation by Hameroff: "Atoms and small particles appear to exist as quantum superpositions of multiple possibilities, but yet we observe discrete, definite objects in our perceived "classical" world" " (p.14 [76]). Notwithstanding this observation in the "classical" world of forms, metamorphosis mediating, Aristotelian-Boolean logic is already superseded by supraduality. This is so particularly when instead of focusing in material structures considered synchronically we do so in considering processes, diachronically, processually, particularly those that arise in perception and cognition, say of the Necker cube, or Gestalt figures. Still to mention, also upon considering the mind, certainly is no object but for itself and for others exercising their theory of the mind making.

To explain the biophysics on which ORC is established and the relations with anaesthesia these authors considered for the main characteristics of the molecules, mainly liquid crystals and in particular water, their polarity and non-polarity properties. However, the nonorientable topologies of liquid crystals and water are entirely disconsidered, despite being crucial to cell physiology, namely to establish its overall coherence. The topological chemistry paradigm and in particular the two-version topology (Huckel and Huckel-Möbius) due to π- -electrons delocalization in aromatics is explicited by their theory, though no reference to topology but for tagging. After all the binding of molecules which is crucial to their model as the anaesthetic binds to tubulin is all about shaping accommodations, a topological issue. As early discussed by Sokolov they can be represented as modifications of the molecules electron distribution and the associated shape modifications and of topology. As Mezey puts it, the electron charge distribution of molecules is "fuzzy" ([118]). The three-dimensional shape of the molecule begs for topology, rather than valence skeleton representations: "molecules are not geometrical but topological objects" (p.8) and molecular shape is dynamical. The shape of a molecule is the molecule's electron distribution shape.

To put in terms of Hameroff & Penrose: "Non-polar anaesthetic binding sites in tubulin (and other proteins) are comprised largely of π- electron resonance clouds of benzene-like organic rings in aromatic amino acids tryptophan, phenylalanine and tyrosine. The basis for organic chemistry, π-electron resonance clouds are delocalized quantum objects, non-polar, but polarizable, and can induce and couple quantum dipoles, participate in quantum optical effects, and oscillate coherently in terahertz frequency ranges" [75]. This is it, in essence and concept. However, upon presenting the main argument for superposition in microtubules they do so in presenting the multiconformations of tubulin as shapes which are conceived either as unsuperposed or superposed interpreted as qubits. This is the basis for their "topological quantum computing" without ever mentioning the Hadamard-Klein Bottle information phase which is central to Quantum Information theory and supports transcending from classical to quantum computations, as already discussed.

These terahertz range frequencies are crucial to the whole range of oscillations of the cytoskeleton as a hierarchical *triplet of triplets* architecture which resonate in a range from hertz to terahertz eliciting topological phases (Bandyopadhyay [14], Gosh et.al. [64], Singh et.al. [134,254-256], Saxena et.al. [246]), and which make of the cytoskeleton a time-crystal. However, the topological anholonomies are still wanting for consideration to the genesis of spacetime consciousness. They claim a "quantum gravity" which would resolve the superposed states each producing a spacetime curvature into a single state, that of the manifest reality, a resolution which -they pledge- is tantamount to the emergence of conscious agency, through the so-called collapse of the wave function. Again, the left hemisphere dual over-rule is rather absolute, as McGilchrist puts it.

27 . Objective reductions and the dynamical reduction program

Basil Hiley showed that the quantum behaviour of particles can be described solely by using a hierarchy of Clifford algebras without recourse of wave functions nor Hilbert space. The essential aspect is the non-commutativity of Clifford algebras and the existence of idempotent elements which substitute the projectors of the standard theory [28]. We have just shown that in Matrix Logic the torsion produced by the commutator of FALSE and TRUE which are not dual in distinction with scalar Boolean logic, is nilpotent due to their status as projectors/idempotents. In the Klein Bottle logophysics non-commutativity is found in the chirality of taking the path around the singularity as in fig 3, either clockwise or anticlockwise produces a change of sign in the torsion closure. Covariantly represented in differential geometry terms, the torsion as a (2,1)-tensor $T_{b\,c}^{a}$ is anticommutative : $T_{bc}^{a} = -T_{cb}^{a}$. Would we change the order of the covariant indices, a change of sign is introduced [33,81-84,264,276].

Elio Conte upon noticing that the fundamental novelty of Quantum Mechanics is the non-commutatitivity of the product of operators of which Quantum Cognition sets to represent the decision process in terms of projectors in Hibert space, proposed instead a formulation using Clifford algebras, just alike Hiley (see Conte [37,38]). In this setting the e_3 idempotent, which he identified as the core element of the formalism, operates precisely as the previous decoherence. This topological decoherence contrasts not only with the quantum to classical physics transitions, but fundamentally with the resolution of quantum superpositions in Hameroff & Penrose ORC OR, where multiple states are reduced to one single state.

As we stated already a geometry with null torsion is such that selfreference and hetero-reference do not operate to its constitution if not banally, by their erasure. The issue is that there is no consciousness, whatever its mode, which is not supported by these two principles. For proof, the mirror-neuron phenomenology introduced by Rizzolatti et.al.. [238,50]. In particular we note that in the mind's resolution of superposition, as shown by Quantum Cognition, the mind entertains to consideration the different possible alternatives, consciously we can say at least of some of the possibilities and unconsciously too. It is the process of topological decoherence which produces the single unsuperposed classical state by action of the Hadamerd-Klein-Bottle-gate, and this appears to operate as if unconsciously. So consciousness appears to be primal rather than appearing as the process of singling out one mind-state alone from a superposition or multiple possible states.

The geometry of quantum states whose state vectors arise from the wave-like, quantum-level objects which evolve deterministically by the Schrodinger equation on state-space, also known as Quantum Geometry, is a projective Hilbert space [20] . Indeed a phase is a component in Quantum Mechanics which cannot be disconsidered, and very much so in logic with imaginary states too (Kauffman [112]). It is precisely this space on which Berry discovered the topological phase. A trajectory of a state-vector under the natural antipodal state-vectors identification produces a Möbius strip.

We already mentioned the Dynamical Reduction Program (DRP) or wave collapse theories as an attempt to complete Quantum Mechanics by accounting for the measuring or observing subject. On the one hand what we observe is discrete though seemingly continuous, while the cognitive process is analogical, continuous though framed in finite intervals, and mostly so when we are in a state of unconsciousness. We already discussed that despite the uncertainty whether perception is ultimately discrete or continuous, the latter can optimally be reconstituted from a discrete signal by sampling as if on a Möbius surface or a Klein Bottle. However, this reconstitution is always under noise, which is conceived as random. We also discussed that both the visual and somatosensory Klein Bottle maps embody these reconstitution under uncertainty.

The DRP approach consists in accepting that the dynamical equation of the standard theory should be modified by the addition of stochastic and nonlinear terms. After all, both the system, the observer and the instruments being applied to elicit the interrelations are all subjected to quantum fluctuations arising from the vacuum. The remarkable fact is that the resulting theory is capable, on the basis of a single dynamics which is assumed to govern all natural processes, to account at the same time for all well-established facts about microscopic systems as described by the standard theory, as well as for the

so-called postulate of wave packet reduction (WPR), which accompanies the interaction of a microscopic system with a measuring device.

Given a Hilbert space of dimension n+1 , since the quantum state vectors are defined up to a phase, this leads to CP(n), the n-dimensional complex projective space of "rays" of quantum vector states whose evolution is deterministic as characterized by the Schroedinger equation for quantum state vectors. Upon measurement a state-vector reduction to a particular eigenstate is eventuated, the so called collapse of the wave function. This process can be described in "objective" agent-free mode by a nonlinear Schroedinger equation on CP(n) which assumes a non-isolation of the system vis-à-vis the environment.

This requires Stochastic Differential Geometry, yet not in configuration space or spacetime [180] but on CP(n) (see Hughston [309], Rapoport [218]). Remarkably, the Schroedinger equation on configuration space or spacetime manifold can be derived from Brownian motion with the drift vectorfield given by the logarithmic gradient of the logarithm of the wavefunction (Rapoport, [201-206,208,210-219,226,227]. They are crucial to Nelson's derivation of the Schroedinger equation in Stochastic Mechanics [172]) in terms of Brownian motions arising from quantum vacuum fluctuations and diffusion processes [174]). The logarithmic singularities of the eikonal- propagating torsion field may undergo quantum jumps [220,221], a primal distinction to the creation of visual experience [220,221,226].These Brownian motions are produced by dynamical geometries with torsion vectorfields or differential form 1-forms, Riemann-Cartan-Weyl connections: they are the basis for Stochastic Differential Geometry [201-204,2010]). The Laplacian operator has an additional to the metric Laplacian a first-order term given by the trace-torsion, the drift of the Brownian motions, so it fits exactly to the Fokker-Planck diffusion operator, or still, the differential generator of the generalized Brownian motion ([200-221,226]).Both the Fokker-Planck operator and the stochastic differential equation which extends the Schroedinger equation to the open case are intrinsic, they do not depend on any coordinate system ([218]). In the latter geometric formulation there is no separation of the quantum system and the apparatus/observer.

This extends to Brownian motion on CP(n) as a stochastic differential equation whose stochastic solution is a diffeomorphism of state-space, while the closed system case evolution produces a diffeomorphism of configuration space. With regards to the metric in the Quantum Geometry it is the Fubini-Study metric, while in configuration space metrics can be either positive-definite for the non-relativistic case or degenerate in the relativistic case (Oron and Hurwitz [180] , respectively (In the case of the Navier-Stokes of fluid-dynamics where the drift-torsion field is the velocity of the fluid, these diffeomorphisms stand for the solution flow of the Navier-Stokes equation [211-217;89]) .

In short, the standard Schroedinger flow on CP(n) is generalized to a random flow which has an additional term, a drift-torsion vector field, given by the gradient of the expectation value of the quantum Hamiltonian and the gradient of the variance of the Hamiltonian, the squared energy quantum uncertainty (Rapoport [218]). It is this torsion-drift term which drives the system to a fixed eigenstate. The noise term of this random flow also proportional to the gradient of the expectation value of the Hamiltonian can be written in terms of the Planck's constant, the gravitational constant and the velocity of light c, in a form that for the superposition of macroscopic systems which differ in little as a proton mass, the collapse to a single eigenstate is almost immediate (Hughston [309], Adler and Horwitz [310]) In this Stochastic Differential Geometry setting, the reduction of the quantum state to a single eigenstate does not require an observer for the system to collapse, so hardly we can attribute this as associated to an emerging consciousness, or a consciousness at all if not that probabilities and their expected values are involved. But then probabilities have a subjective-objective nature, rather than builtin into the quantum system. They have an epistemic function, a mode of knowledge through description, yet supported by a supradual ontology based on the principle of selfreference and hetero-reference.

28. Conclusions

We have presented a geometrical-topological theory which surmounts the paradigmatic divisions produced by the Cartesian Cut, and applied it to the study of consciousness, proposing a novel paradigm. Upon unfolding the presentation, it has been shown that the diverse manifestations of the physical -in

the broad sense- and mental domains are derivatives of the supradual logophysics which is based on the principles of selfreference and hetero-reference, which operates even when an agent is absent. Having said and done, briefness is compelling, a long monograph forthcoming. It has transpired that the transdisciplinarian approach which the supradual logophysics supports, transcending the dual logic hegemonic in science and philosophy, also transcends the somewhat piecemeal knowledge that dualism can afford and manage through the recourse to interdisciplinarity, notwithstanding the courage, competence and brilliance that may be exercised in doing the job. The issue is that a choice of what logic is, means and does, is tantamount to the choice of an ontology fused with an epistemology: This is already the case of dual Aristotelian -Boolean logic which by default, and transparently so, produces fractures which cannot be salvaged if not transcending it, ab initio (Günther, [302], Rapoport [230]). Methodology ultimately is the choice of the logic, in the first place, rather than a decantation of what is left implicit. The usual consideration is that it comes without commitments: if so why the effects of dualism show so much on the contrary and we can still claim blindness?

Rather than a putative theory of quantum gravity for explaining the purported physical source for consciousness or a no less definable panpsychism that can profess little more than its argumentation, an omnipresent supradual logophysics which supports a world of signs and meaning as unseparable of the material-energetic-information domain, offers its cornucopia cum integral meaning (Rapoport,2022).

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